Heavier W boson, dark matter, and gravitational waves from strings in an SO(10) axion model

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Inspired by the recent determination of the W-boson mass by the CDF collaboration, we revisit an $SO(10)$ axion model in which a scalar $SU(2)_L$ triplet field with zero hypercharge is known to acquire a nonzero vacuum expectation value (VEV) through its mixing with the Standard Model Higgs doublet. The triplet VEV provides a sizable contribution to the W mass, which helps in significantly lowering the 7σ discrepancy between the Standard Model prediction and the higher CDF value for m_W. We show that the relatively light triplet mass (∼(1–50) TeV) is compatible with gauge coupling unification and observable proton decay. An unbroken Z_2 gauge symmetry, coupled with the presence of two fermionic 10-plets required to resolve the axion domain wall problem means that both axions and a stable intermediate mass (∼10^9–10^{10} GeV) fermion are plausible dark matter candidates. We also display the gravitational wave spectrum from the intermediate scale topologically stable cosmic strings predicted by the model.

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I. INTRODUCTION

In a recent paper [1] largely concerned with the electroweak monopole in grand unified theories (GUTs), it was briefly noted that a specific $SO(10)$ axion model contains a $SU(2)_L$ scalar triplet field with hypercharge $Y = 0$ that acquires a nonzero vacuum expectation value (VEV) through its mixing with the Standard Model (SM) Higgs doublet. It is well known that this VEV only contributes to the W-boson mass, which makes the $SO(10)$ axion model attractive in light of the recent measurement $m_W = 80.4335 \pm 0.0094$ GeV [2]. The CDF result is about 7σ away from the central value estimated within the SM [3–5], and a large number of papers that can explain this deviation has been proposed [6–60].

In order to realize (better) agreement with the higher value for m_W determined by CDF, the triplet VEV should make a significant contribution to the W mass while maintaining compatibility with the SM ρ parameter. This requires the triplet mass to be of order 10 TeV or so, and we show how this is achieved in the $SO(10)$ axion model while preserving the unification of the SM gauge couplings. In addition to the axion, the model also contains an intermediate-mass fermion dark matter (DM) candidate whose stability is guaranteed by a discrete Z_2 gauge symmetry. This Z_2 symmetry is also responsible for the existence of topologically stable cosmic strings.

The plan of the paper is as follows. In Sec. II we summarize the salient features of the model including the symmetry breaking pattern and the realization of higher m_W compared to the SM. Section III deals with gauge coupling unification and implications for proton decay. Section IV discusses the DM candidates consisting of axions and intermediate-mass neutral fermions. The gravitational wave (GW) spectrum from the intermediate scale cosmic strings is discussed in Sec. V, and we conclude with a summary in Sec. VI.

II. THE MODEL

In this section, we briefly outline the salient features of the $SO(10)$ axion model and refer the reader to Refs. [61,62] for additional details. To start with, we first describe the particle content of the setup and then present all the relevant interactions of these particles. We denote the fermion multiplets present in the model as

$$\psi_{16}^{(i)}(1) \quad (i = 1, 2, 3), \qquad \psi_{10}^{(\alpha)}(-2) \quad (\alpha = 1, 2), \quad (1)$$

and the scalar multiplets as

$$\phi_{10}(-2), \qquad \phi_{45}(4), \qquad \phi_{126}(2), \qquad \phi_{210}(0). \quad (2)$$

Here, the subscripts refer to the dimension of the representations under $SO(10)$, the Peccei-Quinn (PQ) charges ($Q_{\rm PQ}$) are quoted within parentheses, and i and α are the generation indices for the fermions. With the knowledge of the particle spectrum and symmetries of the model, next, we present all the relevant interactions involving these fields. The Yukawa couplings are

$$\psi_{16}^{(i)}\psi_{16}^{(j)}\phi_{10}, \qquad \psi_{16}^{(i)}\psi_{16}^{(j)}\phi_{126}^{\dagger}, \qquad \psi_{10}^{(1)}\psi_{10}^{(2)}\phi_{45}, \quad (3)$$

and the scalar couplings include

$$\phi_{210}\phi_{126}^{\dagger}\phi_{126}^{\dagger}\phi_{45}, \quad \phi_{210}\phi_{126}^{\dagger}\phi_{10}\phi_{45}, \quad \phi_{210}\phi_{126}\phi_{10}. \quad (4)$$

The SM fermions of each family, accompanied by a SM singlet right-handed neutrino, reside in the 16-dimensional representation of $SO(10)$. In addition, two generations of fermionic 10-plets are included to overcome the axion domain wall problem [61,62]. The electroweak sector of the model contains the SM Higgs doublet, which is a linear combination of the two $SU(2)_L$ doublets from ϕ_{10} and two doublets from ϕ_{126}. The three remaining scalar doublets obtain masses of order M_{II} [63].

