Consistent approach to study gluon quasiparticles

Chowdhury Aminul Islam [*]
School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China

Munshi G. Mustafa [†]
Theory Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata 700064, India and Homi Bhabha National Institute, Anushaktinagar, Mumbai, Maharashtra 400094, India

Rajarshi Ray [‡] and Pracheta Singha [§]
Center for Astroparticle Physics & Space Science, Bose Institute, Block-EN, Sector-V, Salt Lake, Kolkata-700091, India and Department of Physics, Bose Institute, 93/1, A. P. C Road, Kolkata - 700009, India

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We discuss a novel approach to estimate the partition function in effective model frameworks when the effective potentials have multiple extrema, so that ascertaining a mean field becomes difficult. Using this approach we present a consistent model to study the thermodynamic properties of gluon quasiparticles as a function of temperature, both in the color confined and the color deconfined phases.

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I. INTRODUCTION

In the strong coupling regime the thermal physics of strongly interacting matter is best described by quantum chromodynamics (QCD) on space-time lattices [1–11]. Deconfinement of quarks and gluons and the chiral symmetry restoration at crossover temperatures $T_c \sim 150$ MeV is now well documented [12–14]. The deconfinement transition in a pure glue system is however found to be of first-order at temperatures $T_d \sim 270$ MeV [15–19]. The thermal average of the Polyakov loop in the fundamental representation $\hat{L}_F$ gives the static quark free energy and is considered as the order parameter [16,17]. Polyakov loop in the deconfined phase also breaks spontaneously the symmetry of the gluon action under Z(3) twists on the gluon fields at the temporal boundary. Similarly, the Polyakov loop in the adjoint representation $\hat{L}_A$ is related to the free energy of a static adjoint color source [20–23]. But it is always invariant under the Z(3) twists of the gluon fields at the physical boundary.

[*]chowdhury.aminulislam@ucas.ac.cn
[†]munshigolam.mustafa@saha.ac.in
[‡]rajarshi@jcbose.ac.in
[§]pracheta.singha@gmail.com

Significant efforts have been put in for building effective models for a spontaneous Z(3) symmetry breaking, with $\hat{L}_F$ as order parameter, using Landau type of polynomial potentials [24–29]. Further models have been built [30–37] by introducing effective $\hat{L}_F$ fields in lieu of background temporal gluon fields in the chiral models like Nambu-Jona-Lasinio (NJL) model [38–43] or chiral sigma models [44–48]. These Polyakov loop enhanced chiral models give simple but insightful description of thermodynamics of strong interactions [49–69,69–82]. In these models the gluon pressure is obtained from the polynomial thermodynamic potential in $\mathrm{Tr}\,\hat{L}_F$. But a more natural alternative seems to be in terms of gluon quasiparticles [83–87], including modifications due to the background $\hat{L}_A$ [88–91]. Here $\hat{L}_A$ is expected to induce statistical confinement of gluons in a similar way as $\hat{L}_F$ does for quarks in the Polyakov enhanced chiral models. But unfortunately the modified statistics result in a negative gluonic pressure below T_d. Various authors have argued for additional terms to preserve overall positivity. But the quasiparticle pressure itself still remains negative. This lacunae may have slowed down further progress in this direction.

Here we argue that the issue lies with the method of obtaining the statistics. Usually the saddle point approximation is employed to obtain the mean value of the Polyakov loop, which is then put back to obtain the thermodynamic potential. We propose a new prescription for obtaining the thermal averages and thermodynamic observables that can solve the issue and reliably predict various observables both below and above T_d.

II. FORMALISM

A. Standard approach

The Polyakov loop in the effective models is written in terms of the background temporal gluon field $\mathcal{A}_0$, in the color diagonal form as

$$\hat{L}_F \sim \exp[i(\mathcal{A}_0^3 T^3 + \mathcal{A}_0^8 T^8)/T]. \tag{1}$$

Here T_3 and T_8 are the diagonal generators of SU(3). Accordingly, in terms of the class parameters θ_1 and θ_2 we have

$$\hat{L}_F = \mathrm{diag}(e^{i\theta_1}, e^{i\theta_2}, e^{-i(\theta_1+\theta_2)}). \tag{2}$$

The normalized character and its complex conjugate are then given as

$$\Phi = \frac{1}{3}\mathrm{Tr}\,\hat{L}_F; \qquad \bar{\Phi} = \frac{1}{3}\mathrm{Tr}\,\hat{L}_F^\dagger. \tag{3}$$

