Nonfactorizable charming loops in FCNC B decays versus B-decay semileptonic form factors

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We compare a nonfactorizable charming-loop correction to an exclusive FCNC B decay given in terms of the three-particle Bethe-Salpeter amplitude (3BS) of the B meson, $\langle 0|\bar{q}(x)G_{\mu\nu}(z)b(0)|B(p)\rangle$, with the corresponding correction to the B-meson semileptonic form factor. In spite of certain similarities, these two corrections are shown to have substantial differences: the form-factor correction is dominated by the *collinear* light-cone configuration of 3BS ($z_\mu = ux_\mu$, $0 < u < 1$, $x^2 = 0$); in contrast, the FCNC amplitude is dominated by a different configuration with *noncollinear* arguments [$x^2 = 0$, $z^2 = 0$, but $(x-z)^2 \neq 0$ (i.e., $z_\mu \neq ux_\mu$)].

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I. INTRODUCTION

Charming loops in rare flavor-changing neutral current (FCNC) decays of the B meson have impact on the B-decay observables [1], providing an unpleasant noise for the studies of possible new physics effects (see, e.g., recent discussions [2–5] and references therein).

A number of theoretical analyses of nonfactorizable (NF) charming loops in FCNC B decays has been published. We mention here those directly related to the discussion of this paper: in Ref. [6], an effective gluon-photon local operator describing the charm-quark loop is calculated as an expansion in inverse charm-quark mass m_c and applied to inclusive $B \to X_s\gamma$ decays (see also Refs. [7,8]); in Ref. [9], NF corrections in $B \to K^*\gamma$ using local operator product expansion (OPE) have been studied; NF corrections induced by the *local* photon-gluon operator have been calculated in Refs. [10,11] in terms of the light-cone (LC) three-particle antiquark-quark-gluon Bethe-Salpeter amplitude (3BS) of the K^* meson [12–14] with two field operators having equal coordinates, $\langle 0|\bar{s}(0)G_{\mu\nu}(0)u(x)|K^*(p)\rangle$, $x^2 = 0$. However, the local OPE for the charm-quark loop in FCNC B decays leads to a power series in $\Lambda_{\rm QCD}m_b/m_c^2$; numerically, this parameter is close to 1. To sum up $O(\Lambda_{\rm QCD}m_b/m_c^2)^n$ corrections, Ref. [15] obtained a *nonlocal* photon-gluon operator describing the charm-quark loop and evaluated its effect making use of 3BS of the B meson in a collinear LC approximation $\langle 0|\bar{s}(x)G_{\mu\nu}(ux)b(0)|B(p)\rangle$, $x^2 = 0$. This approximation was used later for the analysis of other FCNC B decays [16].

The collinear LC configuration is known to provide the dominant 3BS contribution to meson form factors [17,18], in particular, to form factors of semileptonic (SL) B decay induced by the tree-level $b \to u$ weak charged current (CC). So, it may seem attractive to express also the FCNC B-decay amplitude via this collinear LC 3BS of the B meson.

However, the 3BS contribution to the CC B decay and to the FCNC B decay have a qualitative difference. Let us consider the B decay in the B-meson rest frame. In CC B decays, the b quark emits a fast light u quark, which is later hit by a soft gluon and thus keeps moving in the *same* space direction. In FCNC B decays, a fast light s quark and a pair of fast c quarks emitted by the b quark move in the *opposite* space directions. We shall demonstrate that, as the consequences of this difference, the B-meson CC weak form factor is dominated by a *collinear* LC configuration $\langle 0|\bar{q}(x)G_{\mu\nu}(ux)b(0)|B(p)\rangle$, $x^2 = 0$ [19], whereas the FCNC B-decay amplitude is dominated by a *noncollinear* configuration $\langle 0|\bar{q}(x)G_{\mu\nu}(z)b(0)|B(p)\rangle$, $x^2 = 0$, $z^2 = 0$, but $(x-z)^2 \neq 0$ [20,21]. The first application of a noncollinear 3BS to FCNC B decays was presented in Ref. [22].

We study the general properties of the 3BS contributions to the amplitudes of B decays and formulate the conditions necessary for the dominance of the amplitude by a collinear 3BS configuration. We perform the analysis using field theory with scalar quarks/gluons, which is free of technical complications and allows one to focus on the conceptual issues; the generalization of our analysis to QCD is straightforward. Section II demonstrates the technical similarities between the CC and the FCNC amplitudes and their equivalence to the generic diagram of the form-factor topology, and Sec. III studies the conditions under which this generic diagram is dominated by a collinear 3BS configuration. As follows from this analysis, large $O(1)$ corrections to the collinear LC 3BS contribution should emerge in the amplitudes of FCNC B decays. Chapter IV studies in detail the FCNC B-decay amplitude, including the cases of the light u quark and the c quark in the triangle loop, adopting for the latter case the counting scheme $\Lambda_{\rm QCD} m_b/m_c^2 \simeq 1$ [23]. The origin of large $O(1)$ corrections to the collinear LC approximation is identified; namely, we show that a noncollinear 3BS configuration (both x and z on the light cone, but on different axes: x on the $(+)$ axis and z on the $(-)$ axis or vice versa [24]) give parametrically unsuppressed contributions compared to the collinear LC 3BS contribution. So, the full dependence of 3BS of the B meson on the variable $(x-z)^2$ is necessary to properly sum the $(\Lambda_{\rm QCD} m_b/m_c^2)^n$ corrections in FCNC B decays.

II. AMPLITUDES OF B-MESON CC DECAY VS FCNC DECAY

In this section, we show that the 3BS contributions to the CC and FCNC amplitudes may be reduced to the diagram of a generic form-factor topology with an (essential) difference of the location of the heavy-quark field in this diagram.

