Toward interferometry of neutrino electromagnetism

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The long-standing prediction of the Standard Model of elementary particles is that asymmetric neutrino environments cause rotation of linear polarization of electromagnetic wave—the birefringence. We demonstrate that this effect is strongly enhanced if additionally the photon is propagating through refractive medium, which effectively increases the photon exposure to the neutrino medium. Our estimate for an infrared laser beam in 1 m long optical fiber exposed to reactor antineutrino flux results in linear polarization rotation by the angle $\sim 4.6 \times 10^{-39}$ rad. We also derive the proper dependence of the effect on the angle between the directions of photon and neutrino propagation in the laboratory frame. For that purpose, we derive the correct form of the basis of polarization four-vectors, which differs from the one widely used in literature. We also estimate the subleading optical effect of the neutrino medium due to the neutrino dipole magnetic moment, in terms of a variation of the refractive index and its angular dependence. A rough monochromatic approximation points toward the existence of a resonant enhancement of the effect.

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I. INTRODUCTION

The Standard Model of elementary particles extended for neutrino masses (SM) predicts neutrinos, despite being neutral fermions, to exhibit electromagnetic properties. They are classified as the charge radius, electric dipole moment, magnetic dipole moment, and anapole moment [1]. They are result of the screening by quantum fluctuations of neutrinos into virtual charged particles, mainly electrons and W bosons, which are then felt by a photon field. On top of that, the SM predicts a two-photon-fields interaction with neutrinos described effectively by their Rayleigh operator generated at one-loop level [2–7].

The experimentally confirmed existence of the neutrino electromagnetic properties would at first be an independent probe of the SM itself. Second, it would affect whole areas of the astroparticle physics and cosmology, e.g., models of dynamics of compact objects, models of the evolution of the Universe, or properties of the cosmic neutrino relics. Third, it would open a new channel of neutrino detection.

It is a long-standing exercise to estimate the effect of a medium consisting of neutrino (anti)particles with nonzero electromagnetic properties on the photon propagation [2–12]. The medium must be asymmetric in the neutrino and antineutrino densities $n_\nu - \bar{n}_\nu$ to avoid cancellation from these two components. The estimates are based on analyzing the forward scattering of photons on neutrinos, represented by two classes of diagrams. The first class consists of one-particle irreducible diagrams of one-loop order in the SM, underlying the Rayleigh operator; see example in Fig. 1(a). The second class consists of one-particle reducible diagrams which are of two-loop order in the SM, each loop providing an electromagnetic form factor; see diagram in Fig. 1(b).

A significance of individual contributions to the forward-scattering amplitude is assessed by power counting of the suppression factor $\propto 1/M_W^N$ [4]. The leading contribution $N = 2$ is purely parity violating and comes from the diagram in Fig. 1(a). It is responsible for the birefringence, optical activity, or rotary power of the asymmetric neutrino medium, which, e.g., rotates the direction of the linear polarization of the electromagnetic wave. In other words,

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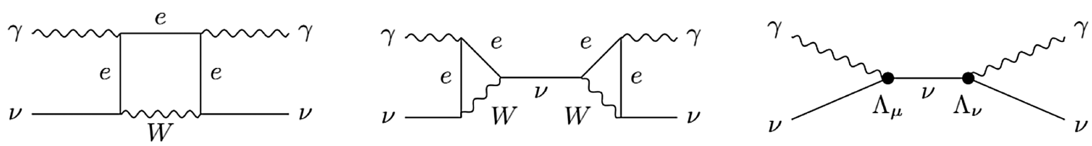

FIG. 1. Feynman diagrams for neutrino-photon forward-scattering (a) one-particle irreducible at one-loop level within the SM providing the dominant contribution to the photon polarization tensor which is parity violating, (b) one-particle reducible at two-loop level within the SM providing the subdominant contribution to the photon polarization tensor, but providing one of the leading parity-conserving contributions, (c) tree level within effective theory of neutrino electrodynamics.

the left- and right-handed polarizations develop different dispersion relations and, consequently, a difference in their phase velocity. The observability of the birefringence relies on the ability to control polarization of the electromagnetic wave. Any kind of averaging over the polarizations wipes out the parity-violating effect. If that is the case, the parity-conserving contributions, which are of the subleading order $N = 4$, become dominant. It is exhibited by the modification of the transverse component of the photon polarization tensor and by the creation of the longitudinal component of the photon polarization tensor. Such effect can be, in principle, observed as the variation of the value of the index of refraction.

