$A_{\rm CP}[D^{0,+}_{(s)} \to V\gamma]$ from a large $\mathcal{O}_8$ contribution

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CP violation in $\Delta A_{\rm CP} = -0.154(29)$, in the $D^0 \to \pi^+\pi^-/K^+K^-$ system, is established, and its central value is 1 order of magnitude above the naive Standard Model (SM) estimate. It remains unclear whether this is due to currently incalculable strong interaction matrix elements or genuine new physics, such as a shift in $\mathcal{O}_8$ with a weak phase. We show that interference of the long-distance (LD) terms with the $\mathcal{O}_8$ matrix element can give rise to $A_{CP}^{D\to V\gamma} = \text{few} \times 10^{-3}$ (for reference values $\text{Im}[C_8^{\rm NP}] \approx 10^{-3}$). In addition, it is pointed out that the ratio of left- to right-handed (photon polarization) LD amplitudes is measurable in time-dependent CP asymmetries. We argue that both theory and experimental consideration favor weak annihilation (WA) as the dominant LD contribution. More definite progress could be achieved by either computing the radiative corrections to WA or the measurement of the charged modes $D^+_{(d,s)} \to (\rho, K^*)^+\gamma$ and $D_s \to \rho^+\gamma$.

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I. INTRODUCTION

A. $A_{\rm CP}$ in $D^0 \to \pi\pi/KK$

CP violation is parametrically suppressed in the charm sector [of order $\mathcal{O}(10^{-4})$]. In 2011, LHCb [1] and CDF [2] reported a value of CP violation in the hadronic system $D^0 \to \pi^+\pi^-/K^+K^-$, $\Delta A_{\rm CP} = -0.65(18)$ with central value considerably above expectation. Since then, CP violation in the charm system has been established [3,4],

$$\Delta A_{\rm CP} = A_{\rm CP}^{K^+K^-} - A_{\rm CP}^{\pi^+\pi^-} = -0.154(29) \times 10^{-2}, \quad (1)$$

at a lower central value. However, the question of whether this is NP or due to hadronic matrix elements considerably above its naive expectation remains unclear and is part of the investigation of this paper. In (1), $A^f_{\rm CP}$ is

$$A^f_{CP} \equiv \frac{\Gamma[D^0 \to f] - \Gamma[\bar{D}^0 \to f]}{\Gamma[D^0 \to f] + \Gamma[\bar{D}^0 \to f]}, \quad (2)$$

a shorthand for the time integrated CP asymmetry, for a case where the final state f is a CP eigenstate. $\Delta A_{\rm CP}$ is a convenient quantity since systematic experimental errors cancel. It is worthwhile to add that if $SU(3)_F$, or more precisely U spin, were a good symmetry then $A_{CP}^{K^+K^-} = -A_{CP}^{\pi^+\pi^-}$. In the quantity $\Delta A_{\rm CP}$ the TDCP asymmetry part cancels. Effects can remain through time-acceptance differences in the π and K system, although the latter is estimated to be small, e.g., [1]. Hence, direct (i.e., time-independent CP asymmetry) is expected to be responsible for the relatively large value of $\Delta A_{\rm CP}$.

Sizeable direct CP asymmetries necessitate large strong (CP even) and weak (CP odd) phase differences in two amplitudes of comparable size (cf. Appendix B). The reason CP violation is believed to be small in the charm system is that the weak phases are suppressed by four powers of the Cabibbo angle (or Wolfenstein parameter $\lambda \approx 0.23$), leading to the naive expectation $\Delta A_{\rm CP} \approx \text{few} \times 10^{-4}$. In the nonleptonic case, the QCD matrix elements, which determine the strong phase as well as the ratio of amplitudes, are difficult to compute from first principles as the size of the charm mass is neither suited to chiral nor heavy quark theory. Advances in lattice QCD open the door to first principles results [5] and should be available in the foreseeable future. Thus, the question of whether the large central value (1), should it remain, is due to NP [6–9] or somewhat unexpected strong dynamics [10–13], such as in the $\Delta I = 1/2$-rule $K \to \pi\pi$ system,[1] is an open question at present.

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[1]It was pointed out quite some time ago [14] that an enhancement of the triplet transition, in the SU(3)-flavor classification, may lead to sizeable CP violation. For example, $A_{CP}^{PP} \approx 0.08 \times 10^{-2}$, which would lead to $|\Delta A_{\rm CP}| \approx 0.16 \times 10^{-2}$, which is not far off the central value in (1).

Taking the viewpoint that the asymmetry is largely due to NP, it turns out that a weak phase in the $|\Delta C| = 1$ chromomagnetic operator,[2]

$$\mathcal{O}_8 \equiv -\frac{g m_c}{8\pi^2} \bar{u}\sigma \cdot G(1+\gamma_5)c,$$
$$\mathcal{O}_8' \equiv -\frac{g m_c}{8\pi^2} \bar{u}\sigma \cdot G(1-\gamma_5)c, \quad (3)$$

($\sigma \cdot G = \sigma_{\mu\nu} G_a^{\mu\nu} \lambda^a/2$), appears to be a promising candidate [15], not contradicting observations such as $D^0 - \bar{D}^0$-mixing. Note that the $\mathcal{O}_8^{(\prime)}$ operators are of the $\Delta I = 1/2$ type and do not fall into the testable $\Delta I = 3/2$ class [18]. Furthermore, $\mathcal{O}_8'$ is the structure which is the less abundant helicity in the SM due left-handedness of the weak interactions; $[C_8'/C_8]|_{\rm SM} \approx m_u/m_c$.

To get an idea of the size of the NP contribution [16], one might resort to naive factorization (NF), e.g., [19]. Slightly extending the notation in [16], one gets

$$\Delta A_{CP}^{\rm NP}|_{\rm NF} \approx -1.8(\mathrm{Im}[C_8^{\rm NP}] - \mathrm{Im}[C_8'^{\rm NP}]) \sin\delta, \quad (4)$$

where δ is the unknown strong phase difference between the KK and $\pi\pi$ rescattering states, which is expected to be sizeable. Note that since the sign of $\sin\delta$ is unknown, in the $D^0 \to \pi\pi/KK$ system, there is additional ambiguity on the $C_8^{(\prime)}$ Wilson coefficients. Since the decay of a $J^P(D^0) = 0^-$ particle into two $J^P(\pi/K) = 0^-$ particles necessitates parity violation, only the γ_5 part in (3) contributes and therefore, results in opposite signs of $\mathrm{Im}[C_8^{\rm NP}]$ and $\mathrm{Im}[C_8'^{\rm NP}]$ in (4), respectively. Now, a value of

$$(\mathrm{Im}[C_8^{\rm NP}] - \mathrm{Im}[C_8'^{\rm NP}]) \sin\delta, \approx 10^{-3} \quad \text{naive factorization (NF)}, \quad (5)$$

could account for the central number in (1). One has to bear in mind that (5) is due to NF and could easily be out by factors of a few. We take

$$\mathrm{Im}[C_8^{(\prime)\rm NP}] = 10^{-3}, \quad (6)$$

as our reference value, which is consistent with [16] after adjusting to the current experimental value (1). This value is at least 2 to 3 orders of magnitude above the SM value for C_8, cf. Appendix C 1, and additionally suppressed by m_u/m_c C_8'.

NP models that could induce such values as in (5) without violating existing constraints are supersymmetric models [16,19,20], leptoquarks [21], Randall-Sundrum flavor anarchy [22], and models of partial compositeness [23], whereas in fourth family models, it seems more difficult to accommodate [12].

[2]Note that this is the sign convention of [15] but opposite to Refs. [16,17].