A. Amplitude of semileptonic B-meson decay induced by weak charged current

To exemplify the essential part of our analysis, we neglect the spins of the B-meson constituents (quarks and gluons are treated as scalar fields) as well as the Lorentz structure of the weak currents. So, instead of the full QCD amplitude describing a SL B-meson weak decay induced by a charged current,

$$A^{\nu}_{\rm SL}(p|q,q') = i\int dx_1 dx_3 e^{iqx_1+iq'x_3}\langle 0|T\{\bar u(x_3) \times \mathcal{O}u(x_3), \bar u(x_1)\gamma^\nu(1-\gamma_5)b(x_1)\}|B_u(p)\rangle, \tag{2.1}$$

where $\mathcal{O}$ is a Dirac matrix, we consider the amplitude

$$A_{\rm SL}(p|q,q') = i\int dx_1 dx_3 e^{iqx_1+iq'x_3} \times \langle 0|T\{u^\dagger(x_3)u(x_3), u^\dagger(x_1)b(x_1)\}|B_u(p)\rangle \tag{2.2}$$

with scalar "quarks" and "gluons." Here, q is the momentum emitted by the weak vertex, and q' is the momentum emitted by the interpolating current of the outgoing state; see Fig. 1. The u- and the b-quark fields are the Heisenberg operators with respect to the strong interaction. We will be interested in the part of the amplitude (2.2) that emerges at the first order of the expansion in the strong coupling and contains the gluon field G. The corresponding Feynman graph is shown in Fig. 1. After some manipulations (and making use of the Fock-Schwinger gauge; see Refs. [17,18]), the part of this amplitude relevant to us may be written as

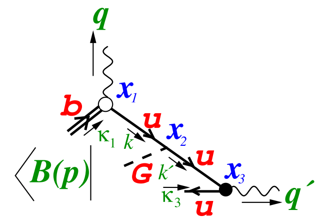

FIG. 1. Momentum notations in the 3BS contribution to the form factor describing weak $b \to u$ semileptonic B decay.

$$A_{\rm SL}(p|q,q') = \int dx_1 dx_2 dx_3 dk dk'\, e^{iqx_1-ik(x_2-x_1)} e^{-ik'(x_3-x_2)+iq'x_3} \times \frac{1}{m_u^2-k^2}\frac{1}{m_u^2-k'^2}\langle 0|u^\dagger(x_3)G(x_2)b(x_1)|B_s(p)\rangle. \tag{2.3}$$

Making use of the transformation properties of field operators under translations, we may shift the coordinate of the gluon field to zero and introduce the new variables through the relations $k=\kappa_1-q$ and $k'=q'-\kappa_3$, $x_1-x_2\to x_1$, and $x_3-x_2\to x_3$. After that, we perform the x_2 integration that leads to the momentum conservation $\delta(p-q-q')$, and the amplitude (2.3) takes the form

$$A_{\rm SL}(p|q,q') = (2\pi)^4\delta(p-q-q')\bar A_{\rm SL}(p|q,q'), \tag{2.4}$$

with

$$\bar A_{\rm SL}(p|q,q') = \int dx_1 dx_3 d\kappa_1 d\kappa_3 e^{i\kappa_1 x_1+i\kappa_3 x_3} \times \frac{1}{m_u^2-(\kappa_1-q)^2}\frac{1}{m_u^2-(q'-\kappa_3)^2} \times \langle 0|u^\dagger(x_3)G(0)b(x_1)|B_s(p)\rangle. \tag{2.5}$$

B. Nonfactorizable part of the amplitude of FCNC B decay

The amplitude of FCNC B_s decay such as, e.g., $B_s\to\gamma^*\gamma^*$ decay, is given by the following expression [25]:

$$A^{\nu\mu}_{\rm FCNC}(p|q,q') = \int dx \exp(iqx)dx_3\exp(iq'x_3) \times \langle 0|T\{\bar c(x)\gamma^\nu c(x), \bar s(x_3)\gamma^\mu s(x_3)\}|B_s)p)\rangle. \tag{2.6}$$

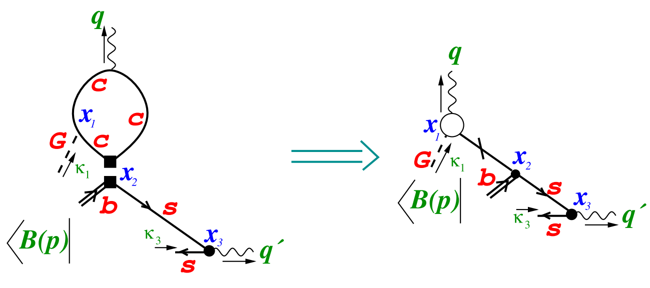

FIG. 2. Feynman diagram describing the 3BS contribution to a NF amplitude of FCNC B decay. The crossed propagator line means that the propagator is replaced by the triangle charming loop $\Gamma_{cc}(\omega_1 p, q)$.

Here, the c- and s-quark operators are the Heisenberg operators with respect to strong and weak interactions. Notice that in the case of an FCNC B decay the b-quark field is not contained in the current operators under the T product but comes into the came at order G_F in weak interaction. The part of the amplitude describing the nonfactorizable contribution of the charming loop emerges at order G_F in weak interaction and at the first order in strong interaction. The contribution of the charming loop is given through the gluon field $G_{\mu\nu}$ [26], so the Feynman diagram describing the FCNC amplitude (2.6) is represented by Fig. 2.

