Small cosmological constant from a peculiar inflaton potential

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We propose a novel scenario to explain the small cosmological constant (CC) by a peculiar inflaton potential. The shape almost satisfies the following conditions: The inflation is eternal if the CC is positive and not eternal if the CC is negative. Although realizing the peculiar shape has a similar amount of fine-tuning as the CC, the shape can be made stable under radiative corrections in the effective theory. By introducing a slowly varying CC from a positive value to a negative value, the dominant volume of the Universe after the inflation turns out to have a vanishingly small CC. The scenario does not require eternal inflation, but the e-folding number is exponentially large, and the inflation scale is low. The Hubble parameter during inflation, $H_{\rm inf}$, is required to be smaller than the present CC scale, and, thus, the CC relaxed during inflation with the low renormalization scale, $\sim H_{\rm inf}$, is safe from the radiative corrections from the standard model particles. The scenario can have a consistent thermal history, but the present equation of state of the Universe is predicted to slightly differ from the one for the ΛCDM model. In a time-varying CC model, CC can be relaxed from $(10^3\ {\rm GeV})^4$, and in a model with a light scalar field scanning the CC during inflation, CC can be relaxed from $(10\ {\rm MeV})^4$.

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I. INTRODUCTION

One of the long-standing theoretical problems in particle theory and cosmology is the fine-tuning of the cosmological constant (CC) [1,2], which is measured as [3]

$$\Lambda_C \simeq 2.2 \times 10^{-3}\ {\rm eV}. \qquad (1)$$

The CC problem should be solved by IR dynamics, because even the QCD contribution $(1\ {\rm GeV})^4/(16\pi^2)$ to the CC should be somehow canceled, which results in an amount of tuning of $\mathcal{O}(10^{-45})$. For this, nontrivial dynamics should happen when the Universe is much colder than the QCD scale.

Aside from the anthropic solution [1,2] (and also works relevant to it [4–6]), there have been several proposals to relax the tuning around the present Universe. Since a vanishing CC is a critical point for the empty Universe to inflate and contract, it was studied in, e.g., Refs. [7,8] and recently in Ref. [9] that a slowly varying scalar field can drive the Universe at around the critical point. Although such a scenario typically predicts an empty Universe, which may be inconsistent with the big bang cosmology, the authors in Ref. [9] showed that the Universe can be reheated by the scalar field in the contracting Universe and discussed that the produced plasma induces a bounce of the Universe to get the standard cosmology.

The inflation paradigm, on the other hand, is widely accepted as a central part of modern cosmology [10–14]. The inflation is driven by a real scalar field, which slow rolls around a pseudoflat direction of the potential. The almost constant potential energy induces an exponential expansion of the spatial volume of the Universe. The slow-roll inflation predicts a flat, homogeneous, and slight anisotropic Universe, which has been confirmed from the cosmic microwave background (CMB) data [15]. The inflationary period is cold. since the Gibbons-Hawking temperature $H_{\rm inf}/(2\pi)$ [16] is Planck scale suppressed compared to the (false) vacuum energy scale of the Universe. In this paper, we focus on the inflationary period to relax the CC.

The quantum diffusion also traps the inflaton in the pseudoflat regime in a probabilistic way. If the typical rate for finishing the inflation in a single Hubble patch is smaller than the expansion rate, there are always volumes that are inflating. Thus, inflation never ends in the entire Universe. This is known as eternal inflation [17–22] (see also [23–25]). In the eternally inflating Universe, due to the infinities, there could be ambiguities in defining the probabilities, which depend on the choice of measures. With certain measures and dynamical CCs, the explanation of the CC during the eternal inflation has been discussed [4,26]. (See also [27] for other attempts relevant to inflation.)

In this paper, we propose an alternative possibility that the CC is relaxed during noneternal inflation with the inflationary Hubble parameter $H_{\rm inf} \ll \Lambda_C$. The basic idea is as follows. First, we assume that the inflation potential is around such a critical point that if the minimum of the potential, i.e., the CC, has a positive value, eternal inflation would occur. If negative, the inflation period would be finite. The inflaton potential shape around such criticality is as tuned as the CC, but we can make it stable under radiative corrections, which is different from the original CC tuning. Second, we assume that the CC decreases from a large and positive value. Since the inflaton potential is around the criticality, most volume of the Universe finishes inflation when the CC crosses zero. Consequently, a small CC is exponentially favored in a probabilistic way. A standard reheating and big bang cosmology start irrelevant of the measure we choose.

As a concrete quantum field theory model, we introduce a scalar field to scan the CC by taking account of the quantum diffusion of the inflaton and the scanning field. We confirm the validity of the scenario analytically in the main part and numerically by solving the Fokker-Planck equation with the termination effect of inflation in Appendix B.

II. ETERNAL, NONETERNAL, AND CRITICAL ETERNAL INFLATION

A. Review on eternal and noneternal inflation

For illustrative purposes, let us consider the following form of the potential for the inflaton ϕ:

$$V = V_\phi[\phi] + V_C, \tag{2}$$

where we separate the CC, V_C, from the dynamical part of the potential, and, thus, V_ϕ has a vanishing minimum value, $V_\phi[\phi_{\rm min}] = 0$. In this review part, we take V_C as a constant, but it will depend on time and space later.

Let us focus on the hilltop of the potential. The Taylor series is given by

$$V_\phi[\phi] = V_0 + V''_\phi \frac{\phi^2}{2} + \mathcal{O}(\phi^3). \tag{3}$$

Without loss of generality, we have taken the hilltop as the origin of ϕ; $V''_\phi(<0)$ is the curvature; V_0 is defined so that the potential minimum is vanishing. There are various models with successful cosmology to get this kind of potential top: modified quartic hilltop inflation, e.g., [28–30], multinatural inflation [31–37], axionlike particle inflation [38–40], or heavy QCD axion inflation [41], etc. As we will see, the inflation scale should be too low for the natural inflation [42,43] and other simple hilltop inflation [13,14] to provide consistent CMB data [15] in this scenario unless there is another inflaton or curvaton [44–46] for explaining the CMB data.

The potential top is so flat that inflation can take place. The eternal inflation may or may not take place depending on the potential shape. The inflationary volume inflates, and the scale factor R increases with

$$R^3 \propto e^{\int 3dtH_{\rm inf}}. \tag{4}$$

Here, $H_{\rm inf} \approx \sqrt{V/3M_{\rm pl}^2}|_{\phi\simeq 0}$ is the Hubble expansion rate.