In general for SU(N_c) the group invariant Haar measure $d\mu$ may be expressed in terms of the distribution of eigenvalues as

$$\int d\mu = \frac{1}{N_c!}\left(\prod_{i=1}^{N_c}\int_0^{2\pi}\frac{d\theta_i}{2\pi}\right)\delta\left(\sum_i \theta_i\right)\prod_{i<j}|e^{i\theta_i} - e^{i\theta_j}|^2 = 1. \tag{4}$$

For SU(3) the corresponding Haar measure is given by

$$\frac{1}{3!}\int_0^{2\pi}\int_0^{2\pi}\frac{d\theta_1}{2\pi}\frac{d\theta_2}{2\pi}\mathrm{Det}_{\mathrm{VdM}}[\theta_1,\theta_2] = 1, \tag{5}$$

where the Vandermonde determinant $\mathrm{Det}_{\mathrm{VdM}}$ is given as [36,88,90]

$$\begin{aligned}\mathrm{Det}_{\mathrm{VdM}} &= 64\sin^2\frac{(\theta_1-\theta_2)}{2}\sin^2\frac{(2\theta_1+\theta_2)}{2}\sin^2\frac{(\theta_1+2\theta_2)}{2} \\ &= 27[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3+\Phi^3) - 3(\bar{\Phi}\Phi)^2].\end{aligned} \tag{6}$$

Correspondingly, in both polynomial and quasiparticle potentials, a Vandermonde term can be included. Thus the n^{th} order polynomial potential becomes

$$\Omega'_{\mathrm{poly}} = [\Omega_{\mathrm{poly}}(\alpha_{i=1\cdots n}(T),\Phi,\bar{\Phi}) + \kappa\ \ln\ [\mathrm{Det}_{\mathrm{VdM}}]]T^4, \tag{7}$$

where α_i, κ are model parameters. The quasiparticle potential becomes

$$\Omega'_{gqp} = \Omega_{gqp} + \kappa\ \ln\ [\mathrm{Det}_{\mathrm{VdM}}]\,T^4, \tag{8}$$

where

$$\begin{aligned}\Omega_{gqp} &= = 2T\int\frac{d^3p}{(2\pi)^3}\ln\ \det\left(1 - \hat{L}_A e^{-\frac{|\vec{p}|}{T}}\right) \\ &= 2T\int\frac{d^3p}{(2\pi)^3}\ln\left(1 + \sum_{n=1}^{8}a_n e^{-\frac{n|\vec{p}|}{T}}\right).\end{aligned} \tag{9}$$

The coefficients a_n for $n = 1\ldots 8$, are

$$\begin{aligned}&a_8 = 1; \quad a_1 = a_7 = 1 - 9\bar{\Phi}\Phi \\ &a_2 = a_6 = 1 - 27\bar{\Phi}\Phi + 27(\bar{\Phi}^3 + \Phi^3) \\ &a_3 = a_5 = -2 + 27\bar{\Phi}\Phi - 81(\bar{\Phi}\Phi)^2 \\ &a_4 = 2[-1 + 9\bar{\Phi}\Phi - 27(\bar{\Phi}^3+\Phi^3) + 81(\bar{\Phi}\Phi)^2].\end{aligned} \tag{10}$$

The adjoint Polyakov loop is given as

$$\begin{aligned}\hat{L}_A = \mathrm{diag}(&1, 1, e^{i(\theta_1-\theta_2)}, e^{-i(\theta_1-\theta_2)}, e^{i(2\theta_1+\theta_2)}, \\ &e^{-i(2\theta_1+\theta_2)}, e^{i(\theta_1+2\theta_2)}, e^{-i(\theta_1+2\theta_2)}),\end{aligned} \tag{11}$$

with the corresponding normalized character,

$$\Phi_A = \frac{1}{N_c^2-1}\mathrm{Tr}\,\hat{L}_A = \frac{1}{8}(9\Phi\bar{\Phi} - 1). \tag{12}$$

Given Ω' from either Eq. (7) or Eq. (8), one can solve for

$$\frac{\partial\Omega'}{\partial\Phi} = 0; \qquad \frac{\partial\Omega'}{\partial\bar{\Phi}} = 0, \tag{13}$$

obtaining the saddle point estimate for the mean fields Φ_{mf} and $\bar{\Phi}_{mf}$, and the mean thermodynamic potential $\Omega' = \Omega'(\Phi_{mf},\bar{\Phi}_{mf})$.