B. $A_{\rm CP}$ in $D^0 \to V\gamma$

The question of whether C_8 values like (6) lead to observable effects elsewhere, or specifically in $D \to V\gamma$, is the subject of this paper. It was pointed out in Ref. [17] that a sizeable direct CP violation in $D^0 \to (\rho^0, \omega)\gamma$ can be induced through $\mathrm{Im}[C_7]$, provided that the LD amplitude carries a strong phase.[3] The latter is necessary as the short distance (SD) contribution of $\mathcal{O}_7$ does not come with a strong phase. Let us emphasize a few points that are either new or improved in our paper as compared with the literature:

- (i) With regards to $A_{\rm CP}$ in $D^0 \to V\gamma$ and [17], our discussion includes the $\mathcal{O}_8$ matrix element, which carries a strong phase. Thus, for (direct) CP violation, no sizeable LD phase is required.
- (ii) We observe that WA is the dominant LD mechanism since it is enhanced over quark loops (QL) by two loop factors. In the neutral modes, this hierarchy is weakened by the color suppression of WA, and we indeed find that in practice $|\mathcal{A}_{\rm QL}|/|A_{\rm WA}|_{D^0}$ could be close to 30% (cf. Appendix A 1).
- (iii) We provide partial radiative corrections in WA in terms of the $D \to \gamma$ form factor; in Sec. III A, this is a new result of this paper.
- (iv) In order to overcome the color suppression, which manifests itself in large scale uncertainties, we emphasize the need for the computation of the *full* radiative corrections for the neutral modes. We motivate the experimental measurement of the charged modes for which the color suppression is not present in practice.
- (v) We observe that TDCP is solely sensitive to long-distance contributions and that its long-distance chirality is measurable in the neutral modes (cf. Sec. III C and also [21,27] for further elaborations).

The paper is organized as follows. In Sec. II, notation is introduced, and the basics of CP violation, specific to the charm sector, is reviewed. Section III is the main part of this paper: the amplitudes are detailed and estimates for direct and time dependent CP violation are given (using the matrix elements of the operator $\mathcal{O}_8^{(\prime)}$ [28]). Conclusions and discussions are presented in Sec. IV. An important part of our work is the discussion of the LD contribution reported in Appendix A. Furthermore, Appendixes B and C contain further material on CP violation in general and specific to the decay in question.

[3]Other channels and effects that were proposed are the electric dipole moment of the nucleon [16,24], CP asymmetry in $D^0 \to \phi \to K^+K^-$ [25] and $D^0 \to V(\to PP) \to K^+K^-$ [26].

II. EFFECTIVE HAMILTONIAN AND AMPLITUDES

A. $|\Delta C| = 1$ Hamiltonian

Following, closely, the notation of [15] we write the effective $\Delta C = 1$ SM Hamiltonian as follows:

$$\mathcal{H}^{\text{eff}} = \lambda_d \mathcal{H}_d + \lambda_s \mathcal{H}_s + \lambda_b \mathcal{H}_{\text{peng}}, \qquad \lambda_D \equiv V^*_{cD} V_{uD}, \quad D = d, s, b, \tag{7}$$

and

$$\begin{aligned} \mathcal{H}_q &= \frac{G_F}{\sqrt{2}} \sum_{i=1}^{2} C_i^q \mathcal{O}_i^q + \text{H.c.}, \qquad q = d, s \\ \mathcal{O}_1^q &= (\bar{u} L_\mu q)(\bar{q} L^\mu c), \qquad \mathcal{O}_2^q = (\bar{u}_\alpha L_\mu q_\beta)(\bar{q}_\beta L^\mu c_\alpha) \\ \lambda_b \mathcal{H}_{\text{peng}} &= \frac{G_F}{\sqrt{2}} (C_7 \mathcal{O}_7 + C_7' \mathcal{O}_7' + C_8 \mathcal{O}_8 + C_8' \mathcal{O}_8' + \cdots), \end{aligned} \tag{8}$$

with $L_\mu \equiv \gamma_\mu (1 - \gamma_5)$ and α, β being color indices. The Hamiltonian $\mathcal{H}_{\text{peng}}$ contains all the SD transitions, including electric (C3) and chromomagnetic (3) operators as well as the four quark operators with a structure different from $\mathcal{O}_{1,2}$. As compared to [15], we have absorbed the λ_b into the Wilson coefficient, which is nonstandard for the SM contribution. Since $\lambda_{d,s} = \mathcal{O}(\lambda)$ and $\lambda_b = \mathcal{O}(\lambda^5)$, where $\lambda \approx 0.226$ [29] is the Wolfenstein parameter, one gets using the unitarity relation,

$$\lambda_d + \lambda_s + \lambda_b = 0, \quad \Rightarrow \lambda_d \approx -\lambda_s, \qquad \lambda_b \approx 0, \tag{9}$$

where the symbol $\approx$ above is to be understood up to corrections of $\mathcal{O}(\lambda^4)$. The fact that the third generation decouples up to $\mathcal{O}(\lambda^4)$ is the reason why in the SM the generic expectation for CP violation is $A_{\text{CP}} \approx \text{few} \times \mathcal{O}(\lambda^4)$ as mentioned in the Introduction.

B. Parametrization of decay rate

We write the amplitude as follows[4]:

$$\begin{aligned} \mathcal{A}[D \to V\gamma] &\equiv \langle V\gamma | \mathcal{H}^{\text{eff}} | D \rangle \\ &= \mathcal{A}_\perp \frac{P_\perp}{2} + \mathcal{A}_\parallel \frac{P_\parallel}{2} \\ &= \mathcal{A}_L \left(\frac{P_\perp + P_\parallel}{4} \right) + \mathcal{A}_R \left(\frac{P_\perp - P_\parallel}{4} \right), \end{aligned} \tag{10}$$

with $P_\perp = 2\epsilon_{\rho\alpha\beta\gamma} \epsilon^{*\rho} \eta^{*\alpha} p^\beta q^\gamma$ and $P_\parallel = 2i\{(p \cdot q)(\eta^* \cdot \epsilon^*) - (\eta^* \cdot q)(p \cdot \epsilon^*)\}$ where $\eta(p)$ and $\epsilon(q)$ are the vector meson and photon polarization tensors and the Levi-Civita convention is settled by $\text{tr}[\gamma_5 \gamma_a \gamma_b \gamma_c \gamma_d] = 4i\epsilon_{abcd}$. It is noted that $\mathcal{A}_{L(R)} \equiv (\mathcal{A}_\perp \pm \mathcal{A}_\parallel)$ correspond to left- and right-handed polarized photons. The rate [30], in our conventions, is given by

$$\Gamma[D \to V\gamma] = \frac{1}{32\pi} m_D^3 \left(1 - \frac{m_V^2}{m_D^2} \right)^3 (|\mathcal{A}_\perp|^2 + |\mathcal{A}_\parallel|^2). \tag{11}$$

[4]The amplitudes $\mathcal{A}_{\perp,\parallel}$ up to phases are often denoted by $\mathcal{A}_{\text{PC,PV}}$ in the literature, e.g., [17,30]. The acronyms PC and PV stand for parity conserving and violating, respectively.

III. CP ASYMMETRIES IN $D \to V\gamma$

The operators (3) consist of $c \to u$ transitions of the FCNC type. In a heavy-to-light transition for which LCSR can make predictions [28] the c quark can pair with a u, d or s quark. This leads to the following possible transitions with CP violation: $D^0 \to (\rho^0, \omega)\gamma$, $D^+ \to \rho^+\gamma$, and $D_s^+ \to K^{*+}\gamma$. The transitions $D^0 \to \bar{K}^{*0}\gamma$ and $D_s^+ \to \rho^+\gamma$ are not of the FCNC type and do not lead to CP violation in our framework (cf. Table I in Appendix A 4 for more info and benchmark values for the rates). Note that it is only for the neutral D^0 system that oscillations and thus, TDCP asymmetries are feasible.