We omit all complications related to Lorentz and spinor structure details and consider scalar quark fields b, c, and s and a scalar gluon field G. We then come to the expression for the nonfactorizable part of the amplitude of FCNC B decay (see Fig. 2),

$$A_{\mathrm{FCNC}}(p|q,q') = (2\pi^4)\delta(p-q-q')\bar{A}_{\mathrm{FCNC}}(p|q,q'), \quad (2.7)$$

with

$$\begin{aligned}\bar{A}_{\mathrm{FCNC}}(p|q,q') = \frac{G_F}{\sqrt{2}}\int dx_1 dx_3 d\kappa_1 d\kappa_3 e^{ix_1\kappa_1 + i\kappa_3 x_3} \\ \times \Gamma_{cc}(\kappa_1,q)\frac{d\kappa_3}{m_s^2-(q'-\kappa_3)^2} \\ \times \langle 0|s^\dagger(x_3)G(x_1)b(0)|B_s(p)\rangle. \qquad (2.8)\end{aligned}$$

Here, $\Gamma_{cc}(\kappa_1,q)$ is the charm-quark triangle diagram, which may be written as a double integral in Feynman parameters (see a detailed discussion in Refs. [15,20,21,25,26])

$$\begin{aligned}&\Gamma_{cc}(\kappa_1,q)\\ &= \frac{1}{8\pi^2}\int_0^1 du\int_0^1 dv\\ &\times\frac{\theta(u+v<1)}{m_c^2-uv(\kappa_1-q)^2-u(1-u-v)\kappa_1^2-v(1-v-u)q^2}. \qquad (2.9)\end{aligned}$$

Important for us is that the quantity is a quadratic function in momentum variables. So, the FCNC amplitude is similar to the SL amplitude, with the light-quark propagator replaced by an "effective" propagator $\Gamma_{cc}(\kappa_1,q)$,

$$\frac{1}{m_u^2-(\kappa_1-q)^2} \to \Gamma_{cc}(\kappa_1,q). \qquad (2.10)$$

The main difference between the SL and the FCNC amplitudes arises from the fact that the heavy b quark in a SL decay amplitude is attached to the *end point* of the line connecting x_1 and x_3, along which the energetic light quarks are propagating, while in the FCNC, the heavy field is attached to the *middle point* of the line connecting x_1 and x_3. We shall see that these features of the SL and FCNC B decays are responsible for a qualitative difference between the configurations of the 3BS of the B meson that provide the dominant contributions in SL B decays and in FCNC B decays.

To demonstrate this difference, the next section considers the general amplitude of the form-factor topology and figures out the properties necessary for the dominance of the collinear 3BS configuration.

III. 3BS CONTRIBUTION TO A GENERIC AMPLITUDE OF THE FORM-FACTOR TOPOLOGY

Let us consider the generic form-factor amplitude $A(p|q,q')$ shown in Fig. 3.

As we have mentioned above, both A_{FCNC} and A_{SL} are reduced to this amplitude, such that each of the FCNC and the SL amplitudes is characterized by a specific (and different) content of the heavy and the light fields $\varphi_{1,2,3}$: in FCNC decays, the field φ_2 is heavy, while $\varphi_{1,3}$ are light; in SL decays, φ_1 is heavy, while $\varphi_{2,3}$ are light. If one considers the case of the 3BS correction to the form factor of a light meson, all fields $\varphi_{1,2,3}$ are light degrees of freedom (light quarks or gluons).

The properties of the set of meson constituents $\varphi 1,2,3$ (i.e., which of these fields are heavy and which are light) are reflected in the properties of the amplitude $\langle 0|\varphi_1(x_1)\varphi_2(x_2)\varphi_3(x_3)|B(p)\rangle$. The goal of our analysis in this section is to figure out the kinematical configuration

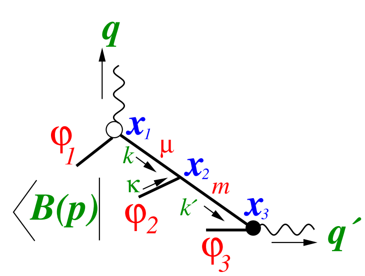

FIG. 3. 3BS contribution to the generic amplitude of the form-factor topology.

of the constituent fields that dominate the amplitude of Fig. 3.

The analytic expression corresponding to the diagram of Fig. 3 has the form

$$A(p|q,q') = \int \frac{dx_1 dx_2 dx_3 dk dk'}{(\mu^2 - k^2)(m^2 - k'^2)} \times e^{iqx_1 - ik(x_2 - x_1) - ik'(x_3 - x_2) + iq'x_3} \times \langle 0|\varphi_1(x_1)\varphi_2(x_2)\varphi_3(x_3)|B(p)\rangle. \quad (3.1)$$

As already mentioned above, the amplitude contains $\delta(p - q - q')$, which may be isolated by making use of the transformation properties of the field operators $\varphi_{1,2,3}$ under translations: one can set one of the arguments of the field operators equal zero and integrate over one of the coordinate differences. For the moment, we will not make use of this property and will keep all three arguments $x_{1,2,3}$ nonzero.

By introducing the Feynman parameter v to combine two propagators in a single propagator squared and after redefinitions of the variables

$$\tilde{k} = k - v\kappa, \quad (3.2)$$

$$x_2 = x_1(1 - v) + x_3 v + z_2, \quad (3.3)$$

the amplitude takes the form convenient for a further analysis:

$$A(p|q,q') = \int_0^1 dv \int d\kappa\, d\tilde{k}\, dx_1\, dz_2\, dx_3 \times \frac{e^{ix_1(\tilde{k}+q) + ix_3(q'-\tilde{k}) + i\kappa z_2}}{[m^2(1-v) + \mu^2 v - \tilde{k}^2 - v(1-v)\kappa^2]^2} \times \langle 0|\varphi_1(x_1)\varphi_2(x_1(1-v) + x_3 v + z_2)\varphi_3(x_3)|B(p)\rangle. \quad (3.4)$$

Here, κ is the momentum transferred in the *central* point x_2, and the variable z_2 measures the deviation of the configuration x_1, x_2, x_3 from the straight line joining the end points x_1 and x_3.

To proceed further, one can attempt to expand $\langle 0|\varphi_1(x_1)\varphi_2(x_1(1-v) + x_3 v + z_2)\varphi_3(x_3)|B(p)\rangle$ in powers of z_2 and obtain in this way a tower of collinear operators of the increasing dimension containing derivatives of φ_2. The expansion in powers of z_2 corresponds to expanding the denominator in powers of κ^2. Such expansion is meaningful if κ is soft compared to the virtualities of the propagators $D(x_1 - x_2)$ and $D(x_2 - x_3)$. Obviously, the amplitude is dominated by a collinear configuration of the 3BS *only* if the momentum transferred in the central vertex is soft compared to the virtualities of the particle propagators along the line $x_1 - x_3$.