The inflaton cannot stay exactly on the hilltop due to the quantum diffusion in the de Sitter space-time. The inflaton undergoes random walks. By neglecting the curvature compared with $H_{\rm inf}$, the random walks follow

$$\langle \Delta\dot{\phi}^2 \rangle \sim \frac{H_{\rm inf}^3}{(2\pi)^2}, \tag{5}$$

where $\dot{X}$ denotes the (cosmic) time derivative of X. Because of the diffusion effect, the classical value of the inflaton field has a probabilistic distribution. If the inflationary regime (or, more precisely, the stochastic regime of ϕ) which is defined by $|\phi| < \phi_{\rm inf}$ has a sufficiently flat potential, the distribution of ϕ during inflation approaches to a constant, $\sim 1/\phi_{\rm inf}$. The inflaton rolls out of the inflationary range at a rate $\dot{\phi}/\phi_{\rm inf} \sim V''_\phi/3H_{\rm inf}$. Thus, we get the probability that ϕ remains in $|\phi| \leq \phi_{\rm inf}$:

$$P \propto e^{\int dtC\frac{V''_\phi}{3H_{\rm inf}}}. \tag{6}$$

Here, the decay rate in the exponent depends on a model-dependent parameter $C = \mathcal{O}(1)$. As a result, we find that the inflating volume $L_{\rm inf}^3$ satisfies

$$L_{\rm inf}^3 \propto P \cdot R^3 \propto e^{\int \left(3H_{\rm inf} + C\frac{V''_\phi}{3H_{\rm inf}}\right)dt}. \tag{7}$$

If the exponent increases with time,[1]

$$3H_{\rm inf} > C\frac{|V''_\phi|}{3H_{\rm inf}} \quad \text{[eternal inflation]}, \tag{8}$$

i.e., the second slow-roll condition, $\eta \lesssim 1$, is satisfied at the top, the inflating volume increases eternally. This is the well-known condition for eternal inflation. It is also known that it may be difficult to discuss the probabilities during the eternal inflation due to the infinities.

If the exponent decreases in time, on the other hand, the total volume decreases:

[1] Strictly speaking, the volume distribution $1/\phi_{\rm inf}$ in this regime may be incorrect, since the difference of Hubble rate within $|\phi| \lesssim \phi_{\rm inf}$ may be important for very long inflation.

$$3H_{\rm inf} < C\frac{|V''_\phi|}{3H_{\rm inf}} \qquad [\text{noneternal inflation}]. \qquad (9)$$

Although we need to tune the conditions for inflation to occur at some Hubble patches, the entire Universe finishes the inflation within a finite e-folding. The cosmic timescale for the termination is $\sim 1/(-3H_{\rm inf} + C|V''_\phi|/(3H_{\rm inf}))$. We note that, even if the inflation is noneternal at the hilltop, the CMB data can be explained due to a finite period of inflation, such as in the inflectionally point inflation [40,47].

B. Critical eternal inflation and fine-tunings

We expect that there is a critical point between the eternal and noneternal regimes by decreasing $V_C + V_0$, and thus $H_{\rm inf}$, with fixed V''_ϕ:[2]

$$3H_{\rm inf} = C\frac{|V''_\phi|}{3H_{\rm inf}} \qquad [\text{critical eternal inflation}]. \qquad (10)$$

At the criticality, the inflation does not end, but the total volume of the inflating Universe does not change. As a consequence, the volume of the Universe after inflation approaches infinity, but the inflating volume is kept finite. We call this kind of inflation *critical eternal inflation*.

In the following, we focus on the possibility that V_ϕ (but not V) almost satisfies the condition for the critical eternal inflation at $V_C \to 0$:

$$3H_{\rm inf}|_{V=V^c_\phi} = C\frac{|V''_\phi|}{3H_{\rm inf}}\bigg|_{V=V^c_\phi} \qquad [\text{inflaton potential at the criticality}]. \qquad (11)$$

Here and hereafter, X^c denotes the parameter or quantity at the criticality.

1. Technical naturalness

The near-criticality condition is realized by tuning the potential shape, while symmetry can stabilize the tuned condition from radiative corrections. For instance, the inflaton may enjoy a discrete shift symmetry:

$$\phi \to \phi + 2\pi f_\phi, \qquad (12)$$

which may imply that the inflaton is a pseudo-Nambu-Goldstone boson with f_ϕ being the decay constant. The potential has a generic form from nonperturbative effects given by

$$V_\phi = \sum_{n=0} \Lambda_n^4 \cos\left(\frac{n\phi}{f_\phi} + \theta_n\right) \qquad (13)$$

with n being the integer and θ_n a relativistic phase. Λ_i^4 is the order parameter that explicitly breaks the continuous shift symmetry to the discrete one. Λ_0^4 is the constant term that makes the potential vanishing at the potential minimum according to our notation (2). We can tune Λ_n^4 and θ_n to get the potential around the criticality. For instance, we have only a single cosine term with $n = 1$; the criticality condition suggests $C/(f^c_\phi)^2 = 6/M^2_{\rm pl}$ and $(\Lambda^c_0)^4 = (\Lambda^c_1)^4 =$ arbitrary. In addition, the inflaton can have sizable derivative coupling to the SM particles to successfully reheat the Universe.

To discuss whether the radiative correction will spoil the criticality, we can estimate the 1PI effective potential. Coleman-Weinberg corrections involving only the derivative couplings to the SM particles do not exist. From dimensional regularization,[3] the leading contribution is [48]

$$V_{\rm CW} \approx \frac{1}{64\pi^2} V''_\phi[\phi]^2 \left(\ln\frac{|V''_\phi|}{\mu^2_{\rm RG}} - 3/2\right), \qquad (14)$$

where $\mu_{\rm RG}$ is the renormalization scale. Since $V_{\rm CW} \sim \mathcal{O}(H^4_{\rm inf}/(8\pi)^2)$ at around the inflationary regime due to the slow-roll condition, the near-criticality condition is technically natural with[4]

$$|V_0 - V^c_0| \gtrsim \frac{H^4_{\rm inf}}{(8\pi)^2}. \qquad (15)$$

As we will see, this quality is enough for our mechanism to work.

We note that this stability under the radiative corrections is guaranteed in the effective theory. Depending on the UV models, the stability may not be guaranteed. Therefore, we describe this stability to be technically natural (such a technical naturalness, or stability under radiative corrections, is used in various studies, especially in the context of the hierarchy problems).

2. Initial condition for inflation

Before ending this section, we also comment on the tuning for the initial condition for the inflaton field to have

[2] To determine the precise value of C, we need the higher-order terms of ϕ in V_ϕ as well as the detailed study on the quantum diffusion during inflation with $H^2_{\rm inf} \sim |V''_\phi|$. As long as there is a fixed value of C for the criticality, our conclusion does not change. The determination of C in a specific model will be studied elsewhere.

[3] We use the regularization scheme that the discrete shift symmetry is maintained. Note that, in any case, the quadratic divergence to the potential is absent due to the symmetry.