The rotary power of the cosmic neutrino background (CNB) has been estimated previously with hopelessly small results. The case of pure CNB medium was initially addressed in [2] and later elaborated in greater detail in [5,6]. The result for visible light wavelengths and the neutrino asymmetry allowed by cosmological models is ~47 orders of magnitude below the rotary power of the intergalactic magnetic fields (IGMF); see the comparative graph in [12]. A strong enhancement has been identified in [4] by including the electron component of intergalactic plasma into the medium, lifting the estimate to ~35 orders of magnitude below the IGMF. Very recently, the authors of [12] reestimated the intergalactic plasma enhancement by including properly thermally averaged three-body photon-neutrino-electron scattering and encountered another ~3 orders of magnitude enhancement, still far below any observability. And yet, we are learning a clear lesson that "slowing down" of photons by letting them pass through the additional component of a medium increases their exposure to neutrinos, enhancing the resulting neutrino effect significantly. Therefore, it is relevant to investigate the effect of optically much denser media.

In this work, we revise the birefringence effect due to the presence of the neutrino asymmetric component within the background of an ordinary refractive medium, e.g., a solid transparent material, pervaded by the neutrino flux. Our goal is to investigate the principal observability of the electromagnetic wave phase shifts due to neutrinos by means of interferometric experiment. An analogous situation of a neutrino medium streaming through plasma with electron component at rest has been examined in [9]. In our analysis we derive directional dependence of the effect given by the angle between photon and neutrino fluxes. In principle, angular variations may help in detecting these subtle effects. We also discuss the subdominant optical effect of parity-conserving variation of the refractive index.

II. PHOTON POLARIZATIONS

In order to make our analysis clear and directly applicable to any conceivable laboratory experiments, we want to stick with the calculation in the rest frame of the refractive medium, characterized by the refractive index n, typically $1 < n < 4$, in which the electromagnetic wave is characterized by the four-momentum

$$k = (\omega, \mathbf{k}) = \omega(1, n\hat{\mathbf{k}}), \qquad k^2 = \omega^2(1 - n^2), \quad (1)$$

where ω is the electromagnetic wave frequency, and $\mathbf{k}$ is its three-momentum in the optically dense refractive medium. The unit three-vector $\hat{\mathbf{k}}$ defines the spatial direction of the electromagnetic wave propagation. In the rest frame of the refractive medium, the neutrino medium is, in general, not in rest. It is rather characterized by the four-velocity

$$u = (u_0, \mathbf{u}) = \gamma(1, \beta\hat{\mathbf{u}}), \qquad u^2 = 1, \quad (2)$$

where β is the speed of the neutrino medium and γ is the corresponding Lorentz factor. The unit three-vector $\hat{\mathbf{u}}$ defines the spatial direction of the neutrino flux. The direction of the electromagnetic wave and direction of the neutrino flux have a general mutual angle θ_{ku},

$$\cos\theta_{ku} = \hat{\mathbf{k}} \cdot \hat{\mathbf{u}}. \quad (3)$$

As the actual neutrino density experienced by the photon beam depends on the angle θ_{ku}, also the resulting optical effect of the neutrino environment should vary with θ_{ku}. This angular dependence is actually carried by the Lorentz invariant K, introduced in former calculations [4,8] and others, interpreted as the magnitude of the photon momentum in the rest frame of the medium.