There are phenomenologically relevant cases where the collinear 3BS configuration indeed dominates the amplitude $A(p|q,q')$:

(i) QCD radiative correction to the $B \to j_1 j_2$ weak form factor. In this case, the field φ_1 is heavy, whereas φ_2 and φ_3 are light (the gluon and the light quark, respectively). If $q^2, q'^2 \ll M_B^2$, $\kappa^2 = O(\Lambda_{\rm QCD}^2)$, the virtualities of the particles propagating along the segments $x_1 - x_2$ and $x_2 - x_3$ are $O(m_b\Lambda_{\rm QCD})$, and $\kappa^2 \simeq \Lambda_{\rm QCD}^2$. The expansion in κ^2 seems meaningful, and the amplitude is dominated by a collinear LC configuration $x_2 = x_1(1-v) + x_3 v$, $x_1^2 \simeq 0$, and $x_3^2 \simeq 0$.

(ii) QCD radiative correction to $M \to j_1 j_2$ form factor, where M is a light meson. In this case, all three fields $\varphi_{1,2,3}$ are light (φ_1 and φ_3 are the light quarks, and φ_2 is the gluon). To make the OPE convergent, we cannot set $q^2 = q'^2 = 0$ but must keep $q^2 \simeq q'^2 \ll -1\ \text{GeV}^2$. In this case, both quark propagators are highly virtual, with virtualities $O(q^2, q'^2)$, the momentum transfer κ is soft, $\kappa^2 = O(\Lambda_{\rm QCD}^2)$, and the collinear 3BS configuration dominates the amplitude.

Unfortunately, the nonfactorizable correction to the FCNC amplitude of the B-meson decay does not fall into this class of processes: φ_1 and φ_3 are light degrees of freedom (the gluon and the light quark, respectively), whereas φ_2 is a heavy quark which carries almost the full momentum of the B meson, $\kappa_2^2 \sim M_B^2$. The momentum κ_2 is thus by far not soft compared to the virtualities of the particles along the line $x_1 - x_3$, and the expansion around the collinear 3BS configuration does not converge: Expanding in powers of κ_2^2 leads to a series in which all terms have the same order of magnitude.

In the next section, we look in more detail at what kind of expansion of the amplitude arises in this case. In particular, we show that a noncollinear 3BS configuration with $(x_1 - x_2)^2 = 0$, $(x_2 - x_3)^2 = 0$, but $(x_1 - x_3)^2 \neq 0$ dominates the FCNC amplitude.

IV. AMPLITUDE OF FCNC *B* DECAY

We start with briefly recalling the general properties of the three-particle (antiquark-quark-gluon) Bethe-Salpeter (BS) wave function of the meson and then obtain the B-decay FCNC amplitude in terms of this BS wave function. To simplify the analytical expressions, we will consider the case $q^2 = q'^2 = 0$.

A. Parametrization of the three-particle BS amplitude

The three-particle BS amplitude may be written in the form (see, e.g., Refs. [14,17,19])[1]

[1]References [14,17,19] provide the expansion of 3BS for collinear arguments $x_1 = vx_3$. A generalization to the case of noncollinear arguments presented here is straightforward.

$$\langle 0|G(x_1)b(x_2)s^\dagger(x_3)|B_s(p)\rangle = \int D\omega e^{-i\omega_1 x_1 p - i\omega_2 x_2 p - i\omega_3 x_3 p} \times [\psi_0(\omega) + \psi_{12}(\omega)x_{12}^2 + \psi_{13}(\omega)x_{13}^2 + \psi_{23}(\omega)x_{23}^2 + \cdots], \quad (4.1)$$

where

$$x_{ij}^2 = (x_i - x_j)^2, \quad D\omega \equiv d\omega_1 d\omega_2 d\omega_3 \delta(1-\omega_1-\omega_2-\omega_3), \quad \psi_i(\omega) \equiv \psi_i(\omega_1,\omega_2,\omega_3). \quad (4.2)$$

Since the b quark is heavy, all functions $\psi_i(\omega)$ in the amplitude (4.1) have support in the end point regions

$$\omega_2 \sim 1 - O(\Lambda_{\rm QCD}/m_b), \qquad \omega_{1,3} \sim O(\Lambda_{\rm QCD}/m_b). \quad (4.3)$$

One of the arguments of the field operators may be set to zero by using the transformation properties of the field operators under translations, so we set the coordinate of the heavy b quark to zero $x_2 = 0$ [in this way, the $\delta(p-q-q')$-function describing the momentum conservation has been singled out from $A_{\rm FCNC}$]. As the next step, we insert this expression in the general formula for the amplitude (3.1), which takes the form

$$\bar{A}(p|q,q') = \int dx_1\, dx_3\, d\kappa_1\, d\kappa_3 e^{i\kappa_1 x_1 + i\kappa_3 x_3} \times \Gamma_{cc}(\kappa_1, q) D_s(\kappa_3 - q')\Psi_p(x_1,x_3), \quad (4.4)$$

where

$$\Psi_p(x_1,x_3) \equiv \langle 0|G(x_1)b(0)s^\dagger(x_3)|B(p)\rangle \quad (4.5)$$

and $D_s(k) = 1/(m_s^2 - k^2 - i0)$. We will not introduce the Feynman parameter v to combine the propagators as was done in Eq. (3.4) but evaluate the amplitude directly. Following the results of the previous section, we are going to demonstrate directly that the FCNC amplitude is indeed not dominated by the collinear field configuration.

