

Improved description of the $e^+e^- \rightarrow p\bar{p}\pi^0$ cross section line shape and more stringent constraints on $\psi(3770)$ and $\psi(4230) \rightarrow p\bar{p}\pi^0$

Ya-Qian Wang^{1,2,*} and Chang-Zheng Yuan^{2,3,†}

¹*Department of Physics, Hebei University, Baoding 071002, China*

²*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*

³*University of Chinese Academy of Sciences, Beijing 100049, China*

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By using the cross sections of $e^+e^- \rightarrow p\bar{p}\pi^0$ measured at the center-of-mass energies from 3.65 to 4.60 GeV, we find that the line shape well agrees with pure continuum production parametrized by a power law function and the significance of both $\psi(3770)$ and $\psi(4230) \rightarrow p\bar{p}\pi^0$ is less than 0.4σ . We set more stringent constraints on the charmless decays of the charmonium state, $\psi(3770)$, and the charmoniumlike state, $\psi(4230)$, to $p\bar{p}\pi^0$ than in previous measurements. The data are also used to estimate the cross sections of $p\bar{p} \rightarrow \pi^0\psi(3770)$ and $p\bar{p} \rightarrow \pi^0\psi(4230)$ that are essential for planning the data taking of the PANDA experiment.

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The charmonium state $\psi(3770)$, lying about 40 MeV above the open charm threshold, is expected to decay dominantly into the Okubo-Zweig-Iizuka allowed final state, $D\bar{D}$, together with a small fraction of hadronic and radiative transitions into final states with a lower mass charmonium state. However, the total cross section $\sigma(e^+e^- \rightarrow \psi(3770))$ measured by counting the total number of inclusive hadronic events is not saturated by $D\bar{D}$ measured by counting the number of charged and neutral $D\bar{D}$ pairs [1,2]. The hadronic transitions and the radiative transitions of the $\psi(3770)$ can be calculated reliably in the quark models [3] and have been measured precisely in experiments [4]. This indicates that noticeable charmless decays of the $\psi(3770)$, i.e., $\psi(3770) \rightarrow$ light hadrons, may exist. On the other hand, people did elaborate studies to search for exclusive charmless decays and failed to find any mode with a large branching fraction [5,6].

The large rate of charmless decays of the $\psi(3770)$ can also be accommodated in theoretical models. Among many theoretical efforts trying to solve the “ $\rho\pi$ puzzle” in J/ψ and $\psi(3686)$ decays observed by the Mark-II experiment [7], the 2S-1D charmonium mixing scenario [8] relates a partial width in $\psi(3770)$ decays into any final state to the corresponding partial widths in J/ψ and $\psi(3686)$ decays

[9], as both $\psi(3686)$ and $\psi(3770)$ are the mixtures of the 2S and 1D charmonium states. As a result, a large $\psi(3770) \rightarrow$ light hadrons is allowed in this model, and the measurement of the $\psi(3770)$ decays can test the model predictions. The process $\psi(3770) \rightarrow p\bar{p}\pi^0$ is one of the possible exclusive channels contributing to the charmless decays of the $\psi(3770)$.

The observation of the charmoniumlike states such as the $X(3872)$ [10], the $Y(4260)$ [11], and the $Z_c(3900)$ [12] indicates that the hadrons are more complicated than the expectations in quark models. The exotic properties of these states such as very close to open charm thresholds and large coupling to hidden-charm final states may suggest they are hadronic states beyond the conventional quark model [13]. Search for their decays into light hadrons will also shed light on their nature. Considering the small coupling to open charm final states, we may even expect a state like the $\psi(4230)$ [14] (the dominant component of the $Y(4260)$ structure) has a larger decay rate to the charmless final state than a charmonium state does. These discussions can be extended to other vector charmoniumlike states such as the $Y(4360)$ and $Y(4660)$ [15].

If indeed $\psi(3770) \rightarrow p\bar{p}\pi^0$ and/or $\psi(4230) \rightarrow p\bar{p}\pi^0$ are observed, one may calculate $\sigma(p\bar{p} \rightarrow \pi^0\psi(3770))$ and $\sigma(p\bar{p} \rightarrow \pi^0\psi(4230))$ by using the cross symmetry as has been done in Ref. [16]. These cross sections serve as an essential input for the PANDA (antiproton annihilations experiment at Darmstadt), which plans to study charmonium and charmoniumlike states produced in $p\bar{p}$ annihilation.