The form of the Lorentz invariant K has its roots in the proper choice of the orthogonal basis of the photon polarization four-vectors. Let us denote, as usually, three polarization four-vectors as e_i, $i = 1, 2, 3$. They satisfy the Lorenz condition

$$k_\mu e_i^\mu = 0. \tag{4}$$

Let us choose the first two of the polarization four-vectors, e_1 and e_2, to be the basis of a two-dimensional subspace of transverse polarizations

$$e_{1,2} = (e_{1,2}^0, \mathbf{e}_{1,2}) = (0, \hat{\mathbf{e}}_{1,2}), \qquad e_{1,2}^2 = -1. \tag{5}$$

The second equality is given by a suitable choice of basis $e_{1,2}^0 = 0$, which makes it explicit that the transverse polarization four-vectors $e_{1,2}$ are one-to-one connected with the polarization unit three-vectors $\hat{\mathbf{e}}$, which are transverse to $\hat{\mathbf{k}}$,

$$\hat{\mathbf{k}} \cdot \hat{\mathbf{e}} = 0. \tag{6}$$

The third polarization four-vector e_3 corresponds to the longitudinal polarization. These three polarizations together with

$$e_0^\mu \equiv \frac{k^\mu}{\sqrt{k^2}} \tag{7}$$

form a complete orthogonal basis if

$$e_\rho \cdot e_\sigma = g_{\rho\sigma}, \tag{8}$$

which embeds the orthogonality (6) and also

$$\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2 = 0. \tag{9}$$

Given e_0^μ, e_1^μ, and e_2^μ by Eqs. (1) and (5), the orthogonality condition (8) determines the four-vector e_3 to be

$$e_3^\mu = \frac{\tilde{u}^\mu}{\sqrt{-\tilde{u}^2}}, \tag{10}$$

where, for the arbitrary neutrino four-velocity u,

$$\tilde{u}^\mu = u^\mu - \frac{(k \cdot u)}{k^2} k^\mu - \frac{(e_1 \cdot u)}{e_1^2} e_1^\mu - \frac{(e_2 \cdot u)}{e_2^2} e_2^\mu. \tag{11}$$

The third and fourth terms on the right-hand side are missing in [8] and all other consequent literature. It leads to the proper angular dependence of the Lorentz invariant K different from the existing literature,

$$K = \sqrt{(k.u)^2 - k^2(1 + (e_1 \cdot u)^2 + (e_2 \cdot u)^2)}. \tag{12}$$

The dependence of K on $(e_{1,2} \cdot u)$ is new. Taking the definitions of $e_{1,2}^\mu$ (5), of k^μ (1), and of u^μ (2), we can write the dot products explicitly

$$(k.u) = \omega\gamma(1 - n\beta\cos\theta_{ku}), \tag{13}$$

$$(e_1.u) = -\gamma\beta\sin\theta_{ku}\cos\phi_{eu}, \tag{14}$$

$$(e_2.u) = -\gamma\beta\sin\theta_{ku}\sin\phi_{eu}, \tag{15}$$

where ϕ_{eu} is the polar angle of $\hat{\mathbf{e}}_1$ within the plane perpendicular to $\hat{\mathbf{k}}$ relative to the projection of $\hat{\mathbf{u}}$ into this plane. Applying this into (12) we come to very simple expression for K,

$$K = \omega\gamma|n - \beta\cos\theta_{ku}|. \tag{16}$$

There are two immediate observations. First, K is always real and non-negative. Second, it can vanish only for $n < 1$; in the case of parallel $\hat{\mathbf{k}}$ and $\hat{\mathbf{u}}$ it vanishes just when $\beta = n$, which results in parallel four-vectors k^μ and u^μ; see (1) and (2). This is exactly expected from the tensor structure of the parity-violation polarization tensor part $\Pi_{P\mu\nu} \propto \varepsilon_{\mu\nu\alpha\beta}k^\alpha u^\beta$ [4]. This is to be compared with the previous expression for K from, e.g., [4,8]. Let us denote it by K',