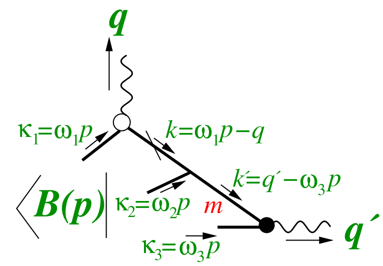

FIG. 4. Momenta values in the 3BS contribution to the amplitude of FCNC B decay. The crossed propagator line means that the propagator is replaced by the triangle charming loop $\Gamma_{cc}(\omega_1 p, q)$.

B. Contribution of the ψ_0 term in 3BS (neglecting all powers of x_{ij}^2)

The term $\propto \psi_0(\omega)$ in 3BS does not contain x_{ij}^2, so its contribution to $A(q,p)$ is calculated easily: the integrals over $x_{1,3}$ give the $\delta(\kappa_1 - \omega_1 p)$ and $\delta(\kappa_3 - \omega_3 p)$, so one ends up with the following expression (see Fig. 4):

$$A_{\psi_0}(q,p) = \int_0^1 d\omega_1 \int_0^{1-\omega_1} d\omega_3 \psi_0(\omega_1,\omega_3) \times \Gamma_{cc}(\omega_1 p, q) D_s((q' - \omega_3 p)^2). \quad (4.6)$$

1. *s-quark propagator*

The denominator of the s-quark propagator takes the form

$$m_s^2 - (\omega_3 p - q')^2 = m_s^2 - \lambda q^2 - (1-\omega_3)q'^2 + (1-\omega_3)\omega_3 M_B^2.$$

For $q^2 = q'^2 = 0$, in the region $\omega_3 \sim \Lambda_{\rm QCD}/m_b$ that dominates the integral, the s quark is highly virtual; its momentum squared is $O(\Lambda_{\rm QCD} m_b)$.

2. *Charm-quark loop*

The analytic expression for the triangle charming loop was already given in (2.9). We now rewrite it in a slightly different way by introducing a new variable $0 < \tau < 1$, $v = (1-u)\tau$:

$$\Gamma_{cc}(\kappa_1, q) = \frac{1}{8\pi^2}\int_0^1 du(1-u)\int_0^1 d\tau \frac{1}{m_c^2 - u(1-u)[\tau(\kappa_1-q)^2 + \kappa_1^2(1-\tau)] - q^2(1-u)^2\tau(1-\tau)}. \quad (4.7)$$

To shorten the formulas, we set hereafter $q^2 = 0$. This expression appears under the convolution Eq. (4.4) with the 3BS of the B meson. Then, the ω_1 integral is peaked near $\omega_1 \sim \Lambda_{\rm QCD}/m_b$, so the gluon is soft: $\kappa_1 = \omega_1 p$ and $\kappa_1^2 \sim O(\Lambda_{\rm QCD}^2) \ll m_c^2$. In (4.7), κ_1^2 may be neglected compared to m_c^2, and we find

$$\Gamma_{cc}(\kappa_1, q) = \frac{1}{8\pi^2}\int_0^1 du(1-u)\int_0^1 d\tau \frac{1}{m_c^2 - u(1-u)\tau(\kappa_1 - q)^2}. \quad (4.8)$$

Taking the τ integral, one comes to a relation very similar to the one obtained in Ref. [15] (up to the appropriate changes related to the spins of the constituents and the Lorentz structures of the currents).

Let us make two remarks:

(i) The obtained expression Eq. (4.6) for the amplitude $A_{\psi_0}(q,p)$ does not "feel" the relative location of the arguments x_i. In particular, ψ_0 is precisely the same function that parametrizes, e.g., the collinear LC configuration discussed in Ref. [15]. Moreover, up to technical complications related to the spins of the B-meson constituents and the Lorentz structure of the currents, the obtained expression corresponds to the approximation considered in Ref. [15]. So, one may say that the A_{ψ_0} approximation to the FCNC amplitude corresponds to the contribution of the collinear LC 3BS of the B meson.

(ii) We have shown in the previous section that the collinear approximation does not dominate the FCNC amplitude, and expanding the amplitude near the collinear configuration should lead to sizeable $O(1)$ corrections to the collinear approximation. The 3BS feels the relative location of the arguments x_i only starting with the terms x_{ij}^2. So, based on the general argument, one expects sizeable corrections, of the order of unity, coming from powers of x_{ij}^2. We shall see that indeed large $O(1)$ corrections emerge from all powers of $(x-z)^2$, whereas terms containing powers of x^2 and z^2 lead to the suppressed contributions.

C. Contributions induced by x_{ij}^2 terms in the three-particle BS amplitude

We now turn to the calculation of the contributions of x_{ij}^2 terms in the 3BS amplitude.

In the problem under consideration, one encounters two heavy-quark scales, m_c and m_b, such that $\Lambda_{\rm QCD}\ll m_c\ll m_b$. Taking into account the real values of the quark masses, one encounters a new parameter of order of unity:

$$\Lambda_{\rm QCD}m_b/m_c^2\simeq 1. \tag{4.9}$$

One needs therefore to sum all powers of the parameter $\Lambda_{\rm QCD}m_b/m_c^2$. We shall see that this task is related to a summation of all corrections of the order $(x_1-x_3)^{2n}$. To calculate the contributions of x_{ij}^2 terms in $\Psi_p(x_1,x_3)$ to the amplitude $\tilde{A}(p|q,q')$ of Eq. (4.4), we proceed as follows:

(i) Let us start with the $x_{1\mu}$ term under the integral. It may be written as

$$x_{1\mu}e^{i\kappa_1x_1}\Gamma_{cc}(\kappa_1,q)d\kappa_1=-i\frac{\partial}{\partial\kappa_1^\mu}e^{i\kappa_1x_1}\Gamma_{cc}(\kappa_1,q)d\kappa_1=ie^{i\kappa_1x_1}\frac{\partial}{\partial\kappa_1^\mu}\Gamma_{cc}(\kappa_1,q)\to i\frac{\partial}{\partial\kappa_1^\mu}\Gamma_{cc}(\kappa_1,q)|_{\kappa_1=\omega_1p}. \tag{4.10}$$

where we have performed the parts integration. The x_1 dependence then remains only in the exponential factors, and x_1 integration may be taken and leads to $\delta(\kappa_1-\omega_1p)$.