As previously mentioned and outlined in Appenidx B, direct CP violation originates in its minimal form by two amplitudes with a weak and strong phase difference. In this work, these two amplitudes are the LD and the NP-enhanced $\mathcal{O}_8$ contributions, respectively.

A. Weak annihilation and O_8 amplitudes

The WA contribution is extensively discussed in Appendix A for which we give an executive summary here. Firstly, it is argued that WA dominates over the QL [cf. Fig. 1 (left) and (center, right)] from a theoretical and experimental viewpoint in Appendixes A 1 and A 2, respectively. In Appendix A 3, we elaborate on making concrete predictions in the neutral modes $D^0 \to V\gamma$, which concern an unfortunate cancellation of Wilson coefficients and highlights the need (for currently) unavailable radiative corrections. This is followed in Appendix A 4 by an overview and comparison of all $D^{0,+} \to V\gamma$ modes. The main and important conclusion of Appendix A 3 is that the situation can considerably be improved by (i) the measurement of the charged modes and or (ii) the computation of the radiative correction to WA.

Here, we discuss aspects related to CP violation. The WA contribution comes with a small weak phase, and the strong phase should not be sizeable either. The former is a direct consequence of the CKM hierarchy. The amplitude is proportional to $\lambda_{d,s} \approx \mathcal{O}(\lambda)$, and its weak phase is of the order of $\mathcal{O}(\lambda^4)$. The strong phase is small in the sense that it originates from radiative corrections to the WA diagram [e.g., Fig. 1 (right)].

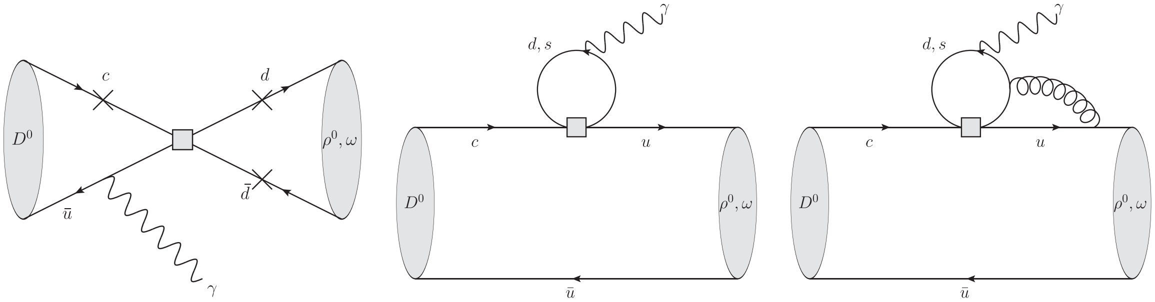

FIG. 1. A selection of LD diagrams for $D^0 \to (\rho^0, \omega)\gamma$. Note that it is the fact that the ρ^0/ω carry both $\bar{d}d$ and $\bar{u}u$ components that makes the same operator $\mathcal{O}_2^{d,s}$ (8) contribute to both (WA and QL) topologies. Left: weak annihilation (WA). Center: quark loop (QL). This contribution vanishes, exactly, for on shell photon, by virtue of gauge invariance as discussed in the text. Right: QL example of $\mathcal{O}(\alpha_s)$ correction. This diagram has a sizeable imaginary part which can be inferred from the computation for $c \to u\gamma$ in Ref. [32].

Restricting ourselves to the LO contribution, as NLO is currently not available, the amplitude is given by ($X = \perp, \|$),[5]

$$\mathcal{A}_X(D^{0(-)} \to V^{0(+)}\gamma) = \kappa_{0(+)} \frac{eG_F}{\sqrt{2}c_V} \lambda_{\rm CKM} a_{2(1)} \frac{m_V f_V}{m_D} V_X^{D^{0(+)}\to\gamma}(m_V^2), \quad (12)$$

in the convention of [33], adapting $D_\mu = \partial_\mu - ieA_\mu$ for the sign of the covariant derivative in this work, $\lambda_{\rm CKM}$ is the product of the appropriate CKM factors and for the decay constants we use the values given in Appendix C of [34]. The factor $\kappa_{0(+)} = 2(1)$ is the empirically motivated scaling factor (cf. Appendix A 3). The functions $V_{\perp,\|}$ are the $D \to \gamma$ form factors which are the only contribution at leading order in the chiral limit in the SM. We use the NLO form factor results [33], evaluating them we obtain the new results,

$$\begin{aligned} &V_\perp^{D_0\to\gamma}(m_\rho^2) \approx -0.55, && V_\|^{D_0\to\gamma}(m_\rho^2) = -0.17, \\ &V_\perp^{D^+\to\gamma}(m_\rho^2) = +0.067, && V_\|^{D^+\to\gamma}(m_\rho^2) = +0.35, \\ &V_\perp^{D_s\to\gamma}(m_\rho^2) \approx +0.11, && V_\|^{D_s\to\gamma}(m_\rho^2) = +0.44. \end{aligned} \quad (13)$$

The continuum threshold of $s_0 = 6\ {\rm GeV}^2$ is well between $(m_D + 2m_\pi)^2 \approx 4.6\ {\rm GeV}^2$ and $(m_D + m_\rho)^2 \approx 6.9\ {\rm GeV}^2$. The Borel parameter is chosen as a compromise value to render the (partonic) OPE convergent and to suppress the continuum contributions in the hadronic contribution. We refrain from an uncertainty analysis as we only aim for rough estimates in order to motivate experimental searches. Moreover, $V_\perp^{D^+\to\gamma}(m_\rho^2) \approx V_\perp^{D^+\to\gamma}(m_\omega^2)$ and $V_\perp^{D_s\to\gamma}(m_\rho^2) \approx V_\perp^{D_s\to\gamma}(m_{K^*}^2)$ hold to sufficient precision. The reference values $a_2 = C_2 + C_1/3 \approx -0.5$ and $a_1 = C_1 + C_2/3 \approx 1$ correspond to the color suppressed and color allowed combination of Wilson coefficients (cf. Appendix A 4 and [35] for further discussion).

We now turn to the $\mathcal{O}_8$ contribution of the chromomagnetic operator (3). Its amplitude is parametrized as follows:

$$\mathcal{A}_i|_8 = \langle V\gamma|\mathcal{H}^{\rm eff}|_8|D\rangle = \frac{G_F}{\sqrt{2}}\left(\frac{em_c}{2\pi^2}\right)\frac{1}{c_V}\begin{cases}(C_8 + C_8')G_1(0) & i = 1\\ (C_8 - C_8')G_2(0) & i = 2\end{cases}, \quad (14)$$

where $\mathcal{H}^{\rm eff}|_8 = \frac{G_F}{\sqrt{2}}(C_8\mathcal{O}_8 + C_8'\mathcal{O}_8')$. Therefore, $G_{1,2}(0)$ corresponds to the matrix elements, with on shell photon $q^2 = 0$,

$$\langle V\gamma|\mathcal{O}_8^{(\prime)}|D\rangle = \left(\frac{em_c}{4\pi^2}\right)\frac{1}{c_V}(G_1(0)P_\perp \pm G_2(0)P_\|), \quad (15)$$

analogous to the penguin matrix element for T_1 and T_2 Eq. (C4), and $e = \sqrt{4\pi\alpha} > 0$ is the electromagnetic charge. In our notation, $G_{1(2)} = G_{\perp(\|)}$ but refrain to do so. Moreover, $G_1^{D^0\to\rho^0\gamma}(0) \approx G_1^{D^0\to\omega\gamma}(0)$, $G_1^{D_s^+\to K^{*+}\gamma}(0) \approx G_1^{D^+\to\rho^+\gamma}(0)$ to sufficient accuracy for our purposes and $G_1(0) = G_2(0)$ holds at twist-2 accuracy [28], which we employ for our estimates.[6] In particular, the imaginary part, relevant for the CP asymmetry, is found to be

$$\mathrm{Im}[G_1^{D_0}(0)] \approx -0.20(8), \quad \mathrm{Im}[G_1^{D^+}(0)] \approx -0.10(4), \quad (16)$$

where numbers were rounded. The values in (16) are sizeable compared to typical estimates $T_1^{D^0}(0) \approx T_1^{D^+}(0) \approx 0.7$ of the $\mathcal{O}_7$ operator as compiled in [17]. The difference

[5] The factor c_V is inserted to absorb trivial factors due to the wave function decomposition $\rho^0(\omega) \sim \frac{1}{\sqrt{2}}(\bar{u}u \mp \bar{d}d)$. $c_V = -\sqrt{2}$ for ρ^0 in $c \to d$, $c_V = \sqrt{2}$ in all other transitions into ω and ρ^0 and $c_V = 1$ otherwise. Note that in the overall CP asymmetry this factor will drop out.