$$K' = \sqrt{(k.u)^2 - k^2} = \sqrt{K^2 - (1 - n^2)\omega^2}, \tag{17}$$

which does not exhibit either of the above properties. There exists a physically reasonable choice of n, β, and θ_{ku} for which K' is even imaginary. For parallel $\hat{\mathbf{k}}$ and $\hat{\mathbf{u}}$, the requirement for vanishing of K' leads to $\beta = n/\sqrt{n^2 - 1}$, being meaningful only for $n > 0$, which in no way leads to parallel four-vectors k^μ and u^μ. The expression for K and K' coincides in a special case of lightlike photon, $k^2 = 0$, or in the case of having no preferred inertial frame other than the rest frame of the neutrino medium, in which case one can choose $u = (1, 0, 0, 0)$ and the new terms $\propto (e_{1,2}.u)^2$ vanish.

Having chosen the proper basis of polarization four-vectors e_i^μ, $i = 1, 2, 3$, there exists a set of corresponding projectors onto them; these are the transverse projector $P_\mathrm{T}^{\mu\nu}$ onto transverse modes given by $e_{1,2}^\mu$ and the longitudinal projector $P_\mathrm{L}^{\mu\nu}$ onto the longitudinal mode given by e_3^μ,

$$P_{\mathrm{T}\nu}^{\mu} e_{1,2}^\nu = e_{1,2}^\mu, \qquad P_{\mathrm{T}\nu}^{\mu} e_3^\nu = 0, \tag{18}$$

$$P_{\mathrm{L}\nu}^{\mu} e_3^\nu = e_3^\mu, \qquad P_{\mathrm{L}\nu}^{\mu} e_{1,2}^\nu = 0. \tag{19}$$

On top of that, there is a parity-violating generator of the transverse polarization rotation $P_\mathrm{P}^{\mu\nu}$,

$$P_{\mathrm{P}\nu}^{\mu} e_{1,2}^\nu = \pm \mathrm{i} e_{2,1}^\mu. \tag{20}$$

These tensors have the explicit form given, e.g., in [8],

$$P_{\mathrm{L}}^{\mu\nu} = \frac{\tilde{u}^{\mu}\tilde{u}^{\nu}}{\tilde{u}^{2}}, \tag{21}$$

$$P_{\mathrm{T}}^{\mu\nu} = g^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{k^{2}} - \frac{\tilde{u}^{\mu}\tilde{u}^{\nu}}{\tilde{u}^{2}}, \tag{22}$$

$$P_{\mathrm{P}}^{\mu\nu} = \frac{\mathrm{i}}{K}\varepsilon^{\mu\nu\alpha\beta}k_{\alpha}\tilde{u}_{\beta}, \tag{23}$$

keeping in mind that $\tilde{u}^{\mu}$ and K are defined according to (11) and (16). From this point on, the analysis follows according to formerly established lines described, e.g., in [4,8]. The photon polarization tensor can be decomposed as

$$\Pi^{\mu\nu} = \Pi_{\mathrm{T}}P_{\mathrm{T}}^{\mu\nu} + \Pi_{\mathrm{L}}P_{\mathrm{L}}^{\mu\nu} + \Pi_{\mathrm{P}}P_{\mathrm{P}}^{\mu\nu}. \tag{24}$$

It is the parity-violating polarization function Π_{P} that is responsible for the birefringence effect.

III. BIREFRINGENCE

The leading term in the $1/M_W^2$ expansion of the neutrino-induced correction to the polarization tensor $\Pi^{\mu\nu}(k)$ comes from the one-loop diagram [Fig. 1(a)] and contributes exclusively to the parity-violating polarization function $\Pi_{\mathrm{P}}(k^2, K)$ responsible for the birefringence effect. The polarization function $\Pi_{\mathrm{P}}(k^2, K)$, under the assumption $k^2 \ll m_e^2 \ll M_W^2$, has the form [4]

$$\Pi_{\mathrm{P}}(k^2, K) = \frac{\sqrt{2}G_F\alpha}{3\pi}\frac{k^2}{m_e^2}(n_{\nu} - n_{\bar{\nu}})K, \tag{25}$$

