Gluon fragmentation into $B_c^{(*)}$ in NRQCD factorization

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The universal fragmentation functions of gluon into the flavored quarkonia B_c and (polarized) B_c^* are computed within NRQCD factorization framework at the lowest order in velocity expansion and strong coupling constant. It is mandatory to invoke the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi renormalization program to render the NRQCD short-distance coefficients UV finite in a pointwise manner. The calculation is facilitated with the sector decomposition method, with the final results presented with high numerical accuracy. This knowledge is useful to enrich our understanding toward the large-p_T behavior of $B_c^{(*)}$ production at LHC experiment.

I. INTRODUCTION

Like parton distribution functions (PDFs), fragmentation functions (FFs) are process-independent functions that encapsulate the nonperturbative hadronization effect and play a central role in QCD phenomenology of collider physics. In accordance with the celebrated QCD factorization theorem [1], the inclusive production rate of an identified hadron H at large transverse momentum in high energy collision experiment is dictated by the fragmentation mechanism:

$$
\begin{aligned}
d\sigma[A + B &\to H(p_T) + X] \\
&= \sum_i d\hat{\sigma}[A + B \to i(p_T/z) + X] \otimes D_{i\to H}(z,\mu) \\
&\quad + \mathcal{O}(1/p_T^2),
\end{aligned}
\tag{1}
$$

where A, B represent two colliding particles, $d\hat{\sigma}$ denotes the inclusive rate for producing the parton i, $D_{i\to H}(z)$ is the fragmentation function for the parton i into H, which characterizes the probability for i to transition into any final state containing the hadron H specifying the fractional light-cone momentum z with respect to the parent parton i. The sum in (1) is extended over all species of partons, $i = q, \bar{q}, g$.

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Similar to the PDFs, the scale dependence of fragmentation functions is also governed by the celebrated Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation. Taking the gluon fragmentation into the hadron H as example, the DGLAP equation reads

$$
\frac{d}{d\ln\mu^2} D_{g\to H}(z,\mu) = \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ig}(\xi, \alpha_s(\mu)) D_{i\to H}\left(\frac{z}{\xi}, \mu\right),
\tag{2}
$$

where μ is interpreted as the renormalization scale, and $P_{ig}(\xi)$ are the corresponding splitting kernels. Once this FF is deduced at some initial scale μ_0 by any means, one then deduces its form at any other scale μ by solving the evolution equation (2).

The fragmentation functions for light hadrons such as π, K, p, …, are hopelessly nonperturbative objects, which can only be extracted from experimental data. On the contrary, it was realized in the mid-1990s that FFs for heavy quarkonia need not be genuinely nonperturbative entities, which nevertheless can be largely understood in perturbative QCD by the virtue of asymptotic freedom [2,3]. This philosophy is systematically embodied in the modern nonrelativistic QCD (NRQCD) factorization framework [4], that is, the quarkonium FFs can be expressed as the sum of products of short-distance coefficients (SDCs) and long-distance yet universal NRQCD matrix elements, with the series organized by the velocity expansion [2,3]. Since the nonperturbative NRQCD matrix elements are merely numbers rather than functions, the profiles of the quarkonia FFs are largely determined by perturbation theory, therefore NRQCD factorization approach is endowed with strong predictive power.

During the past three decades, a number of fragmentation functions for S-wave/P-wave charmonia/bottomonia have been investigated in the context of NRQCD factorization approach, typically at lowest order in α_s (for a very incomplete list, see [2,3,5–25]). With the advance of higher-order calculational technique, the SDCs associated with some S-wave quarkonium FFs have recently been calculated through the next-to-leading order (NLO) in α_s [14,19,26–29].

Unlike J/ψ and Υ, the B_c meson is the unique heavy quarkonium which is composed of two different heavy flavors: the b and $\bar{c}$ quarks. It is interesting to understand the production mechanism of this special heavy meson in hadron collision environment. The LO fragmentation functions for $b/\bar{c} \to B_c^{(*)}$ was computed long ago. The NLO perturbative correction has also been recently available. Due to the rich gluon content inside the proton in small x region, it is also of great phenomenological incentive to study the $g \to B_c^{(*)}$ fragmentation functions to predict their production rates at LHC. To produce a B_c meson in this case, one has to first create a pair of $c\bar{c}$ and another pair of $b\bar{b}$, therefore the perturbative order of this LO fragmentation process is already comparable with that of the NLO QCD correction to $b/\bar{c} \to B_c^{(*)}$. Thus, the computation of the $g \to B_c^{(*)}$ fragmentation functions at LO is already rather challenging technically, which have never been considered before. The aim of this work is to fill this gap, by computing the fragmentation functions for

$g \to B_c^{(*)}$ at lowest order in velocity expansion and α_s, by invoking the sector decomposition technique widely used in the area of multiloop computation.

The paper is organized in the following way. We first present the definition of the fragmentation functions for $g \to B_c^{(*)}$ following Collins and Soper, and the desired NRQCD factorization formula in Sec. II. In Sec. III, we illustrate the matching procedure, the strategy of loop integral computation and the subtraction of the UV divergences, and present the numerical results for the short-distance coefficients and $g \to B_c^{(*)}$ fragmentation functions. Finally, we summarize in Sec. IV.