[6] In fact, the ratio of the WA to the $G_1(0)$ form factor is well approximated by $R = r_\rho/r_\omega$ where $r_X = (f_X^\perp)/(m_X f_X^\|)$ is the ratio of the tensor to the vector decay constant. Information on this ratio exists only sparsely in the literature. Similar remarks apply to the $D_s^+ \to K^{*+}$ and $D^+ \to \rho^+$-transitions.

in the numerical value of neutral and charged matrix elements in Eq. (16) originate from different charges of the valence quarks of the mesons. Using the reference value for $\mathrm{Im}[C_8^{(\prime)}]$, the relevant ratios are around

$$\frac{|\mathcal{A}_{\perp,\parallel}|_8}{\mathcal{A}_{\perp,\parallel}|_{\mathrm{LD}}|} = \text{few} \times 10^{-3}, \tag{17}$$

and thus, the scale for direct CP violation is set at the subpercent level for the reference value (6).

B. Direct CP violation

Since the photon polarization is not easy to measure in practice a slightly inclusive rate $\Gamma[D \to V\gamma] = \Gamma[D \to V\gamma_L] + \Gamma[D \to V\gamma_R]$ is measured. We parametrize the corresponding amplitudes as follows:

$$\mathcal{A}_{L,R} = \mathcal{A}_\perp \pm \mathcal{A}_\parallel = l_{L,R} e^{i\delta_{L,R}} + g_{L,R} e^{i\Delta_{L,R}} e^{i\phi_{L,R}}, \tag{18}$$

with

$$\begin{aligned} l_{L(R)} &= |l_\perp \pm l_\parallel|, \qquad l_{\perp,\parallel} \equiv \mathcal{A}_{\perp,\parallel}|_{\mathrm{LD}} \\ g_{L(R)} e^{i\Delta_{L(R)}} &= \frac{G_F}{\sqrt{2}} \left(\frac{e m_c}{2\pi^2}\right) \frac{1}{c_V} |C_8^{(\prime)}| 2 G_{L,R}(0) \\ G_{L,R}(0) &= |G_{1,2}(0)| e^{i\Delta_{L,R}}, \\ C_8 &= |C_8| e^{i\phi_L} \quad C_8' = |C_8'| e^{i\phi_R}, \end{aligned} \tag{19}$$

where $\Delta_{L,R}$, $\delta_{L,R}$, and $\phi_{L,R}$ are the strong and the weak phase of (14), respectively, leaving the quantities $l_{L(R)}, g_{L(R)}$ real valued. In the equation above, we have made use of $G_1(0) = G_2(0)$, found at leading twist [28], implying that $\mathcal{O}_8$ and $\mathcal{O}_8'$ solely contribute to the left- and right-handed amplitude, respectively, and in addition, leads to $\Delta_L = \Delta_R$. The latter is not true when the contribution due to $\mathrm{Im}[C_7^{(\prime)}]$ is included, in which case, the formulas for $g_{L,R}$ have to be modified according to Eq. (B3) in Appendix B 2.

In the case where the two photon polarizations are not distinguished, the formula for CP violation is slightly more complicated than the one given in Eq. (B2). The general formulas and a derivation, including TDCP asymmetries, can be found in the Appendix of Ref. [36], for example. Using the corresponding standard formulas for the amplitude (18) yields

$$\begin{aligned} A_{CP}(D^0 \to V\gamma) &= \frac{-4}{n} (g_L l_L \sin(\Delta_L - \delta_L) \sin(\phi_L) \\ &\quad + \{L \leftrightarrow R\}), \\ n &\equiv 2(l_L^2 + 2(g_L l_L \cos(\Delta_L - \delta_L) \cos(\phi_L) g_L^2) \\ &\quad + \{L \leftrightarrow R\}). \end{aligned} \tag{20}$$

Assuming $l_{L(R)} \gg g_{L(R)}$ and imposing $\Delta \equiv \Delta_L = \Delta_R$, one gets

$$\begin{aligned} &A_{\mathrm{CP}}(D^0 \to V\gamma) \\ &\approx \frac{-2}{l_L^2 + l_R^2} (g_L l_L \sin(\Delta - \delta_L) \sin(\phi_L) + \{L \leftrightarrow R\}). \end{aligned} \tag{21}$$

In the absence of a computation, and in view of the chiral suppression at leading order, we set the LD phases $\delta_{L,R}$ (18) to zero in the remaining formulas, but it will be taken into account in the error budget. This allows us to express A_{CP} in terms of the quantities discussed at the beginning of the paper,

$$\begin{aligned} &A_{\mathrm{CP}}(D^0 \to V\gamma) \\ &= \frac{-4}{l_L^2 + l_R^2} \frac{G_F}{\sqrt{2}} \left(\frac{e m_c}{2\pi^2}\right) \frac{\mathrm{Im}[G_1(0)]}{c_V} (l_L \mathrm{Im}[C_8] + l_R \mathrm{Im}[C_8']). \end{aligned}$$

This formula, modulo notation, reduces to A_{CP} (B2) for $l_\perp = l_\parallel$ (i.e., $l_R = 0$).

With $m_c = 1.3$ GeV, Eqs. (5), (13), and (16), we get for the neutral transitions,

$$A_{\mathrm{CP}}(D^0 \to (\rho^0, \omega)\gamma) = \pm(3.0 \mathrm{Im}[C_8^{\mathrm{NP}}] + 1.6 \mathrm{Im}[C_8^{\prime\mathrm{NP}}]), \tag{22}$$

where the difference between ρ_0 and ω due to mass and decay constants is negligible compared to the estimated uncertainty of about 50% (to be discussed further below). In going from (21) to (22), we have used the fact that the imaginary part of C_8^{SM}, which contains the CKM prefactors, is negligible with respect to the values (5). For the charged transitions, we get

$$\begin{aligned} &A_{CP}(D^+_{(d,s)} \to (\rho, K^*)^+\gamma) \\ &= (4.6, 3.3) \mathrm{Im}[C_8^{\mathrm{NP}}] - (3.1, 2.0) \mathrm{Im}[C_8^{\prime\mathrm{NP}}], \end{aligned} \tag{23}$$

where we recall our reference value $\mathrm{Im}[C_8^{(\prime)\mathrm{NP}}] = 10^{-3}$ (6). Again the uncertainty is estimated to be about 50%. Note that the different sensitivity of ΔA_{CP}, $A_{\mathrm{CP}}(D^0 \to (\rho^0, \omega)\gamma)$ and $A_{\mathrm{CP}}(D^+_{(d,s)} \to (\rho^+, K^{*+})\gamma$ with respect to $\mathrm{Im}[C_8]$ and $\mathrm{Im}[C_8']$ gives a handle to discriminate between the individual contributions of the two chromomagnetic operators.