again keeping in mind the definition of K (16). The quantities $n_{\nu(\bar{\nu})}$ are the (anti)neutrino medium rest frame densities. The expression (25) vanishes for $k^2 \to 0$ in accord with the Gell-Mann theorem [13]. In the same line as in [4], where the intergalactic plasma effect has been included in (25) simply by taking $k^2 = \omega_p^2 \neq 0$ (with ω_p being the plasma frequency), we are fixing k^2 to include the effect of the refractive medium in terms of the refractive index by setting $k^2 = \omega^2(1 - n^2) \neq 0$. We are aware of the fact that the proper treatment should rely on the calculation presented in [12], in which the thermal effect of the electron medium is properly implemented. Based on the experience from [12], we expect that our result, based on (25), might underestimate the effect by just a smaller number of orders of magnitude.

As shown in [4], from diagonalization of the photon full propagator, which includes both the refractive medium effect in the form of $\Pi_T \sim k^2 = \omega^2(1 - n^2)$ and the neutrino medium effect in the form of Π_P given by (25), the dispersion formula for the two transverse modes has the form

$$k_{\pm}^2 \equiv \omega^2 - |\mathbf{k}_{\pm}|^2 = \Pi_T \pm \Pi_P. \tag{26}$$

Here we have introduced momenta $|\mathbf{k}_{\pm}|$, which represents magnitude of the actual photon wave vectors that includes both refractive and neutrino medium effects. The rotary power as the linear polarization rotation angle per unit length is then expressed in terms of the actual photon momenta $\mathbf{k}_{\pm}$ as

$$\frac{\phi}{l} = \frac{1}{2}(|\mathbf{k}_{-}| - |\mathbf{k}_{+}|) \simeq \frac{\Pi_P}{2\omega n}. \tag{27}$$

After combining all the pieces (1), (16), and (25) into the expression (27) for rotary power we get our key result,

$$\frac{\phi}{l} = \frac{G_F\alpha}{3\sqrt{2}\pi}\frac{\omega^2(1 - n^2)}{m_e^2}\gamma(n_{\nu} - n_{\bar{\nu}})\left|1 - \frac{\beta}{n}\cos\theta_{ku}\right|. \tag{28}$$

Using the formula (28), we can make a numerical estimate of the effect in the conceivable physical situation. A linearly polarized infrared laser beam of $\omega \sim 1$ eV would pass through a silicon optical fiber with the refractive index $n \sim 3.5$ in the vicinity of a nuclear reactor approximated as a pointlike source of antineutrino flux of $f = 10^{21}\,\mathrm{s}^{-1}$ (here we take into account also slow antineutrinos). The $l = 1$ m long optical fiber is placed to a $d = 5$ m distance from the reactor core in the perpendicular direction to the antineutrino flux, so $\cos\theta_{ku} = 0$. The local density of antineutrinos in the optical fiber is then $\gamma(n_{\nu} - n_{\bar{\nu}}) = f/(4\pi c d^2) \approx 10^4$ cm^{-3} and the resulting angle, by which the linear polarization of the laser beam is rotated, is $\phi \sim 4.6 \times 10^{-39}$ rad. This appears to be an undetectably small quantity. On the other hand, it should be compared with the previous results [12], where a comparable magnitude of the same effect has been achieved by letting pass the astronomical photons through the intergalactic void filled with plasma electrons and relic neutrino medium over the distance of a Hubble radius $l = l_H \sim 10^{26}$ m. Notice in (28) the dependence of the effect on the refractive index factor $(1 - n^2) \sim \mathcal{O}(1 - 10)$, which is 26 orders of magnitude larger than the corresponding quantity ω_p^2/ω^2 describing the electron component of the intergalactic plasma characterized by the plasma frequency $\omega_p \sim 10^{-13}$ eV. It is the optical density that compensates the need for Hubble distances down to laboratory scales to get the effect of the same magnitude.