II. FRAGMENTATION FUNCTION AND ITS NRQCD FACOTRIZATION

In literature there are several different strategies to extract the quarkonium FFs. Among them, the most systematic approach is to start from the gauge-invariant operator definition for the fragmentation functions pioneered by Collins and Soper long ago [30] (Note that this definition was first used by Ma to compute the quarkonium FFs in NRQCD [6]). One great virtue of this operator-based approach is to render the renormalization program transparent. According to the operator definition given in [30], the g-to-$B_c^{(*)}$ fragmentation function is expressed as (see also [15,17]):

$$D_{g \to B_c^{(*)}}(z,\mu) = \frac{-g_{\mu\nu} z^{D-3}}{2\pi k^+ (N_c^2-1)(D-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+ x^-}$$
$$\times \langle 0|G_c^{+\mu}(0)\Phi^\dagger(0,0,\mathbf{0}_\perp)_{cb} \sum_X |B_c^{(*)}(P)+X\rangle \langle B_c^{(*)}(P)+X|\Phi(0,x^-,\mathbf{0}_\perp)_{ba} G_a^{+\nu}(0,x^-,\mathbf{0}_\perp)|0\rangle. \quad (3)$$

$D = 4 - 2\varepsilon$ signifies the space-time dimensions, and μ is the renormalization scale associated with this nonlocal composite operator. $G_{\mu\nu}$ is the matrix-valued gluon field-strength tensor in the adjoint representation of $SU(N_c)$. Here it is convenient to adopt the light-cone coordinates. Any four-vector $A^\mu = (A^0, A^1, A^2, A^3)$ can be recast in the light-cone format $A^\mu = (A^+, A^-, \mathbf{A}_\perp)$, with $A^\pm \equiv \frac{1}{\sqrt{2}}(A^0 \pm A^3)$ and $\mathbf{A}_\perp \equiv (A^1, A^2)$. We assume to work in a frame where the B_c meson is moving along the z direction with the $+$-momentum P^+, Moreover, $k^+ = P^+/z$ denotes the $+$-component momentum of injected by the gluon field strength operator. The symbol X indicates collectively those unobserved light hadrons accompanying the B_c.

The eikonal factor $\Phi(0, x^-, \mathbf{0}_\perp)$ in (3) is a path-ordered exponential of the gluon field, inserted to ensure the gauge invariance of the FF:

$$\Phi(0, x^-, \mathbf{0}_\perp)_{ba} = \mathsf{P} \exp\left[ig_s \int_{x^-}^\infty dy^- A^+(0^+, y^-, \mathbf{0}_\perp) \right]_{ba}, \quad (4)$$

where A^μ designates the matrix-valued gluon field in the adjoint representation, g_s is the QCD coupling constant, and P denotes the path-ordering.

The key observation underlying the NRQCD factorization that, prior to forming a B_c state via soft interaction, the b and $\bar{c}$ quarks have to be first created with slow relative momentum, also within small distance $\sim 1/m_{c,b}$, which necessarily involves hard momentum transfer, thus can be studied in perturbation theory owing to asymptotic freedom of QCD. At the lowest order in velocity expansion, NRQCD factorization [4] allows one to refactorize the $g \to B_c^{(*)}$ FFs into the product of the SDCs and the long-distance NRQCD matrix elements:

$$D_{g \to B_c}(z, \mu) = d_{g \to B_c}(z, \mu) \frac{\langle 0|\mathcal{O}_1^{B_c}(^1S_0)|0\rangle}{M^3} + \cdots, \quad (5a)$$

$$D_{g \to B_c^*}(z, \mu) = d_{g \to B_c^*}(z, \mu) \frac{\langle 0|\mathcal{O}_1^{B_c^*}(^3S_1)|0\rangle}{M^3} + \cdots, \quad (5b)$$

where $M = m_b + m_c$, D is the dimension of space-time, $d_{g \to H}(z, \mu)$ is the desired SDCs for $H = B_c, B_c^*$, and the corresponding NRQCD production operators are defined by

$$\mathcal{O}_1^{B_c}(^1S_0) = \chi_c^\dagger \psi_b \sum_X |B_c + X\rangle\langle B_c + X|\psi_b^\dagger \chi_c, \quad (6a)$$

$$\mathcal{O}_1^{B_c^*}(^3S_1) = \chi_c^\dagger \boldsymbol{\sigma}\psi_b \sum_X |B_c^* + X\rangle \cdot \langle B_c^* + X|\psi_b^\dagger \boldsymbol{\sigma}\chi_c, \quad (6b)$$

where ψ_b and $\chi_c^\dagger$ are the NRQCD field operators that annihilate a b quark and a $\bar{c}$ quark, respectively.

Using the vacuum saturation approximation, these vacuum NRQCD matrix elements can be approximated by the radial wave function at the origin for $B_c^{(*)}$ in phenomenological potential model:

$$\langle\mathcal{O}_1^{B_c}(^1S_0)\rangle \simeq \frac{N_c}{2\pi}|R(0)|^2,$$

$$\langle\mathcal{O}_1^{B_c}(^3S_1)\rangle \simeq (D-1)\frac{N_c}{2\pi}|R(0)|^2. \quad (7)$$

III. COMPUTATION OF THE FRAGMENTATION FUNCTIONS

A. Matching procedure

We can proceed to calculate the SDCs $d_1^H(z)$ by the standard perturbative matching technique, i.e., by replacing the physical H state in (5) with the free quark pair $b\bar{c}$ carrying the appropriate quantum number. Concretely speaking, one replaces B_c by a fictitious meson $b\bar{c}(^1S_0^{(1)})$, and replaces B_c^* by the quark state $b\bar{c}(^3S_1^{(1)})$. Computing the left side of (5) using perturbative QCD, combined with the following vacuum matrix elements in perturbative NRQCD:

$$\langle\mathcal{O}_1^{b\bar{c}}(^1S_0)\rangle = 2N_c, \qquad \langle\mathcal{O}_1^{b\bar{c}}(^3S_1)\rangle = 2N_c(D-1), \quad (8)$$

we can directly solve the SDC $d_1^H(z)$, order by order in α_s.

The fragmentation function defined in (3) is manifestly gauge-invariant. In practical calculation, we specialize to Feynman gauge for simplicity. We use a private *Mathematica* code to automatically generate the Feynman diagrams and the associated cut amplitudes that correspond to the perturbative fragmentation function defined in (3). The Feynman rules for the eikonal propagator and vertex [30], as well as those for conventional QCD propagators and vertices are also implemented with the aid of the package QGRAF [31]. There

are seven diagrams on each side of the cut, so in total 49 cut diagrams at lowest order in α_s. A simplifying feature is that the gluon propagator cannot be attached to the eikonal line at this perturbative order. For concreteness, in Fig. 1 we exhibit a typical LO cut diagram associated with the perturbative FF $D_{g \to b\bar{c}}$.