In the quasiparticle picture this mean field approach gives satisfactory results for $T > T_d$ [88–91]. Below T_d $\Phi_{mf} = \bar{\Phi}_{mf} = 0$, and the thermodynamic potential becomes

$$\begin{aligned}\Omega_{gqp}(\Phi_{mf},\bar{\Phi}_{mf}\to 0) = 2T\int\frac{d^3p}{(2\pi)^3}[&\ln(1 - e^{(-\frac{3|\vec{p}|}{T})})^2 \\ &+ \ln(1 - e^{(-\frac{|\vec{p}|}{T}+\frac{2\pi i}{3})}) \\ &+ \ln(1 - e^{(-\frac{|\vec{p}|}{T}-\frac{2\pi i}{3})})].\end{aligned} \tag{14}$$

The last two terms are positive, resulting in an overall temperature dependent negative pressure for $T < T_d$.

In Ref. [88] the authors proposed a hybrid approach including glueballs implemented as dilaton fields, resulting in an *overall* positive pressure below T_d. In Ref. [89], an additional pure matrix interaction term was used along with Ω'_{gqp}, and a Weiss mean field analysis was done. In a related Weiss averaging procedure [56], a model parametrization was invoked similar to the polynomial potential. In Ref. [91], a bag term was introduced along with Ω'_{gqp} and a thermodynamic consistency analysis was done. Truly each of these additional terms may have significant physical inputs. But the negativity of gluon quasiparticle pressure below T_d remains unaddressed.

B. Alternate approach

Here we propose to use a matrix model for the background Polyakov loop. We begin with the corresponding partition function given as

$$Z_{\mathrm{PL}} = \int \mathcal{D}\theta_1 \mathcal{D}\theta_2 \exp\left[-\frac{1}{T}\int d^3x \Omega_{gqp}[\theta_1(\mathbf{x}), \theta_2(\mathbf{x})]\right]$$
$$= \int \prod_{\mathbf{x}} \frac{1}{24\pi^2} d\theta_1(\mathbf{x}) d\theta_2(\mathbf{x}) \mathrm{Det}_{\mathrm{VdM}}$$
$$\times \exp\left[-\frac{1}{T}\int d^3x \Omega_{gqp}[\theta_1(\mathbf{x}), \theta_2(\mathbf{x})]\right], \quad (15)$$

where Ω_{gqp} is given in Eq. (9). Additional terms as in Refs. [88,89,91] could be introduced but are not relevant for the physics discussed here. As we shall see that in our approach, even this simple Z_{PL} is sufficient to describe the pure gauge lattice field theory data quite satisfactorily.

The Polyakov loop is an oscillating function of θ_1 and θ_2, and so will be the thermodynamic potential. Hence the configurations away from the saddle point may have a significant measure. In fact below T_d, where $\langle\Phi\rangle = 0$, configurations with $|\Phi| \sim 1/3$ is most preferred [90]. Here instead, we compute the partition function Z_{PL}, by numerically integrating over all the finite periodic interval of the θ_1 and θ_2 fields. The difficulty is with taking the $V \to \infty$ limit, and this is why the saddle point analysis is the usual choice. However a simplification arises by noting that the effective action contains no derivatives of the θ_1 and θ_2 fields. Therefore the configuration space can be split up into $N \to \infty$ independent and equivalent points, such that the partition function becomes

$$Z_{\mathrm{PL}} = z_{\mathrm{PL}}^N, \quad (16)$$

where

$$z_{\mathrm{PL}} = \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \mathrm{Det}_{\mathrm{VdM}} \exp\left[-\frac{v}{T}\Omega_{gqp}[\theta_1, \theta_2]\right], \quad (17)$$

and v is a parameter with the dimension of volume. This is the only free parameter and may be suitably related with T_d, the only physical scale in the finite temperature SU(3) pure gauge field theory. We assume $v = (\beta_1/T_d)^3$, where β_1 is some constant. We further scale out the momentum variable as $|\tilde{\vec{p}}| = |\vec{p}|/T$, whereby the partition function may be expressed in terms of the scaled temperature T/T_d as

$$z = \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \mathrm{Det}_{\mathrm{VdM}}$$
$$\times \exp\left[-2\left(\frac{\beta_1 T}{T_d}\right)^3 \int \frac{d^3\tilde{p}}{(2\pi)^3} \ln\left(1 + \sum_{n=1}^{8} a_n e^{-n|\tilde{\vec{p}}|}\right)\right]. \quad (18)$$