Let us turn to the discussion of the uncertainty. The major uncertainty comes from the estimate of the $\mathcal{O}_8$ matrix elements, which we estimate to be around 35% [28]. Then there is the phase of the WA contribution, $\delta_{L,R}$, for which we assign an uncertainty $|\delta_{L,R}| = 45°$ based on the estimate that the radiative corrections of WA could be of equal size as leading order with maximal 90° phase. Note that more than 90% degrees itself is again unrealistic since the rate suggests that the interference is not destructive. In summary, this leads to an uncertainty of approximately 30%. Amongst the LD contributions, the combination $l_L^2 + l_R^2$ is taken from experiment, but the ratio l_L/l_R that we took from [33] could have uncertainties, say, at the 20% level. Adding the three sources discussed above in quadrature, as they would seem uncorrelated, we get about 50% uncertainty. A few additional remarks are in order. In Appendix C 2, we estimate the SM contribution to be of the order of 10^{-4}, which is negligible. Furthermore, we refrain from including at this point the uncertainty due to the C_7 effect discussed in Appendix C 3. We would like to mention though that it cannot be excluded, depending on the model and the LD phase, that the C_7 and C_8 effect conspire to cancel significantly in the CP asymmetry.

C. Time-dependent *CP* violation

As a result of D^0-$\bar{D}^0$ oscillations, CP asymmetries are time dependent for the neutral meson, giving rise to novel features. In particular, TDCP asymmetries do not necessitate a strong phase difference in the two amplitudes. Thus, in principle, we have to adjust the amplitudes to include the C_7 effect, from $g_{L,R}$ Eqs. (18), (19) to $\tilde{g}_{L,R}$ (B3) as detailed in Appendix B 2. Indications are though that these effects are overshadowed by the dominance of the LD amplitudes $l_{L,R}$.

Important mixing parameters of the D^0-$\bar{D}^0$ system are the mass and width difference, the mixing phase ϕ_D as well as the ratio $|p/q|$ of the parameters p and q translating between the flavor and mass eigenstates. The latest HFAG values [4] are

$$x_D = \frac{\Delta m_D}{\Gamma} = 0.409(48) \times 10^{-2}, \qquad \left|\frac{p}{q}\right|_D \approx 1,$$
$$y_D = \frac{\Delta\Gamma_D}{2\Gamma} = 0.719(113) \times 10^{-2},$$
$$\phi_D \approx -13(13)(4)°[-6(11)(4)°], \tag{24}$$

where $\Gamma = (\tau_{D^0})^{-1}$ is the inverse lifetime of the D^0 mesons and Δm_D and $\Delta\Gamma_D$ are the difference of the heavy and the light D^0 meson mass and width, respectively. Above we did set the value for $|p/q| = 1$ as both the no direct CP allowed and direct CP allowed value are compatible with 1 within very small uncertainties. The value for ϕ_D we quote both values no direct CP allowed and direct CP allowed (in brackets). Assuming $|p/q|_D = 1$, the TDCP asymmetry assumes the following form:

$$A_{\rm CP}(D \to V\gamma)[t] = \frac{S \sin(\Delta m_D t) - C \cos(\Delta m_D t)}{\cosh(\frac{\Delta\Gamma_D}{2} t) - H \sinh(\frac{\Delta\Gamma_D}{2} t)}, \tag{25}$$

where the convention $A_{\rm CP}(0) = -C$ is somewhat awkward but standard. The formulas for S and H are given in Appendixes B 2 and C from the previous section. Let us define the LD chirality asymmetry (ratio) by

$$\chi_{\rm LD} \equiv \frac{l_\perp^2 - l_\parallel^2}{l_\perp^2 + l_\parallel^2} = \frac{2 l_L l_R}{l_L^2 + l_R^2} \in [-1, 1]. \tag{26}$$

With values as in (13), we get $\chi_{\rm LD} \approx 0.8(1)$. Thus, if we assume $\chi_{\rm LD} \gg 10^{-2}$, $l_{L,R} \gg \tilde{g}_{L,R}$, which both seem true, and once more set $\delta_{L,R} = 0$, we get an interesting expression for for H and S,

$$H[S] \approx \frac{2 l_L l_R}{l_L^2 + l_R^2} \cdot (-\xi \cos[\sin](\phi_D)) = \chi_{\rm LD} \cdot (-\xi \cos[\sin](\phi_D)), \tag{27}$$

which directly measures the ratio of the LD chirality structure times the cosine and sine of the mixing angle of the D^0 system. The variable $\xi = \pm 1$ is the CP eigenvalue of the V-meson whose values can be found in Appendix B 2. With $\xi(\rho^0, \omega) = 1$, we get

$$H[D^0 \to (\rho^0, \omega)\gamma] \approx -0.8(1) \cos(\phi_D),$$
$$S[D^0 \to (\rho^0, \omega)\gamma] \approx -0.8(1) \sin(\phi_D). \tag{28}$$

Let us emphasize once more that this relation is valid in the case where a left- and right-handed amplitude are comparable in size and dominate all the other contributions.

The experimental tractability of S and/or H depends on the angle ϕ_D. Should ϕ_D (24), that is to say $\sin \phi_D$, turn out to be sizeable then S could be measured as for $B \to K^*\gamma$ at the B factories. If $\cos \phi_D$ is sizeable, which is what the value in (24) indicates, then one would need to focus on H. The latter might be measured, in analogy to $B_s \to \phi\gamma$ case [36], in the rates $D^0 \to (\rho^0, \omega)\gamma$ and the one for $\bar{D}^0$ without flavor-tagging, which has experimental advantages, though it has to be added that the relatively small width difference in the D^0 system, $y_D/y_{B_s} \approx 0.1$, means that roughly a hundred times more data have to be accumulated to achieve the same precision on H in the D^0—as in the B_s system. We further refer the reader to the works [21,27], where some of these ideas have been extended to baryon decays $\Lambda_c \to p\gamma$ and 1^+ final state mesons ($D^0 \to K_1\gamma$), respectively. The 1^+-modes combined with the 1^--modes have the potential to discriminate between LD and SD *per se* [37].

IV. DISCUSSION AND CONCLUSIONS

Partly building up on ideas in [17] we have shown how $\mathrm{Im}[C_8]$ and $\mathrm{Im}[C_8']$ become observable in CP asymmetries in $D \to V\gamma$. Setting the LD phases $\delta_{L,R} = 0$ (18), in the absence of a computation, we got (22) and (23),

$$
\begin{aligned}
&A_{\rm CP}(D^0 \to (\rho^0,\omega)\gamma) \\
&\quad \approx \pm(3.0{\rm Im}[C_8^{\rm NP}] + 1.6{\rm Im}[C_8^{\prime\rm NP}]), \\
&A_{\rm CP}(D^+_{(d,s)} \to (\rho, K^*)^+\gamma) \\
&\quad \approx (4.6, 3.3){\rm Im}[C_8^{\rm NP}] - (3.1, 2.0){\rm Im}[C_8^{\prime\rm NP}], \qquad (29)
\end{aligned}
$$

where we recall our reference values ${\rm Im}[C_8^{(\prime)\rm NP}] = 10^{-3}$ (6). Uncertainties are in the 50% range, cf. Sec. III B. The SM contribution is negligible, down by an order of magnitude (cf. Appendix C 2). A useful aspect is that the Wilson coefficients of the two chiralities of the chromomagnetic operator enter with different sensitivity in (29), which has discrimination potential.

The chirality of the photon is an interesting aspect and deserves some discussion in comparing it to the b sector. In $b \to (d,s)\gamma$ transitions, the left-handed amplitude dominates over the right-handed amplitude as a result of the large b-quark mass and the $V - A$ interactions. This pattern might be broken by physics beyond the SM and can be measured in TDCP asymmetries [38]. The situation in $D^0 \to V\gamma$ is rather different. Whereas it is still true that the left-handed amplitude is larger than the right-handed amplitude, e.g., (13) it is not very significant since the c-quark mass is smaller. This neither-nor situation has consequences.