The cut-amplitude structure of fragmentation function stems from the insertion of the asymptotic out states in (3). Consequently, the corresponding cut-line phase space integration measure reads [15,17]

$$d\Phi_n = 4\pi M_{B_c}\delta\left(k^+ - P^+ - \sum_{i=1}^n k_i^+\right)\prod_{i=1}^2 \frac{dk_i^+}{2k_i^+}\frac{d^{D-2}k_{i\perp}}{(2\pi)^{D-1}}\theta(k_i^+), \quad (9)$$

where k_i ($i = 1, 2$) stands for the momentum of the ith on-shell quark line ($\bar{b}$ and c) that passes through the cut. The integration over k_i^+ can be transformed into a parametric integration in a finite interval, but the integration over the transverse momentum $k_{i,\perp}$ are completely unbounded, i.e., from $-\infty$ to $+\infty$. This feature may persuade us that integration over $k_{i,\perp}$ could be regarded as loop integration in $D - 2$-dimensional spacetime, with $D = 4 - 2\varepsilon$. Throughout this work, we adopt the dimensional regularization to regularize occurring UV divergences.

To project the $b\bar{c}$ pair onto the intended color/spin/orbital/color states, it is convenient to employ the covariant projector technique to expedite the calculation. At the lowest order in velocity expansion, it is legitimate to partition the quark momenta inside the fictitious $B_c^{(*)}$ state commensurate to their mass ratio: $p_{\bar{c}} = rP$, $p_b = \bar{r}P$, with $r \equiv m_c/(m_b + m_c)$ and $\bar{r} \equiv m_b/(m_b + m_c)$. One then make the following substitutions in the quark amplitude
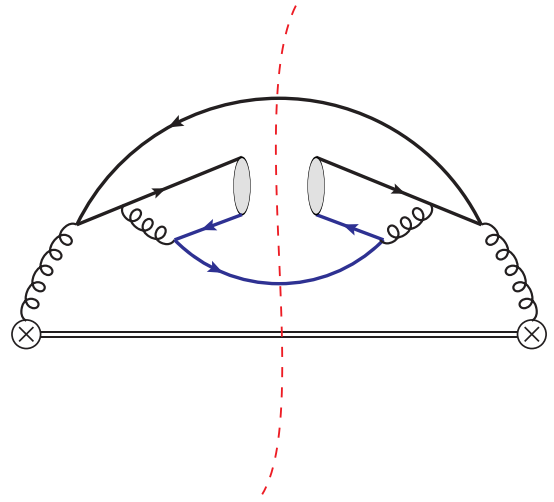

FIG. 1. Representative cut Feynman diagram for perturbative gluon fragmentation function $D_{g \to b\bar{c}}(z)$ at LO in α_s. The cap represents the gluonic field strength operator $G_a^{+\nu}$, and double line signifies the eikonal line.

to project out the desired contributions from the $b\bar{c}(^1S_0^{(1)})$ and $b\bar{c}(^3S_1^{(1)})$ [32]:

$$v(p_{\bar{c}})\bar{u}(p_b) \rightarrow \frac{1}{\sqrt{8m_bm_c}}(\not{p}_{\bar{c}} - m_c)\gamma_5(\not{p}_b + m_b) \otimes \frac{1_c}{\sqrt{N_c}},$$

$$\text{for } {}^1S_0^{(1)} \tag{10a}$$

$$v(p_{\bar{c}})\bar{u}(p_b) \rightarrow \frac{1}{\sqrt{8m_bm_c}}(\not{p}_{\bar{c}} - m_c)\not{\epsilon}^*(\not{p}_b + m_b) \otimes \frac{1_c}{\sqrt{N_c}},$$

$$\text{for } {}^3S_1^{(1)} \tag{10b}$$

with $p_b^2 = m_b^2$, $p_{\bar{c}}^2 = m_c^2$, and $\epsilon^\mu(P)$ designates the polarization vector for the fictitious B_c^* meson.

With the aid of the above projectors (10), we employ the packages FeynCalc/FormLink [33,34] to conduct the Dirac/color trace operation. We also use the package APART [35] to simplify the amplitude by the method of partial fraction to make the integrand simpler.

B. Phase space integration and DGLAP renormalization

It is appealing to use some modern multiloop technique such as the reverse unitarity method and integration by parts (IBP) to deal with phase space integration in (9). The gluon fragmentation into $c\bar{c}(^3S_1^{(1)})$ and $c\bar{c}(^1P_1^{(1)})$ have been analytically calculated in this manner [23,24,28]. Nevertheless, it turns out to be a quite subtle issue to use these techniques to deal with our case. In this work, similar to [27,29], we choose to apply the *sector decomposition* method [36,37] to evaluate the phase space integration as given in (9). Consequently, we will present our final results in an entirely numerical manner. In our opinion, the approach adopted in this work appears to be more amenable to automated calculation, and yield more accurate numerical predictions than the complicated subtraction approach first developed in [19] (see also [26,38,39]).

We first combine all the propagators in a cut diagram using Feynman parametrization, then accomplish two-loop integration over $k_{1,2\perp}$ in $D-2$-dimensional spacetime. We are then left with multifold integrals over Feynman parameters, which can be numerically calculated by the package FIESTA [40] which is based on the sector decomposition algorithm [36,37]. This method is typically useful with many finite multivariable parametric integrals as output, with various UV poles explicitly isolated.