The scaled pressure can be expressed in terms of the scaled temperature as

$$p/T^4 = \left(\frac{T}{V}\ln Z_{\mathrm{PL}}\right)/T^4 = \frac{1}{Nv} N \ln z_{\mathrm{PL}}/T^3$$
$$= \ln z_{\mathrm{PL}}/(\beta_1 T/T_d)^3. \quad (19)$$

III. RESULTS

A. Parameter fitting

The variation of p/T^4 with T/T_d obtained from Eq. (19) is completely consistent throughout the range of temperatures (Fig. 1). We chose $\beta_1 \sim 2$. However for quantitative agreement with data from lattice SU(3) field theory, we consider a temperature dependent effective mass $[m_g(T)]$ for the gluon quasiparticles. Effects of such constant mass was studied in [88], while a temperature dependent ansatz was used in [89], following the studies in [92]. We substitute $|\tilde{\vec{p}}|$ with $\tilde{E}_g = \sqrt{|\tilde{\vec{p}}|^2 + \tilde{m}_g(T)^2}$ in Eq. (18), where $\tilde{m}_g(T) = m_g(T)/T$. The lattice data [93] for pressure were solved for the scaled masses from the pressure equation $p/T^4|_{\mathrm{model}} = p/T^4|_{\mathrm{lattice}}$ to an accuracy of 10^{-6} or better. The momentum rescaling is undefined for $T \to 0$. Also the numerical uncertainties were insignificant only above $T/T_d \sim 0.45$. Therefore the partition function at a given T/T_d was normalized with the one at $T/T_d = 0.45$.

The extracted scaled masses $\tilde{m}_g(T) = m_g(T)/T$ are shown in Fig. 2 as data points. The functional dependence

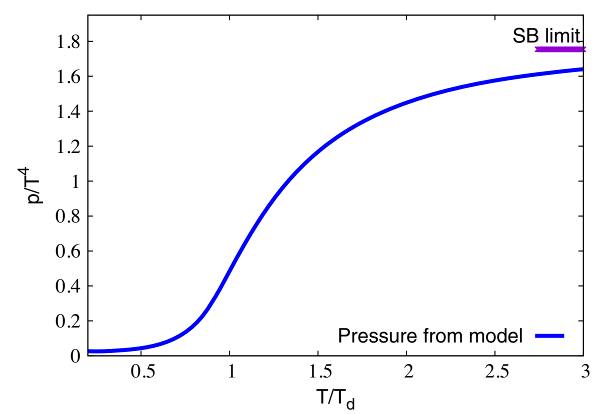

FIG. 1. Scaled pressure of gluon quasiparticles as function of scaled temperature.

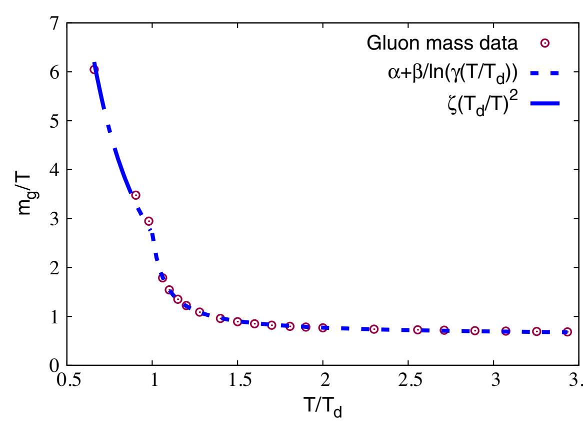

FIG. 2. Scaled mass of gluon quasiparticles as function of scaled temperature.

TABLE I. Model parameters.

Fitted				Chosen
α	β	γ	ζ	$v(\mathrm{GeV}^{-3})$
0.548 ± 0.006	0.174 ± 0.005	1.083 ± 0.004	2.70 ± 0.07	$(0.5T_d)^{-3}$

of $\tilde{m}_g(T)$ has an abrupt change close to $T/T_d = 1$, and may have a functional form,

$$m_g(T)/T = \alpha + \beta/\ln(\gamma T/T_d), \quad \text{for } T/T_d > 1 \tag{20}$$

$$= \zeta(T_d/T)^2, \quad \text{for } T/T_d < 1. \tag{21}$$

With T_d arbitrary and assuming $v = (2/T_d)^3$, the functional fit is shown in Fig. 2. The parameters α, β and γ are obtained by least square fit using the "gnuplot" software and summarized in Table I.

Given the mass parametrization a comparative plot of the scaled pressure obtained in our model *vis-à -vis* the lattice QCD data are shown in Fig. 3.

