

Relativistic composite-particle theory of the gravitational form factors of the pion: Quantitative results

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(Received 14 January 2022; accepted 22 August 2022; published 12 September 2022)

We use a version of the instant-form relativistic quantum mechanics of composite systems to obtain the gravitational form factors of the pion in a common approach to its electroweak and gravitational properties. In the preceding work [Phys. Rev. D **103**, 014029 (2021)], we formulated the mathematical background, presented the principal scheme of calculation and testified the obtained qualitative results to satisfy the general constraints given by the principles of the theory of hadron structure. In the present work, we give the detailed calculation of the gravitational form factors in the large range of momentum transfer, their static limits, and the slopes at zero value, the mean-square mass and mechanical radii of the pion. Now we take into account the gravitational structure of the constituent quarks. We show that the results are almost insensitive to the type of model two-quark wave function in a close analogy to the case of the pion electromagnetic form factor. We present a correct calculation of the form factor D and corresponding matrix element of the energy-momentum tensor, going beyond the scope of the modified impulse approximation. Most of the parameters that we use for the calculation had been fixed even earlier in our works on the pion electromagnetic form factors. The only free parameter is the D -term of the constituent quark, which we fix by fitting the result for the slope at zero of the normalized to pion D -term form factor D of the pion, to a chosen experimental value.

DOI: [10.1103/PhysRevD.106.054013](https://doi.org/10.1103/PhysRevD.106.054013)

I. INTRODUCTION

The understanding of the gravitational structure of hadrons is a fundamental problem of particle physics. To consider this problem, one needs to involve specific mathematical objects: the gravitational form factors (GFFs) of hadrons [and, in general, the energy-momentum tensor (EMT)]. These functions are the focus of investigation in numerous recent works (see, e.g., the reviews [1–5]). The mathematical background of these investigations was laid in the 1960s [6–8], but became of large use only during the last decade.

It is the formalism of Ref. [7] that we use in our investigations. In the preceding work [9], we formulated the mathematical statements and described the principal

scheme of the method; we testified that the derived gravitational characteristics of the pion satisfy the constraints given by the general principles of the theory of hadron structure. In the present work, we refine the approach to give a detailed technique of the calculation, thus transforming it into a quantitative method. The refinement includes three main points. First, we take into account the gravitational structure of the constituent quarks. Second, we analyze different types of the two-quark wave functions in the pion. Third, we formulate a minimal way to overstep our modified impulse approximation (MIA) for correct description of the pion form factor D . Note that the pion EMT contains two form factors, and the second form factor A is described well in the frame of MIA.

The majority of papers concerning closely the problem under consideration were reviewed in [9] and we do not repeat the review in the present paper. Here we discuss only some recent results. Although the papers published during last year are dealing mainly with the proton, they show the modern trends and perspectives in general. At present, there is no possibility to obtain directly the data on gravitational characteristics, including the D -term. The information about the GFFs is usually extracted from the hard-exclusive

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processes described in terms of unpolarized generalized parton distribution (GPD) [10,11]. The GPD gives the information on the space distribution of strong forces that act on quarks and gluons inside hadrons. In this connection, in works [12–15] the processes of deeply virtual Compton scattering (DVCS) on nucleons were investigated. In Ref. [12], a new technique of artificial neural networks was used for the reduction of model dependence of the data handling. The distribution of shear forces inside the proton was obtained for the first time in [13] and also the proton form factor D was constructed by fitting of three parameters in the multipole-type decomposition. The kinematic corrections to the cross section, appearing as a consequence of nonuniqueness of description of the photon in the final state on the light cone, is discussed in Ref. [14]. It is shown that these corrections can be significant in the kinematic regions of future experiments. The experimental study of DVCS is realized in timelike region for the first time in [15].

In another set of works (Refs. [16–20]), the gravitational characteristics of hadrons are extracted from the data on near-threshold vector mesons photoproduction. In Ref. [16], the mass radius is calculated not only for the proton, but for the deuteron, too. The analysis of the possible sources of difference between charge and mass radii of the hadrons, systematically obtained in numerous recent calculations, is given in [19].

The low-energy chiral effective field theory was used in [21] to calculate the ρ -meson GFFs and in [22] to describe the mechanical stresses inside the nucleon.

Constituent-quark models are exploited in Refs. [23–25]. In particular, the spin-orbital correlations in a pion in a quark model on the light cone are studied in [23] in terms of GPD and of generalized transverse momentum distributions. In [24], the proton GFFs are derived in a light-front quark-diquark model. The authors of [25] study general problems of the interpretation of the experimental data and theoretical methods of calculation for nucleon GFFs. The investigation is focused on space distributions of the energy, shear stresses, momentums, and angular momentums in different frame systems. The authors use the quark bag model in the large- N_c limit in various cases—from nonrelativistic to ultrarelativistic.

As to nonhadronic systems, the form factor D , the distribution of matter, and internal stresses in the electron are obtained in [26] in the one-loop approximation of QED.

It seems obvious that, because of extreme weakness of the gravitational interaction at hadron scale, the information on GFFs presently can be extracted from electroweak processes only. So, a theory that describes electroweak and gravitational properties simultaneously, based on the unique foundations, and uses common model parameters, is welcome. The approach that we present here possesses these features.

We use a particular variant of the instant-form Dirac [27] relativistic quantum mechanics (RQM) (see, e.g., [28–31]) extended for composite systems. The approach was successfully used to describe the pion electromagnetic form factor (see, e.g., [32–34]). We have shown [9] that the pion GFFs can be derived in the same formalism using the same approximations and the same model parameters, adding only one new parameter fixed by fitting the slope at zero of the normalized to pion D -term form factor D of pion.

In the present paper, we take into account the gravitational structure of the constituent quarks and different forms of the quark-antiquark wave function. It is worth noting that including the gravitational structure of constituents in our relativistic model means an implicit accounting for gluons. Their degrees of freedom are incorporated in the parameters describing constituent quarks.

Let us emphasize that it was just the account of the structure of the constituents [32] that made it possible to predict the behavior of the pion charge form factor at intermediate and high momentum transfers. We have shown that, when two model parameters were fixed by the charge mean-square radius and the lepton decay constant, then the form factors only weakly depended on the choice of model interaction of quarks in the pion. The curves corresponding to different interactions but one and the same value of quark mass were agglomerated into narrow groups, or bunches. The chosen group of curves with the constituent mass $M = 0.22$ GeV had predicted, with surprising accuracy, the values of the pion charge form factor that were measured a decade later in JLab experiments. By the way, this value of the constituent-quark mass has been admitted and confirmed by other authors (from the well-known work [35] to a recent result [36]).

Let us list some other advantages of our model with fixed parameters. The obtained pion charge form factor at high momentum transfer coincides with the QCD predictions in the ultraviolet limit, reproducing correctly not only the functional form of the QCD asymptotics, but also the numerical coefficient [37–39] (analogous results were obtained for the kaon [40]). The method allows for an analytic continuation of the pion electromagnetic form factor from the spacelike region to the complex plane of momentum transfer squares and gives an adequate description of the pion form factor in the timelike region [41]. Interesting results of the approach in the case of its generalization to vector mesons, which required adding the anomalous magnetic moment of quark, were obtained for the ρ meson [42,43].