To study the resonance contribution in the $e^+e^- \rightarrow p\bar{p}\pi^0$ channel, the process $\psi(3770) \rightarrow p\bar{p}\pi^0$ can hardly be

*whyaqm@hbu.edu.cn

†yuancz@ihep.ac.cn

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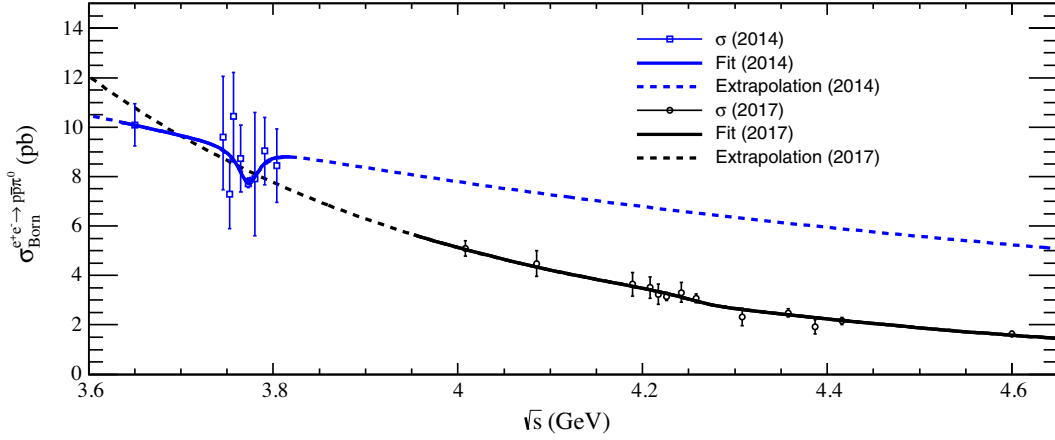


FIG. 1. Extrapolation of the BESIII fit results, the 2014 [19] and 2017 [21] analyses give very different estimation of the continuum amplitude.

determined without a good knowledge of the continuum production [17]. The continuum cross sections of $e^+e^- \rightarrow p\bar{p}\pi^0$ are calculated in the energy range from threshold up to $\sqrt{s} = 4.2$ GeV, applying the conservation of the hadron electromagnetic currents and the P-invariance of the hadron electromagnetic interaction [18]. We focus on the study of $e^+e^- \rightarrow p\bar{p}\pi^0$ in this paper, and aim to extract both the resonance and the continuum contributions.

The process of $e^+e^- \rightarrow p\bar{p}\pi^0$ is studied in the vicinity of the $\psi(3770)$ by the BESIII experiment in 2014 [19] (labeled with “2014” hereinafter). Two indistinguishable solutions [20] for the cross section of $\psi(3770) \rightarrow p\bar{p}\pi^0$ are extracted, and the maximum cross section of $p\bar{p} \rightarrow \psi(3770)\pi^0$ is expected at a center-of-mass energy (CME) of 5.26 GeV using a constant decay amplitude approximation [16]. The same process is studied in the energy range $\sqrt{s} = 4.008\text{--}4.600$ GeV with the BESIII data taken at 13 CMEs [21] (labeled with “2017” in the context), and no significant resonance is observed.

The above two analyses study the $e^+e^- \rightarrow p\bar{p}\pi^0$ process from $\sqrt{s} = 3.65\text{--}4.60$ GeV, and in principle the data can be combined to extract a better estimation of the physics quantities. By combining the measurements, we benefit not only from a better description of the continuum contribution but also from improvement on the fit result in the vicinity of the $\psi(3770)$ and the $\psi(4230)$. In a word, besides feeding the PANDA experiment by estimating the cross sections $\sigma(p\bar{p} \rightarrow \pi^0\psi)$, the results could also provide further insights into the puzzling question on the

mechanisms of non- $D\bar{D}$ transitions for $\psi(3770)$ and shed light on the understanding of the $\psi(4230)$ and other vector charmoniumlike states.