One can think of increasing the effect by some orders of magnitude via changing some of the parameters. First, the effect is linear in the local neutrino density. Approaching closer toward the core of the nuclear reactor and increasing its power would help, conceivably, each by one order of magnitude, for the price of exposing the experimental setup to an extreme thermal and radiation environment. Second, the effect is linear in the optical fiber length. It is conceivable to design a $l = \mathcal{O}(100\text{ km})$ optical fiber.

However, the optical stability of the optical fiber against thermal and vibration noise at the required level is extremely sensitive to its length, already $l = 1$ m is beyond current technological capabilities. A double-link laser set up using two or more laser sources of different wavelengths simultaneously in the same optical fiber would provide a set of experimental data, whose linear combination might suppress the noise while keep the signal, provided that the noise and signal have distinct dispersion. For increasing the effective optical fiber length, unfortunately, the fiber-based Fabry Pérot cavity, which can significantly enhance the effective photon path without increasing the noise, does not bring any advantage, because the sign of the effect, i.e., of the angle ϕ, is given by the sense of the photon propagation so that the reflected laser beam would get exactly opposite linear polarization rotation. Third, the effect is quadratic in ω of the electromagnetic wave. Using harder photons, e.g., x rays with $\omega = 1$–100 keV may bring a factor $10^6 - 10^{10}$, however, correspondingly the refractive index drops to unity with inverse squared ω so that $1 - n^2 \propto \omega^{-2}$, which consequently cancels out the advantage coming from the shorter wavelength. On top of that, for harder photons, the attenuation increases.

IV. VARIATION OF REFRACTIVE INDEX

In this last part of our work, we would like to address briefly the subdominant effect of the order of $1/M_W^4$ that becomes the leading effect in the situation, when the birefringence discussed above is averaged away for some reason, such as where the laser beam is not polarized, the polarization is destroyed along the propagation due to the imperfection of the optical fiber, or the Fabry Pérot cavity resonator is used.

There are many Feynman diagrams contributing to the forward scattering at the $1/M_W^4$ order. Two of them are depicted in Figs. 1(a) and 1(b). We, however, discuss Fig. 1(c), coming from the effective neutrino electromagnetic theory [1], which is induced by nonzero neutrino electromagnetic coupling Λ_μ, which corresponds, e.g., to the neutrino dipole magnetic moment. In the SM, the dipole magnetic moment of the Dirac neutrino is predicted from the vertex loop in Fig. 1(b) to be $\propto 1/M_W^2$ and among other neutrino electromagnetic couplings to be the strongest one, though with its value $\mu_\nu \sim 10^{-19}\mu_{\mathrm{B}}$ it is still $\sim$7 orders of magnitude below current experimental capabilities.

Following the analogous calculation of [4], we calculate the part of the photon polarization tensor induced by the electromagnetic coupling $\Pi_{\mu\nu}^{(\Lambda)}$ and consider only the medium-induced part, which we indicate by the prime $\Pi_{\mu\nu}^{(\Lambda)\prime}$,

$$\Pi_{\mu\nu}^{(\Lambda)\prime}(k) = \mathrm{i}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\mathrm{Tr}\{S'(p,u)[T_{\mu\nu}(p,k) + T_{\nu\mu}(p,-k)]\}, \tag{29}$$

where

$$S'(p,u) = (\not{p} + m_\nu)2\pi\mathrm{i}\delta(p^2 - m_\nu^2) \times [\theta(p\cdot u)f(p\cdot u) + \theta(-p\cdot u)\bar{f}(-p\cdot u)], \tag{30}$$

$$T_{\mu\nu}(p,k) = \frac{\Lambda_\mu(\not{p} + \not{k} + m_\nu)\Lambda_\nu}{(p+k)^2 - m_\nu^2}, \tag{31}$$

where f and $\bar{f}$ are the distribution functions of neutrinos and antineutrinos, respectively. The integral (29) can be conveniently calculated in the neutrino medium rest frame by Lorentz transforming it first, so that $p\cdot u \to p_0$ and $k \to k_u$ becomes the photon four-momentum in the neutrino medium rest frame. After the integration, we Lorentz transform the result back to the laboratory frame. In what follows, we keep in mind the physical setup of the laser beam in an optical fiber exposed to reactor antineutrinos and simplify significantly our calculation by considering the antineutrino stream as perfectly collinear and monochromatic, specified in the neutrino medium rest frame as