B. Thermodynamic observables

Given p/T^4 as a function of T/T_d, other thermodynamic observables like entropy density (s), energy density (ϵ), specific heat (c_V) and speed of sound (v_s) may be obtained as

$$s/T^3 = \frac{1}{T^3}\frac{\partial p}{\partial T} = \frac{1}{(T/T_d)^3}\frac{\partial[(p/T^4)(T/T_d)^4]}{\partial(T/T_d)}, \tag{22}$$

$$\epsilon/T^4 = s/T^3 - p/T^4, \tag{23}$$

$$c_V/T^3 = \frac{1}{T^3}\frac{\partial \epsilon}{\partial T} = \frac{1}{(T/T_d)^3}\frac{\partial[(\epsilon/T^4)(T/T_d)^4]}{\partial(T/T_d)}, \tag{24}$$

$$v_s^2 = \frac{\partial p}{\partial \epsilon} = \frac{\partial p}{\partial T}\Big/\frac{\partial \epsilon}{\partial T} = \frac{s/T^3}{c_V/T^3}. \tag{25}$$

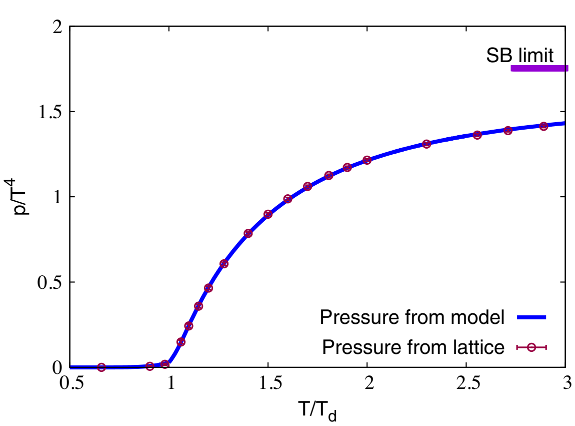

FIG. 3. Temperature variation of scaled pressure (lattice data from [93]).

The scaled interaction measure [Fig. 4(a)] is expected to capture the deviation of thermal system from that of a relativistic noninteracting gas of gluons. However for $T \ll T_d$ due to the heavy effective mass of the gluon quasiparticles as well as the confinementlike interactions both p/T^4 and ϵ/T^4 are insignificant, and so is the interaction measure. With increasing T/T_d, both m_g/T and the confinement effect decrease, thereby increasing the interaction measure. A turnover occurs for $T/T_d > 1$ inside the gluonic phase where the measure gradually decreases towards relativistic ideal gas limit.

A more direct observable for transition from the nonrelativistic confined phase to the relativistic gluonic phase is the conformal measure $(\epsilon - 3p)/\epsilon$, which varies from 1 to 0 between the two phases respectively [Fig. 4(b)]. This behavior follows the general trend of m_g/T. At $T/T_d = 1$ there is a sudden gap arising out of the sudden changes in m_g/T and the deconfining effects.

For a conformal theory in d dimensions, $\epsilon = d.p$ and $c_V/T^3 = (1+d)\epsilon/T^4$. In Fig. 5(a) we show a direct comparison of scaled specific heat with $\frac{4\epsilon}{T^4}$. The gap in the ϵ/T^4 gives an estimate of the latent heat of transition.

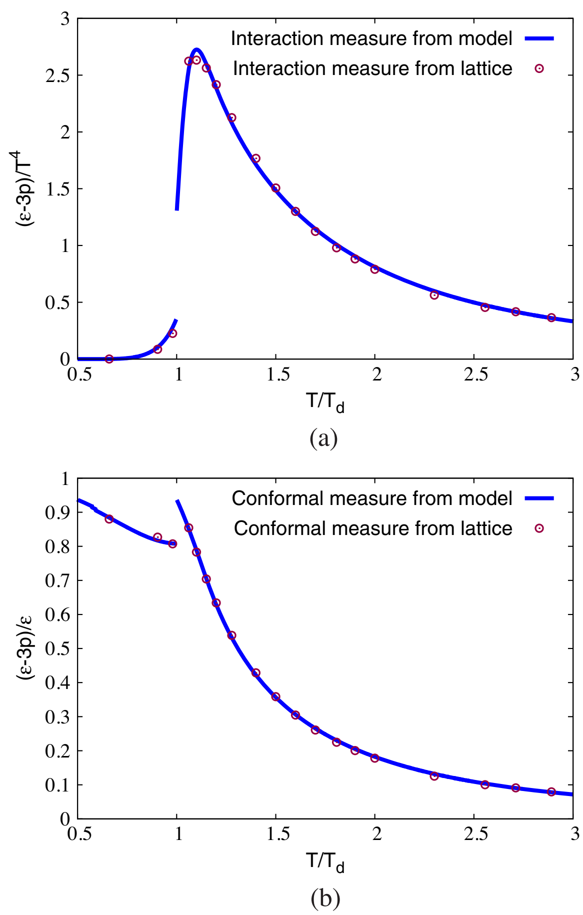

FIG. 4. Temperature variation of (a) scaled interaction measure and (b) scaled conformal measure (lattice data from [93]).