At this stage some remarks about the so-called quality problem in axion models are in order. If one makes the rather arbitrary assumption that Planck scale suppressed operators are present in axion models, it then follows that they must not be permitted to spoil the axion resolution of the strong CP problem. We are therefore lead to the conclusion that the coefficients accompanying the potentially dangerous operators, dimension five (e.g., $\phi_{45}^4\phi_{210}$, $\phi_{45}^{\dagger}\phi_{45}^3\phi_{210}$) and some higher ones in our case, must be adequately suppressed. Clearly, such operators do not arise in the renormalizable $SO(10)$ framework, and their occurrence in the presence of gravity has not been convincingly demonstrated. Indeed, it has been suggested that wormhole tunneling may give rise to $U(1)_{\rm PQ}$ symmetry violating effects with exponentially suppressed coefficients, and they only become important for $f_a \gtrsim 10^{17}$ GeV [64–66]. In our model the axion decay constant f_a is, of course, orders of magnitude smaller than 10^{17} GeV and the problem is therefore avoided.

Be that as it may, perhaps a more elegant approach for resolving the axion quality problem is to assume that a suitable discrete gauge symmetry effectively behaves as $U(1)_{\rm PQ}$. Discrete gauge symmetries routinely arise from the four dimensional compactification of higher dimensional superstring theories, and the first examples based on this idea have been discussed in Ref. [67].

Finally, as shown in Ref. [68], it is possible that $U(1)_{\rm PQ}$ may appear as an accidental symmetry in $SO(10)$ models supplemented by a suitable continuous gauge symmetry. In this case too the axion quality problem is suitably ameliorated.

For definiteness, we employ a specific symmetry breaking pattern of $SO(10)$ shown in Eq. (5) which, among other things, also allows a light $SU(2)_L$ scalar triplet from ϕ_{45} that remains compatible with the unification of the SM gauge couplings. Note that the induced VEV of the scalar triplet arises from the coupling $\phi_{10}\phi_{10}\phi_{45}$.

$$\begin{aligned}
&SO(10)\times U(1)_{\rm PQ} \xrightarrow[M_U]{\langle 210(0)\rangle} \\
&SU(2)_L\otimes SU(2)_R\otimes SU(4)_C\times U(1)_{\rm PQ} \xrightarrow[M_I]{\langle (1,1,15)\in 210(0)\rangle} \\
&SU(2)_L\otimes SU(2)_R\otimes SU(3)_C\otimes U(1)_{B-L}\times U(1)_{\rm PQ} \xrightarrow[M_{II}]{\langle (1,3,1,-2)\in(1,3,10)\in\overline{126}(-2)\rangle} \\
&SU(3)_C\otimes SU(2)_L\otimes U(1)_Y\otimes \mathbb{Z}_2\times U(1)'_{\rm PQ} \xrightarrow[f_a]{\langle\{(1,1,0)\in(1,3,1)+(1,1,0)\in(1,1,15)\}\in 45(4)\rangle} \\
&SU(3)_C\otimes SU(2)_L\otimes U(1)_Y\otimes \mathbb{Z}_2 \xrightarrow[m_W]{\langle (1,2,\pm\frac{1}{2})\in 10(-2)\rangle} SU(3)_C\otimes U(1)_Q\otimes \mathbb{Z}_2. \qquad (5)
\end{aligned}$$

We employ two-loop renormalization group equations (RGEs) to estimate the GUT scale (M_U) and the two gauge symmetry breaking intermediate-scales M_I and M_{II}. We find it instructive and useful to distinguish the two latter scales from the axion symmetry breaking scale f_a ($\leq M_{II}$). The remnant anomalous global symmetry after M_{II} is $U(1)'_{\rm PQ}$, which is generated by $Q'_{\rm PQ} = 5Q_{\rm PQ} - 3(B-L) + 4T_R^3$, where T_R^3 is the diagonal generator of $SU(2)_R$. The $U(1)'_{\rm PQ}$ symmetry is broken by the VEV of (1,1,15) and (1,3,1) in 45(4) at the scale f_a. The fermions from ψ_{10} acquire masses during this symmetry breaking which we assume are all of the same order of magnitude,

$m_{\mathrm{DM}} = y_{45}\langle\phi_{45}\rangle$. The lightest neutral fermion from the 10-plets along with the axion can account for the observed dark matter relic density of the universe [69] (see Ref. [62] for details).