(ii) Similarly, the $x_{3\mu}$ term under the integral may be handled leading to $\delta(\kappa_3-\omega_3p)$:

$$x_{3\mu}e^{i\kappa_3x_3}D_s(\kappa_3-q')d\kappa_3=-i\frac{\partial}{\partial\kappa_3^\mu}e^{i\kappa_3x_3}D_s(\kappa_3-q')d\kappa_3=ie^{i\kappa_3x_3}\frac{\partial}{\partial\kappa_3^\mu}D_s(\kappa_3-q')\to i\frac{\partial}{\partial\kappa_3^\mu}D_s(\kappa_3-q')|_{\kappa_3=\omega_3p}. \tag{4.11}$$

Making use of these formulas, we obtain the factors in the integrands that describe the relative contributions to $A_{\psi_{ij}}(p,q)$ of the $x_{ij}^2\psi_{ij}$ terms compared to the ψ_0 contribution to $A_{\psi_0}(p,q)$ (i.e., the factors given below are to be compared with ψ_o):

$$x_1^2\Lambda_{\rm QCD}^2\psi_{12}\colon\ \Lambda_{\rm QCD}^2\psi_{12}\frac{8u^2(1-u)^2(\kappa_1-q\tau)^2}{([m_c^2-u(1-u)[(\kappa_1-q)^2\tau+\kappa_1^2(1-\tau)]]^2}\bigg|_{\kappa_1=\omega_1p}\sim\frac{\Lambda_{\rm QCD}^2\omega_1M_B^2}{m_c^4}\psi_{12}\sim\frac{\Lambda_{\rm QCD}^3m_b}{m_c^4}\psi_{12}; \tag{4.12}$$

$$x_3^2\Lambda_{\rm QCD}^2\psi_{23}\colon\Lambda_{\rm QCD}^2\psi_{23}\frac{8(\kappa_3-q')^2}{[m_s^2-(\kappa_3-q')^2]^2}\bigg|_{\kappa_1=\omega_1p}\sim\frac{\Lambda_{\rm QCD}^2\omega_3M_B^2}{(\omega_3M_B^2)^2}\psi_{23}\sim\frac{\Lambda_{\rm QCD}}{m_b}\psi_{23}; \tag{4.13}$$

$$x_1x_3\Lambda_{\rm QCD}^2\psi_{13}\colon\ \frac{2u(1-u)(\kappa_1^\mu-q^\mu\tau)}{m_c^2-u(1-u)[(\kappa_1-q)^2\tau+\kappa_1^2(1-\tau)]}\frac{2(\kappa_3-q')^\mu}{m_s^2-(\kappa_3-q')^2}\bigg|_{\kappa_{1,3}=\omega_{1,3}p}\sim\Lambda_{\rm QCD}\frac{q'q}{m_c^2m_b}\psi_{13}\sim\frac{\Lambda_{\rm QCD}m_b}{m_c^2}\psi_{13}. \tag{4.14}$$

Taking into account the adopted scaling, $\Lambda\ll m_c\ll m_b$, and $\Lambda m_b/m_c^2\simeq 1$, we see that the factors describing the relative contributions of powers of x_1^2 and x_3^2 are small (i.e., the dominant contribution comes from the region where x_1 and x_3 are on the light cone, $x_1^2=0$ and $x_3^2=0$), whereas, as expected from the general arguments, each term of the form $(x_1-x_3)^{2n}$ in $\Psi_p(x_1,x_3)$ leads to the contribution to $A(p,q)$ of the same order as $A_{\psi_0}(p,q)$. So, the knowledge of the full functional dependence of $\Psi_p(x_1, x_3)$ on the variable $(x_1 - x_3)^2$ is essential for a proper resummation of large $\Lambda_{\mathrm{QCD}} m_b/m_c^2$ corrections to the amplitude $A(p, q)$ of FCNC B decay.

(iii) We would like to remark on what happens if one considers the contribution of the light u quark instead of the c quark in the loop. First, in this case, one cannot set $q^2 = 0$ but has to keep $q^2 \leq -1$ GeV^2 to validate the perturbative calculation of the u-quark loop. Second, the scaling relations change

$$x_1^2 \Lambda_{\mathrm{QCD}}^2 \psi_{12}\colon \sim \frac{\Lambda_{\mathrm{QCD}}}{m_b} \psi_{12}; \tag{4.15}$$

$$x_3^2 \Lambda_{\mathrm{QCD}}^2 \psi_{23}\colon \sim \frac{\Lambda_{\mathrm{QCD}}}{m_b} \psi_{23}; \tag{4.16}$$

$$x_1 x_3 \Lambda_{\mathrm{QCD}}^2 \psi_{13}\colon \sim \psi_{13}. \tag{4.17}$$

Obviously, the statement that all $(x_1 - x_3)^{2n}$ terms provide the $O(1)$ contributions applies to both cases of the c and the u quarks in the triangle diagram.