$$f = 0, \qquad \bar{f} = (2\pi)^3 n_{\bar{\nu}}\delta^{(3)}(\mathbf{p}), \tag{32}$$

where $n_{\bar{\nu}}$ is the density of antineutrino medium in its rest frame. Then we first perform the p_0 integration with the help of $\delta(p^2 - m_\nu^2)$,

$$\Pi_{\mu\nu}^{(\Lambda)\prime}(k_u) = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\frac{\bar{f}}{2|p_0|}\frac{\mathrm{Tr}_{\mu\nu}(k_u) + \mathrm{Tr}_{\nu\mu}(-k_u)}{4(k_u\cdot p)^2 - k_u^4}\bigg|_{p_0=-\epsilon}, \tag{33}$$

where $\epsilon \equiv \sqrt{\mathbf{p}^2 + m_\nu^2}$ and

$$\mathrm{Tr}_{\mu\nu}(k) = \mathrm{Tr}\{(\not{p} + m_\nu)\Lambda_\mu(\not{p} + \not{k} + m_\nu)\Lambda_\nu\}(k^2 - 2(k\cdot p)). \tag{34}$$

Because of the monochromatic approximation (32), the three-momentum integration is trivial and basically it comprises the substitution in the integrand $p^\mu \to (m_\nu, 0, 0, 0) \equiv m_\nu u_u^\mu$, where u_u^μ is the antineutrino medium four-velocity in its rest frame. Finally, we Lorentz transform the result back to the laboratory frame, so that we remove the u subscripts k_u and u_u. The final result is

$$\Pi_{\mu\nu}^{(\Lambda)\prime}(k) = \frac{1}{m_\nu}\frac{\mathrm{Tr}_{\mu\nu}(k,u) + \mathrm{Tr}_{\nu\mu}(-k,u)}{4m_\nu^2(k\cdot u)^2 - k^4}n_{\bar{\nu}}. \tag{35}$$

For the coupling of the dipole magnetic moment, we have $\Lambda_\mu = \mu_\nu k^\alpha \sigma_{\alpha\mu}$ and the resulting contribution to the photon transverse polarization function is

$$\Pi_{\mathrm{T}}^{(\mu)\prime}(k) = 8\mu_\nu^2 m_\nu k^4 \frac{2 + (\mathbf{k}\cdot\mathbf{u})^2/|\mathbf{k}|^2}{k^4 - 4m_\nu^2(k\cdot u)^2}n_{\bar{\nu}}. \tag{36}$$

Applying this result to the photon propagation in the refractive medium, i.e., using (1) and (2), we can make the estimate of the variation of the index of refraction due to the neutrino medium

$$\Delta n \approx \frac{\Pi_{\mathrm{T}}^{(\mu)'}}{2\omega^2} \simeq -\frac{\mu_\nu^2 \gamma n_{\bar{\nu}}}{E_\nu}(1-n^2)^2 \frac{\sin^2\theta_{ku}}{(1-n\beta\cos\theta_{ku})^2 - \Gamma}, \quad (37)$$