As was mentioned above, we need to overstep the frame of MIA for calculating correctly the pion form factor D . Here we present a possible minimal extension based on the nonrelativistic limit of MIA. In the relativistic series expansion of the ill-defined terms of the form factor, we preserve the main contributions only.

In the present paper, we extend our model with non-pointlike quarks to obtain the pion GFFs while making an essential assumption, which looks natural. We suppose that the functional forms of electroweak and gravitational form factors of constituent quark are the same. Also, we suppose that values of the charge and mass radii of quark and the slope at zero of the normalized form factor D are all equal. On one hand, this gives an opportunity to add to the model only one new parameter. On the other hand, the results of calculation are rather reasonable and plausible.

We fix the new free parameter, the quark D -term (instead of the quark anomalous magnetic moment in the vector meson case of electroweak processes), by fitting the slope at zero of the normalized to the pion D -term form factor D of pion to the value obtained in Ref. [44]. The scheme of narrow bunches of curves, corresponding to different wave functions, works in the case of pion GFFs, too. As we show below, it is possible to choose the quark D -term as the main characteristic of the bunch and so derive the value of the free parameter.

Other model parameters, which describe the mass and the gravitational structure of the constituent quarks and the quark interaction in the pion, are directly transported from our electroweak model. It is important to emphasize that the interval of the values of the quark D -term obtained by this fixation gives the value of the slope at zero of the normalized form factor D of the pion [44] for any quark interaction (for each of three two-quark wave functions) without additional variation of the previous values of the parameters.

This set of parameters—those inherited from the electroweak calculation plus the quark D -term—allow us to describe all other gravitational characteristics of the pion: the GFFs in a large range of momentum transfers, their static limits, including the pion D -term, and the mean-square mass radius. So, the results of calculations of the present paper can be considered as firm predictions in the following sense. If a value of, for example, the slope at zero of the normalized to D -term form factor D of pion extracted from a future experiment is admitted by the community of specialists, then other gravitational characteristics can be predicted following our prescription. The success demonstrated previously by our method inspires hope on its validity in describing the pion GFFs that can be obtained in future experiments.

Let us remind the reader that the first estimations of the pion GFFs were extracted from the data of the Belle Collaboration program KEKB [44,45]. Today, the real perspective of GPD estimation for light mesons, including the pion, is connected with the future experiments on the SuperKEKB collider. In general, we look forward for the experiments at the Electron-Ion Collider [46,47], Chinese Electron-Ion Collider [48,49], and Large Hadron-Electron Collider [50].

Moreover, it is possible that our unified approach can construct another bridge between electroweak and gravitational properties of composite systems, complementary to GPD.

The rest of the paper is organized as follows. In Sec. II we present the main basic points of Instant-Form (IF) Relativistic Quantum Mechanics (RQM) and the equations for the pion GFFs. Then in Sec. III we describe the gravitational structure of the constituent quarks and present the details of calculation. In Sec. IV we give the results of the calculation of the pion GFFs up to 10 GeV², their static limits, and the mean-square mass and mechanical radii. We briefly discuss the results and conclude in Sec. V.

II. MAIN BASIC POINTS OF IF RQM AND THE CALCULATION OF THE PION GFFs

Let us recall briefly some important features of the IF RQM algebraic structure. The basic point is the direct realization of the Poincaré algebra on the set of dynamical observables of a composite system (see the reviews [28–31]). In this context, the fundamental property of the Poincaré algebra, as compared to the algebra of the Galilean group, is as follows. The adding of the operator of constituent interaction to the total energy operator (that is, to zero component of total momentum) requires the inclusion of the interaction also in the operators of other observables to preserve the algebraic structure. Different forms of relativistic Dirac dynamics correspond to different ways of realizing the interaction, including which are characterized by different kinematic subgroups, namely, subgroups of interaction-independent observables. The kinematic subgroup in the case of IF RQM contains rotations and translations of three-dimensional space. From the point of view of the principles underlying the RQM theory, it occupies an intermediate position between local quantum field theory and nonrelativistic quantum mechanical models. In particular, constituents of the composite system are assumed to lie on the mass shell, and the corresponding wave function is defined as the eigenfunction of the complete set of commuting operators. In the case of IF RQM, this set is

$$\hat{M}_I^2(\text{or } \hat{M}_I), \quad \hat{J}^2, \quad \hat{J}_3, \quad \hat{\vec{P}}, \quad (1)$$

where $\hat{M}_I$ is the mass operator for the system with interaction, $\hat{J}^2$ is the operator of the square of the total angular momentum, $\hat{J}_3$ is the operator of the projection of the total angular momentum on the z axis, and $\hat{\vec{P}}$ is the operator of the total momentum.

In the IF RQM operators $\hat{J}^2$, $\hat{J}_3$, and $\hat{\vec{P}}$ coincide with corresponding operators for the composite system without interaction and only the term $\hat{M}_I^2(\hat{M}_I)$ is interaction dependent.

To solve the eigenfunction problem for the set (1) it is necessary to choose an appropriate basis in the Hilbert space of the state of the composite system. In the case of the system of two constituent quarks, one can use, first, the basis of individual spins and momenta (see [9] for details),

$$|\vec{p}_1, m_1; \vec{p}_2, m_2\rangle = |\vec{p}_1 m_1\rangle \otimes |\vec{p}_2 m_2\rangle, \quad (2)$$

where $\vec{p}_1$ and $\vec{p}_2$ are the 3-momenta of particles, and m_1 and m_2 are the projections of spins to the z axis.

Second, it is possible to use the basis in which the motion of the center of mass of two particles is separated,

$$|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle, \quad (3)$$

where $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = s$, $\sqrt{s}$ is the invariant mass of the system of two particles, l is the orbital momentum in the center-of-mass system, $\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2 = S(S+1)$, S is the total spin in c.m., J is the total angular momentum, and m_J is the projection of the total angular momentum.

The bases (2) and (3) are linked by the Clebsch-Gordan decomposition of a direct product (2) of two irreducible representations of the Poincaré group into irreducible representations (3) [31].

In the basis (3), three out of four operators in the complete set (1) (except $\hat{M}_I$) are diagonal. So, the two-quark wave function for the pion in the basis (3) has the following form:

$$\langle \vec{P}, \sqrt{s} | \vec{p}_\pi \rangle = N_C \delta(\vec{P} - \vec{p}_\pi) \varphi(s), \quad (4)$$

where $\vec{p}_\pi$ is the pion 3-momentum. Here we do not exploit the implicit form of the normalization constant N_C that can be found in [9]. The zero values of pion quantum numbers are omitted in the notation of basis vectors (3).