At this stage, it is important to point out that the above breaking chain allows a light $SU(2)_L$ scalar triplet from ϕ_{45} that is compatible with the unification of the gauge couplings. We now shed light on this scalar triplet and describe its role in raising the W-boson mass above the SM prediction as suggested by the CDF result. We can write the scalar triplet interaction that arises from the term $\phi_{10}\phi_{10}\phi_{45} + \mathrm{H.c.}$ of Eq. (4) as

$$-\lambda m_T {H_{10}^u}^{\mathrm{T}} i\sigma_2 T^i \sigma_i H_{10}^d + \mathrm{H.c.} \tag{6}$$

Here $T^i \equiv (1,3,0)$ is the complex triplet scalar from ϕ_{45}, and $H_{10}^u(\equiv(1,2,\frac{1}{2})) \oplus H_{10}^d(\equiv(1,2,-\frac{1}{2}))$ arise from the bidoublet $(2,2,1) \in \phi_{10}$. As a result of electroweak breaking and the presence of this term, a nonzero VEV is induced for the scalar triplet

$$v_T = \sqrt{2}\lambda v_{10}^u v_{10}^d / m_T, \tag{7}$$

where $\langle T^3\rangle = v_T/\sqrt{2}$ and $\langle H_{10}^d\rangle = v_{10}^d/\sqrt{2}$, $\langle H_{10}^u\rangle = v_{10}^u/\sqrt{2}$. The induced triplet VEV modifies the W-boson mass such that

$$m_W^2 = \frac{g^2}{4}(v_{\mathrm{SM}}^2 + 4v_T^2), \quad \text{with} \quad g_{2L}(m_Z) \equiv g, \tag{8}$$

and $v_{\mathrm{SM}} = 246$ GeV is the SM VEV. This means (as also shown in Eq. (11) of Ref. [21]) that the electroweak mixing angle and Z-boson mass remain unaltered, and the change in the ρ parameter is solely due to the W-mass anomaly. Following Eq. (8), the ρ-parameter can be expressed as

$$\rho = 1 + 4(v_T/v_{\mathrm{SM}})^2. \tag{9}$$

The experimental value of ρ in this case is 1.00219 ± 0.00044 (see Ref. [21]) which, in turn, implies that the central value of the triplet VEV is $v_T = 5.7561$ GeV.

Before closing this section, we would also like to make a few remarks about the topological defects in this model. The $SO(10)$ breaking at M_U to $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C$ yields a topologically stable monopole that subsequently turns into a superheavy monopole carrying a single unit of Dirac magnetic charge as well as some color magnetic charge. This monopole is inflated away within a suitable inflationary setting as shown, for instance, in Refs. [70,71]. The second breaking yields a stable monopole significantly lighter than M_U that carries two quanta of Dirac charge as well as color charge. Depending on the magnitude of the symmetry breaking scale M_I versus H_{inf}, the Hubble parameter during inflation, this monopole with mass $\sim 10 M_I$ may be present in our galaxy at an observable level.

As previously mentioned the unbroken Z_2 gauge symmetry implies the presence of topologically stable cosmic strings whose mass scale is determined by the second intermediate scale M_{II}. We will discuss these strings and their gravitational wave emission in Sec. V. Finally, for completeness let us note that the axion strings in this model appear after inflation and form a string-wall system at the QCD phase transition. The strings are superconducting and the loops emit axions and perhaps even the intermediate scale fermion dark matter.

TABLE I. Electroweak observables at m_Z [3].

Z-boson mass, m_Z	91.1876(21) GeV
Strong fine structure constant, α_{3C}	0.1179(10)
Fermi coupling constant, G_F	$1.1663787(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$
Electroweak mixing angle, $\sin^2\theta_W$	0.23121(4)

III. UNIFICATION SOLUTIONS

We aim to obtain unification solutions compatible with the electroweak observables (see Table I) in terms of the unified gauge coupling (g_U), intermediate scales (M_I and M_{II}), and unification scale (M_U) for different choices of m_{DM} and triplet scalar mass (m_T). We minimize the χ^2 defined at m_Z and given by

$$\chi^2 = \sum_{i=1}^{3} \frac{(g_i^2 - g_{i,\mathrm{exp}}^2)^2}{\sigma_{g_{i,\mathrm{exp}}^2}^2}, \tag{10}$$

where g_i ($i = Y, 2L, 3C$) are the SM gauge couplings at m_Z obtained through the RGEs starting from the unified gauge coupling at the unification scale and $g_{i,\mathrm{exp}}$ are their experimental values. We compute the β-coefficients as outlined in Refs. [72–74]. The one- and two-loop β-coefficients governing the renormalization group evolution of the gauge couplings at different stages are given in Table II.