Upon summing all 49 cut diagrams, we find the total cut amplitude still contains a single uncanceled pole, whose coefficient varies with the momentum fraction z. This pole is clearly of UV origin, from the large $k_{1,2\perp}$ integration region in (9). This is a clear sign that the bare fragmentation function requires an additional operator renormalization, following the DGLAP paradigm [20,30]:

$$D_{g\rightarrow H}^{\overline{\text{MS}}}(z,\mu)$$
$$= D_{g\rightarrow H}(z,\mu) - \sum_{q=b,\bar{c}}\frac{1}{\epsilon}\int_z^1\frac{dy}{y}P_{qg}(y)D_{q\rightarrow H}(z/y,\mu), \tag{11}$$

where $P_{qg}(y)$ represents the splitting kernel for $g \rightarrow q$:

$$P_{qg}(y) = \frac{\alpha_s(\mu)}{2\pi}T_F[y^2 + (1-y)^2] + \mathcal{O}(\alpha_s^2), \tag{12}$$

with $T_F = 1/2$.

The heavy-quark-to-$B_c^{(*)}$ fragmentation functions at LO in α_s are known long ago [5]. Recasting the results in the NRQCD factorization language, we have

$$D_{b\rightarrow B_c}(z,\mu) = d_{b\rightarrow B_c}(z,\mu)\frac{\langle 0|\mathcal{O}_1^{B_c}(^1S_0)|0\rangle}{M^3} + \cdots, \tag{13a}$$

$$D_{b\rightarrow B_c^*}(z,\mu) = d_{b\rightarrow B_c^*}(z,\mu)\frac{\langle 0|\mathcal{O}_1^{B_c^*}(^3S_1)|0\rangle}{M^3} + \cdots, \tag{13b}$$

$$D_{b\rightarrow B_c^{*T}}(z,\mu) = d_{b\rightarrow B_c^{*T}}(z,\mu)\frac{\langle 0|\mathcal{O}_1^{B_c^*}(^3S_1)|0\rangle}{M^3} + \cdots. \tag{13c}$$

where the second equation implies the polarization-summed B_c^*, and the last one implies the transversely polarized B_c^* (denoted by B_c^{*T} henceforth)[1] For the $\bar{c}$ fragmentation into $B_c^{(*)}$, one simply makes the exchange $r \leftrightarrow \bar{r}$ in above expressions.

The corresponding short-distance coefficients in (13) at LO in α_s read [25]:

$$d_{b\rightarrow B_c}(z) = \frac{\alpha_s^2 C_F^2}{12N_c^2}\frac{z(1-z)^2}{r^2(1-\bar{r}z)^6}$$
$$\times [6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2$$
$$- 2\bar{r}(6 - 19r + 18r^2)z^3 + 3\bar{r}^2(1 - 2r + 2r^2)z^4], \tag{14a}$$

$$d_{b\rightarrow B_c^*}(z,\mu) = \frac{\alpha_s^2 C_F^2}{12N_c^2}\frac{z(1-z)^2}{\bar{r}^2(1-rz)^6}$$
$$\times [2 - 2(3-2r)z + 3(3 - 2r + 4r^2)z^2$$
$$- 2\bar{r}(4 - r + 2r^2)z^3 + \bar{r}^2(3 - 2r + 2r^2)z^4], \tag{14b}$$

$$d_{b\rightarrow B_c^{*T}}(z,\mu) = \frac{\alpha_s^2 C_F^2}{12N_c^2}\frac{z(1-z)^2}{\bar{r}^2(1-rz)^6}$$
$$\times [2 - 2(3-2r)z + (9 - 4r + 10r^2)z^2$$
$$- 2\bar{r}(4 + r)z^3 + 3\bar{r}^2z^4]. \tag{14c}$$

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[1]One can readily deduce the longitudinally polarized B_c^* fragmentation through $D_{b\rightarrow B_c^{*L}}(z) = D_{b\rightarrow B_c^*}(z) - D_{b\rightarrow B_c^{*T}}(z)$.

For our purpose, we need compute these LO quark fragmentation functions to $\mathcal{O}(\epsilon)$ in (11). Note in (11) the UV pole is subtracted in accordance with the $\overline{\text{MS}}$ procedure.

After implementing the DGLAP renormalization program, we then obtain the pointwise UV-finite SDCs for the g-to-$B_c^{(*)}$ FFs (defined in the $\overline{\text{MS}}$ scheme). It is convenient to divide them into two parts:

$$d_{g \to H}(z, \mu) = \alpha_s^3 [c_0^H(z) \ln \frac{\mu^2}{M^2} + c_1^H(z)], \qquad (15)$$

with the logarithmic part given by

$$\alpha_s^3 c_0^H(z) = \sum_{q=b,\bar{c}} \int_z^1 \frac{dy}{y} P_{qg}(y) d_{q \to H}(z/y). \qquad (16)$$

The logarithmic coefficient function of $c_0^H(z)$ can be obtained in closed form. Here we consider three cases, with $H = B_c$, B_c^*, and B_c^{*T}:

$$\begin{aligned}
c_0^{B_c}(z) = \frac{C_F^2}{48\pi N_c^2 r^2} \Bigg\{ & 12z(1-z)\ln z - \frac{2}{\bar{r}^5}[2r(5 - 6r + 3r^2) + 3\bar{r}(1 - 6r + 10r^2 - 8r^3 + 2r^4)z - 6\bar{r}^5 z^2]\ln\frac{1 - \bar{r}z}{r} \\
& + \frac{1}{15\bar{r}^5(1 - \bar{r}z)^5 r}[4(6 + 40r^2 - 105r^3 + 90r^4 - 31r^5) - \bar{r}(168 - 255r + 1760r^2 - 3255r^3 + 2460r^4 - 758r^5)z \\
& + \bar{r}^2(528 - 1650r + 6925r^2 - 9870r^3 + 6375r^4 - 1678r^5)z^2 \\
& - 5\bar{r}^3(192 - 876r + 2759r^2 - 3123r^3 + 1659r^4 - 342r^5)z^3 \\
& + 5\bar{r}^4(216 - 1224r + 3109r^2 - 2814r^3 + 1182r^4 - 174r^5)z^4 \\
& - \bar{r}^5(744 - 4755r + 10075r^2 - 7200r^3 + 2250r^4 - 180r^5)z^5 + 6\bar{r}^6(48 - 325r + 585r^2 - 315r^3 + 60r^4)z^6 \\
& - 6\bar{r}^7(8 - 55r + 85r^2 - 30r^3)z^7]\Bigg\} + (r \leftrightarrow \bar{r}),
\end{aligned} \qquad (17a)$$