Since the amplitudes themselves are LD dominated the TDCP asymmetries are not sensitive to novel right-handed currents. However, TDCP asymmetries measure the LD chirality asymmetry $\chi_{\rm LD}$ (26) and thus, can provide interesting information on LD dynamics and could serve as validation criteria for theoretical tools. Let us add that the feasibility of the measurement depends on the definite value of the mixing phase ϕ_D (as commented on at the end Sec. III C).

On the speculative side, it is of course possible that NP contributes to SM or non-SM operators of the WA-type, $O^d_{1,2}$ (8)[7] possibly with new weak phases. Allowing for the latter and parametrizing a strong phase for the yet to be computed $\mathcal{O}(\alpha_s)$ corrections $l_{L(R)} \to l_{L(R)}e^{i\Phi_{L(R)}}$, one gets

$$
H[S] = \chi_{\rm LD} \cdot (-\xi \cos[\sin](\phi_D - \Phi_L - \Phi_R)\cos(\delta_L - \delta_R)), \qquad (30)
$$

and of course, $\chi_{\rm LD}$ is then affected by the NP and needs reevaluation.

At last, let us give an outlook and hint how the current work could be improved. On the experimental side, the measurement of the branching ratios of the three charged modes $D^+_d \to \rho^+\gamma$ and $D^+_s \to (\rho, K^*)^+\gamma$ would be helpful. This is the case since the corresponding Wilson coefficients are not color suppressed at LO and would thus allow us to assess the matrix elements themselves. On the theoretical side, it would benefit from $\mathcal{O}(\alpha_s)$ correction of the WA contributions. In particular, the radiative corrections would allow an estimate of the strong phase and the inclusion of the C_7 effect [17]. The prominence of WA in the isospin asymmetry in $b \to s$ processes provides yet another motivation for their reassessment. Furthermore, it might be interesting to extend this work from $D \to V\gamma$ to $D \to V\ell^+\ell^-$ as the latter might be easier to deal with at the LHCb, where the photon final state remains challenging at present.

In conclusion, charm physics is theoretically challenging and the situation with regards to new physics remains inconclusive in the sense that it is far from impossible that new physics is lurking in this sector. Charm physics in $b \to u$ and non-FCNC modes therefore deserves further study in our opinion.

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APPENDIX A: LONG-DISTANCE AMPLITUDES

This appendix is devoted to aspects of WA, which we argue is the dominant mechanism.

1. Theory: Weak annihilation vs quark loops

One may distinguish two types of LD contributions according to whether the quark level transitions is $c\bar{u} \to d\bar{d}$ or $c \to ud\bar{d}(s\bar{s})$. They can be generated by the weak operators $\mathcal{O}^{d,s}_{1,2}$ (8), for instance. From the viewpoint of quarks and gluons, the first type is known as WA (Fig. 1, left) and the quark loop (QL) (Fig. 1, center, right). The WA contributions have been computed $B, D \to V\gamma$ and $D \to V\gamma$ in [32,39,40] at $\mathcal{O}(\alpha_s^0)$. Note that the QL of the type shown in Fig. 1(center, right) are evaluated in an $1/m_c(1/m_b)$ expansion for $c(b) \to u(d,s)\gamma$, although in principle one could compute them in the exclusive case with LCSR, which is not based on a $1/m_c(1/m_b)$ expansion.

We advocate that WA dominates over QL for the following reason. QL and WA are generated by the same weak operator, $\mathcal{O}^{d,s}_{1,2}$ and $\mathcal{O}^d_{1,2}$ (8), respectively, yet the QL is down by two loops with respect to WA. This is the case because the single QL Fig. 1 (center) vanishes by gauge invariance. The reason therefore is that the photon polarization $\Pi_{\mu\nu}(q) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$ vanishes for $q^2 = 0$ when contracted with the photon polarization tensor.

[7]Note that in [15], it is the GIM combination (9), $O^d_{1,2} - O^s_{1,2}$, which is severely constrained through ϵ'/ϵ in new weak phases but not the individual operators $O^{d(s)}_{1,2}$ of down and strange *per se*.

Note that in addition, there is a GIM suppression of the QL, though not very effective for the matrix elements [32]. This suggests a natural hierarchy WA $\gg$ QL in the types of charm transitions discussed in this paper.[8]

Some confirmation can be found in B physics. That is taking numbers from [43] for WA and QL one gets $|\mathcal{A}_{\rm QL}/\mathcal{A}_{\rm WA}|_{B^-\to\rho^-\gamma} \approx 2\times 10^{-2}$.[9] To be more precise, for $\mathcal{A}_{\rm QL}$ we have taken the charm loop contribution where the gluon is radiated into the final state vector meson.[10]

Does this hierarchy remain intact for D physics? Despite the obvious fact that the $\alpha_s(m_c)$ expansion and the $1/m_c$ expansion are less trustworthy, it seems hard to see how a 2 order of magnitude hierarchy can be overthrown. Taking the contribution Fig. 1(right) for the QL from [32], which does rely on $1/m_c$-expansion, and the estimates of [33], one gets a number, $|\mathcal{A}_{\rm QL}/\mathcal{A}_{\rm WA}| \approx 2\times 10^{-2}$, which is somewhat accidentally close to the one for the $B^-\to\rho^-\gamma$.

2. Experiment: Weak annihilation vs quark loops

Let us turn to experiment. The known branching fractions are given by [29,31][11]

$$\mathcal{B}(D^0\to\{\rho^0,\phi,\bar{K}^{*0}\}\gamma) = \{1.77(32), 2.81(19), 41(7)\}\times 10^{-5}, \quad \text{(A1)}$$

with respective uncertainties of $\{17,7,17\}\%$, respectively, and the hierarchy is based on the Wolfenstein suppression of $\{\lambda,\lambda,\lambda^0\}$ at the amplitude level. A crucial feature is that only the $D^0\to\rho^0\gamma$ amplitude allows for a QL topology. Hence, by comparing the branching fraction rescaled by CKM-factors and wave function decomposition, one would expect to find values compatible with $SU(3)$-flavor symmetry. Let us check and define the auxiliary quantity,

$$x_V \equiv \frac{c_V^2}{\lambda_{\rm CKM} f_V^2 m_V^2}(|\mathcal{A}_\perp|^2 + |\mathcal{A}_\parallel|^2), \quad \text{(A2)}$$

where $c_{\rho^0} = \sqrt{2}$ (and unity otherwise) compensates for the wave function decomposition of the $\rho^0 \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$. Using (A1), one then gets

$$\frac{1}{x_\phi}\{x_{\rho^0}, x_\phi, x_{\bar{K}^{*0}}\} \approx \{1.65, 1.00, 0.97\}. \quad \text{(A3)}$$

Now, for $D^0\to\phi\gamma$ and $D^0\to\bar{K}^{*0}\gamma$ the major effect of $SU(3)$ seems to be carried by the decay constants. The $D^0\to\rho^0\gamma$ channel differs and indicates a 28% correction ($1.28^2\approx 1.65$) in the amplitude. We identify the following possible reasons therefore:

(1) It could be that after all, the doubly loop suppressed QL contribution is sizeable. One could argue that the color suppression of the WA process at LO essentially acts like a loop suppression. At the charm scale, a one loop correction could easily amount to 20%.

(2) The ρ^0 is a broad state, and it could be that it was not treated uniformly in the experiments of $\rho^0\to e^+e^-$ from where the decay constant is extracted versus the $D^0\to\rho^0\gamma$ measurement *per se*. Compare [34], where this aspect is stressed in the context of the form factor computation.

(3) There is just a single measurement of this mode, and confirmation by another facility would be most helpful. Although the Belle measurements of the other modes is in line with previous measurements (even though the K^* node is slightly higher).