[4] This criticality is expected to exist in, e.g., the multinatural inflation models [31–41] which are stable under radiative correction thanks to a discrete shift symmetry. As argued in Ref. [40], the eternal inflation and noneternal inflation both exist in parameter regimes which are continuously connected.

inflation. A conservative estimation on the field range for the inflation may be [49] $|V'_\phi| < H^3_{\rm inf}$, which is the region where the classical motion is smaller than the quantum diffusion. This gives $\phi_{\rm inf} \sim \frac{H^3_{\rm inf}}{V''_\phi}$. We have to set the inflaton field to be within this range, $|\phi| < \phi_{\rm inf}$, as the initial condition, which requires a certain amount of tuning. If the inflation lasts long enough, this tuning can be compensated by the expanding volume. In fact, initial tuning for inflation as precise as $e^{-10^{\mathcal{O}(10-100)}}$ can be compensated when our mechanism works (e.g., Fig. 3).[5] This is also explicitly shown in Fig. 2 in the next section. When the initial condition is not tuned within the stochastic regime of ϕ, the inflation soon ends and the volume at the end-of-inflation boundary is suppressed.

III. RELAXING CC DURING INFLATION

In the following, we use the previously discussed inflaton potential at the criticality to relax the CC by considering a time-varying CC. This time-varying CC turns the eternal inflation into noneternal, which avoids the measure problem. In this section, our inflation will be noneternal.

A. Relaxing CC by a generic time-varying CC

To present the idea and to provide model-independent conditions for the mechanism, let us first assume that V_C decreases slowly in time but it is a constant in space coordinate, i.e., $V_C = V_C[t]$. We take $V_C = V_C^i > 0$ and $|\phi| < \phi_{\rm inf}$ initially. We assume for a while that $V_\phi = V_\phi^c$ for an illustrative purpose. We will come back to relax this assumption. For simplicity of analysis, let us focus on the regime $|V_C| \ll V_\phi^c[0]$ throughout this paper. Then, we can expand

$$H_{\rm inf} \approx H^c_{\rm inf} + \frac{V_C[t]}{6M^2_{\rm pl} H^c_{\rm inf}}. \tag{16}$$

The spatial volume escaping from the ϕ-stochastic regime is produced at a rate

$$\frac{d}{dt} L^3_{\rm end} \sim C \frac{|V''_\phi|}{3H_{\rm inf}} L^3_{\rm inf}[t] \propto \exp\left[\int^t dt' \frac{V_C[t']}{M^2_{\rm pl} H^c_{\rm inf}}\right]. \tag{17}$$

We note that the leading term in Eq. (16) is canceled between the expansion rate and the "decay" rate in Eq. (7), due to the requirement (11). Since V_C decreases in time, V_C and time have one-to-one correspondence. Defining $\kappa \equiv \frac{d}{dt} V_C (< 0)$, we obtain

$$\frac{d}{dV_C} L^3_{\rm end} \propto |\kappa^{-1}[V_C]| \exp\left[\int_{V_C^i}^{V_C} dx \frac{\kappa^{-1}[x] x}{M^2_{\rm pl} H^c_{\rm inf}}\right]. \tag{18}$$

This gives the differential distribution of the CC for the Universe finishing the inflation. If we can approximate κ to be a constant, we obtain

$$\frac{d}{dV_C} L^3_{\rm end} \propto \exp\left[-\frac{V_C^2}{2|\kappa| M^2_{\rm pl} H^c_{\rm inf}}\right]. \tag{19}$$

Interestingly, this is a normal distribution which is peaked at $V_C = 0$ with a variance of

$$\sigma \equiv \sqrt{M^2_{\rm pl} H^c_{\rm inf} |\kappa|}. \tag{20}$$

σ should be the typical value of V_C in the Universe. We get the variance because the e-fold to end the inflation is $\Delta N_{\rm end} \sim M^2_{\rm pl} (H^c_{\rm inf})^2/(V_C)$ during which V_C still changes. The variance σ can be also estimated from $V_C \sim |\kappa \Delta N_{\rm end}/H^c_{\rm inf}|$. The timescale to end the inflation is estimated as

$$\Delta N_{\rm end} \sim \kappa^{-1/2} M_{\rm pl} (H^c_{\rm inf})^{3/2} \sim \frac{M^2_{\rm pl} (H^c_{\rm inf})^2}{\sigma}. \tag{21}$$

The e-folds also represent the timescale that the difference of V_C by σ affects the distribution of a via different $H_{\rm inf}$.[6] We can relax the CC to the desired value if

$$\sigma \lesssim \Lambda_C^4. \tag{22}$$

This is the case V_C varies so slow that $\kappa \lesssim \Lambda_C^8/(M^2_{\rm pl} H^c_{\rm inf})$.

So far, we have assumed $V_\phi = V_\phi^c$ for simplicity. When V_ϕ is slightly away from V_ϕ^c by fixing $(V_\phi^c)'' = V''_\phi$, the deviation $V_0 - V_0^c > 0$ (< 0) would change the center value of the distribution (18) and bias the cosmological constant to a negative (positive) value. Therefore, we need

$$|V_0 - V_0^c| \lesssim \Lambda_C^4. \tag{23}$$

In addition, ϕ diffusion can change the typical energy of the inflaton potential by $(V_\phi^c)''(\phi^c_{\rm inf})^2$. Requiring this around or smaller than the CC, we get

$$\frac{3H^4_{\rm inf}}{(2\pi)^2} \lesssim \Lambda_C^4. \tag{24}$$

Thus, a model-independent bound on the inflation scale is obtained: $V_0 \lesssim (4\ \text{TeV})^4$. In other words, the Gibbons-Hawking temperature should be $\lesssim \Lambda_C$. Once Eq. (24) is

[5]In addition, there are several mechanisms to set the correct initial condition [13,14,38,50].

[6]The relation may be understood similarly to the uncertainty principle. If we would like to measure the Hubble expansion rate with an extremely good precision ($\sigma \to 0$), we need extremely large e-folds ($\Delta N_{\rm end} \to \infty$).

satisfied, we get the technically natural parameter region with $H_{\rm inf}^4/(8\pi)^2 \lesssim |V - V_0^c| \lesssim \Lambda_C^4$.

In other words, if there is no other contribution, our mechanism naturally predicts the CC of

$$|\Lambda_C^4| \sim \max\left[\sigma, \frac{3H_{\rm inf}^4}{(2\pi)^2}, |V_0^c - V_0|\right] \tag{25}$$

with $|V_0^c - V_0| \gtrsim \frac{H_{\rm inf}^4}{(8\pi)^2}$ for a technical naturalness.

B. Relaxing CC by a scalar field

To have a slowly time-varying V_C, we may introduce a dynamical field a, which slow rolls during the inflation by ϕ:

$$V_C = V_C[a]. \tag{26}$$

In this case, we should take account of a dynamics to check whether it spoils our previous discussion. To slow roll, a has a very flat potential due to an approximate continuous shift symmetry. We assume again that $V_\phi = V_\phi^c$ for illustrative purpose.