The wave function of intrinsic motion is the eigenfunction of the operator $\hat{M}_I^2(\hat{M}_I)$ and, in the case of two particles of equal masses, is (see, e.g., [33])

$$\begin{aligned} \varphi(s(k)) &= \sqrt[4]{s} k u(k), \quad s = 4(k^2 + M^2), \\ \int u^2(k) k^2 dk &= 1, \end{aligned} \quad (5)$$

where $u(k)$ is a model quark-antiquark wave function of the pion and M is the mass of the constituent quarks.

Let us construct now the pion EMT in IF RQM. Using the general method of the relativistic-invariant parametrization of the matrix elements of the local operators we have obtained in [9] the following form:

$$\langle \vec{p}_\pi | T_{\mu\nu}^{(\pi)}(0) | \vec{p}_{\pi'} \rangle = \frac{1}{2} G_{10}^{(\pi)}(t) K'_\mu K'_\nu + G_{60}^{(\pi)}(t) [t g_{\mu\nu} - K_\mu K_\nu], \quad (6)$$

where $G_{10}^{(\pi)}$ and $G_{60}^{(\pi)}$ are gravitational form factors, $g_{\mu\nu}$ is the metric pseudotensor, and

$$K_\mu = (p_\pi - p'_\pi)_\mu, \quad K'_\mu = (p_\pi + p'_\pi)_\mu.$$

We present the decomposition of the lhs of (6) in terms of the basis (3) as a superposition of the same tensors as in the rhs of (6), and so obtain (see [9] for details) the pion gravitational form factors in the following form of the functionals, given on two-quark wave functions [(4) and (5)]:

$$\begin{aligned} G_{i0}^{(\pi)}(t) &= \int d\sqrt{s} d\sqrt{s'} \varphi(s) \tilde{G}_{i0}(s, t, s') \varphi(s'), \\ i &= 1, 6, \end{aligned} \quad (7)$$

where $\tilde{G}_{i0}(s, t, s')$, $i = 1, 6$ are the Lorentz-invariant regular distributions.

To calculate the invariant distributions on the rhs of (7), one can use MIA. Let us discuss this problem in more detail. Consider the commonly used standard impulse approximation (IA). In general, the EMT of a composite system has the following form [9]:

$$T = \sum_k T^{(k)} + \sum_{k < m} T^{(km)} + \dots, \quad (8)$$

where the first term presents the sum of one-particle EMTs, the second term presents the sum of two-particle EMTs, and so on. The first sum describes the scattering of a projectile by each independent constituent, the second sum describes the scattering by two constituents simultaneously, and so on. The standard IA leaves in (8) only the first term,

$$T \approx \sum_k T^{(k)}. \quad (9)$$

To construct the pion GFFs, we use a modified impulse approximation that we first formulated earlier (see, e.g., Refs. [33,34] and the review [31]). In contrast to the baseline impulse approximation, MIA is formulated in terms of the reduced matrix elements, that is, form factors, and not in terms of the operators themselves. So, in MIA there appears important objects—the free gravitational form factors presenting the gravitational characteristics of systems without interaction.

Consider the system of two free constituent quarks [9]. Note, that in [51] it was shown that the form factor D is zero in the case of pointlike free fermions. In contrast, our constituent quarks have all properties of realistic particles with an internal structure that is described by a set of form factors including form factor D .

The corresponding relation based on the Clebsch-Gordan decomposition of the EMT of a system of non-interacting fermions with total quantum numbers of pion $J = l = S = 0$ has the following form [9]:

$$\begin{aligned}
& \langle P, \sqrt{s} | T_{\mu\nu}^{(0)}(0) | P', \sqrt{s'} \rangle \\
&= \sum \int \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}} \frac{d\vec{p}'_1}{2p'_{10}} \frac{d\vec{p}'_2}{2p'_{20}} \langle P, \sqrt{s} | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle \\
&\quad \times [\langle \vec{p}_1, m_1 | \vec{p}'_1, m'_1 \rangle \langle p_2, m_2 | T_{\mu\nu}^{(u)}(0) | p'_2, m'_2 \rangle \\
&\quad + \langle \vec{p}_2, m_2 | \vec{p}'_2, m'_2 \rangle \langle p_1, m_1 | T_{\mu\nu}^{(\bar{d})}(0) | p'_1, m'_1 \rangle] \\
&\quad \times \langle \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 | P', \sqrt{s'} \rangle, \quad (10)
\end{aligned}$$

where $\langle P, \sqrt{s} | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle$ is the Clebsch-Gordan coefficient, and the sums are over the variables m_1, m_2, m'_1 , and m'_2 . On the lhs, zero discrete quantum numbers in state vectors are ignored.

Using the general method of parametrization of local-operator matrix elements [7,9], we write the matrix element on the lhs as

$$\begin{aligned}
& \langle P, \sqrt{s} | T_{\mu\nu}^{(0)}(0) | P', \sqrt{s'} \rangle \\
&= \frac{1}{2} G_{10}^{(0)}(s, t, s') A'_\mu A'_\nu + G_{60}^{(0)}(s, t, s') [tg_{\mu\nu} - A_\mu A_\nu], \quad (11)
\end{aligned}$$

where $G_{i0}^{(0)}(s, t, s')$, $i = 1, 6$ are free two-particle GFFs,

$$\begin{aligned}
A_\mu &= (P - P')_\mu, \quad A^2 = t, \\
A'_\mu &= \frac{1}{(-t)} [(s - s' - t)P_\mu + (s' - s - t)P'_\mu].
\end{aligned}$$

The method gives for the one-particle matrix elements on the rhs of (10) the form

$$\begin{aligned}
& \langle p, m | T_{\mu\nu}^{(q)}(0) | p', m' \rangle \\
&= \sum_{m''} \langle m | D_w^{1/2}(p, p') | m'' \rangle \langle m'' | (1/2) g_{10}^{(q)}(t) K'_\mu K'_\nu \\
&\quad + i g_{40}^{(q)}(t) [K'_\mu R_\nu + R_\mu K'_\nu] + g_{60}^{(q)}(t) [tg_{\mu\nu} - K_\mu K_\nu] | m' \rangle, \quad (12)
\end{aligned}$$

$q = u, \bar{d}$, $D_w^j(p, p')$ is the transformation operator from the small group, the matrix of three-dimensional rotation, and $g_{i0}^{(u, \bar{d})}$, $i = 1, 4, 6$ are the constituent-quark GFFs. Their links with conventional notations are given later in Sec. III,

$$\begin{aligned}
K_\mu &= (p - p')_\mu, \quad K'_\mu = (p + p')_\mu, \\
R_\mu &= \epsilon_{\mu\nu\lambda\rho} p^\nu p'^\lambda \Gamma^\rho(p'). \quad (13)
\end{aligned}$$

Here $\Gamma^\rho(p')$ is a well-known four-vector of spin (see, e.g., [7,9,31,34]), and $\epsilon_{\mu\nu\lambda\rho}$ is the absolutely antisymmetric pseudotensor of rank 4, $\epsilon_{0123} = -1$.