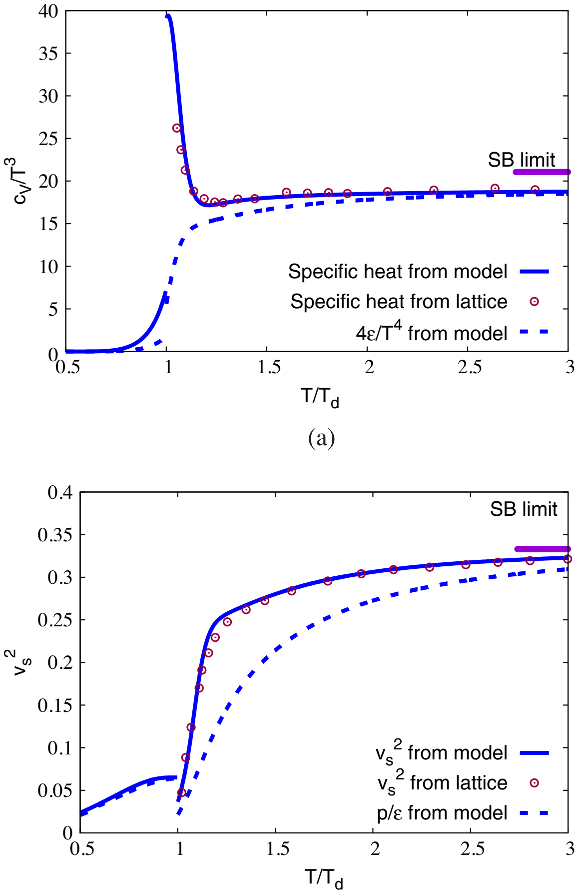

FIG. 5. Temperature variation of (a) scaled specific heat and (b) squared speed of sound (lattice data from [1]).

The scaled specific heat is both discontinuous and divergent right at $T/T_d = 1$. The agreement with lattice data could be ascertained only for $T/T_d > 1$ from the measurements reported in Ref. [1].

Finally we present the behavior of the squared speed of sound (v_s^2) which is supposed to be an important transport coefficient determining the hydrodynamic evolution in the heavy-ion collisions [94]. In the conformal limit $v_s^2 = p/\epsilon = 1/3$. A comparison of our estimation for the speed of sound along with the ratio p/ϵ and measurements on the lattice [1] is shown in Fig. 5(b). Note that p/ϵ is 3 times the additive inverse of the conformal measure. Reflection of such a variation is seen in the $v_s^2 - T$ curve. The softest equation of state is supposed to be at $T/T_d = 1$, where v_s^2 drops towards zero.

Thus our model results for various sensitive thermodynamic observables are in excellent numerical agreement with the lattice data. In fact the parametrizations obtained by fitting data from Ref. [93] (Fig. 4), makes excellent predictions for the data from Ref. [1] (Fig. 5).

C. Order parameter

As discussed earlier, $\langle\Phi\rangle$ is expected to vanish in the Z(3) symmetric confined phase. In the deconfined phase the system may be in any one of the spontaneously chosen ground states. For numerical implementation, choosing the ground state requires some biasing. For example with a source term for Φ towards one of the ground states, one can consider the sequential limits $V \to \infty$ and source term going to zero. However we have already simplified the $V \to \infty$ limit. Consequently the thermal averages of various operators are obtained as

$$\langle\hat{O}[\Phi,\bar{\Phi}]\rangle = \frac{1}{z}\int d\theta_1 d\theta_2 \mathrm{Det}_{\mathrm{VdM}} O[\Phi,\bar{\Phi}] \times \exp\left[-2\left(\frac{2T}{T_d}\right)^3 \int \frac{d^3\tilde{p}}{(2\pi)^3} \ln\left(1+\sum_{n=1}^{8} a_n e^{-n\tilde{E}_g}\right)\right]. \tag{26}$$

Since the averages are obtained by considering all the Z(3) states, $\langle\Phi\rangle$ is trivially zero, both in the Z(3) symmetric and symmetry broken phases. We are thus left only with the option of saddle point solution giving $\langle\Phi\rangle \simeq \Phi_{mf}$. The Φ_{mf} is shown in Fig. 6 and shows the characteristics of a first order phase transition.