Since the potential is extremely flat, we can expand it around any field value. In general, the leading term for a a linear term is

$$V_C = V_C' a, \tag{27}$$

where $V_C[0] = 0$ is obtained via a field redefinition $a \to a + \text{const}$. Let us take $a[0] = a^i (> 0)$ as the initial condition at $t = 0$, and, thus, $V_C' > 0$ for our mechanism to work.

Then, a undergoes the slow roll with the classical motion $a^{\rm cl}[t] \approx -tV_C'/3H_{\rm inf} + a_i$. V_C rolls down to ~ 0 at a timescale

$$\Delta N_{\rm slowroll}[V_C^i] \sim \frac{3(H_{\rm inf}^c)^2 V_C^i}{(V_C')^2}, \tag{28}$$

which will be the longest timescale in this scenario. We obtain $\kappa \approx -\frac{V_C'^2}{3H_{\rm inf}^c}$, $\sigma \simeq \frac{|V_C'| M_{\rm pl}}{\sqrt{6}}$, and

$$\Delta N_{\rm end} \simeq \sqrt{6}\frac{(H_{\rm inf}^c)^2 M_{\rm pl}}{|V_C'|}. \tag{29}$$

In this model, we have various additional constraints from the quantum diffusion of a. (The following results relevant to the quantum diffusion are numerically checked by solving the Fokker-Planck equation. See Appendix B.) As ϕ, a undergoes random walks around the trajectory of the classical motion

$$\Delta a^2[t] \equiv \langle (a - a_{\rm cl})^2 \rangle \simeq t\frac{H_{\rm inf}^3}{(2\pi)^2}. \tag{30}$$

Here, we have assumed that, at the beginning, all the inflating Universe has $a = a_i$. Then, we get the distribution function of a as

$$f[a, t] \propto e^{-\frac{(a - a_{\rm cl}[t])^2}{2\Delta a^2[t]}}. \tag{31}$$

Notice that this can be obtained when the contribution to $H_{\rm inf}$ from the quantum diffusion is neglected. Since classical motion dominates over the quantum diffusion at the timescale $\Delta N_{\rm diffuse} \sim \frac{9H_{\rm inf}^6}{(2\pi)^2 (V_C')^2}$, we need

$$\Delta N_{\rm end} \gg \Delta N_{\rm diffuse} \tag{32}$$

so that the inflation volume is not sensitive to the quantum diffusion.

With this condition satisfied, we can obtain the volume distribution (see Appendix B for the derivation by the Fokker-Planck equation and its more accurate numerical solution)

$$\begin{aligned} \partial_{V_C} L_{\rm inf}^3[t] \propto \partial_a L_{\rm inf}^3[t] &\propto P[a, t] \cdot R^3[a, t] \cdot f[a, t] \\ &\propto e^{t\frac{V_C' a}{M_{\rm pl}^2 H_{\rm inf}^c} - \frac{(a - a_{\rm cl}[t])^2}{2\Delta a^2}}. \end{aligned} \tag{33}$$

At $a = a_i$ we get $\partial_a L_{\rm inf}^3[t]|_{a=a_i} \propto e^{[\frac{V_C' a_i}{M_{\rm pl}^2 H_{\rm inf}^c} - \frac{(V_C')^2}{9H_{\rm inf}^5/(2\pi)^2}]t}$. To avoid the inflating volume at $a = a_i$ from dominating over the Universe, and thus to evade eternal inflation, we need

$$\frac{(V_C')^2}{9(H_{\rm inf}^c)^5/(2\pi)^2} \gtrsim \frac{V_C^i}{M_{\rm pl}^2 H_{\rm inf}^c}. \tag{34}$$

This gives

$$V_C^i \lesssim (10\ \text{keV})^4 \left(\frac{1\ \text{GeV}^4}{V_0}\right)^2 \left(\frac{V_C'}{10^{-66}\ \text{GeV}}\right)^2. \tag{35}$$

This can be also obtained by requiring that the diffusion $\sqrt{\Delta a^2}$ is smaller than σ when $a_{\rm cl}$ crosses zero: $V_C' \sqrt{N_{\rm slowroll}} \frac{H_{\rm inf}^0}{2\pi} \lesssim \sigma$. Thus, for the inflation scale, $V_0^{1/4} \sim$ MeV, GeV, 100 GeV, we can relax the tuning of the cosmological constant by $V_C^i/\Lambda_C^4 \sim 10^{50}, 10^{26}, 10^{10}$ with $V_C' = 10^{-66}$ GeV, which is around the experimental bound as will be explained.

The parameter region in $H_{\rm inf} - V_C'$ plane is shown in Fig. 1. The contours denote $\max(V_C^i/\Lambda_C^4)$, i.e., the maximal amount of the relaxation (35). In the lower gray region, Eq. (32) is not satisfied and our estimation is invalid. In the upper green region, the slow roll is too fast and $\sigma > \Lambda_C^4$.

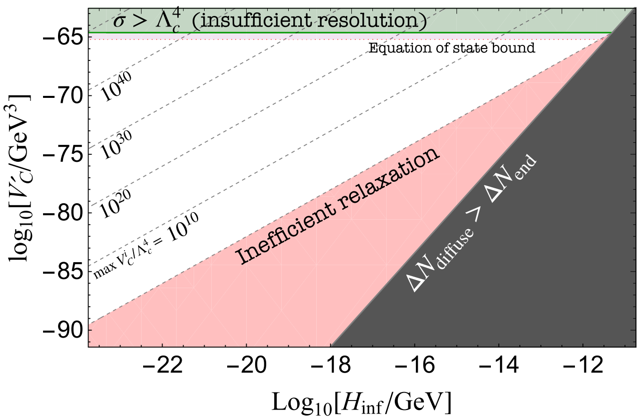

FIG. 1. The contours of max V_C^i/Λ_C^4 in the $H_{\rm inf} - V_C'$ plane. This is the most efficient relaxation without eternal inflation. In the lower gray region, our estimation is invalid. In the upper green region, the slow roll is too fast, and the resolution of the relaxation mechanism is worse than Λ_C^4. The purple region above the dotted line represents the current bound for the slow-rolling a. The pink region below the lowest dashed contour cannot have $V_C^i > \Lambda_C^4$.

The pink region below the lowest dashed contour cannot have $V_C^i > \Lambda_C^4$. Here, we take $H_{\rm inf}^c > 2\times 10^{-23}$ GeV in the figure so that the reheating temperature of the Universe, assuming instantaneous reheating, is larger than 10 MeV, which is favored for a successful big bang nucleosynthesis.