We show the RGE running of the gauge couplings in Fig. 1 for a unification solution with $m_T = 10$ TeV,

TABLE II. One- and two-loop beta coefficients for the renormalization group evolution of the gauge couplings at different stages of gauge symmetry.

$\mathcal{G}_{2_L 2_R 4_C} \times U(1)_{PQ}$	$\mathcal{G}_{2_L 2_R 3_C 1_{B-L}} \times U(1)_{PQ}$
$\begin{pmatrix} 4 \\ \frac{32}{3} \\ \frac{5}{3} \end{pmatrix}, \begin{pmatrix} 108 & 51 & \frac{525}{2} \\ 51 & \frac{884}{3} & \frac{1245}{2} \\ \frac{105}{2} & \frac{249}{2} & \frac{3551}{6} \end{pmatrix}$	$\begin{pmatrix} -\frac{2}{3} \\ 0 \\ -\frac{17}{3} \\ \frac{41}{6} \end{pmatrix}, \begin{pmatrix} \frac{142}{3} & 9 & 12 & \frac{3}{2} \\ 9 & 66 & 12 & \frac{27}{2} \\ \frac{9}{2} & \frac{9}{2} & -\frac{2}{3} & \frac{7}{6} \\ \frac{9}{2} & \frac{81}{2} & \frac{28}{3} & \frac{187}{6} \end{pmatrix}$
$\mathcal{G}_{3_C 2_L 1_Y \mathbb{Z}_2} \times U(1)'_{PQ}$	$\mathcal{G}_{3_C 2_L 1_Y \mathbb{Z}_2}$ (Triplet)
$\begin{pmatrix} -\frac{17}{3} \\ -\frac{7}{6} \\ \frac{163}{30} \end{pmatrix}, \begin{pmatrix} -\frac{2}{3} & \frac{9}{2} & \frac{41}{30} \\ 12 & \frac{245}{6} & \frac{3}{2} \\ \frac{164}{15} & \frac{9}{2} & \frac{667}{150} \end{pmatrix}$	$\begin{pmatrix} -7 \\ -\frac{5}{2} \\ \frac{41}{10} \end{pmatrix}, \begin{pmatrix} -26 & \frac{9}{2} & \frac{11}{10} \\ 12 & \frac{49}{2} & \frac{9}{10} \\ \frac{44}{5} & \frac{27}{10} & \frac{199}{50} \end{pmatrix}$

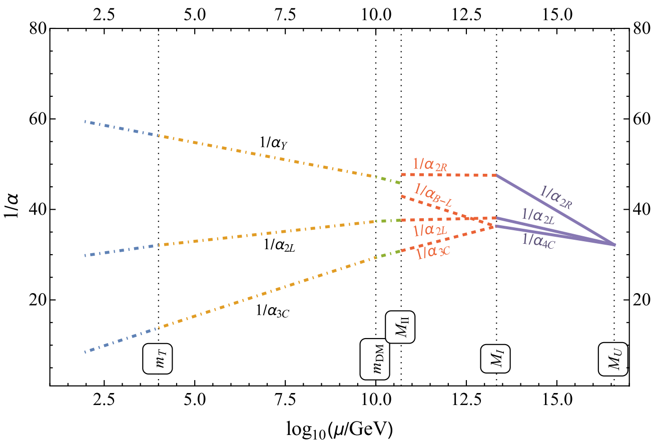
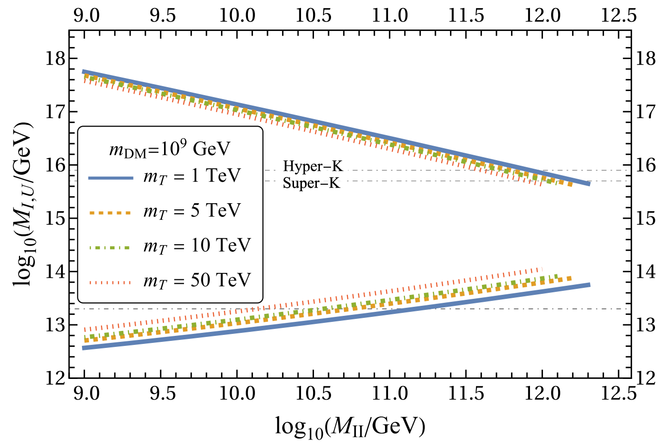


FIG. 1. Renormalization group evolution of the gauge couplings for a unification solution with $m_T = 10$ TeV, $m_{\rm DM} = 10^{10}$ GeV, $M_{II} = 5.0 \times 10^{10}$ GeV, $M_I = 2.15 \times 10^{13}$ GeV, $M_U = 3.8 \times 10^{16}$ GeV, and $g_U = 0.624$.