and

$$\begin{aligned}
c_0^{B_c^*}(z) = \frac{C_F^2}{144\pi N_c^2 r^2} \Bigg\{ & 12(z(1-z) + 4rz^2)\ln z - \frac{6}{\bar{r}^5}[2 + 2r^3 - \bar{r}(1 + 6r - 10r^2 + 8r^3 - 2r^4)z - 2\bar{r}^5(1 - 4r)z^2]\ln\frac{1 - \bar{r}z}{r} \\
& + \frac{1}{5\bar{r}^4(1 - \bar{r}z)^5 r}[2(12 + 17r + 2r^2 + 17r^3 + 12r^4) - (168 - 65r + 370r^2 - 585r^3 + 470r^4 - 238r^5)z \\
& + \bar{r}(528 - 710r + 3045r^2 - 4440r^3 + 3245r^4 - 1038r^5)z^2 \\
& - 5\bar{r}^2(192 - 436r + 1695r^2 - 2373r^3 + 1605r^4 - 414r^5)z^3 \\
& + 5\bar{r}^3(216 - 654r + 2383r^2 - 3186r^3 + 1970r^4 - 434r^5)z^4 \\
& - \bar{r}^4(744 - 2645r + 9165r^2 - 11580r^3 + 6390r^4 - 1140r^5)z^5 \\
& + 6\bar{r}^5(48 - 185r + 615r^2 - 725r^3 + 340r^4 - 40r^5)z^6 - 2\bar{r}^6(24 - 95r + 305r^2 - 330r^3 + 120r^4)z^7]\Bigg\} \\
& + (r \leftrightarrow \bar{r}),
\end{aligned} \qquad (17b)$$

$$\begin{aligned}
c_0^{B_c^{*T}}(z) = \frac{C_F^2}{144\pi N_c^2 r^2} \Bigg\{ & 8z(1 - (1 - 4r)z)\ln z - \frac{4}{\bar{r}^5}[2(1 + r) - \bar{r}(1 + 8r - 12r^2 + 8r^3 - 2r^4)z - 2\bar{r}^5(1 - 4r)z^2]\ln\frac{1 - \bar{r}z}{r} \\
& + \frac{2}{15\bar{r}^4(1 - \bar{r}z)^5 r}[2(12 + 22r + 47r^2 - 28r^3 + 7r^4) - (168 + 5r + 840r^2 - 1575r^3 + 880r^4 - 198r^5)z \\
& + \bar{r}(528 - 490r + 4145r^2 - 6630r^3 + 4045r^4 - 968r^5)z^2 \\
& - 5\bar{r}^2(192 - 356r + 1951r^2 - 2871r^3 + 1755r^4 - 402r^5)z^3 \\
& + 5\bar{r}^3(216 - 564r + 2527r^2 - 3492r^3 + 2038r^4 - 430r^5)z^4 \\
& - \bar{r}^4(744 - 2335r + 9275r^2 - 12060r^3 + 6450r^4 - 1140r^5)z^5 \\
& + 6\bar{r}^5(48 - 165r + 605r^2 - 735r^3 + 340r^4 - 40r^5)z^6 - 2\bar{r}^6(24 - 85r + 295r^2 - 330r^3 + 120r^4)z^7]\Bigg\} \\
& + (r \leftrightarrow \bar{r}),
\end{aligned} \qquad (17c)$$

TABLE I. Numerical values of nonlogarithmic coefficient functions $c_1^H(z)$ introduced in (15), for $H = B_c$, B_c^*, and B_c^{*T}.

z	B_c	B_c^*	B_c^{*T}	z	B_c	B_c^*	B_c^{*T}
0.05	0.4331101	0.9824315	0.6589912	0.55	0.2280349	0.0994856	0.0695023
0.10	0.4867564	0.8066066	0.5452066	0.60	0.1886068	0.0776735	0.0543129
0.15	0.4963966	0.6547864	0.4451087	0.65	0.1509164	0.0603226	0.0421736
0.20	0.4839731	0.5271039	0.3600146	0.70	0.1155837	0.0461855	0.0322497
0.25	0.4590161	0.4211974	0.2888957	0.75	0.0832644	0.0342556	0.0238663
0.30	0.4265974	0.3343628	0.2302324	0.80	0.0547023	0.0238138	0.0165418
0.35	0.4265974	0.3343628	0.2302324	0.85	0.0308205	0.0145346	0.0100606
0.40	0.3502768	0.2074592	0.1438627	0.90	0.0128651	0.006692	0.0046142
0.45	0.3095808	0.1625875	0.1130914	0.95	0.0024351	0.0014133	0.0009705
0.50	0.26858	0.1272286	0.088722	0.99	0.0000291	0.0000185	0.0000127

Notice all of c_0^H are regular near $z = 0$.

$$c_0^{B_c}(z) \xrightarrow{z \to 0} \frac{C_F^2}{180 \pi N_c^2 \bar{r}^5 r^3} [\bar{r}(6 + 6r + 46r^2 - 59r^3 + 31r^4)$$
$$+ 15r^2(5 - 6r + 3r^2) \ln r] + (r \leftrightarrow \bar{r}), \qquad (18a)$$

$$c_0^{B_c^*}(z) \xrightarrow{z \to 0} \frac{C_F^2}{360 \pi N_c^2 \bar{r}^5 r^3} [\bar{r}(12 + 17r + 2r^2 + 17r^3 + 12r^4)$$
$$+ 30r(1 + r^3) \ln r] + (r \leftrightarrow \bar{r}), \qquad (18b)$$

$$c_0^{B_c^{*T}}(z) \xrightarrow{z \to 0} \frac{C_F^2}{360 \pi N_c^2 \bar{r}^5 r^3} [\bar{r}(12 + 22r + 47r^2 - 28r^3 + 7r^4)$$
$$+ 30r(1 + r) \ln r] + (r \leftrightarrow \bar{r}). \qquad (18c)$$