We would think that point 1 is the most likely explanation, but ultimately we cannot tell.

3. LCSR vs weak annihilation from experiment

In this section, we pose the question whether LCSR can accommodate the $D^0\to V\gamma$ branching fractions quoted above. There are two parts to it, the matrix elements and the Wilson coefficients. In the neutral case, both are problematic at LO due to scale uncertainties. Let us discuss them one by one.

a. Matrix elements at leading order (in LCSR)

In the SM at LO in α_s WA is given by the so-called initial state radiation [cf. Fig. 1 (left)] as the emission from the final state is suppressed by the (light) quark masses. The former is then simply given by the $V_{\perp,\parallel}^{D\to\gamma}(m_V^2)$ transition form factor. This conclusion is also true in QCD

[8]In the approach in [30], the two transitions are modeled with hadronic data. We identify WA and QL with the pole (P) and the vector-meson dominance (VMD) part, respectively, The comparable numbers for P and VMD are not in line with the arguments above. (We further note that in [30] the P part receives no contribution in $\mathcal{A}_\parallel(\leftrightarrow A_{\rm PV})$, which is not reflected in the LCSR computation [33,39,41].) A possible issue is that the signs of the couplings of the VMD models are not known, that is to say only their absolute values can be inferred from experiment. Thus, the formalism might overestimate the contributions as it cannot capture cancellations, which gauge invariance suggests to be present. A similar point of view has been taken in [42] by one of the authors of [30] in Chap. 3.1.3.

[9]WA is Cabibbo suppressed with respect to QL in B physics. In comparing the WA and QL processes/diagrams, we, of course, do not take CKM hierarchies into account, especially because they are not present in the charm decays we are interested in.

[10]Note that WA for $B^0\to\rho^0\gamma$ is accidentally small because of cancellations between tree-level and penguin four quark operator contributions. We do not expect the same to take place for $D^0\to(\rho^0,\omega)\gamma$ since those cancellations are between tree and penguin four quark operator contributions and the latter are tiny in D physics.

[11]For $D^0\to\rho^0\gamma$, we have taken the value from Belle [31] as this is the single measurement, and it is somewhat unclear to us why [29] quotes $1.82(32)\times 10^{-5}$.

TABLE I. The Wolfenstein parameter is $\lambda \approx 0.23$, and the acronym cs stands for color suppressed. Decays 3 and 6 are not of the FCNC type, in the sense that they can be directly written in terms of tree W exchange. The O_8 column indicates whether they have O_8 matrix elements and are thus potentially CP violating in the context of this paper. The experimental value in the first row is for the ρ^0 case, as for the ω, only a bound exists at present [29]. Additionally, we refer the reader to Tables I and II in [21], where a comparison between different studies for the branching fractions has been made, which includes QCD factorization applied to charm decays.

No.	Decay	FCNC	Transition	O_8	CKM	cs	$\mathcal{B}(D \to V\gamma)$
1	$D^0 \to \rho^0(\omega)\gamma$	$c \to u$	$c\bar{u} \to d\bar{d}$	Yes	λ^1	yes	$1.77(32) \times 10^{-5}$ [31]
2	$D^0 \to \phi\gamma$	$c \to u$	$c\bar{u} \to s\bar{s}$	No	λ^1	Yes	$2.81(19) \times 10^{-5}$ [29]
3	$D^0 \to \bar{K}^{0,*}\gamma$	no	$c\bar{u} \to s\bar{d}$	No	λ^0	Yes	$4.1(7) \times 10^{-4}$ [29]
4	$D^+ \to \rho^+\gamma$	$c \to u$	$c\bar{d} \to u\bar{d}$	Yes	λ^1	No	6.4×10^{-6} this work
5	$D^+_s \to K^{*+}\gamma$	$c \to u$	$c\bar{s} \to u\bar{s}$	Yes	λ^1	No	1.7×10^{-5} this work
6	$D^+_s \to \rho^+\gamma$	no	$c\bar{s} \to u\bar{d}$	No	λ^0	No	2.1×10^{-4} this work

factorization. For the actual form factors, we take the analytic results of a NLO LCSR computation into account [33].[12,13] The values are collected in the main text in (13) as they constitute a new result.

b. Size of the (effective) Wilson coefficients

Let us consider the operators,

$$O^0 = \bar{c}\gamma_\mu(1-\gamma_5)D\bar{D}\gamma_\mu(1-\gamma_5)u,$$
$$O^+ = \bar{c}\gamma_\mu(1-\gamma_5)u\bar{D}\gamma_\mu(1-\gamma_5)D, \qquad \text{(A4)}$$

(with $D = d, s$) that govern the weak annihilation transition of $D^{0(+)}$ at LO in α_s. They relate to the combination of Wilson coefficients denoted by

$$H^{\text{eff}} \propto a_{2(1)}O^{0(+)}, \quad a_2 = C_1/3 + C_2, \quad a_1 = C_1 + C_2/3 \qquad \text{(A5)}$$

and are referred to as color suppressed and allowed, respectively. The values at the charm scale are $C_1 \approx 1.2$ and $C_2 \approx 0.4$, e.g., [10] (cf. [35] for a more elaborate discussion and analysis) which lead to $a_2 \approx 0$ and $a_1 \approx 1$. Their values at the electroweak scale are of course $C_1 = 1$ and $C_2 = 0$ and $a_2 = 1/3$ and $a_1 = 1$, respectively. One concludes that the renormalization group running for a_1 is moderate and can be trusted much to the contrary to a_2. Its value at the charm scale is absurdly small [compare $a_2(m_b) \approx 0.2$]. With such a steep running, it is clear that the radiative corrections are large. The value of $a_2 \approx -0.5$ in [35,41], as used in the main text, is meant to model this effect. Such values were fitted to experiment in other contexts.

This unsatisfactory situation could be improved by computing the radiative corrections to the WA matrix elements and/or measuring the charged modes, where radiative corrections can be expected to be more moderate. However, even for the charged modes, there is a twist in that for the photon emitted from the charged meson, which is the dominant process at LO, there is a large suppression between the charm and strange quark contribution [33]. This puts even more pressure on the community to compute WA at next-leading order in α_s.

c. Branching fractions and LCSR amplitude

One may subject the amplitude (12) with Wilson coefficients and form factors as described above to experiment. We do so by comparing $\mathcal{B}(D^0 \to \phi\gamma)$ (suitable as the ϕ is narrow), which shows that the LCSR predictions [33,41] differ by about a factor of 2. This is not a small effect, yet not impossible in view of experimental and theoretical uncertainties. Being pragmatic, we scale the neutral modes by the factor $\kappa_0 = 2$ (and $\kappa_+ = 1$ in the absence of better knowledge).[14] Such procedures are not ideal, but there is no other way at present.

4. The $D^+_{(s)} \to V\gamma$ branching fractions

Here, we give an overview of the main $D \to V\gamma$ modes collected in Table I. The neutral ones are all measured but not the charged ones. Here, we give reference

[12] Note this of course does not mean that WA is covered at NLO as it would involve the connection of a gluon between initial and final state quarks, which is laborious task.

[13] Of course it is interesting to compare to the earlier computation [41], which is though LO, whereas [33] includes radiative corrections and further higher twist corrections. The results in $V_\perp$ are comparable to [41] but a little lower. There are significant differences for $V_\parallel$ especially in the charged case. Differences are due to higher twist terms and in the charged case where the difference is largest, a factor of 3, this is further due to the nonsubtraction of the contact term.