(iv) Finally, let us emphasize the following feature of the $B \to j_1 j_2$ amplitude. If in Eq. (4.14) we keep the leading term only and neglect all corrections $O(\Lambda_{\mathrm{QCD}}/m_b)$, then the contributions of ψ_{13} simplifies considerably and takes the form similar to the contribution of ψ_0 (4.6). Moreover, the contribution of the full 3BS Eq. (4.1), including all powers of $(x_1 - x_3)^{2n}$ ($\psi_{13}^{(1)} \equiv \psi_{13}$),

$$\Psi_p(x_1, x_3) = \int d\omega_1 d\omega_3 e^{-i\omega_1 x_1 p - i\omega_3 x_3 p} \times \left[\psi_0(\omega_1, \omega_3) + \sum_{n=1} \psi_{13}^{(n)}(\omega_1, \omega_3) x_{13}^{2n} + O(x_1^2, x_3^2)\right], \tag{4.18}$$

to a FCNC $B \to j_1 j_2$ amplitude may be written with $O(\Lambda_{\mathrm{QCD}}/m_b)$ accuracy in a simple form [27]:

$$A(q, p) = \int_0^1 d\omega_1 \int_0^{1-\omega_1} d\omega_3\, \psi_{\mathrm{eff}}(\omega_1, \omega_3) \times \Gamma_{cc}(\omega_1 p, q)\, D_s\left((q' - \omega_3 p)^2\right), \tag{4.19}$$

with

$$\psi_{\mathrm{eff}}(\omega_1, \omega_3) = \psi_0(\omega_1, \omega_3) + \frac{4}{M_B^2} \frac{\partial^2}{\partial\omega_1 \partial\omega_3} \psi_{13}(\omega_1, \omega_3) + \ldots. \tag{4.20}$$

Here, the dots stand for the contributions of higher functions $\psi_{13}^{(n)}$, $n \geq 2$. The expression (4.20) as well as the contributions of $\psi_{13}^{(n)}$ may be easily obtained by making use of the relation [27]

$$x_1 x_3 = \frac{2}{M_B^2} x_1 p x_3 p. \tag{4.21}$$

The latter relation is valid to $O(\Lambda_{\mathrm{QCD}}/m_b)$ accuracy as soon as x_1^2 and x_3^2 are near the LC, $x_1^2 = O(1/\Lambda_{\mathrm{QCD}} m_b)$ and $x_3^2 = O(1/\Lambda_{\mathrm{QCD}} m_b)$.

As said above, in general, the invariant amplitudes appearing in 3BS are functions of five independent variables: $x_1 p$, $x_3 p$, x_1^2, x_3^2, and $(x_1 - x_3)^2$. However, if $x_1^2 = 0$ and $x_3^2 = 0$, the variable $(x_1 - x_3)^2$ is not an independent variable anymore and is reduced to the combination of the variables $x_1 p$ and $x_3 p$, Eq. (4.21).

V. CONCLUSIONS

We performed a detailed comparison of the contributions coming from three-particle BS amplitude of the B meson

$$\langle 0|\bar{s}(x) G(z) b(0)|B(p)\rangle \tag{5.1}$$

to (i) B-meson weak decay form factor that describes the CC B-decay amplitude and to (ii) nonfactorizable part of FCNC B-decay amplitude. Both amplitudes are related to the same diagram of the form-factor topology with, however, a different location of the heavy-quark field. In the CC amplitude, the heavy b quark is located at the end point of the line along which fast light quarks propagate in the diagram, whereas in the FCNC amplitude, the heavy b quark hits the middle of this line. As the result, the dominant contributions to these two amplitudes come from different 3BS configurations:

(i) The dominant contribution to a CC amplitude comes from the collinear configuration when both vertices x and z lie along the same light-cone direction $z_\mu = u x_\mu$, $x^2 = 0$ and $z^2 = 0$. Therefore, a collinear 3BS $\langle 0|\bar{s}(x) G(ux) b(0)|B(p)\rangle$ is sufficient for the calculation of the 3BS correction to the form factor.

(ii) The dominant contribution to a FCNC amplitude comes from a different configuration when both vertices x and z lie on the light cone, but along the different light-cone directions such that $z^2 = x^2 = 0$, but $xz \neq 0$. Therefore, a noncollinear 3BS of the B meson, $\langle 0|\bar{s}(x) G(z) b(0)|B(p)\rangle$, with $x^2 = z^2 = 0$, but $x - z)^2 \neq 0$, is necessary for a reliable calculation of the 3BS correction to the FCNC form factor.

We point out that a simple physics picture lies beyond these results: one considers the B-meson rest frame, and in this rest frame, the b quark is almost at rest.

In a CC decay, the b quark decays in a fast lepton pair with momentum q and a fast light quark which moves, say, along the + light-cone direction. At the point z, it is hit by a soft gluon and continues to move practically along the same direction before it reaches the point x where it emits the momentum q'. So, we come to the well-known result that the 3BS correction to the B-decay CC form factor is dominated by the collinear light-cone configuration $x^2 = 0$, $z^2 = 0$, and $z_\mu = ux_\mu$.

In a FCNC amplitude, the situation is different: a resting b quark emits a fast s quark in one space direction and a fast pair of charmed quarks[2] in an opposite space direction. If we translate this into light-cone directions and assign the direction of the s quark as $(+)$, then the c-quark pair moves along the $(-)$ LC direction. At point x, the s quark emits the momentum q'. The point x thus lies on the LC along its $(-)$ direction. The fast c-quark pair is hit by the soft gluon at the point z and continues to move up to the point z' where it emits the momentum q. Both z and z' lie along the $(+)$ LC direction. We see that $x^2 = z^2 = 0$, but, in general, $(x-z)^2 \neq 0$.

Notice that this argument does not say yet that all $O((x-z)^{2n})$ terms in 3BS of the B meson lead to $O(1)$ contributions compared to the contribution of the ψ_0 term.

The analysis of Sec. IV showed that for the case of the c quark in the loop the dominant contributions to the FCNC amplitude come from the region $x^2 = z^2 = 0$, and $\Lambda_{\rm QCD}^2(x-z)^2 \simeq \Lambda_{\rm QCD} m_b/m_c^2 = O(1)$. In this case, all $O((x-z)^{2n})$ terms in the 3BS of the B meson lead to $O(1)$ contributions compared to the contribution of the ψ_0 term. The same result holds for the light quark in the triangle instead of the c quark.