Substituting (11) and (12) into (10), multiplying both sides by scalars, first by $A'_\mu A'_\nu$ and second by $g_{\mu\nu}$, and performing the integrations and summations, we obtain the system of equations for the free two-particle form factors

that enter (11). MIA means replacing the invariant distributions on the rhs of (7) with free two-particle form factors from Eqs. (10) and (11). The physical meaning of MIA is equivalent to that of the universally accepted IA (9) because the free two-particle form factors are given in terms of one-particle currents (10) and (12). So, we obtain the expressions for the pion gravitational form factors in MIA. For convenience, we present them in a slightly different form than in [9],

$$\begin{aligned}
G_{10}^{(\pi)}(t) &= \frac{1}{2} [g_{10}^{(u)}(t) + g_{10}^{(\bar{d})}(t)] G_{110}^{(\pi)}(t) \\
&\quad + [g_{40}^{(u)}(t) + g_{40}^{(\bar{d})}(t)] G_{140}^{(\pi)}(t), \quad (14)
\end{aligned}$$

$$\begin{aligned}
G_{60}^{(\pi)}(t) &= \frac{1}{2} [g_{10}^{(u)}(t) + g_{10}^{(\bar{d})}(t)] G_{610}^{(\pi)}(t) \\
&\quad + [g_{40}^{(u)}(t) + g_{40}^{(\bar{d})}(t)] G_{640}^{(\pi)}(t) \\
&\quad + [g_{60}^{(u)}(t) + g_{60}^{(\bar{d})}(t)] G_{660}^{(\pi)}(t). \quad (15)
\end{aligned}$$

Here $g_{i0}^{(q)}(t)$, $q = u, \bar{d}$, $i = 1, 4, 6$ are the GFFs of the constituent quarks, also introduced previously in (12).

To calculate the form factors on the rhs of Eqs. (14) and (15), we use the MIA [9] and write them in terms of the integrals

$$G_{1i0}^{(\pi)}(t) = \int d\sqrt{s} d\sqrt{s'} \varphi(s) G_{1i0}^{(0)}(s, t, s') \varphi(s'), \quad (16)$$

$$G_{6k0}^{(\pi)}(t) = \int d\sqrt{s} d\sqrt{s'} \varphi(s) G_{6k0}^{(0)}(s, t, s') \varphi(s'). \quad (17)$$

Here $i = 1, 4, k = 1, 4, 6$, and $G_{i0}^{(0)}(s, t, s')$ and $G_{6k0}^{(0)}(s, t, s')$ are components of the so-called free GFFs that describe the system of two free particles with total quantum numbers of pion [9]; $\varphi(s)$ is the pion wave function in the sense of RQM (5), and s' and s are the invariant masses of the free two-particle system in the initial and final states, respectively.

Now we have under integrals in (16) and (17) (compare [9]),

$$\begin{aligned}
G_{110}^{(0)}(s, t, s') &= -\frac{R(s, t, s')(-t)}{\lambda(s, t, s')} [(4M^2 - t)\lambda(s, t, s') \\
&\quad + 3t(s + s' - t)^2] \cos(\omega_1 + \omega_2), \quad (18)
\end{aligned}$$

$$\begin{aligned}
G_{140}^{(0)}(s, t, s') &= -3M \frac{R(s, t, s')(-t)^2}{\lambda(s, t, s')} \\
&\quad \times \xi(s, t, s')(s + s' - t) \sin(\omega_1 + \omega_2), \quad (19)
\end{aligned}$$

$$\begin{aligned}
G_{610}^{(0)}(s, t, s') &= \frac{1}{2} R(s, t, s') [(s + s' - t)^2 \\
&\quad - (4M^2 - t)\lambda(s, t, s')/(-t)] \cos(\omega_1 + \omega_2), \quad (20)
\end{aligned}$$

$$G_{640}^{(0)}(s, t, s') = -\frac{M}{2} R(s, t, s') \xi(s, t, s') (s + s' - t) \sin(\omega_1 + \omega_2), \quad (21)$$

$$G_{660}^{(0)}(s, t, s') = R(s, t, s') \lambda(s, t, s') \cos(\omega_1 + \omega_2), \quad (22)$$

where

$$R(s, t, s') = \frac{(s + s' - t)}{2\sqrt{(s - 4M^2)(s' - 4M^2)}} \frac{\vartheta(s, t, s')}{[\lambda(s, t, s')]^{3/2}},$$

$$\xi(s, t, s') = \sqrt{-(M^2 \lambda(s, t, s') + ss't)}.$$

ω_1 and ω_2 are the Wigner spin-rotation parameters

$$\omega_1 = \arctan \frac{\xi(s, t, s')}{M[(\sqrt{s} + \sqrt{s'})^2 - t] + \sqrt{ss'}(\sqrt{s} + \sqrt{s'})},$$

$$\omega_2 = \arctan \frac{\alpha(s, s') \xi(s, t, s')}{M(s + s' - t) \alpha(s, s') + \sqrt{ss'}(4M^2 - t)},$$

$\alpha(s, s') = 2M + \sqrt{s} + \sqrt{s'}$, $\vartheta(s, t, s') = \theta(s' - s_1) - \theta(s' - s_2)$, and θ is the Heaviside function,

$$s_{1,2} = 2M^2 + \frac{1}{2M^2} (2M^2 - t)(s - 2M^2) \mp \frac{1}{2M^2} \sqrt{(-t)(4M^2 - t)s(s - 4M^2)},$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc).$$

Recall that the form factors $G_{k0}^{(0)}(s, t, s')$, $k = 1, 6$ describe gravitational features of a system of two particles without interaction. Free two-particle form factors are regular generalized functions (distributions) given by the corresponding functionals, defined on the space of test functions depending on the variables (s, s') . The functionals, in turn, are a function of the variable t , a square of momentum transfer. This variable is to be considered as a parameter.

In the frames of MIA, the pion GFFs are functionals (14)–(17), generated by the free two-particle GFFs of (11) on test functions, which are the products of the two-quark wave functions [see (14)–(22)]. There is a following difficulty in calculating the form factor $G_{60}^{(\pi)}$ (an analog of the form factor D) in MIA. Our expressions (20)–(22) show that $G_{60}^{(0)} \sim 1/t$ when $t \rightarrow 0$. A consequence of the mentioned singularity in $G_{60}^{(0)}$ is the singularity in the pion form factor $G_{60}^{(\pi)}$ at $t \rightarrow 0$. So, for a correct description of the pion form factor D we must overflow MIA and add to the free form factor $G_{60}^{(0)}(s, t, s')$ an invariant term,

$$G_{60}^{(0)}(s, t, s') \rightarrow G_{60}^{(0)}(s, t, s') + G_{60}^{(a)}(s, t, s'). \quad (23)$$

Strictly speaking, the calculation of this additional term should include the construction of a theory of interaction of graviton with two or more quarks simultaneously. However, we avoid this difficult problem, choosing a simple “minimal” extension, based on the fact that the nonrelativistic limit of $G_{60}^{(0)}(s, t, s')$ is not singular. It is important to note that the proximity of relativistic and nonrelativistic descriptions of GFFs at $t \rightarrow 0$ is largely adopted [3]. In fact, in our minimal variant, the dynamical part of possible additional contribution, the analog of meson exchange currents in nuclear physics, remains out of scope.