C. Numerical results

It is quite challenging, if possible, from the sector decomposition approach to deduce the analytic expressions for those nonlogarithmic coefficient functions $c_1^H(z)$ in (15). We are content to knowing their numerical values to very high precision within relatively short time. For
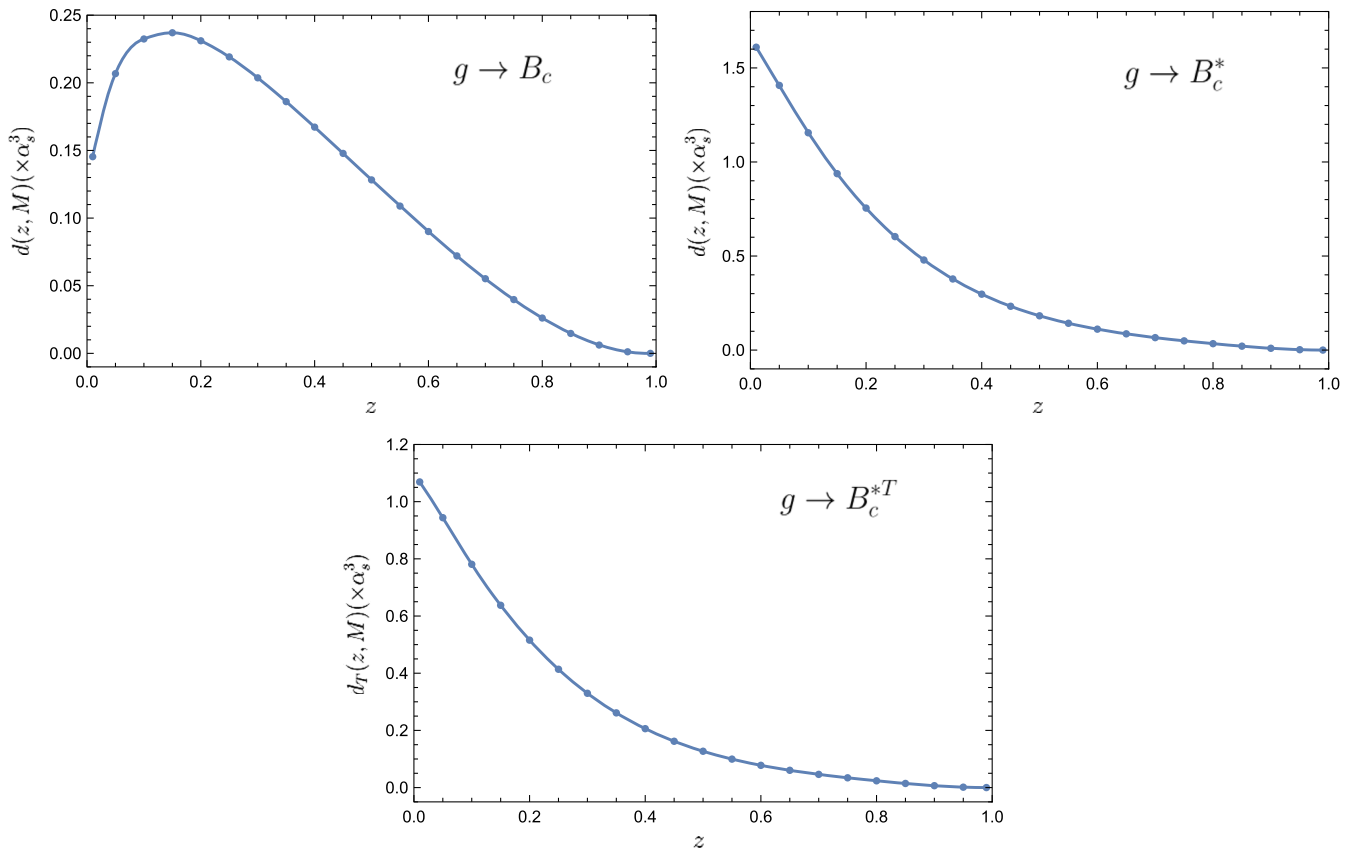