Both the BESIII analyses fit the cross sections with the formula

$$\sigma(m) = \left| \sqrt{\sigma_{\text{con}}} + \sqrt{f_{\sigma_{\psi}}} \frac{M\Gamma}{m^2 - M^2 + iM\Gamma} e^{i\phi} \right|^2, \quad (1)$$

where $\sigma_{\text{con}} = C/s^\lambda$ represents the continuum amplitude with the unknown exponent λ and a constant C ; the resonance ψ has a fixed mass M and total width Γ [4]; the factor $f_{\sigma_{\psi}} = \sigma(e^+e^- \rightarrow \psi \rightarrow p\bar{p}\pi^0)$ is the peak cross section of the resonance ψ ; the parameter ϕ describes the relative phase between the continuum and resonance amplitudes.

The two analyses study the same final state but at different energy ranges. The cross sections are fitted with the same model but with different resonances. The parameters (C, λ) describing the continuum contribution are expected to be the same. However, an obvious discrepancy exists between the two best fits in the regions dominated by the continuum amplitude, as shown in Fig. 1 and indicated by the very different common parameters listed in Table I.

We try to do a combined fit to the measurements presented in the two analyses [19,21]. Table II shows a compilation of the cross sections. We do a least χ^2 fit with

TABLE I. Fit parameters in the two BESIII analyses [19,21]. The resonance parameters of the $\psi(3770)$ and $\psi(4230)$ are fixed in the fits.

Data	\sqrt{s} (GeV)	C ($\text{GeV}^{2\lambda}\text{pb}$)	λ	M_{ψ} (GeV)	Γ_{ψ} (GeV)
2014	3.650–3.804	$(0.4 \pm 0.6) \times 10^3$	1.4 ± 0.6	3.77315	0.0272
2017	4.008–4.600	$(5.4 \pm 5.3) \times 10^5$	4.2 ± 0.4	4.251	0.12

TABLE II. $e^+e^- \rightarrow p\bar{p}\pi^0$ Born cross sections measured by the BESIII experiment [19,21]. The first errors are statistical and second ones systematic. $\frac{1}{|1-\Pi|^2}$ is the vacuum polarization factor to obtain the dressed cross sections [22].

\sqrt{s} (GeV)	$\sigma(e^+e^- \rightarrow p\bar{p}\pi^0)$ (pb)	$\frac{1}{ 1-\Pi ^2}$
3.650	$10.09 \pm 0.84 \pm 0.16$	1.0196
3.746	$9.60^{+2.45}_{-2.12} \pm 0.16$	1.0596
3.753	$7.28 \pm 1.38 \pm 0.12$	1.0573
3.757	$10.44 \pm 1.77 \pm 0.17$	1.0564
3.765	$8.73 \pm 1.35 \pm 0.14$	1.0558
3.773	$7.71 \pm 0.09 \pm 0.13$	1.0598
3.780	$7.92^{+2.66}_{-2.31} \pm 0.13$	1.0611
3.791	$9.03 \pm 1.35 \pm 0.15$	1.0592
3.804	$8.44 \pm 1.48 \pm 0.14$	1.0573
4.008	$5.09 \pm 0.18^{+0.26}_{-0.24}$	1.004
4.085	$4.47 \pm 0.46^{+0.27}_{-0.21}$	1.052
4.189	$3.64 \pm 0.43^{+0.18}_{-0.19}$	1.056
4.208	$3.52 \pm 0.39^{+0.17}_{-0.22}$	1.057
4.217	$3.24 \pm 0.37 \pm 0.18$	1.057
4.226	$3.15 \pm 0.08 \pm 0.14$	1.056
4.242	$3.30 \pm 0.36^{+0.19}_{-0.15}$	1.056
4.258	$3.08 \pm 0.10^{+0.14}_{-0.15}$	1.054
4.308	$2.32 \pm 0.33^{+0.15}_{-0.10}$	1.053
4.358	$2.48 \pm 0.11^{+0.13}_{-0.12}$	1.051
4.387	$1.92 \pm 0.26 \pm 0.10$	1.051
4.416	$2.16 \pm 0.10^{+0.10}_{-0.11}$	1.053
4.600	$1.63 \pm 0.08 \pm 0.08$	1.055

$$\chi^2 = \sum_{i=1}^N \frac{(\sigma_i^{\text{meas}} - \sigma^{\text{fit}}(m_i))^2}{(\Delta\sigma_i^{\text{meas}})^2},$$

where $\sigma_i^{\text{meas}} \pm \Delta\sigma_i^{\text{meas}}$ is the dressed cross section from experimental measurement, and $\sigma^{\text{fit}}(m_i)$ is the cross section value calculated from the model below with the parameters

obtained from the fit. Here m_i is the CME that corresponds to the i th one of all the N energy points. We only use the statistical errors in our fits since the systematic errors ($\sim 6.5\%$) for all the data points are correlated.