A prediction of this scenario is that the CC is time varying, which leads to the equation of state $w \approx \dot{a}^2\rho_c^{-1} - 1 \approx \frac{V_C'^2}{9H_0^2}\rho_c^{-1} - 1$, with ρ_c being the critical density of the Universe. To be consistent with the equation of state [51], $w < -0.95(95\%{\rm C.L.})$, we obtain

$$|V_C'| \lesssim 0.6 \times 10^{-65}\ {\rm GeV}^3. \tag{36}$$

Interestingly, this bound is close to the theoretical bound $\sigma \lesssim \Lambda_C^4 \to |V_C'| \lesssim 2\times 10^{-65}\ {\rm GeV}^3$ (see Appendix A for a discussion that $\sigma \sim \Lambda_C^4$ is favored). In particular, the experimental precision will be improved in the Euclid CMB mission [52], Rubin observatory [53], and DESI [54], which may probe the scenario.

In any case, if the inflation scale is low enough, we can obtain V_C^i as large as $V_C^i \sim 10^{40\text{–}50}\Lambda_C^4$. In this case, a MeV-GeV scale inflation is required [40,55]. Since the total e-fold $\Delta N_{\rm slowroll}$ is exponentially large, even very light particles due to an approximate shift symmetry reach the equilibrium distribution during the inflation with the energy density of $3H_{\rm inf}^4/(2\pi)^2$ [29,56,57]. Since this is much smaller than Λ_C^4, they cannot be the dominant dark matter. On the other hand, axion dark matter can be produced via mixing with axionic inflaton especially if the light axions are at equilibrium distribution [50,58], or it can be produced from inflaton decay [59,60]. Baryogenesis is also possible due to the inflaton decay with higher-dimensional operators that are baryon number violating while a proton is stabilized by a parity [61].

C. An alternative view of the mechanism as a summary

Let us summarize the mechanism so far from a schematic discussion. In this part, we investigate the inflationary boundaries and trajectories in ϕ, a field space (a should be replaced to be V_C to apply to the generic time-varying scenario; in this case, the eternal inflation constraint may not apply). The boundaries as well as trajectories are shown in Fig. 2 in the $\phi - a$ plane. Above the gray line, eternal inflation takes place due to the a diffusion. We avoid this region by setting the bound of Eq. (35). Thus, the a classical motion is always dominant, i.e., a always slow-rolls, and it does not jump to a positive a direction. Thus, the cosmic time t has a one-to-one correspondence to a or V_C value by approximating $H_{\rm inf} = H_{\rm inf}^c$. The blue dashed line denotes the end-of-inflation boundary; i.e., below the line, the slow-roll condition of ϕ is violated.[7] The tuning at the criticality can be seen that the $a = 0$ line almost touches the lower bound of the blue dashed line. The two black vertical lines denote $\pm\phi_{\rm inf} = \mathcal{O}(H_{\rm inf})$, defined in the last paragraph in Sec. II. Between the two lines, the ϕ motion is dominated by the random walk, while a motion is classical. This is the stochastic region of ϕ that we have focused on.

In the stochastic region of ϕ, we have used the ϕ-model-independent form of the escaping rate $CV_\phi''/(3H_{\rm inf})$ and the expansion rate $H_{\rm inf} = \sqrt{(V_\phi(0) + V_C[a])/3M_{\rm pl}^2}$ to describe the system. The escaping rate is derived by assuming an equilibrium flat distribution in $|\phi| < \phi_{\rm inf}$. The reasons that the microscopic ϕ motion is irrelevant are as follows.

(i) The difference of the ϕ value in the range $|\phi| \ll \phi_{\rm inf}$ contributes to the energy density by at most $H_{\rm inf}^2\phi_{\rm inf}^2, m_\phi^2\phi_{\rm inf}^2 \sim H_{\rm inf}^4$. The first (second) term comes from the kinetic (mass) term. The error of the approximation neglecting those contributions of the escaping rate or expansion rate is $\lesssim H_{\rm inf}^4/\sqrt{V_\phi[0]M_{\rm pl}}$. Since we focus on the regime $H_{\rm inf}^4 \ll \Lambda_C^4$, this error does not change our conclusions. The motion of ϕ in this range is negligible.

(ii) Because of the small field range of $|\phi| < \phi_{\rm inf} \sim \mathcal{O}(1)H_{\rm inf}$, the ϕ distribution approaches to a flat one within $\mathcal{O}(1)$ e-folds, which is much shorter than the typical timescale $\Delta N_{\rm end} = 10^{\mathcal{O}(10)}$. In the timescale of our interest, we can safely average possible ϕ values according to the flat distribution. The possible error to the escaping rate may be a difference by a factor of $e^{-\Delta N_{\rm efold}/\mathcal{O}(1)}$, which is the probability that the equilibrium distribution is not reached.

(iii) There is no direct interaction between ϕ and a.

[7]Here, we have assumed the quadratic potential of ϕ to present this line for illustrative purpose, i.e., $a \propto \phi^2$ that represents the constant second slow-roll parameter. In a realistic case, by requiring the fitting of the CMB data, this boundary will be modified. Our discussion does not change.

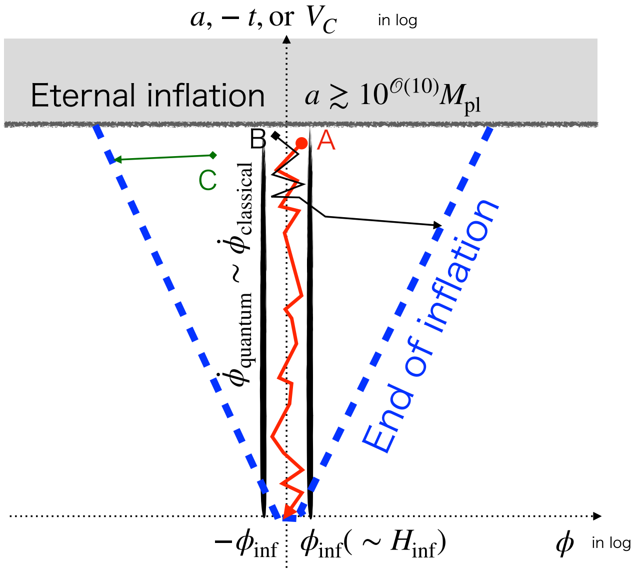

FIG. 2. Possible trajectories of coarse-grained fields in the $\phi - a$ (or V_C) plane. Above the gray line, eternal inflation takes place due to the a diffusion, which we do not consider (this condition may not needed when we consider generic time-dependent V_C). The blue dashed line denotes the end-of-inflation boundary; i.e., below the line, the slow-roll condition of ϕ is violated. We require the blue dashed line to touch the $a = 0$ line by tuning the ϕ potential height. The two black vertical lines denote $\pm\phi_{\rm inf}$. Between the two lines, the ϕ motion is dominated by the quantum diffusion. Trajectories A, B, and C have different initial conditions and ϕ diffusion (see the main text). Among the three trajectories, A, which ends at $V_C \propto a \approx 0$, has the longest inflationary period. Thus, the volume at the end-of-inflation boundary exponentially dominates over B and C.