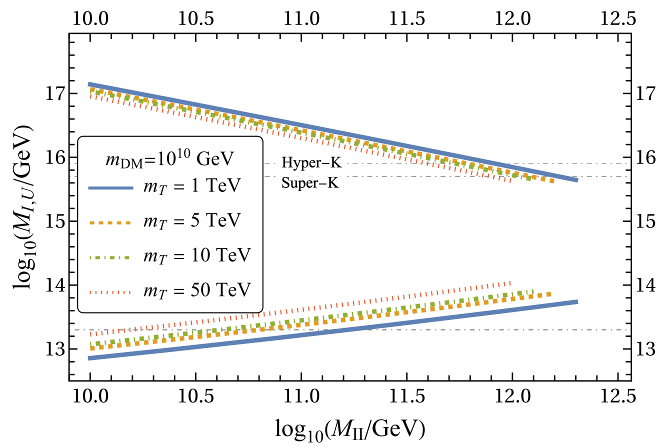
$m_{\rm DM} = 10^{10}$ GeV, $M_{II} = 5.0 \times 10^{10}$ GeV, $M_I = 2.15 \times 10^{13}$ GeV, $M_U = 3.8 \times 10^{16}$ GeV, and $g_U = 0.624$.

Next, in Table III we show the unification solutions for two typical values 10^9 GeV and 10^{10} GeV of $m_{\rm DM}$ with $m_T = \{1, 5, 10, 50\}$ TeV for each value of $m_{\rm DM}$. In Fig. 2, we have plotted the unification scale (M_U) and first intermediate scale (M_I) as functions of the second intermediate scale (M_{II}) for different choices of $m_{\rm DM}$ and m_T.

The nonobservation of proton decay in the Super-Kamiokande (Super-K) experiment has pushed the partial lifetime bound for the decay channel $p \to e^+\pi^0$ to be above 2.4×10^{34} yrs [75], which constrains the unification scale $M_U \gtrsim 5.3 \times 10^{15}$ GeV. On the other hand, the Hyper-Kamiokade (Hyper-K) experiment has 3σ discovery potential to probe the channel $p \to e^+\pi^0$ with partial lifetime 10^{35} yrs [76] which corresponds to $M_U \simeq 7.5 \times 10^{15}$ GeV. We have indicated the Super-K lower limit and the Hyper-K sensitivity in Fig. 2. There are unification solutions that are compatible with the Super-K bound and a part of them will be probed by the Hyper-K experiment as can be seen in Fig. 2. The monopoles produced during the symmetry



(a) $m_{\rm DM} = 10^9$ GeV.



(b) $m_{\rm DM} = 10^{10}$ GeV.

FIG. 2. Variation of unification scale (M_U) and first intermediate breaking scale (M_I) with the second intermediate breaking scale (M_{II}) for different choices of the triplet scalar mass (m_T) and dark matter mass ($m_{\rm DM}$). The horizontal dot-dashed lines at $\log_{10}(M_U/\text{GeV})$ equal to 15.7 and 15.9 are the lower bound on M_U from the Super-Kamiokande experiment and the sensitivity of the proposed Hyper-Kamiokande experiment respectively. The horizontal dot-dashed line at $\log_{10}(M_I/\text{GeV}) = 13.3$ is the lower bound from the MACRO experiment within the inflationary scenario driven by the Coleman-Weinberg potential of a real GUT singlet [71].

TABLE III. Unification solutions for the unification scale M_U, intermediate scales M_I and M_{II}, and unified coupling g_U for different choices of $m_{\rm DM}$ and m_T.

$m_{\rm DM}$	m_T (TeV)	$\log_{10}(\frac{M_U}{\rm GeV})$	$\log_{10}(\frac{M_I}{\rm GeV})$	$\log_{10}(\frac{M_{II}}{\rm GeV})$	g_U
10^9 GeV	1	$\{17.75, 15.65\}$	$\{12.57, 13.75\}$	$\{9.0, 12.3\}$	$\{0.679, 0.605\}$
	5	$\{17.68, 15.63\}$	$\{12.71, 13.88\}$	$\{9.0, 12.2\}$	$\{0.671, 0.600\}$
	10	$\{17.65, 15.66\}$	$\{12.77, 13.91\}$	$\{9.0, 12.1\}$	$\{0.668, 0.600\}$
	50	$\{17.58, 15.63\}$	$\{12.91, 14.04\}$	$\{9.0, 12.0\}$	$\{0.661, 0.596\}$
10^{10} GeV	1	$\{17.14, 15.65\}$	$\{12.86, 13.74\}$	$\{10.0, 12.3\}$	$\{0.649, 0.600\}$
	5	$\{17.07, 15.62\}$	$\{13.01, 13.87\}$	$\{10.0, 12.2\}$	$\{0.642, 0.596\}$
	10	$\{17.03, 15.65\}$	$\{13.07, 13.90\}$	$\{10.0, 12.1\}$	$\{0.639, 0.596\}$
	50	$\{16.96, 15.63\}$	$\{13.23, 14.03\}$	$\{10.0, 12.0\}$	$\{0.633, 0.592\}$