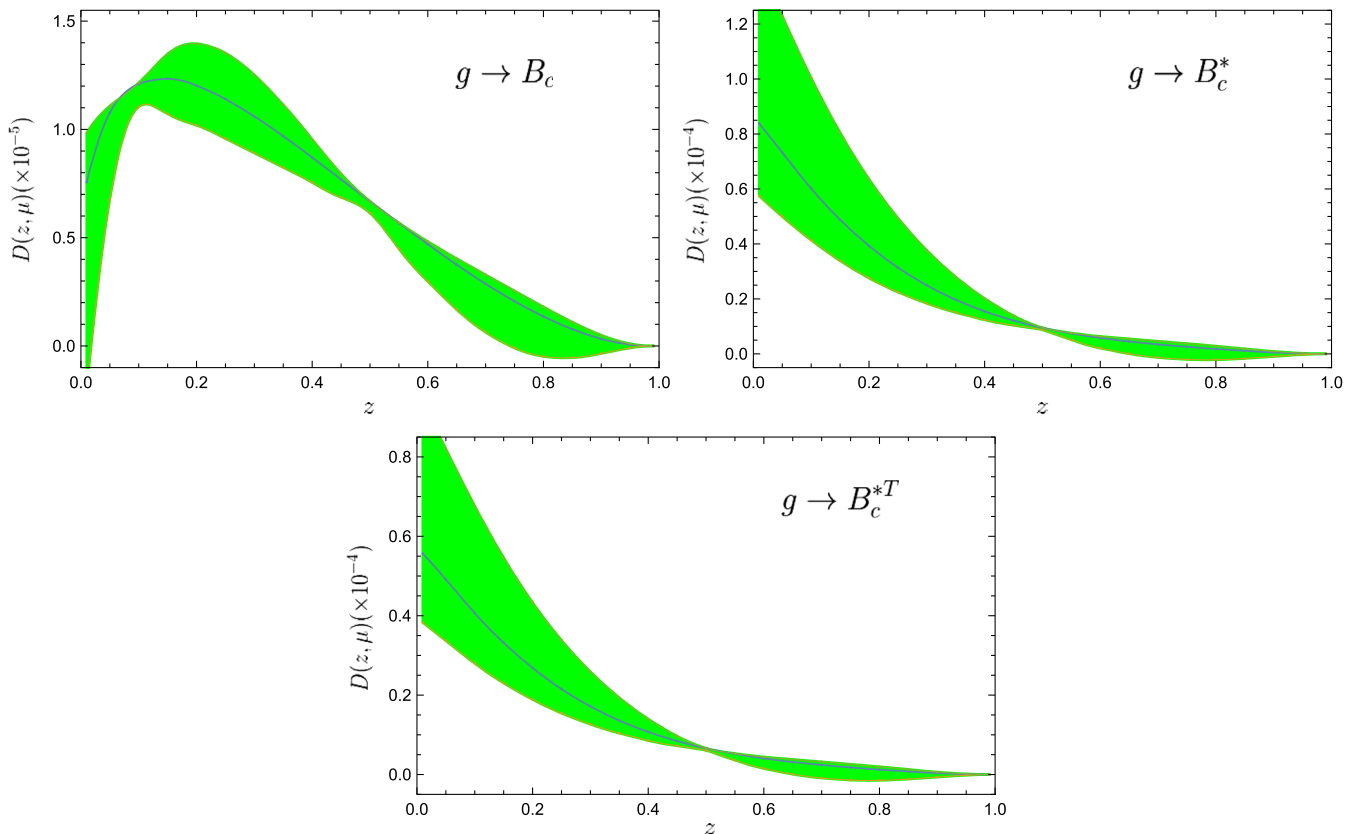FIG. 2. Profiles of nonlogarithmic coefficient functions $c_1^H(z)$ defined in (15).

FIG. 3. The profiles of the fragmentation functions $D_{g \to H}(z, \mu)$ with $m_c = 1.5 \text{GeV}$ and $m_b = 4.8$ GeV, and $|R(0)|^2 = 1.642 \text{ GeV}^3$. The uncertain band is obtained by sliding μ from $M/2$ to $2M$, with the central line taken at $\mu = M = m_b + m_c$.

numerical investigation, we adopt the following input parameters:

$$m_c = 1.5 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \quad \alpha_s(M) = 0.199, \quad (19)$$

and $|R(0)|^2 = 1.642 \text{ GeV}^3$ [41].

For the reader's convenience, in Table I we have enumerated the values of $c_1^H(z)$ for some typical values of z. We also plot these functions in Fig. 2.

In Fig. 3, we also plot the various fragmentation functions $D_{g \to H}(z)$ for $H = B_c, B_c^*$ and B_c^{*T} as function of z. We observe that these functions are regular near $z = 0$.

IV. SUMMARY

In summary, in this work we have computed the g-to-$B_c^{(*)}$ fragmentation functions within NRQCD factorization framework, at the lowest order in velocity expansion and α_s. We start from the Collins and Soper's gauge-invariant operator definition, which makes the renormalization program transparent. We have employed an automated approach based on sector decomposition strategy to conduct the phase space integral in dimensional regularization. It turns out that, within tolerable amount of time, this method yields better numerical accuracy than the conventional NLO subtraction method. After implementing DGLAP renormalization procedure, we obtain the UV finite NRQCD short-distance coefficient functions in a pointwise manner. We hope that our results are helpful to strengthen our understanding of $B_c^{(*)}$ production at large-p_T at the LHC experiment.

Note added.—After this work was finished and while we were preparing the manuscript, a preprint has appeared very recently [42], which also computes the gluon-to-$B_c^{(*)}$ fragmentation functions in NRQCD factorization approach, albeit using the conventional subtraction method.

APPENDIX: THE NUMERICAL COMPARISON BETWEEN OUR WORK AND REF. [42]

In [42], the authors present the NLO $g \to B_c^{(*)}$ fragmentation functions as

$$
D_{g \to B_c(B_c^*)}^{\mathrm{NLO}}(z, \mu_F)
$$
$$
= \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{4M^2} \int_z^1 \left[\sum_{Q=\bar{b},c} P_{Qg}(y) D_{Q \to B_c(B_c^*)}^{\mathrm{LO}}(z/y) \right] \frac{dy}{y}
$$
$$
+ \frac{\alpha_s^3 |R_S(0)|^2}{M^3} f(z). \tag{A1}
$$

They also present polynomial interpolations of the non-logarithmic part $f(z)$ of the short-distance coefficients of the fragmentation functions for conventions of phenomenological applications, which are

$$
f(z) = -61.309z^8 + 269.569z^7 - 491.879z^6 + 484.568z^5
$$
$$
- 280.805z^4 + 97.865z^3 - 20.199z^2 + 1.888z
$$
$$
+ 0.300, \tag{A2}
$$

for $g \to B_c$, and

$$
f(z) = -300.413z^8 + 1282.128z^7 - 2253.159z^6
$$
$$
+ 2105.108z^5 - 1123.528z^4 + 336.215z^3
$$
$$
- 45.194z^2 - 3.034z + 1.870, \tag{A3}
$$

for $g \to B_c^*$. They are related to the expressions in our convention as

$$
f(z) = \frac{N_c}{2\pi} [2c_0^{B_c}(z) \ln 2 + c_1^{B_c}(z)], \tag{A4}
$$

for B_c, and

TABLE II. Comparisons between the numerical values of $f(z)$ introduced in (A1), for B_c and B_c^*.