Then, whether a trajectory of the coarse-grained fields moves out of the stochastic regime of ϕ is probabilistic. The probability was calculated analytically so far (and confirmed numerically in Appendix B). By focusing on this stochastic range of ϕ, we have shown that the largest volume that finishes the inflation has a vanishingly small cosmological constant.[8] This can also be found in Fig. 2. The trajectory of B (black arrow from a square) is shorter than A (red arrow from a circle) in the a direction. Since the inflationary time or the e-folds for each trajectory corresponds to the a excursion in the $|\phi| < \phi_{\rm inf}$ regime, A experiences exponentially larger e-folds than B. Namely, the volume at the end of inflation for trajectory A is exponentially larger than that for B. In general, given a set of initial field values satisfying $|\phi| < \phi_{\rm inf}$ and $a > 0$, the trajectory with the longest a excursion, or, equivalently, the largest volume at the end of inflation, approaches to $a(V_C) \approx 0$.

[8]We have neglected the e-folds after the trajectory diffuses out of the stochastic range (see the justification in the next paragraph).

The reason that our mechanism is insensitive to the other regime of $|\phi| \gg \phi_{\rm inf}$ is as follows. Out of the two vertical black solid lines, both fields evolve following the classical motion. In particular, above the blue dashed line, both fields slow roll. Since the ϕ direction is much steeper than the a direction, the slow roll is mostly in the ϕ direction. Once the trajectory is out of the $|\phi| < \phi_{\rm inf}$ regime, it soon reaches the end-of-inflation boundary within a few e-folds $\ll \Delta N_{\rm end}$. We can neglect the expansion in this period compared to the exponentially large $\Delta N_{\rm end} = 10^{\mathcal{O}(10)}$ which is the typical timescale for the dynamics of $|\phi| < \phi_{\rm inf}$. This discussion applies not only to the case that the trajectory diffuses out of the ϕ-stochastic regime (trajectory B), but also to the case that the fields are initially in the slow-roll regime, e.g., trajectory C (green arrow from a diamond). Thus, the trajectory of C at the end-of-inflation boundary has a suppressed volume distribution, which is negligible compared to that of A or B. This corresponds to what we have discussed at the end of Sec. II. We argued that the long inflation compensates for the tuning of the initial condition. This is why the finely tuned initial value of ϕ for the A trajectory is preferred in the end.

IV. CONCLUSIONS AND DISCUSSION

We have shown that if the inflaton potential has a specific form, and if the CC is time varying, the CC can be relaxed during inflation. The price to pay was the tuning, which can be made technically natural, to realize the peculiar inflaton potential. This potential drives inflation when CC is positive, while it ends when CC is negative. The resulting Universe is filled by a landscape of the CC with a normal distribution peaked around zero. The time-varying CC, if it persists until today, leads to a deviation of the equation of state of the Universe and can be searched for in the future. In particular, if the measured CC is around the variance of the distribution, the equation of state is predicted to differ from −1 by $\mathcal{O}(1–10)\%$. In a time-varying CC model, the CC can be relaxed from $(10^3\ {\rm GeV})^4$, and in a slow-rolling scalar model, the CC can be relaxed from $(10\ {\rm MeV})^4$.

Let us recall some comments on the fine-tuning of our proposal compared with the conventional inflation models. In general, realistic inflation models have three kinds of fine-tunings: the inflaton potential shape to satisfy the slow-roll conditions, the tuning for the initial conditions, including the metric, inflaton field value, and the homogeneity over a Hubble patch, and the tuning of the cosmological constant. The initial conditions' tunings are compensated by the inflation volume. In the context of conventional hilltop inflation, the initial homogenous field value ϕ within the range $|\phi| < \phi_{\rm inf}$ may be preferred so that inflation lasts eternally, although there is a measure problem. In our case, the initial condition of $|\phi| < \phi_{\rm inf}$ is also preferred due to very long inflation, but it is not eternal thanks to the time-varying CC.

We would like to emphasize that the total amount of the other two tunings for our scenario can be smaller than conventional inflation models with a similar inflation scale. The requirement of criticality, which has a similar amount of tuning of the CC, includes the tuning for the slow-roll conditions of the ϕ potential and reduces the tuning of the CC by the same amount. As a result, the tuning in total may be less severe than conventional cases in which tunings are required for explaining the slow-roll conditions and the CC. We also remind that the tuning can be made technically natural.

ACKNOWLEDGMENTS

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APPENDIX A: SECOND INFLATION BY a

In the main part, we have focused on the inflation driven by ϕ and its termination. We found that most of the Universe finishing the ϕ inflation has the CC almost zero by assuming that ϕ has a peculiar potential form and the CC is time varying.

To obtain a consistent CC, we need a small enough $|V'_C|$. On the other hand, the small $|V'_C|$ can drive second inflation by a if the slow-roll condition

$$\epsilon(a) \simeq \frac{M_{\rm pl}^2}{2}\left(\frac{V'_C}{V}\right)^2 \ll 1 \tag{A1}$$

is satisfied. When the first inflation ends with $\tilde{V}_C^i \gg \Lambda_C^4$ by chance, $V \simeq \tilde{V}_C^i \gtrsim \Lambda_C^4$. Since $\sigma \simeq \frac{|V'_C| M_{\rm pl}}{\sqrt{6}} \lesssim \Lambda_C^4$ in our scenario, the second inflation has to take place.[9] When V_C decreases to $\sim\sigma$, the second inflation ends, and then the empty Universe starts to contract. This Universe may be difficult to have a consistent cosmology unless $\tilde{V}_C^i \lesssim \Lambda_C^4$.

Here, let us estimate the volume distribution of the empty Universe. The timescale of the second inflation can be obtained by solving the slow-roll equation

$$\Delta N_{\rm slowroll}^{\rm 2nd}[\tilde{V}_C^i] \simeq \frac{1}{2}\frac{(\tilde{V}_C^i)^2}{(V'_C)^2 M_{\rm pl}^2}. \tag{A2}$$

Therefore, the volume produced at t satisfying $V_C[t] \simeq \tilde{V}_C^i$ increases exponentially by

$$\Delta \log L_{\rm inf,2nd}^3 \sim 3\Delta N_{\rm slowroll}^{\rm 2nd}[\tilde{V}_C^i] \sim \frac{3}{2}\left(\frac{\tilde{V}_C^i}{V'_C M_{\rm pl}}\right)^2 \tag{A3}$$

[9]The first inflation reheats the Universe, but soon V_C dominates the Universe. The timescale for the matter- or radiation-dominated Universe can be neglected compared with the inflationary timescales.

due to the second inflation by a. On the other hand, the volume undergoing the first inflation also exponentially increases by

$$\Delta \log L_{\rm inf,1st}^3 \sim \frac{3}{2}\left(\frac{2\tilde{V}_C^i}{(V'_C)^2}\frac{(V_0 - V_0^c)}{M_{\rm pl}^2} + \left(\frac{\tilde{V}_C^i}{V'_C M_{\rm pl}}\right)^2\right). \tag{A4}$$

The second term is the same as Eq. (A3). The empty Universe should be subdominant compared to the Universe with a consistent cosmology. Thus, we need $V_0 - V_0^c \geq 0$. However, this leads to the negative central value of the CC [see the discussion around Eq. (23)]. Therefore, to explain the size of the CC, we need other contributions. A candidate is $\sigma \sim \Lambda_C^4$ from Eq. (25) in the main part, which predicts the current equation of state differing from -1 by $\mathcal{O}(1\text{–}10)\%$.[10]

We can alternatively have a positive CC if the V_C^i is close to the upper bound of the contours in Fig. 1. In this case, we have checked numerically that the distribution of a gets broadened due to the expansion effect (see Appendix B). Also, many light particles can contribute to the positive CC and further relax the CC (see Appendix C).