The structure of the form factor $G_{60}^{(0)}(s, t, s')$, which is the analog of form factor D in the case of two free particles, is defined by the following relation, obtained using (10)–(12):

$$2t(A'_\mu A'^\mu) G_{60}^{(0)}(s, t, s') = A \left\{ \frac{1}{2} [g_{10}^{(u)}(t) + g_{10}^{(\bar{d})}(t)] [(\tilde{K}'_\mu \tilde{K}'^\mu)(A'_\mu A'^\mu) - (\tilde{K}'_\mu A'^\mu)^2] \cos(\omega_1 + \omega_2) + [g_{40}^{(u)}(t) + g_{40}^{(\bar{d})}(t)] (\tilde{K}'_\mu A'^\mu) (\tilde{R}_\mu A'^\mu) \sin(\omega_1 + \omega_2) + 2[g_{60}^{(u)}(t) + g_{60}^{(\bar{d})}(t)] t(A'_\mu A'^\mu) \cos(\omega_1 + \omega_2) \right\}, \quad (24)$$

where $\tilde{K}'_\mu, \tilde{R}_\mu$ —four-vectors in (12) appearing after integration and summation in (10), $A = A(s, t, s')$ —a multiplier defined by normalization of the Clebsch-Gordan coefficients in (10).

The first two terms on the rhs have purely relativistic origin and are zero in the nonrelativistic limit; they do not depend on the quark form factors $g_{60}^{(q)}$ from (12) (an analog of the quark form factor D) and exactly these terms contain the singularity. So, to advance in the simplest way, we require that the additional term in (23) has zero for the nonrelativistic limit and contributes only to the two first terms in (24), compensating the singularity. An important point, in our opinion, is the fact that the mentioned dangerous terms depend on the vectors of one-particle parametrization on the rhs of (10) and (12), while the additional form factor in (23), according to its meaning, contributes to the rhs of (10), which contains the one-particle currents. So, it seems natural that $G_{60}^{(a)}(s, t, s')$ deforms the terms with the vectors of parametrization of one-particle currents (13) and (24).

We define the additional term in (23) to compensate the main contribution of the relativistic series expansion of scalar products in the first two terms in (24). So, the divergent terms become closest to their nonrelativistic limit. The series expansion is carried out using as parameters the quantities $k/M, k'/M$, and $\sqrt{(-t)}/M$ [the variable k is

defined in (5)]. Thus, we choose for $G_{60}^{(a)}(s, t, s')$ the following relation:

$$\begin{aligned} 2(-t)(A'_\mu A'^\mu)G_{60}^{(a)}(s, t, s') \\ = A \left\{ \frac{1}{2} [g_{10}^{(u)}(t) + g_{10}^{(\bar{d})}(t)] [(\tilde{K}'_\mu \tilde{K}'^\mu)(A'_\mu A'^\mu) - (\tilde{K}'_\mu A'^\mu)^2] \right. \\ \left. - [(\tilde{K}'_\mu \tilde{K}'^\mu)(A'_\mu A'^\mu) - (\tilde{K}'_\mu A'^\mu)^2]_{lt} \right\} \cos(\omega_1 + \omega_2) \\ + [g_{40}^{(u)}(t) + g_{40}^{(\bar{d})}(t)] [(\tilde{K}'_\mu A'^\mu)(\tilde{R}_\mu A'^\mu) \\ - [(\tilde{K}'_\mu A'^\mu)(\tilde{R}_\mu A'^\mu)]_{lt} \sin(\omega_1 + \omega_2), \end{aligned} \quad (25)$$

where $[\dots]_{lt}$ are the main terms of the corresponding relativistic series.

Because of the relations (23)–(25), only the main terms of the relativistic expansion remain in diverging terms of (24) so making them finite. It is seen from Eqs. (23)–(25) that the chosen scheme for going beyond MIA does not contain arbitrariness. Taking into account the additional term contracted with the use of (23)–(25) leads to the following replacements in Eqs. (20) and (21):

$$\begin{aligned} (\tilde{K}'_\mu \tilde{K}'^\mu)(A'_\mu A'^\mu) - (\tilde{K}'_\mu A'^\mu)^2 \\ = (4M^2 - t)\lambda(s, t, s')/(-t) - (s + s' - t)^2 \\ \rightarrow 4M^2(-t), \end{aligned} \quad (26)$$

$$\begin{aligned} (\tilde{K}'_\mu A'^\mu)(\tilde{R}_\mu A'^\mu) \\ = -\xi(s, t, s')(s + s' - t) \rightarrow -8M^2\tilde{\xi}(s, t, s') \end{aligned} \quad (27)$$

$$\tilde{\xi}(s, t, s') = M\sqrt{[(s' - 4M^2) - \tilde{s}_1][\tilde{s}_2 - (s' - 4M^2)]}, \quad (28)$$

where

$$\begin{aligned} \tilde{s}_1 &= \left(\sqrt{s - 4M^2} - \sqrt{(-t)} \right)^2, \\ \tilde{s}_2 &= \left(\sqrt{s - 4M^2} + \sqrt{(-t)} \right)^2. \end{aligned} \quad (29)$$

After this procedure is carried out, the purely relativistic terms of (17) take the form

$$\begin{aligned} G_{6k0}^{(\pi)}(t) &= \int d\sqrt{s}d\sqrt{s'}\varphi(s)G_{6k0}^{(R)}(s, t, s')\varphi(s'), \\ k &= 1, 4. \end{aligned} \quad (30)$$

The new form factors $G_{6k0}^{(R)}$, $k = 1, 4$ are obtained using Eqs. (20), (21), and (26)–(29),

$$G_{610}^{(R)}(s, t, s') = \frac{1}{2}\tilde{R}(s, t, s')4M^2t\cos(\omega_1 + \omega_2), \quad (31)$$

$$G_{640}^{(R)}(s, t, s') = -\frac{M}{2}\tilde{R}(s, t, s')8M^2\tilde{\xi}(s, t, s')\sin(\omega_1 + \omega_2), \quad (32)$$

where $\tilde{\xi}(s, t, s')$ is defined by (28), and the functions $\tilde{R}(s, t, s')$ and $R(s, t, s')$ [in (18)–(22)] differ in cutting function,

$$\vartheta(s, t, s') \rightarrow \tilde{\vartheta}(s, Q^2, s') = \theta(s' - \tilde{s}_1) - \theta(s' - \tilde{s}_2). \quad (33)$$

So, to summarize, after the described minimal extension of MIA is carried out, we calculate the pion form factor $G_{60}^{(\pi)}$ using Eqs. (15), (17), (30), and (30)–(32).