where $\Gamma = \frac{\omega^2(1-n^2)^2}{4E_\nu^2} \sim 10^{-12}$ for the reactor antineutrino energy $E_\nu = \gamma m \sim \mathcal{O}(\mathrm{MeV})$ and for the visible light frequency $\omega \sim \mathcal{O}(\mathrm{eV})$. For these conditions, we are getting an extremely tiny correction to the refractive index $\Delta n \sim 1.2 \times 10^{-65}\Theta(\theta_{ku})$, where the function $\Theta(\theta_{ku})$ carries the angular dependence, which provides a typical factor of $\mathcal{O}(1)$. Interestingly, however, the angular factor may be the source of a resonant enhancement if the angle θ_{ku} is tuned to be close to $\cos\theta_{ku} \sim 1/(n\beta)$. Similar resonances are typical for the media composed of neutral particles with dipole magnetic moment. The angular factor in the expression (37), due to the assumption of perfect collinearity and monochromaticity of the neutrino flux, could reach unlimited values, which is, of course, an unphysical consequence of unrealistic approximation. To analyze properly the resonant enhancement is the subject of future work. Now we just speculate that the values of the resonant enhancement up to the level $\sim\Gamma^{-1}$ could be potentially accessible if the collinearity and angular stability of the neutrino flux with respect to the photon direction could be tuned down to the level of the angular spread $\Delta\cos\theta_{ku} < \sqrt{\Gamma}$.

Notice that the resonance within the angular factor, in principle, appears in two regimes: The first regime is the ultrarelativistic regime, characterized by $\Gamma \to 0$ and $\beta \to 1$, in which the resonance is achieved by tuning the angle θ_{ku}. The second regime is the nonrelativistic regime, characterized by $\Gamma \sim \mathcal{O}(1)$ and $\beta \to 0$, in which the resonance is achieved by tuning the ratio of neutrino and photon energies. Interestingly, $\Gamma \sim \mathcal{O}(1)$ for ω from the visible-light range, or similar, is only accessible thanks to lightness of neutrinos, so that one can arrange $m_\nu \sim \omega$. If neutrinos were heavier, harder photons would be needed, for the high price of losing the advantage of the refractive index being $(1-n^2) \sim \mathcal{O}(1)$. Hypothetically, if the CNB were much colder than currently predicted, $T \ll \beta_\odot m_\nu \ll 1.95$ K, so that the peculiar motion of Earth with respect to the universal grid, characterized by the speed $\beta_\odot \sim 10^{-4}$, would produce pretty monochromatic neutrino flux through the detector, then one could, at least in principle, tune ω to probe the resonance. In reality, however, the thermal motion of the CNB neutrinos totally smears out the resonance of this kind.

V. CONCLUSIONS

In the present work, we have addressed a novel idea of the interferometric detection of neutrino electromagnetic properties in laboratory. For that purpose, we have revised the formula describing the birefringence effect for the case of neutrino flux streaming through an ordinary transparent refractive medium, such as an optical fiber. Though the estimate of the magnitude of the effect turned out to be undetectable by current interferometric technologies, the result exhibits a significant reduction of the length scale of the phenomenon down to the laboratory scales, compared to the cosmological scales addressed in the existing literature so far. Additionally, within very rough monochromatic approximation, we have derived and estimated the subdominant optical effect, the variation of the refractive index, induced by the neutrino dipole magnetic moment for the same environment. The approximation points toward the existence of a resonant enhancement of the effect, whose advantage would be accessible only under a delicate tuning of the monochromaticity and directional uniformity of the neutrino flux. To estimate the resonant enhancement factor for realistic neutrino fluxes, analysis beyond the rough approximation used here is necessary, which is the subject of future work.

In order to properly derive the angular dependence of the present optical effects, we have revised the description, used in the related literature, of the photon polarizations in medium in the general Lorentz frame. We identified a discrepancy from the description used in the existing literature leading to distinct angular dependencies of the effects. We have presented simple arguments in favor of our description based on directional dependence of the birefringence, inferred from a general covariant form of the parity-violating photon polarization tensor component.

The results of this work suggest the interferometry of neutrino electromagnetism to be far from current technological capabilities. They are, however, presented here to trigger interest in this research direction. The existence and nature of the possible resonance requires thorough theoretical investigation. Sophisticated experimental setups, which boost the interferometric effects further by orders of magnitude, may be revealed. New technological concepts, such as the optical-fiber-based atomic interferometry, may shrink the gap between the theory and experiment in the foreseeable future. On top of that, the present ideas, concepts, and results are, in principle, applicable to the case of an asymmetric dipolar candidate for dark matter (see, e.g., [14]), something that certainly deserves proper attention.

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