First, we fit the cross sections from 3.65 to 4.60 GeV with the continuum amplitude only. The fitted result is shown as the red curve in Fig. 2, and the fitted parameters are listed in Table III. The goodness-of-fit is $\chi^2/\text{NDF} = 13.0/20$ (NDF is the number of degrees of freedom), corresponding to a confidence level of 88%, a very good fit.

We then fit the cross sections considering possible resonance contributions. The continuum amplitude and the resonance [$\psi(3770)$ or $\psi(4230)$] amplitudes are added coherently,

$$\sigma(m) = |\sqrt{\sigma_{\text{con}}} + BW(m)e^{i\phi}|^2,$$

where $BW(m) = \frac{\sqrt{12\pi\Gamma_{e^+e^-}\Gamma_{\text{tot}}\mathcal{B}(\psi \rightarrow p\bar{p}\pi^0)}}{m^2 - M^2 + iM\Gamma_{\text{tot}}}$ is a Breit-Wigner function to describe the resonance amplitude with a mass M , total width Γ_{tot} , electronic partial width $\Gamma_{e^+e^-}$, and the branching fraction $\mathcal{B}(\psi \rightarrow p\bar{p}\pi^0)$. The continuum term σ_{con} is defined the same as before. In the fits, both the mass and the total width of the resonances are fixed according to the PDG [4], and the product $\Gamma_{e^+e^-} \times \mathcal{B}(\psi \rightarrow p\bar{p}\pi^0)$ is a free parameter. The continuum term σ_{con} and the relative phase between the continuum and the resonance amplitudes, ϕ , are also float parameters.

The $\psi(3770)$ or the $\psi(4230)$ resonance amplitude is added to the fit. The fit results are shown as the solid green and dashed blue curves in Fig. 2 and listed in Table III. The goodness-of-fit, χ^2/NDF , is 12.3/18 and 12.8/18 for $\psi(3770)$ and $\psi(4230)$, corresponding to a confidence level of 83% and 80%, respectively. The statistical significance of the $\psi(3770)$ is 0.36σ and that for the $\psi(4230)$ is 0.11σ , by comparing the differences in the χ^2 and in the NDF. Two solutions are found in each fit, one of which has a

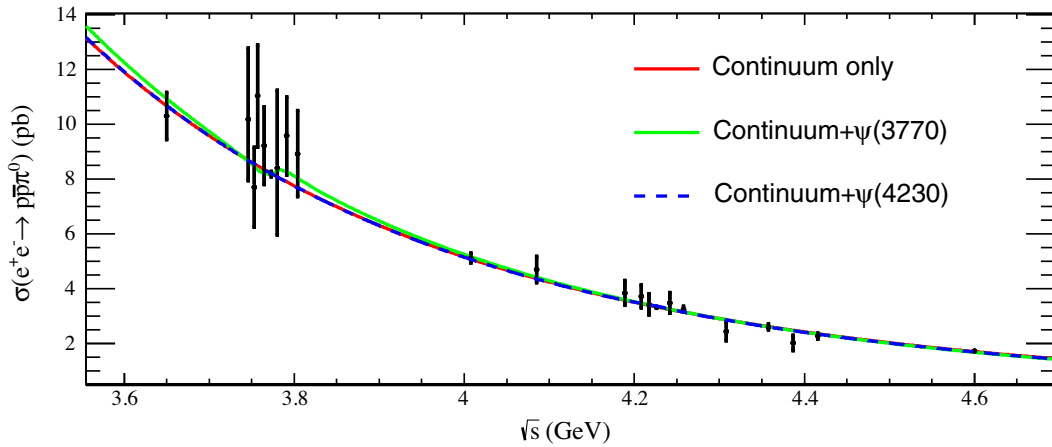


FIG. 2. Combined fits to the $e^+e^- \rightarrow p\bar{p}\pi^0$ cross sections [19,21]. The red curve is the fit with continuum amplitude only, the green curve is the fit with coherent sum of the continuum and the $\psi(3770)$ amplitudes, and the dashed blue curve is the fit with coherent sum of the continuum and the $\psi(4230)$ amplitudes.