APPENDIX B: SOLUTIONS TO THE FOKKER-PLANCK EQUATION

Let us explain the dynamics of a during the critical eternal inflation more systematically. To this end, we assume that $H_{\rm inf}$ does not change over $\Delta N_{\rm diffuse}$ and estimate the distribution for the ultralight field a, whose mass can be neglected. The evolution of the classical motion of a is described by the Langevin equation

$$\dot{a} = -\frac{1}{3H_{\rm inf}}V'_C(a) + f(\vec{x}, t), \tag{B1}$$

where $V(a)$ is the potential for a and the dot and prime represent the derivative with respect to the cosmic time t and a, respectively. $f(\vec{x}, t)$ satisfies

$$\langle f(\vec{x}, t_1) f(\vec{x}, t_2)\rangle = \frac{H_{\rm inf}^3}{4\pi^2}\delta(t_1 - t_2), \tag{B2}$$

[10]This may be slightly in tension with the current CMB data, but the Hubble parameter itself may have $\mathcal{O}(10\%)$ tension between the early and late measurements [62], which may require certain physics beyond the standard model (BSM) to explain it. In certain BSM models, it may be consistent or even better to have a slow-roll scalar to further reduce the Hubble parameter at late time [63]. Alternatively, the change of the equation of state is suppressed if a couples to (dark) particles in the Universe. Then, the matter effect easily gives a friction for a [9,64–67]. Such a light field then can mediate force and can be tested phenomenologically if it couples to the SM matter or its spin [68,69] and if it couples to both SM matter's spin and dark matter [70]. The friction is not important during inflation, since the (dark) matter is absent there.

where $\langle\cdots\rangle$ represents the stochastic average that includes the short-wavelength modes. The corresponding Fokker-Planck equation can be derived as [71,72]

$$\frac{\partial\mathcal{P}(a,t)}{\partial t}=\frac{1}{3H_{\rm inf}}\frac{\partial}{\partial a}(V'_C\mathcal{P}(a,t))+\frac{H_{\rm inf}^3}{8\pi^2}\frac{\partial^2\mathcal{P}(a,t)}{\partial a^2}, \qquad \text{(B3)}$$

where $\mathcal{P}(a,t)$ denotes the probability distribution for the coarse-grained field a in a Hubble patch. This equation describes the time evolution of $\mathcal{P}(a,t)$ within the timescale Δt, satisfying $\Delta N_{\rm end}>\Delta t H_{\rm inf}$. By assuming a constant V'_C, we can solve this equation with an initial condition $P(a,0)=\delta(a-a_i)$ as

$$\hat{\mathcal{P}}(a,t,a_i)=\frac{1}{N_t}\exp\left(-\frac{(a-a_{\rm cl}[t,a_i])^2}{2\Delta a[t]^2}\right). \qquad \text{(B4)}$$

Here, $a_{\rm cl}\equiv -t\frac{V'_C}{3H_{\rm inf}}+a_i$ and $\Delta a^2\equiv tH_{\rm inf}^3/(2\pi)^2$, and $N_t\equiv\sqrt{2\pi\Delta a^2}$ is the normalization factor. With a general initial distribution $\mathcal{P}(a,0)$, the solution can be obtained from

$$\mathcal{P}(a,t)=\int da_i\hat{\mathcal{P}}(a,t,a_i)\mathcal{P}(a_i,0). \qquad \text{(B5)}$$

Let us consider $t\ll\Delta N_{\rm end}/H_{\rm inf}$, where we have to take account of the backreaction from the inflaton sector. Then, it is convenient to follow the volume distribution $\mathcal{L}^3[a,t]$. This is usually considered for the inflaton field [73–76]. A similar effect was discussed in Ref. [56] for estimating the validity for the estimation of the axion abundance from inflationary equilibrium distribution [29,56]. Here, we use the evolution equation of the inflating volume by taking into account both the inflationary expansion and the terminating probability for the inflation:

$$\frac{\partial\mathcal{L}^3[a,t]}{\partial t}\approx\left(3H_{\rm inf}+C\frac{V''_\phi}{3H_{\rm inf}}\right)\mathcal{L}^3[a,t]+\frac{\partial}{\partial a}\left(\frac{V'_C}{3H_{\rm inf}}\mathcal{L}^3(a,t)+\frac{H_{\rm inf}^{3/2}}{8\pi^2}\frac{\partial H_{\rm inf}^{3/2}\mathcal{L}^3(a,t)}{\partial a}\right), \qquad \text{(B6)}$$

where $\mathcal{L}^3[a,t]=\partial_a L_{\rm inf}^3[t]$ is defined in Eq. (33). The first term denotes the Hubble expansion minus the rate to end the inflation due to the ϕ dynamics discussed in Sec. II A.

We can solve the equation numerically. In Fig. 3, we show the solution of the inflating volume $\log\mathcal{L}^3[a,t]$ with starting from a normal distribution $\mathcal{L}^3[a,t]=\frac{1}{H_{\rm inf}^3}\frac{1}{\sqrt{2\pi\sigma_{\rm ini}^2}}e^{\frac{-(a-a_i)^2}{2\sigma_{\rm ini}^2}}$ with a variance $\sigma_{\rm ini}=M_{\rm pl}/\sqrt{8}$. Here, the initial volume is taken to be a Hubble volume $1/H_{\rm inf}^3$. $V_C^i=2\times10^{16}\Lambda_C^4$, $V'_C=10^{-68}$ GeV^3, and $H_{\rm inf}=10^{-19}$ GeV are taken for a sample set of parameters satisfying the conditions we have