III. THE MODEL OF QUARK GRAVITATIONAL STRUCTURE AND DETAILS OF CALCULATION

In [9] we used the general method of the relativistic-invariant parametrization of the matrix elements of the local operators established in Ref. [7] for systems with arbitrary spin. Although now we consider actually the pion, we preserve for reasons of convenience the notations of the preceding paper. The pion GFFs in this parametrization are connected with the commonly used (see, e.g., [3]) following relations:

$$A^{(\pi)}(t) = G_{10}^{(\pi)}(t), \quad D^{(\pi)}(t) = -2G_{60}^{(\pi)}(t), \quad (34)$$

where $t = (p_\pi - p'_\pi)^2$, and p'_π, p_π are the pion 4-momenta in the initial and the final states, respectively. $G_{10}^{(\pi)}$ and $G_{60}^{(\pi)}$ are given by Eqs. (14)–(19), (22), (30)–(32).

The quark GFFs in our approach are connected with GFFs that are commonly used for particles with spin 1/2 (see, e.g., [3]) by the following relations [9]:

$$g_{10}^{(q)}(t) = \frac{1}{\sqrt{1-t/4M^2}} \left[\left(1 - \frac{t}{4M^2} \right) A^{(q)}(t) + 2\frac{t}{4M^2} J^{(q)}(t) \right], \quad (35)$$

$$g_{40}^{(q)}(t) = -\frac{1}{M^2} \frac{J^{(q)}(t)}{\sqrt{1-t/4M^2}}, \quad (36)$$

$$g_{60}^{(q)}(t) = -\frac{1}{2} \sqrt{1 - \frac{t}{4M^2}} D^{(q)}(t). \quad (37)$$

We assume that the GFFs of u and $\bar{d}$ quarks are equal:

$$g_{i0}^{(u)}(t) = g_{i0}^{(\bar{d})}(t), \quad i = 1, 4, 6.$$

To define the explicit form of the quark GFFs, we recall our calculation of the electroweak structure of the pion. We derived the functional form of the electromagnetic form factor of the quark from the behavior of our charge form factor of pion at $(-t) \rightarrow \infty$ [32], which turned out to coincide with that of QCD, as mentioned above,

$$f_q(t) = \frac{1}{1 + \ln(1 - \langle r_q^2 \rangle t/6)}, \quad (38)$$

where $\langle r_q^2 \rangle$ is a mean-square charge radius of the constituent quark. The charge form factor of the constituent coincides with the function (38) and the magnetic form factor is equal to this function multiplied by the corresponding magnetic moment [32]. Now we admit a similar definition for the quark GFFs (35)–(37) in terms of (38),

$$\begin{aligned} A^{(q)}(t) &= f_q(t), & J^{(q)}(t) &= \frac{1}{2}f_q(t), \\ D^{(q)}(t) &= D_q f_q(t), \end{aligned} \quad (39)$$

where D_q is the D -term of the constituent quark. These equations give standard static limits (see, e.g., [3]),

$$A^{(q)}(0) = 1, \quad J^{(q)}(0) = \frac{1}{2}, \quad D^{(q)}(0) = D_q. \quad (40)$$

We set the parameter $\langle r_q^2 \rangle$ in (38) to be equal to the mass mean square radius (MSR) of the quark and to define the slope of the quark form factor D at zero. The corresponding actual value is given by the following form [52–55]:

$$\langle r_q^2 \rangle \simeq \frac{0.3}{M^2}. \quad (41)$$

For the calculation of the pion GFFs according to (14)–(17), (30), and (34) we use in (5) consequently one of the following model wave functions:

$$u(k) = 2(1/(\sqrt{\pi}b^3))^{1/2} \exp(-k^2/(2b^2)), \quad (42)$$

$$u(k) = 16(2/(7\pi b^3))^{1/2}(1 + k^2/b^2)^{-3}, \quad (43)$$

$$u(k) = 4(2/(\pi b^3))^{1/2}(1 + k^2/b^2)^{-2}. \quad (44)$$

We use the parameter b in (42)–(44) fixed previously in the works [9,32,56] on the electroweak properties of the pion. The different types of functions correspond to the different types of confinement. So, in the model (42), quadratic confinement is carried out; the model (43), as our calculations of the electroweak properties of a pion in the work [32] show, gives predictions very close to the model with linear confinement [57], and finally, the wave function (44) corresponds to a confinement weaker than a linear one. We calculate the pion GFFs using the same value of the constituent-quark mass $M = 0.22$ GeV, as in our previous works (see, e.g., [32]).

So, now only one free parameter remains—the D -term of the constituent quark D_q in the form factor [(39) and (40)]. In Sec. IV we fix this free parameter and present the gravitational properties of the pion calculated taking into account the constituent-quark gravitational structure.

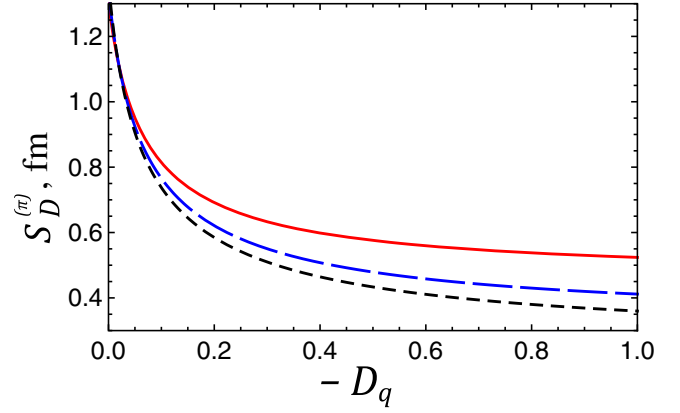


FIG. 1. The slope at zero of the normalized to pion D -term form factor D of pion as a function of the D -term of the constituent quark D_q calculated with various model wave functions (42)–(44). Full line (red), with wave function (42); dashed line (blue), with (43); short-dashed line (black), with (44).

IV. THE CALCULATION OF THE PION GFFs AND THEIR STATIC MOMENTS

To calculate numerically the pion GFFs we need, first of all, to fix the remaining free parameter D_q which enters the pion form factor D . Namely, we use the static moments of $D^{(\pi)}(t)$. For the time being, there is no strict experimental estimation of the value $D^{(\pi)}(0)$ [3]. However, the first data for the slope at zero of the normalized to pion D -term form factor D of pion extracted from the process $\gamma^*\gamma \rightarrow \pi^0\pi^0$ were presented in Ref. [44],

$$S_D^{(\pi)} = (0.82 - 0.88) \text{ fm}. \quad (45)$$

We use this estimation to fix D_q taking into account the following definition [3]:

$$(S_D^{(\pi)})^2 = \frac{6}{D^{(\pi)}(t)} \frac{dD^{(\pi)}}{dt} \Big|_{t=0}. \quad (46)$$

As all but one parameter of the model were fixed previously in the works on electroweak structure of the pion, then $S_D^{(\pi)}$ (46) is a function of the D -term of the constituent quark D_q only. This function is given in Fig. 1 for three types of model quark interaction in pion (42)–(44).