TABLE III. Combined fit results for the three configurations. The subscripts of $\psi(3770)$ and $\psi(4230)$ indicate the two solutions of the fits. The errors are statistical only. There is an additional 6.5% systematic error in $\Gamma_{e^+e^-} \times \mathcal{B}$ and C due to the common systematic error in the cross section measurements.

Fit	χ^2/NDF	$\Gamma_{e^+e^-} \times \mathcal{B}$ (eV)	ϕ ($^\circ$)	C ($10^5 \text{ GeV}^{2\lambda}$ pb)	λ
Continuum	13.0/20	3.07 ± 0.58	3.96 ± 0.07
$\psi(3770)_1$	12.3/18	$(0.28 \pm 1.40) \times 10^{-3}$	340 ± 54	4.1 ± 1.8	4.07 ± 0.15
$\psi(3770)_2$	12.3/18	0.875 ± 0.017	270.1 ± 2.4	4.1 ± 1.8	4.07 ± 0.15
$\psi(4230)_1$	12.8/18	$(0.88 \pm 4.20) \times 10^{-5}$	78 ± 202	3.22 ± 0.83	3.98 ± 0.10
$\psi(4230)_2$	12.8/18	0.820 ± 0.016	268.7 ± 1.1	3.23 ± 0.84	3.98 ± 0.10

well-determined magnitude and interferes with the continuum amplitude destructively ($\phi \sim 270^\circ$), and the other solution comes with large uncertainty and agrees with zero with statistical uncertainties. The two solutions of $\Gamma_{e^+e^-} \times \mathcal{B}$ may differ by several orders of magnitude for both $\psi(3770)$ and $\psi(4230)$.

By combining the two BESIII measurements, the fit parameters, especially those describing the continuum are further constrained. Our first fit without any resonance indicates in another way the extremely small significance of the $\psi(3770)$ or $\psi(4230)$. We notice that the parameters C and λ are relatively stable in all the fits within uncertainties. The theoretical predictions on the continuum amplitude vary significantly in both the shape and the magnitude with different parametrizations (“new” and “old”) [18]. The new one favored by the BESIII data follows a power law with $C = 2.1 \times 10^3 \text{ GeV}^{2\lambda}$ and $\lambda = 3.58$ in the CME from 3.6 to 4.2 GeV [18]. However, the absolute cross sections, reflected by C , is still below the BESIII data by two orders of magnitude. The λ , describing the slope of the line shape, is sensitive to the choice of parametrization and can be further constrained by our combined fit.

Although not significant, the central value of the branching fraction of $\psi(3770) \rightarrow p\bar{p}\pi^0$ is also extracted to be either $(1.1 \pm 5.4 \pm 0.1 \pm 0.1^{\Gamma_{e^+e^-}}) \times 10^{-6}$ or $(0.33 \pm 0.01 \pm 0.03 \pm 0.03^{\Gamma_{e^+e^-}})\%$ using $\Gamma_{e^+e^-} = (0.262 \pm 0.018) \text{ keV}$ from the PDG [4], where the errors are statistical, systematic, and from the uncertainty of quoted $\Gamma_{e^+e^-}$ [4]. Very recently, the authors of Ref. [23] obtained $\Gamma_{e^+e^-} = (0.19 \pm 0.04) \text{ keV}$ for the $\psi(3770)$ by taking into account the contributions of the mixed $\psi(3770)$ and $\psi(3686)$ resonances. With this $\Gamma_{e^+e^-}$, the corresponding branching fraction is calculated as $(1.5 \pm 7.4 \pm 0.1 \pm 0.4^{\Gamma_{e^+e^-}}) \times 10^{-6}$ or $(0.46 \pm 0.01 \pm 0.03 \pm 0.10^{\Gamma_{e^+e^-}})\%$. All the results are within their large uncertainties either from our fit or from the $\Gamma_{e^+e^-}$. Obviously, the branching fraction of $\psi(3770) \rightarrow p\bar{p}\pi^0$ highly depends on the choice of the $\Gamma_{e^+e^-}$ in our calculation, and any reliable $\Gamma_{e^+e^-}$ can be taken as input to extract the branching fraction. The two solutions suggest either noticeable or negligible charmless decays of $\psi(3770) \rightarrow p\bar{p}\pi^0$. As one of the charmless decays of the $\psi(3770)$, the branching fraction can also be estimated considering the mixing with the $\psi(3686)$ due to tensor