Unlike Φ, Φ_A is invariant under the Z(3) transformation. Therefore no complication arises in the evaluation of its thermal expectation value. The temperature dependence of $\langle\Phi_A\rangle$ is shown in Fig. 7. Though a discontinuity corresponding to the one present in m_g/T appears at $T/T_d = 1$, the variation indicates a crossover rather than a phase transition. This is further confirmed from the temperature variation of the thermal derivative of $\langle\Phi_A\rangle$ (inset of Fig. 7), having a gap at $T/T_d = 1$ and an inflection point at some $T/T_d > 1$. This can be attributed to the fact that within this current model framework, quasigluons have a finite mass at temperatures below T_d. However a clear understanding of the relation between the fundamental and adjoint representations of the Polyakov loop is still to be investigated.

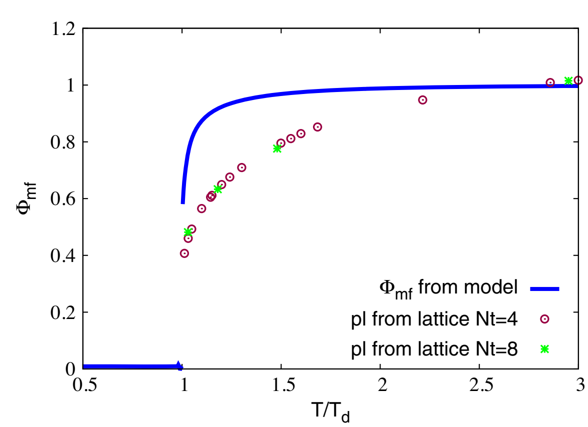

FIG. 6. Thermal evolution of Φ_{mf} (lattice data from [23]).

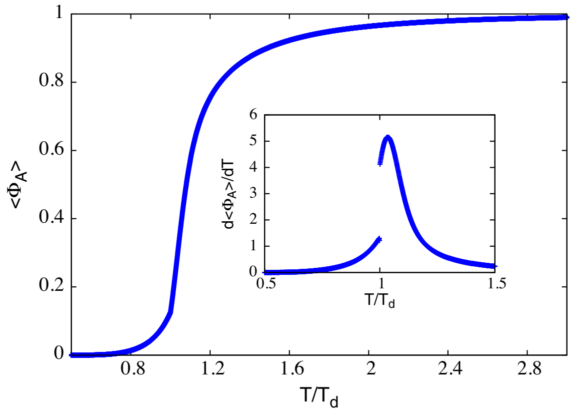

FIG. 7. Thermal evolution of $\langle\Phi_A\rangle$ and $d\langle\Phi_A\rangle/dT$ (inset).

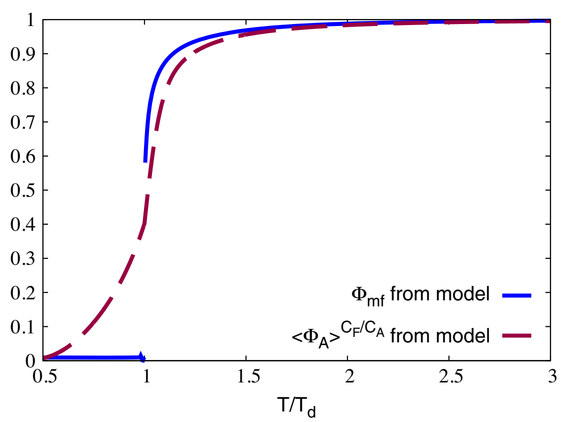

FIG. 8. Casimir scaling as a function of T/T_d (see [23] for notations).

Given that Φ is renormalization dependent on the lattice, it may not agree with model results (as seen in Fig. 6). On the lattice the Polyakov loop in various representations have been found to follow Casimir scaling above $T/T_d = 1$ [23]. We find $\langle\Phi\rangle$ and $\langle\Phi_A\rangle$ do follow the scaling reasonably well for $T/T_d > 1$ (Fig. 8). Below $T/T_d = 1$, the scaling breaks because $\langle\Phi_A\rangle$ is nonzero. This is natural as the quasigluons have finite mass. and the situation resembles the crossover observed in $\langle\Phi\rangle$ in the presence of low mass quarks in models as well as on lattice [95]. Possible effects of additional terms including effects of glueballs, bag pressure or Polyakov loop interaction terms discussed earlier may contribute to further understanding the behavior of $\langle\Phi_A\rangle$. These possibilities may be explored elsewhere.