The results presented in the figure show that for each of the model wave functions there is an interval of the values of D_q which gives the interval (45) of the values of $S_D^{(\pi)}$. Moreover, there exists the interval of the variable D_q , for which $S_D^{(\pi)}$ (46) falls in the interval (45) for all the wave functions (42)–(44), without any additional variation of the parameters. This interval is

$$D_q = -(0.0715 - 0.0709). \quad (47)$$

TABLE I. The results of calculating the static gravitational moments of the pion with different model wave functions (42)–(44). The mass of constituent quarks is $M = 0.22$ GeV, their MSR is given in (41), and the D -term D_q is taken from (47). The deviation from the given values of $S_D^{(\pi)}$ when the parameter D_q is varied in the interval (47) is about 0.1%.

Model	(42)	(43)	(44)
b , GeV	0.3500	0.6131	0.4060
$A^{(\pi)'}(0)$, GeV $^{-2}$	1.074	1.134	1.217
$\langle r^2 \rangle_{\text{mass}}^{1/2}$, fm	0.50	0.52	0.53
$-D^{(\pi)}(0)$	0.704–0.700	0.653–0.649	0.620–0.617
$-D^{(\pi)'}(0)$, GeV $^{-2}$	2.331–2.327	1.978–1.976	1.773–1.771
$S_D^{(\pi)}$, fm	0.88	0.84	0.82
$\langle r^2 \rangle_{\text{mech}}^{1/2}$, fm	0.162–0.163	0.153–0.154	0.133–0.134

Note that this interval is rather narrow ($< 0.5\%$). The existence of this interval recalls the fact that when describing the electroweak properties of pion in RQM we obtained the same values of the physical characteristics of the constituent quark for all model wave functions [32]. Now we add the quark D -term to the previous set (the mass, the charge MSR, and the anomalous magnetic moment).

Now, with (47), all the parameters are fixed and we do calculate the pion GFFs. Note that the pion form factor A , [(34) and (14)] does not depend on D_q and is determined by the parameters obtained previously; so, it is predicted. The standard condition $A^{(\pi)}(0) = 1$ is fulfilled automatically for arbitrary values of parameters, if the quark form factors satisfy Eq. (40).

The results of calculation of the static gravitational moments of the pion are presented in Table I. The mass MSR is defined following the relation [3]

$$\langle r^2 \rangle_{\text{mass}} = 6 \frac{1}{A^{(\pi)}(t)} \frac{dA^{(\pi)}}{dt} \Big|_{t=0}. \quad (48)$$

In the last line of the table, we present the results for the pion mechanical radius using the standard expression

$$\langle r^2 \rangle_{\text{mech}} = 6 \frac{D^{(\pi)}(0)}{\int_{-\infty}^0 D^{(\pi)}(t') dt'}. \quad (49)$$

For the calculation, it is important to have the form factor D in the large range of momentum transfer including its asymptotics. This is a complicated problem, which is out of the scope of this paper. It will be considered elsewhere. A thorough correct full-fledged approach can be realized in the spirit Refs. [37–40]. The GFF obtained in the present paper predictably gives underestimated values of the mechanical radius as compared with mass and electromagnetic radii. We believe that the detailed calculation that is in progress will give the ultimate truth.

Note that the deviations of the values $S_D^{(\pi)}$ from that given in Table I when the parameter D_q is changed in the interval (47) is approximately 0.1%.

Let us make some remarks about our values of the pion D -term. As is well known, in theories with broken chiral symmetry (see, e.g., [58,59]) the pion D -term is equal to -1 . However, if the EMT contains only the contribution of quarks, without taking gluons into account, then its value is approximately $\simeq -0.75$, obtained in Ref. [44] in agreement with the soft pion theorem $D = -1$, given that the extracted value does not include the gluon contribution [3,60].

Our values given in Table I are close to this value. The same is true about the comparison of our slope at zero of the pion D -form factor with that of chiral theory, where $-D^{(\pi)'}(0) = 2.40$ GeV $^{-2}$. The results for pion GFFs satisfy the well-known relation (see, e.g., [3]) $-D^{(\pi)'}(0) > A^{(\pi)'}(0)$. In addition, our values of $A^{(\pi)'}(0)$ are close to the estimations given, for example, in the same review,

$$A^{(\pi)'}(0) = (1.33 - 2.02) \text{ GeV}^{-2}. \quad (50)$$

Let us compare the mass and charge MSRs of the pion. The experimental pion charge MSR is (see, e.g., [44])

$$\sqrt{\langle r^2 \rangle_{\text{ch}}} = 0.672 \pm 0.008 \text{ fm}. \quad (51)$$

The mass MSR was estimated in [44] and the following interval of the values was obtained:

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = (0.32 - 0.39) \text{ fm}. \quad (52)$$

Our result for the mass radius (see Table I) lays much closer to the charge radius (51) than (52). Curiously enough, our difference between the mass and the charge radii is the same as obtained in the work [18] for the proton ($\Delta R_{\text{cm}} = 0.1709 \pm 0.0304$ fm). We recall that our result for $\langle r^2 \rangle_{\text{mass}}^{1/2}$ (48) is a direct consequence of our model of electroweak properties of the pion without new fittings.

The results of calculation of the pion form factors A and D at low and intermediate momentum transfers for the wave functions (42)–(44) and the quark structure (35)–(41) are presented in Figs. 2 and 3. For comparison, the results for GFFs as extracted [44] from the experimental data are given, too. Note that the slope of our curves is close to that of the experimental one at large Q^2 . The results for the lower and higher bounds of D_q from (47) are presented, but in fact cannot be separated from one another in the figures.

The pion GFFs, shown in Figs. 2 and 3, calculated for the values of the quark D -term D_q in the interval (47), form a rather narrow bunch corresponding to the quark mass $M = 0.22$ GeV. The curves remain agglomerated in the bunch up to squares of momentum transfer ~ 10 GeV 2 . The schema of bunches works anew.