breaking at the scale M_I should be partially inflated to satisfy the lower bound on the monopole flux $2.8 \times 10^{-16}\ \mathrm{cm^{-2}\,s^{-1}\,sr^{-1}}$ [77]. In the inflationary scenario driven by the Coleman-Weinberg potential of a real GUT singlet, the monopoles undergo a sufficient number of e-foldings to comply with the MACRO bound for $M_I \gtrsim 2 \times 10^{13}$ GeV [71] which is compatible with a good part of the unification solutions as shown in Fig. 2.

IV. AXION AND INTERMEDIATE SCALE FERMION DARK MATTER

In this section we investigate the scenario of axion and the lightest neutral component of the 10-plet as DM candidates in the model such that

$$\Omega_{\rm Total}h^2 = \Omega_a h^2 + \Omega_{10}h^2, \tag{11}$$

where $\Omega_a h^2$ and $\Omega_{10}h^2$ denote the axion and fermion relic densities respectively. It is interesting to point out that in this model axions can be produced by two different mechanisms, namely (a) the misalignment mechanism [78–81] and (b) the decay of axionic strings [81,82]. The relic axion abundance produced by the misalignment mechanism is expressed as [81]

$$\Omega_a^{\rm mis}h^2 \simeq 0.236\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6}\langle\theta^2 f(\theta)\rangle, \tag{12}$$

where θ denotes the misalignment angle that lies in the interval $[-\pi,\pi]$ [83]. The function $f(\theta)$ contains the anharmonicity of the axion potential, and $\langle\theta^2 f(\theta)\rangle$ evaluated in the interval $[-\pi,\pi]$ turns out to be around 8.77 [81]. As previously discussed in Sec. II, the decay of $U(1)_{\rm PQ}$ strings also contributes significantly in the production of axions and hence cannot be ignored. This contribution to the relic density can be expressed as [81]

$$\Omega_a^{\rm str}h^2 \simeq 0.34\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6}. \tag{13}$$

The total axion relic density is thus given by

$$\Omega_a h^2 = \Omega_a^{\rm mis}h^2 + \Omega_a^{\rm str}h^2 \simeq 2.41\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6}. \tag{14}$$

In Fig. 3, we show how $\Omega_a h^2$ varies with f_a, with the black dashed line denoting the Planck limit [69] on the relic DM abundance. With $f_a \simeq 8 \times 10^{10}$ GeV, the axion saturates the observed DM relic density.

For f_a smaller than 8×10^{10} GeV, the contribution from the fermionic DM component should be taken into account. We do not aim to discuss the production mechanism of the fermion DM but provide, instead, a rough analytical estimate of its abundance ($Y_{\rm DM} = n_{\rm DM}/s$). The relic density of the fermion DM can be expressed as

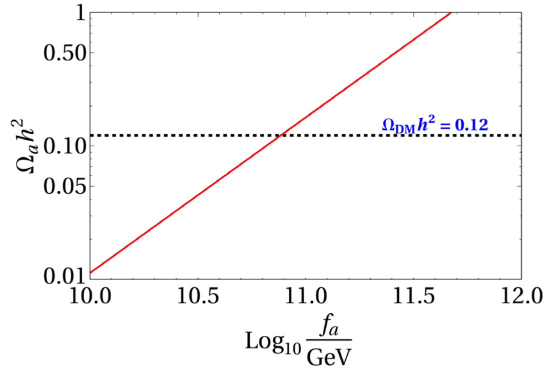


FIG. 3. Variation of the axion relic density with the axion decay constant f_a. The black dashed line corresponds to $\Omega_{\rm DM}h^2 = 0.12$.

$$\Omega_{\rm Total}h^2 - \Omega_a h^2 = \frac{m_{\rm DM}Y_{\rm DM}s_0}{\rho_c}, \tag{15}$$

where $s_0 \simeq 2890\ \mathrm{cm^{-3}}$ is the present entropy density and $\rho_c \simeq 1.05 \times 10^{-5}\ \mathrm{GeV\,cm^{-3}}$ is the present day critical density. Using Eq. (15), we find that

$$Y_{\rm DM} \simeq 4.36 \times 10^{-10}(\Omega_{\rm Total}h^2 - \Omega_a h^2)\left(\frac{\mathrm{GeV}}{m_{\rm DM}}\right). \tag{16}$$