	Reference [42]	This paper	Reference [42]	This paper
z	B_c	B_c	B_c^*	B_c^*
0.01	0.316955	0.311591	1.835470	1.81726
0.05	0.354525	0.345893	1.640940	1.58443
0.10	0.360975	0.349615	1.357450	1.30676
0.15	0.348498	0.339127	1.100280	1.07055
0.20	0.330147	0.322512	0.892590	0.873933
0.25	0.310450	0.303014	0.730806	0.712408
0.30	0.290177	0.282165	0.603119	0.581293
0.35	0.269025	0.260711	0.498891	0.476112
0.40	0.246834	0.238991	0.411930	0.392658
0.45	0.223890	0.217120	0.340068	0.326996
0.50	0.200723	0.195094	0.283027	0.27545
0.55	0.177753	0.172845	0.240056	0.234592
0.60	0.155024	0.150285	0.208377	0.201238
0.65	0.132160	0.127345	0.182997	0.172455
0.70	0.108600	0.104023	0.157953	0.145607
0.75	0.084061	0.080458	0.128606	0.118483
0.80	0.059079	0.057079	0.094120	0.089603
0.85	0.035393	0.034871	0.058776	0.058875
0.90	0.015828	0.015809	0.030327	0.028856
0.95	0.003240	0.003244	0.013084	0.006414
0.99	-0.00136946	0.000041	-0.000867	0.000087

$$
f(z) = \frac{3N_c}{2\pi} [2c_0^{B_c^*}(z) \ln 2 + c_1^{B_c^*}(z)], \tag{A5}
$$

for B_c^*. In Table II, we list the numerical values of $f(z)$ in our calculations and those by employing the polynomial interpolations (A2) and (A3) given in [42], for comparison. One can see that the interpolated expressions in (A2) and (A3) agree with our numerical results reasonably well.

[1] J. C. Collins, D. E. Soper, and G. F. Sterman, Adv. Ser. Dir. High Energy Phys. **5**, 1 (1989).
[2] E. Braaten, K. m. Cheung, and T. C. Yuan, Phys. Rev. D **48**, 4230 (1993).
[3] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993).
[4] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).
[5] E. Braaten, K. m. Cheung, and T. C. Yuan, Phys. Rev. D **48**, R5049 (1993).
[6] J. P. Ma, Phys. Lett. B **332**, 398 (1994).
[7] E. Braaten and T. C. Yuan, Phys. Rev. D **50**, 3176 (1994).
[8] P. L. Cho and M. B. Wise, Phys. Rev. D **51**, 3352 (1995).
[9] J. P. Ma, Nucl. Phys. **B447**, 405 (1995).
[10] J. P. Ma, Phys. Rev. D **53**, 1185 (1996).
[11] E. Braaten and T. C. Yuan, Phys. Rev. D **52**, 6627 (1995).
[12] K. m. Cheung and T. C. Yuan, Phys. Rev. D **53**, 3591 (1996).
[13] C. f. Qiao, F. Yuan, and K. T. Chao, Phys. Rev. D **55**, 5437 (1997).
[14] E. Braaten and J. Lee, Nucl. Phys. **B586**, 427 (2000).
[15] G. T. Bodwin and J. Lee, Phys. Rev. D **69**, 054003 (2004).
[16] W. l. Sang, L. f. Yang, and Y. q. Chen, Phys. Rev. D **80**, 014013 (2009).
[17] G. T. Bodwin, U. R. Kim, and J. Lee, J. High Energy Phys. 11 (2012) 020.
[18] Y. Q. Ma, J. W. Qiu, and H. Zhang, Phys. Rev. D **89**, 094029 (2014).

[19] P. Artoisenet and E. Braaten, J. High Energy Phys. 04 (2015) 121.

[20] G. T. Bodwin, H. S. Chung, U. R. Kim, and J. Lee, Phys. Rev. D **91**, 074013 (2015).

[21] X. Gao, Y. Jia, L. Li, and X. Xiong, Chin. Phys. C **41**, 023103 (2017).

[22] R. Sepahvand and S. Dadfar, Phys. Rev. D **95**, 034012 (2017).

[23] P. Zhang, Y. Q. Ma, Q. Chen, and K. T. Chao, Phys. Rev. D **96**, 094016 (2017).

[24] F. Feng, S. Ishaq, Y. Jia, and J. Y. Zhang, Phys. Rev. D **102**, 014038 (2020).

[25] D. Yang and W. Zhang, Chin. Phys. C **43**, 083101 (2019).

[26] P. Artoisenet and E. Braaten, J. High Energy Phys. 01 (2019) 227.

[27] F. Feng, Y. Jia, and W. L. Sang, Eur. Phys. J. C **81**, 597 (2021).

[28] P. Zhang, C. Y. Wang, X. Liu, Y. Q. Ma, C. Meng, and K. T. Chao, J. High Energy Phys. 04 (2019) 116.

[29] F. Feng and Y. Jia, arXiv:1810.04138.

[30] J. C. Collins and D. E. Soper, Nucl. Phys. **B194**, 445 (1982).

[31] P. Nogueira, J. Comput. Phys. **105**, 279 (1993).

[32] A. Petrelli, M. Cacciari, M. Greco, F. Maltoni, and M. L. Mangano, Nucl. Phys. **B514**, 245 (1998).

[33] R. Mertig, M. Bohm, and A. Denner, Comput. Phys. Commun. **64**, 345 (1991).

[34] F. Feng and R. Mertig, arXiv:1212.3522.

[35] F. Feng, Comput. Phys. Commun. **183**, 2158 (2012).

[36] T. Binoth and G. Heinrich, Nucl. Phys. **B585**, 741 (2000).

[37] T. Binoth and G. Heinrich, Nucl. Phys. **B680**, 375 (2004).

[38] X. C. Zheng, C. H. Chang, T. F. Feng, and X. G. Wu, Phys. Rev. D **100**, 034004 (2019).

[39] X. C. Zheng, C. H. Chang, and X. G. Wu, Phys. Rev. D **100**, 014005 (2019).

[40] A. V. Smirnov, Comput. Phys. Commun. **185**, 2090 (2014).

[41] E. J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995).

[42] X. C. Zheng, C. H. Chang, and X. G. Wu, J. High Energy Phys. 05 (2022) 036.