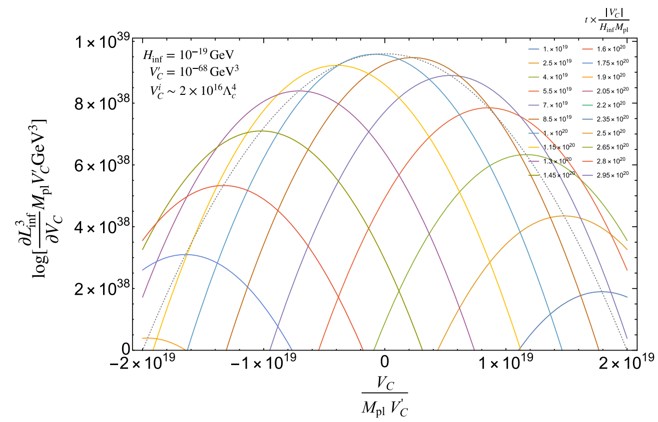

FIG. 3. The solution of the Fokker-Planck equation at several t for the solid lines. $V_C^i=2\times10^{16}\Lambda_C^4$, $V'_C=10^{-68}$ GeV^3, and $H_{\rm inf}=10^{-19}$ GeV are taken. The initial distribution is taken as a normal distribution with a variance of $V'_CM_{\rm pl}/\sqrt{8}$. The dotted line represents the maximum of the distribution at different t. These parameter choices satisfy the conditions we have discussed in the main part.

discussed in the main part. We have checked numerically that $\log\mathcal{L}^3[a,t]$ becomes the largest, and, thus, the inflationary volume $\mathcal{L}^3[a,t]$ becomes exponentially largest at $|V_C|<\Lambda_C^4$, in the whole integration time t.

In Fig. 4, we take $V_C^i=10^{26}\Lambda_C^4$ with other parameters unchanged from Fig. 3. This does not satisfy Eq. (35) in the main part. The inflationary volume is favored at larger V_C as we have explained intuitively and analytically in the main part. Our mechanism does not work in this region.

We did not take $\sigma_{\rm ini}$ much smaller than $M_{\rm pl}$ to have a delta function due to a numerical limitation. Because of the numerical power, we have directly checked the inequality (35) in the main part within a few orders of magnitude. On the other hand, we have checked that the mechanism does (not) work if $\sigma_{\rm ini}\lesssim(\gtrsim)\mathcal{O}(1)\sigma$ with $V_C^i=\mathcal{O}(1)\sigma$. This is an analytically equivalent condition to Eq. (35) [see the discussion below Eq. (35)].

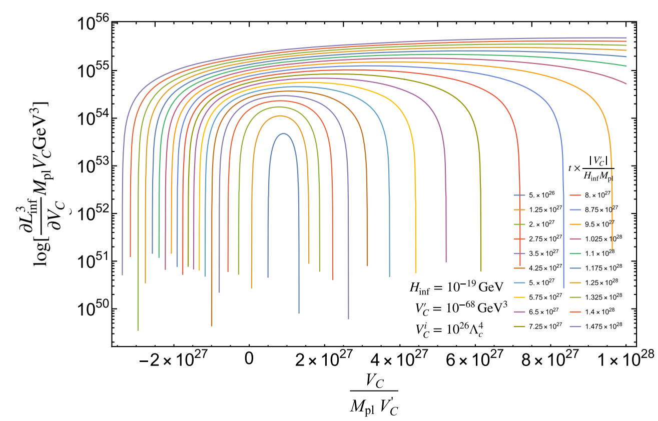

FIG. 4. The same as Fig. 3 except for $V_C^i=10^{26}\Lambda_C^4$, which does not satisfy Eq. (35).

APPENDIX C: SMALL CC FROM MANY SCALARS

We have shown that a single scalar and an inflaton for ultralow-scale inflation can relax the CC by $10^{40\text{–}50}$. We may introduce other particles or interactions to increase the inflation scale or further relax the CC.

One possible extension of the model is to consider N light particles a_α ($\alpha = 1 \ldots N$) whose potential is given by V_α. The vacuum energy is given by $V_C = \sum_\alpha V_\alpha$. We would not obtain a further relaxation of the CC if $V'_\alpha(>0)$ were a constant. This is because the system is equivalent to a single slow-roll field with the linear potential satisfying

$$V'_{C,\mathrm{eff}} = \sqrt{\sum_{\alpha=1}^{N} (V'_\alpha)^2}. \tag{C1}$$

In other words, all the discussed conditions and constraints apply to the single slow-roll field in the direction of $-\vec{V}'_\alpha$. However, in a realistic system, a potential V_α should have a minimum. Thus, the slow roll of a_α will end when it rolls down and settle into the minimum $a_\alpha = a_\alpha^{\min}$ at which $V'_\alpha[a_\alpha^{\min}] = 0$ but with nonvanishing mass $m_\alpha^2 = V''_\alpha[a_\alpha^{\min}] > 0$.

For concreteness, let us assume that $V_\alpha^i - V_\alpha(a_\alpha^{\min}) = \mathcal{O}(V_\alpha^i)$ is the same order as $V_\alpha^i \sim V_\beta^i = \tilde{\Lambda}^4 (> 0)$ and $V'_\alpha \gg V'_{\alpha+1} > 0$ at the beginning. Then, a_1 soon slowly rolls to the minimum. When this happens, the total potential energy V_C decreases by $\mathcal{O}(\tilde{\Lambda}^4)$ from $V_C^i = N\mathcal{O}(\tilde{\Lambda}^4)$. After the stabilization of a_1, the effective linear potential has $V'_{C,\mathrm{eff}(1)} = \sqrt{\sum_{\alpha=2}^{N} (V'_\alpha)^2} \sim V'_2$. This process recursively takes place until $V_C = \mathcal{O}(\tilde{\Lambda}^4)$, which crosses zero during the slow roll along the direction $\sim a_{n+1}$. We can neglect the timescales for the stabilization of $a_{\alpha\le n}$ compared to the slow-roll timescale of a_{n+1} because of our assumption $V'_\alpha \gg V'_{\alpha+1}$.

At the last slow roll, $V'_{C,\mathrm{eff}(n)} \sim V'_{n+1}$, $V_C \sim \mathcal{O}(\tilde{\Lambda}^4)$. Therefore, we can use the main part discussion with the replacement of $V'_C \to V'_{n+1}$ and $V_C^i \to \mathcal{O}(\tilde{\Lambda}^4)$. For instance, with $H_{\mathrm{inf}} \sim 10^{-18}$ GeV, $V'_{n+1} = 10^{-66}$ GeV, and $n \sim 4 \times 10^{24}$, we can relax the CC by 10^{48} by assuming $V_C^i = N\mathcal{O}(\tilde{\Lambda}^4) \sim n\mathcal{O}(\tilde{\Lambda}^4) < V_0$. We also take into account the equilibrium distribution contribution of $a_{\alpha\le n}$ around the minimum. Every a (which is light enough) settled into the minimum forms the equilibrium distribution if $m_\alpha \ll H_{\mathrm{inf}}$. Thus, the energy density is probabilistic with a typical value around $n \times 3H_{\mathrm{inf}}^4/(2\pi)^2$. We need $n \times 3H_{\mathrm{inf}}^4/(2\pi)^2 \lesssim \sigma$ to prevent the fluctuation of the vacuum energy contributing the volume distribution.

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