In Fig. 4, we show the variation of the asymptotic yield of the fermion DM with its mass for three different values of the axion decay constant f_a. The solid red line that corresponding to $f_a = 10^{10}$ GeV suggests that around 91% of the total relic density of the DM is composed of intermediate mass scale fermions, with the remaining 9% coming from axions. As expected, making f_a larger increases the axion contribution to the total DM relic density (see Fig. 3), and the corresponding contribution from the fermion DM has to be reduced. This can be seen from the red dashed ($f_a = 5 \times 10^{10}$ GeV) and red dot-dashed ($f_a = 7 \times 10^{10}$ GeV) lines in Fig. 4.

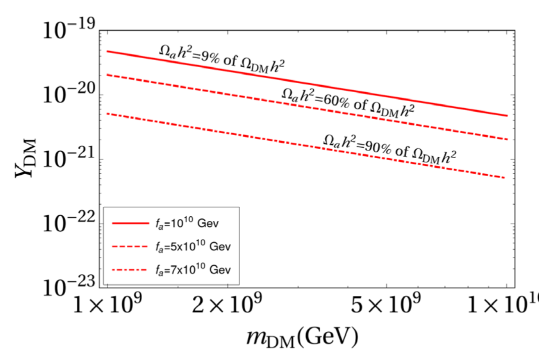


FIG. 4. Variation of the fermion DM yield versus its mass for three different values of f_a: 10^{10} GeV (red solid), 5×10^{10} GeV (red dashed), and 7×10^{10} GeV (red dot-dashed).

V. GRAVITATIONAL WAVES FROM COSMIC STRING LOOPS

The spontaneous symmetry breaking at M_{II} generates local cosmic strings which are topologically stable. The dimensionless tension of the strings is given by

$$G\mu = \frac{1}{8}\left(\frac{M_{II}}{m_{\rm Pl}}\right)^2, \tag{17}$$

where G is Newton's gravitational constant and $m_{\rm Pl}$ is the reduced Planck mass. The strings intercommute and form loops that decay by emitting GWs. We estimate the gravitational wave spectra following the burst method described in Refs. [84–86]. To this end, we need the loop distribution function $n(l,t)$ (the number density of loops per unit loop length l) at the time of GW emission. This is given in the different cosmic epochs in Refs. [87,88] (also see the Supplemental Material of Ref. [89]).

In the radiation dominated universe, we have

$$n_r(l,t) = \frac{0.18}{t^{3/2}(l+\Gamma G\mu t)^{5/2}}\Theta(0.1t - l), \tag{18}$$

where $\Gamma \simeq 50$. In the matter dominated universe there are two contribution. For the loops that are remnants from the radiation era

$$n_{rm}(l,t) = \frac{0.18 t_{\rm eq}^{1/2}}{t^2(l+\Gamma G\mu t)^2}\Theta(0.18 t_{\rm eq} - l - \Gamma G\mu(t - t_{\rm eq})), \tag{19}$$

where $t_{\rm eq}$ is the equidensity time, and for the loops that are produced during the matter dominated era

$$n_m(l,t) = \frac{0.27 - 0.45(l/t)^{0.31}}{t^2(l+\Gamma G\mu t)^2}\Theta(0.18t - l)\,\Theta(l + \Gamma G\mu(t - t_{\rm eq}) - 0.18 t_{\rm eq}). \tag{20}$$

Assuming cusp domination, the waveform at frequency f and redshift z is given by

$$h(f,l,z) = g_{1c}\frac{G\mu l^{2/3}}{(1+z)^{1/3} r(z)} f^{-4/3}, \tag{21}$$

where $g_{1c} \simeq 0.85$ [89] and r is the proper distance

$$r(z) = \int_0^z \frac{dz'}{H(z')}, \tag{22}$$

with H being the Hubble parameter. For the burst rate per unit space-time volume we have

$$\frac{d^2R}{dz dl} = N_c H_0^{-3}\phi_V(z)\frac{2n(l,t(z))}{l(1+z)}\left(\frac{\theta_m(f,l,z)}{2}\right)^2\Theta(1-\theta_m), \tag{23}$$

where H_0 is the present value of the Hubble parameter,

$$\theta_m(f,l,z) = \left[\frac{\sqrt{3}}{4}(1+z) f l\right]^{-1/3} \tag{24}$$

is the beam opening angle, and

$$\phi_V(z) = \frac{4\pi H_0^3 r^2}{(1+z)^3 H(z)}. \tag{25}$$

We have taken $N_c = 2.13$ as in Ref. [85].