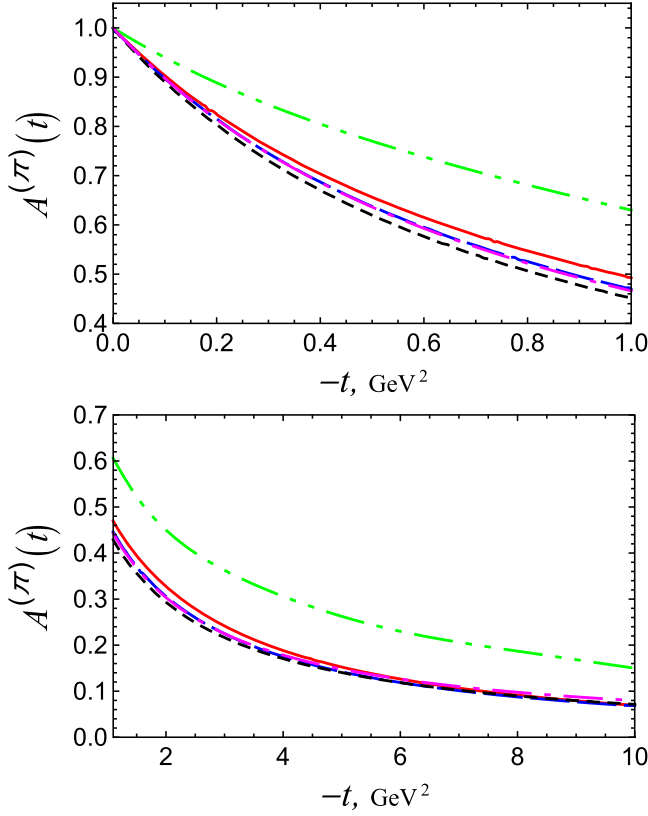


FIG. 2. The pion form factor A calculated for the quark structure defined by (35)–(41) and different model wave functions (42)–(44) with b given in Table I, $M = 0.22$ GeV. Full line (red), with wave function (42); dashed line (blue), with (43); short-dashed line (black), with (44); dot-dashed line (magenta), the results for the fitting (53); double-dot-dashed line (green) presents the result of the extraction of the form factor A given in Ref. [44].

As the dependence of the pion GFFs on the choice of model wave functions (42)–(44) is weak, it seems to us reasonable to construct an averaged fit for these form factors. We take the fitting function in the following form, which is used, for example in [61]:

$$F(t) = \frac{F(0)}{(1 - t/a)^n}. \quad (53)$$

Least-squares fitting gives the following values of parameters. For the form factor $A^{(\pi)}$, we obtain $F(0) = 1$, $a = 0.882$ GeV², and $n = 1.007$, and for the form factor $D^{(\pi)}$, we obtain $F(0) = -0.657$, $a = 0.0949$ GeV², and $n = 0.293$. The value of the parameter $F(0)$ for $D^{(\pi)}$ is equal to the average over models value of the pion D -term (see Table I) and the slopes of the form factors A and D at zero—to the corresponding averaged slopes. Obviously, there is no such correspondence for the pion mechanical radius [see the remarks above, after (49)]. The results of the fitting are shown in Figs. 2 and 3.

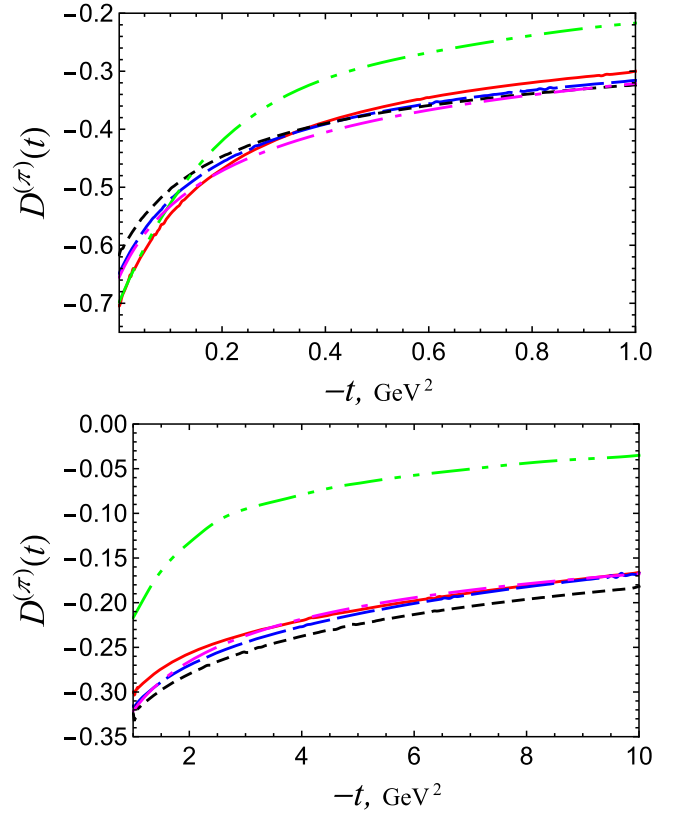


FIG. 3. The pion form factor D [(34), (15), (17), (30)] calculated for the quark structure defined by (35)–(41) and different model wave functions (42)–(44) with b given in Table I, $M = 0.22$ GeV. Two bound values of D_q from the interval (47) were used for the calculation, but the obtained curves are inseparable. Full line (red), with wave function (42); dashed line (blue), with (43); short-dashed line (black), with (44); dot-dashed line (magenta), the results for the fitting (53); double-dot-dashed line (green) presents the result of the extraction of the form factor D given in Ref. [44].

V. CONCLUSIONS

In this work, we extend our relativistic theory of composite-particle systems that describe simultaneously their electroweak and gravitational properties to calculate the gravitational characteristics of the pion. The approach is based on a version of the instant-form relativistic quantum mechanics. In the preceding work [9], we formulated the mathematical base and described the principal schema of deriving the gravitational characteristics in the approach. In the present work, we give a detailed technique of calculation, thus transforming the approach into a quantitative method. The extension includes three main points: the constituent quarks are no longer pointlike, we analyze different types of the quark interaction in the pion, and we formulate a minimal way to overstep the frame of our MIA for correct description of the pion form factor D .

We derive the equations for the pion GFFs, the mass radius, and the slope at zero of the normalized to

pion D -term form factor D of pion in terms of parameters describing the quark structure. Most of the parameters have been fixed even earlier in our works on the pion electromagnetic form factors. The only free parameter is the D -term of the constituent quark, which we fix by fitting the result for the slope at zero of the normalized to pion D -term form factor D of pion to a chosen experimental value [44]. The results for the form factor A and for the mass MSR of the pion do not depend on this new parameter and so are direct predictions of our approach. As a whole, the obtained results for the gravitational characteristics of the pion can be considered as firm predictions if calculated with a new future value of the mentioned slope.

Using these parameters, we calculate the pion GFFs, their derivatives, and static moments. We present the pion GFFs in the range of low and intermediate momentum transfers up to $\sim 10 \text{ GeV}^2$. At large Q^2 the slope of our curves is close to that of [44]. The curves for different model quark-antiquark wave functions are agglomerated to form bunches corresponding to the quark mass $M = 0.22 \text{ GeV}$, as it is in the electroweak case. The fits of the calculated pion GFFs, averaged over and

obtained with three different model wave functions, are constructed. The values of the static moments, mass MSR, and the D -term of the pion are consistent with the values given in the literature.

It seems obvious that, because of extreme weakness of the gravitational interaction at hadron scale, the information on GFFs presently can be extracted from electroweak processes only. So, we believe that our theory, which describes electroweak and gravitational properties simultaneously, based on the unique foundations and which exploits common model parameters, will be useful. Moreover, it is possible that our unified approach can construct another bridge between electroweak and gravitational properties of composite systems, complementary to GPD.

ACKNOWLEDGMENTS

The work was supported in part by the Ministry of Science and Higher Education of the Russian Federation (Grant No. 0778-2020-0005) (A. F. K.).

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