forces and coupling to charmed meson pairs [8]. This S- and D-wave charmonium mixing model accounts for the “ $\rho\pi$ puzzle” by converting expected $\psi(3686) \rightarrow \rho\pi$ to corresponding $\psi(3770)$ partial widths. Taking the J/ψ and the $\psi(3686)$ decays into the same final state as input, the branching fraction of $\psi(3770) \rightarrow p\bar{p}\pi^0$ is predicted in the range $[(2.9 \pm 4.9) \times 10^{-7}, (15.3 \pm 1.5) \times 10^{-5}]$ assuming the mixing angle $\theta = (12 \pm 2)^\circ$. One of our fit results falls in the range of this prediction, while the other solution is beyond the range by one order of magnitude. If we use another mixing angle $\theta = -(27 \pm 2)^\circ$, the model predicts the branching fraction to be in the range $[(2.4 \pm 1.9) \times 10^{-7}, (2.9 \pm 0.3) \times 10^{-5}]$. Compared to the results with $\theta = 12^\circ$, the range becomes narrower, and the maximum decay rate of $\psi(3770) \rightarrow p\bar{p}\pi^0$ is much smaller.

The PANDA experiment produces neutral state with any conventional quantum numbers (J^{PC}) through $p\bar{p}$ reactions. While exotic states is also available in association with an additional meson, e.g., $p\bar{p} \rightarrow \pi^0 X$, where X is an exotic hybrid or a charmonium [16,24]. The branching fraction of $\psi(3770) \rightarrow p\bar{p}\pi^0$ can be taken as input for the constant decay amplitude model [16] to calculate the cross section of $p\bar{p} \rightarrow \pi^0\psi(3770)$. The maximum cross sections predicted by the model is $\sigma(p\bar{p} \rightarrow \pi^0\psi(3770)) = (0.04 \pm 0.20 \pm 0.01 \pm 0.01^{\Gamma_{e^+e^-}}) \text{ nb}$ or $(118 \pm 6 \pm 12 \pm 12^{\Gamma_{e^+e^-}}) \text{ nb}$ at $\sqrt{s} = 5.26 \text{ GeV}$, both agree with previous results [19]. The cross sections are sensitive to the $\Gamma_{e^+e^-}$ in the same way as the branching fraction of $\psi(3770) \rightarrow p\bar{p}\pi^0$. If we use $\Gamma_{e^+e^-} = (0.19 \pm 0.04) \text{ keV}$ [23], the above cross section becomes $(0.05 \pm 0.27 \pm 0.01 \pm 0.02^{\Gamma_{e^+e^-}}) \text{ nb}$ or $(164 \pm 7 \pm 13 \pm 37^{\Gamma_{e^+e^-}}) \text{ nb}$, which are larger by 25% and 39% compared with the results with $\Gamma_{e^+e^-}$ from the PDG [4]. The production rate of $p\bar{p} \rightarrow \pi^0\psi(4230)$ reaches the peak at $\sqrt{s} = 5.9 \text{ GeV}$, which exceeds the energy of PANDA by 0.4 GeV. The cross sections at $\sqrt{s} = 5.5 \text{ GeV}$ is predicted as $\sigma(p\bar{p} \rightarrow \pi^0\psi(4230)) = \frac{1}{\Gamma_{e^+e^-}} (29.4 \pm 0.59 \pm 5.3) \text{ nb} \cdot \text{keV}$ or $\frac{1}{\Gamma_{e^+e^-}} (0.32 \pm 1.6 \pm 0.06) \text{ nb} \cdot \text{eV}$, where the currently unknown electron partial width of the $\psi(4230)$ is expected in the future.

In summary, we do a combined fit to the $e^+e^- \rightarrow p\bar{p}\pi^0$ cross sections measured by the BESIII experiment and obtain a better description of the continuum production of

$e^+e^- \rightarrow p\bar{p}\pi^0$. It shows that the interference effect between continuum and the resonance amplitudes plays a very important role in determining the resonance contributions in this mode. BESIII has collected data at much more energy points from 4.130 to 4.946 GeV, and the cross sections are expected to be further measured and better examined in the future [25].

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