The GW background is given by

$$\Omega_{GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 \int_{z_*}^{z(t_F)} dz \int dl\, h^2(f,l,z)\frac{d^2R}{dz dl}, \tag{26}$$

where t_F is the time when loop formation starts and the lower limit z_* in the integral in Eq. (26) leaves out the infrequent bursts from the stochastic background [85] so that

$$\int_0^{z_*} dz \int dl \frac{d^2R}{dz dl} = f. \tag{27}$$

We have taken the integration limit on l to be from 0 to $2t$ ($3t$) for the radiation (matter) domination. The various Heaviside Θ functions will anyway control the upper and lower integration limits during numerical evaluations.

The gravitational wave spectra for the breaking scales $M_{II} \in [10^{9.5}, 10^{12.3}]$ GeV are shown in Fig. 5. They satisfy the present PPTA bound [90] and can be probed in various

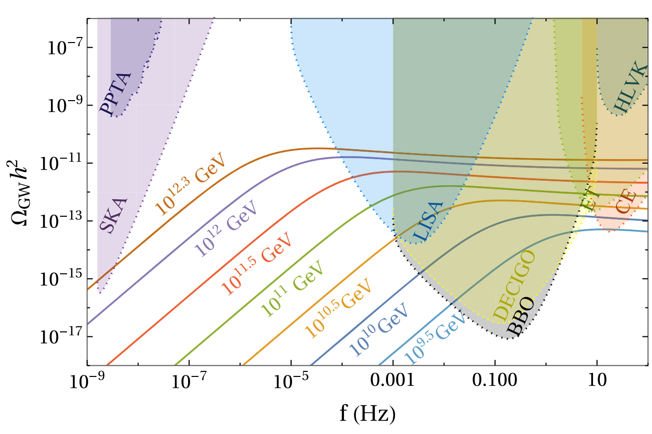


FIG. 5. Gravitational wave spectra from cosmic strings generated during the symmetry breaking at $\log_{10}(M_{II}/{\rm GeV}) = [9.5, 12.3]$. The sensitivity curves [104,105] for PPTA [90] and various proposed experiments, namely, SKA [91,92], CE [93], ET [94], LISA [95,96], DECIGO [97], BBO [98,99], HLVK [106], etc., are shown on the plot.

proposed experiments, including SKA [91,92], CE [93], ET [94], LISA [95,96], DECIGO [97], and BBO [98,99]. We have assumed, without loss of any generality, that the network of the string loops is present in the horizon from a very early time $t_F = 10^{-25}$ sec. In fact, in an inflationary universe driven by the Coleman-Weinberg potential of a real GUT-singlet [100,101], inflation ends at a cosmic time 8.3×10^{-37} sec and the phase transitions occur during inflation only if the corresponding symmetry breaking scales $\gtrsim 10^{13}$ GeV [102]. Therefore, the strings in the present case are produced after the end of inflation during the inflaton oscillation [71]. Needless to say, the new radiation temperature dominates over the Hawking temperature from the inflaton oscillations soon after inflation. Consequently, the Ginzburg criterion [103] for a phase transition is governed by the radiation temperature which approaches the reheat temperature (see Ref. [71]) at the reheat time $t_r \simeq 2.3 \times 10^{-25}$ sec. The smaller loops formed during the inflaton oscillation era do not contribute to the gravitational wave background within the frequency range of nHz to kHz. Therefore, we can safely take $t_F = 10^{-25}$ sec to compute the GW spectra.

VI. SUMMARY

We have discussed how the recent measurement of m_W by CDF can be readily incorporated into a well-motivated axion model based on $SO(10)$ grand unification. No *ad hoc* additional symmetries are imposed, in line with the spirit of the Standard Model. The axion symmetry breaking scalar field contains an $SU(2)_L$ triplet component that acquires a nonzero VEV through its mixing with the SM doublet. We show how the unification of the SM gauge couplings is preserved with an appropriate symmetry-breaking pattern of $SO(10)$. The proton lifetime is estimated to lie within the reach of future experiments. The model contains two 10-plets of fermions that are introduced to resolve the axion domain wall problem. An unbroken Z_2 gauge symmetry from $SO(10)$ ensures the presence of a stable intermediate-mass fermion from these 10-plets which, in addition to the axion, is a plausible dark matter candidate. The Z_2 symmetry also yields topologically stable intermediate scale cosmic strings whose gravitational wave spectrum we have also provided.

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