IV. PRELIMINARY EXPLORATION INCLUDING QUARKS

The results discussed in the previous sections for the pure glue model is expected to hold true in the presence of infinitely heavy quarks. For practical purposes it should hold true even for a system of quarks whose masses are much higher than the temperature scales. Here we make a preliminary discussion on the presence of heavy quarks in the present model. Neglecting effects of chiral physics altogether and incorporating the Polyakov loop modified quark quasiparticle contribution we have the additional potential,

$$\Omega_{qqp} = 2N_f T \int \frac{d^3\tilde{p}}{(2\pi)^3}\{\ln[3(\Phi + \overline{\Phi}e^{-\tilde{E}_q})e^{-\tilde{E}_q} + 1 + e^{-3\tilde{E}_q}] + \ln[3(\bar{\Phi} + \Phi e^{-\tilde{E}_q})e^{-\tilde{E}_q} + 1 + e^{-3\tilde{E}_q}]\}, \tag{27}$$

where, $\tilde{E}_q = \sqrt{\tilde{p}^2 + (\frac{m_q}{T})^2}$, m_q being the quark mass. N_f is number of quark flavors, which we shall consider to be 2. The full potential will be the sum of Ω_{qqp} and Ω_{gqp} [given in Eq. (9)] and to be used in Eq. (15). In this preliminary study with quarks we assume all parameters of Ω_{gqp} to remain unchanged from those obtained in the preceding sections, except that we now specify $T_d = 270$ MeV. We shall discuss the behavior of the Φ_{mf} as a function of temperature for various quark masses. Again for simplicity we shall use Ω_{qqp} for various m_q, some of which are smaller than T.

The effect of introducing the quarks can be seen from Fig. 9. For $m_q = 3$ GeV the results are identical with the pure gauge results. With reducing masses we find the corresponding deconfinement temperature, as well as the gap of Φ_{mf} to decrease. Subsequently between 1.65 Gev $< m_q <$ 1.8 GeV the transition goes over to a crossover. These results are commensurate with the findings in the literature [96]. The variation of Φ_{mf} with T shows a dimple at the $T = T_d$ of the pure glue model, which is nothing but an artifact of the discontinuity of the variation of m_g with T. This can be taken care of in a detailed analysis with dynamical quarks that will be explored elsewhere.

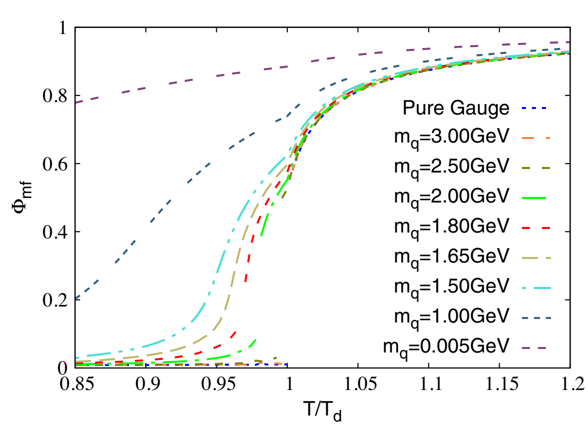

FIG. 9. Comparison of Φ_{mf} with varying quark masses.

V. DISCUSSION

The gluon quasiparticle models are usually found to become inconsistent in the confined phase. We identified the problem to lie with the saddle point method. To overcome the problem we discussed a novel prescription of obtaining the thermodynamic observables by a pseudo path integral formalism. Essentially instead of considering only the saddle point solution for the field variable, all possible field variables are considered with their appropriate thermal weight functions. By implementing this approach we predicted a variety of sensitive thermodynamic quantities to a high level of accuracy. In addition, we observed that while the temperature variation of Φ indicates a first order phase transition, that of Φ_A is almost like a crossover. The latter seems natural as it is similar to thermal variation of Φ when the quark masses are finite in chiral models. However the deeper connection between the different representations of the Polyakov loop in our model would need further investigation. A preliminary study including quarks with heavy masses give consistent results with existing literature.

We conclude that the quasiparticle model presented here is the most consistent one for studies of color deconfinement and gluon thermodynamics from cosmology to heavy-ion collision experiments.

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