In conclusion, let us recall that the idea of going from local OPE for the FCNC amplitude to a nonlocal OPE was motivated by the necessity to sum up large $(\Lambda_{\rm QCD} m_b/m_c^2)^n$ corrections to the FCNC amplitude. However, keeping only the collinear LC part of the 3BS of the B meson and neglecting all terms of the order $(x-z)^{2n}$ leads to the resummation of a part of these large $(\Lambda_{\rm QCD} m_b/m_c^2)^n$ corrections, whereas another source of the corrections of the same order of magnitude remains unaccounted. So, the full dependence of 3BS of the B meson on the variable $(x-z)^2$ is necessary to properly sum the $(\Lambda_{\rm QCD} m_b/m_c^2)^n$ corrections.

[2]Obviously, the picture does not change if the c quark in the triangle is replaced by the u quark.

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[1] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Penguins with charm and quark-hadron duality, Eur. Phys. J. C **61**, 439 (2009).

[2] M. Ciuchini, M. Fedele, E. Franco, A. Paul, and L. Silvestrini, Lessons from the $B^{0,+} \to K^{*0,+}\mu^+\mu^-$ angular analyses, Phys. Rev. D **103**, 015030 (2021).

[3] M. Ciuchini, M. Fedele, E. Franco, A. Paul, and L. Silvestrini, New physics without bias: Charming penguins and lepton universality violation in $b \to s\ell^+\ell^-$ decays, arXiv:2110.10126.

[4] D. Guadagnoli, B discrepancies hold their ground, Symmetry **13**, 1999 (2021).

[5] M. Algueró, B. Capdevila, A. Crivellin, and J. Matias, Disentangling lepton flavour universal and lepton flavour universality violating effects in $b \to s\ell^+\ell^-$ transitions, Phys. Rev. D **105**, 113007 (2022).

[6] M. B. Voloshin, Large $O(m_c^{-2})$ nonperturbative correction to the inclusive rate of the decay $B \to X_s\gamma$, Phys. Lett. B **397**, 275 (1997).

[7] Z. Ligeti, L. Randall, and M. B. Wise, Comment on non-perturbative effects in $\bar{B} \to X_s\gamma$, Phys. Lett. B **402**, 178 (1997).

[8] G. Buchalla, G. Isidori, and S. J. Rey, Corrections of order $\Lambda_{\rm QCD}^2/m_c^2$ to inclusive rare B decays, Nucl. Phys. **B511**, 594 (1998).

[9] A. Khodjamirian, R. Ruckl, G. Stoll, and D. Wyler, QCD estimate of the long distance effect in $B \to K^*\gamma$, Phys. Lett. B **402**, 167 (1997).

[10] P. Ball and R. Zwicky, Time-dependent CP asymmetry in $B \to K^*\gamma$ as a (quasi) null test of the standard model, Phys. Lett. B **642**, 478 (2006).

[11] P. Ball, G. W. Jones, and R. Zwicky, $B \to V\gamma$ beyond QCD factorisation, Phys. Rev. D **75**, 054004 (2007).

[12] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Radiative decay $\Sigma^+ \to p\gamma$ in quantum chromodynamics, Nucl. Phys. **B312**, 509 (1989).

[13] P. Ball and V. Braun, Higher twist distribution amplitudes of vector mesons in QCD: Twist—4 distributions and meson mass corrections, Nucl. Phys. **B543**, 201 (1999).

[14] P. Ball, Theoretical update of pseudoscalar meson distribution amplitudes of higher twist: The Nonsinglet case, J. High Energy Phys. 01 (1999) 010.

[15] A. Khodjamirian, T. Mannel, A. Pivovarov, and Y.-M. Wang, Charm-loop effect in $B \to K^{(*)} l^+ l^-$ and $B \to K^* \gamma$, J. High Energy Phys. 09 (2010) 089.

[16] N. Gubernari, D. van Dyk, and J. Virto, Non-local matrix elements in $B_{(s)} \to \{K^{(*)}, \phi\} \ell^+ \ell^-$, J. High Energy Phys. 02 (2021) 088.

[17] V. M. Braun and I. Halperin, Soft contribution to the pion form-factor from light cone QCD sum rules, Phys. Lett. B **328**, 457 (1994).

[18] A. Khodjamirian, T. Mannel, and N. Offen, Form-factors from light-cone sum rules with B-meson distribution amplitudes, Phys. Rev. D **75**, 054013 (2007).

[19] H. Kawamura, J. Kodaira, C.-F. Qiao, and K. Tanaka, B-meson light cone distribution amplitudes in the heavy quark limit, Phys. Lett. B **523**, 111 (2001); Phys. Lett. B **536**, 344(E) (2002).

[20] A. Kozachuk and D. Melikhov, Revisiting nonfactorizable charm-loop effects in exclusive FCNC B-decays, Phys. Lett. B **786**, 378 (2018).

[21] D. Melikhov, Charming loops in exclusive rare FCNC B-decays, EPJ Web Conf. **222**, 01007 (2019).

[22] Q. Qin, Yue-Long Shen, Chao Wang, and Yu-Ming Wang, Deciphering the long-distance penguin contribution to $\bar{B}_{d,s} \to \gamma\gamma$ decays, arXiv:2207.02691.

[23] S. J. Lee, M. Neubert, and G. Paz, Enhanced non-local power corrections to the $B \to X_s \gamma$ decay rate, Phys. Rev. D **75**, 114005 (2007).

[24] M. Benzke, S. J. Lee, M. Neubert, and G. Paz, Factorization at subleading power and irreducible uncertainties in $\bar{B} \to X_s \gamma$ decay, J. High Energy Phys. 08 (2010) 099.

[25] A. Kozachuk, D. Melikhov, and N. Nikitin, Rare FCNC radiative leptonic $B_{s,d} \to \gamma l^+ l^-$ decays in the Standard Model, Phys. Rev. D **97**, 053007 (2018).

[26] W. Lucha and D. Melikhov, The puzzle of the $\pi \to \gamma\gamma^*$ transition form factor, J. Phys. G **39**, 045003 (2012).

[27] Q. Qin, Yue-Long Shen, Chao Wang, and Yu-Ming Wang (private communication).