[14] We note that the $D^0 \to \rho\gamma$ in QCD factorization would necessitate $\kappa_0 \approx 3$ when inspecting Table I in [21]. Since the same effective Wilson coefficients are assumed in our and their work, this means that our LCSR result is ca 50% larger than the QCD factorization contribution. This is well within the expected ballpark since already for b physics, the correction in $1/m_b$ are sizeable cf. Sec. 5 [28].

values using the values in (13), taken from [33] [and use $\kappa_+ = 1$ in (12) and $a_1 = 1$ as by above). With this input, we get the values quoted in Table I. The hierarchies are easily understood in our approximation, where the basic amplitude is degenerate,

$$\mathcal{B}(D^+ \to \rho^+\gamma) = \frac{\tau_{D^+}}{\tau_{D_s}} \mathcal{B}(D_s \to K^{*+}\gamma) = \lambda^2 \frac{\tau_{D^+}}{\tau_{D_s}} \mathcal{B}(D_s \to \rho^+\gamma). \quad (A6)$$

Above $\lambda \approx 0.266$, is the Wolfenstein parameter and the ratio of lifetimes is $\frac{\tau_{D^+}}{\tau_{D_s}} \approx 2$ because of the pion's wave function decomposition. The uncertainty at the amplitude level is easily 50% (improvable by an NLO computation as already mentioned a few times).

APPENDIX B: FORMULAS FOR *CP* VIOLATION

1. Formulas for direct *CP* violation

In this appendix, we collect some formulas which are useful throughout the text. We shall parametrize an amplitude as follows:

$$\mathcal{A}(D^0 \to f) = A_a e^{i\delta_a} e^{i\phi_a} + A_b e^{i\delta_b} e^{i\phi_b}, \quad (B1)$$

with weak (*CP* odd) phases ϕ and strong (*CP* even) phases δ separated to leave $A_{a,b}$ real. Note that in the SM, the decomposition (B1) is sufficient as one might use unitarity (9) to eliminate one amplitude to arrive at two amplitudes. Using the notation $\Delta \equiv \frac{A_a}{A_b}$, $\delta(\phi)_{ab} = \delta(\phi)_a - \delta(\phi)_b$, the *CP* asymmetry becomes

$$A_{\rm CP}[D^0 \to f] = \frac{-2\sin(\delta_{ab})\sin(\phi_{ab})\Delta}{1 + 2\Delta\cos(\delta_{ab})\cos(\phi_{ab}) + \Delta^2} \overset{\Delta \ll 1}{\approx} -2\sin(\delta_{ab})\sin(\phi_{ab})\Delta. \quad (B2)$$

In the second line, we have assumed a hierarchy between the amplitudes, which is the case for $D^0 \to (\rho^0, \omega)\gamma$ as studied in this paper.

2. Formulas for TDCP violation

The replacement due to the relevance of $\mathcal{O}_7$ as described in Sec. III C is as follows:

$$g_L e^{i\delta} e^{i\phi_L} \to \tilde{g}_L e^{i\Delta_L} e^{i\Phi_L} = \frac{G_F}{\sqrt{2}} \left(\frac{e m_c}{2\pi^2}\right) \frac{1}{c_V} [C_8(2G_1(0)) + C_7(2T_1(0))], \quad (B3)$$

and for g_R is given by the following replacements: $L \to R$ and $C_8, C_7 \to C_8', C_7'$. Note that unlike before we cannot assume a common strong phase as the ratios C_8/C_7 and C_8'/C_7' might not necessarily be the same. This is why the strong phase Δ carries a chirality label. The symbol Φ denotes the weak phase. The formulas for H and S in (25) are given, including a derivation, in the Appendix of Ref. [36][15] and take the following form:

$$\begin{aligned} H[S] = \frac{-4\xi}{n} (& l_L l_R \cos(\delta_L - \delta_R) \cos[\sin](\phi_D) \\ & + \tilde{g}_L \tilde{g}_R \cos(\Delta_L - \Delta_R) \cos[\sin](\phi_D - \Phi_L - \Phi_R) \\ & + (\tilde{g}_L l_R \cos(\Delta_L - \delta_R) \cos[\sin](\phi_D - \Phi_L) \\ & + \{L \leftrightarrow R\})), \end{aligned}$$

with

$$n \equiv 2(l_L^2 + 2(g_L l_L \cos(\Delta_L - \delta_L)\cos(\phi_L) g_L^2) + \{L \leftrightarrow R\}),$$

and where ξ is the *CP* eigenvalue of V. For $V = \{\rho, \omega, \phi, \bar{K}^*(\bar{K}_S \pi^0)\}$. the eigenvalue is $\xi = 1$, and for $V = \bar{K}^*(\bar{K}_L \pi^0)$, it is $\xi = -1$.

APPENDIX C: $A_{\rm CP}(D^0 \to V\gamma)$ OTHER THAN THROUGH $C_8^{\rm NP}$

For our discussion, it is convenient to write the amplitude as follows:

$$\mathcal{A} \approx \lambda_d e^{i\delta_d} A_d + \lambda_s e^{i\delta_s} A_s + \lambda_b e^{i\delta_b} A_b, \quad (C1)$$

which is similar to (B1) with the exception that the unitarity relation (9) has not been used and that the weak phases are contained within $\lambda_{d,s,b}$ (7). As argued in Appendix A, we expect the lion's share of A_d to be covered by WA which has, presumably, a small strong phase that we shall neglect ($\delta_d \to 0$). We assume a Wolfenstein parametrization up to order $\mathcal{O}(\lambda^5)$ which fulfils, e.g. [44],

$$\mathrm{Im}[\lambda_d] = 0, \quad \mathrm{Im}[\lambda_s] = A^2\lambda^5\eta, \quad \mathrm{Im}[\lambda_b] = -A^2\lambda^5\eta, \quad (C2)$$

where A, ρ, and η are the other three Wolfenstein parameters and $A^2\lambda^5\eta \approx 1.4 \times 10^{-4}$, Eq. (C2). The fact that $|\mathrm{Im}[\lambda_{b,s}]| \approx 1.4 \times 10^{-4}$ indicates small *CP* asymmetries,[16] of that order.

Thus, it remains to identify contributions with sizeable strong phases $\delta_{s,b}$ and amplitudes $A_{s,b}$ for which we see two

[15]Note that the different sign of H as with respect to [36]. This originates from the fact that $\Delta\Gamma_s$ in that reference is the light minus the heavy decay rate rather than the other way around as is assumed in the D^0 system. The reason for this difference in convention is that in each case, $\Delta\Gamma$ is chosen to be positive. Of course this sign is experimentally unobservable as only $\Delta\Gamma \times H^{2n+1}$ for integer n is observable.

[16]One might be tempted to say that if WA dominates by another 2 order of magnitudes, then this implies that the *CP* asymmetry is automatically below 10^{-5}. This is not correct as in this way of thinking the absolute value of λ_b should be factored into A_b and then $\mathrm{Im}[\lambda_b/|\lambda_b|] \approx \mathcal{O}(1)$ is not small any more.

major sources. First, the matrix element of $\mathcal{O}_8$, e.g., (16) [28] and second, the matrix element of $\mathcal{O}_2^{d,s}$ [32] [cf. Fig. 1 (right) for a contribution] giving rise, effectively, to an $\mathcal{O}_7$ operator. The latter as well as its matrix element analogous to (15) are defined and parametrized respectively, as follows:

$$\mathcal{O}_7^{(\prime)} \equiv -\frac{m_c e}{8\pi^2}\bar{u}\sigma_{\mu\nu}F^{\mu\nu}(1\pm\gamma_5)c \qquad \text{(C3)}$$

$$\langle V\gamma|\mathcal{O}_7^{(\prime)}|D\rangle = \left(\frac{em_c}{4\pi^2}\right)\frac{1}{c_V}(T_1(0)P_\perp \pm T_2(0)P_\parallel). \qquad \text{(C4)}$$

1. Effective Wilson coefficients $C^{\mathrm{eff}}_{7,8}(m_c)$

Let us state that we do not intend to give a critical review of the treatment of Wilson coefficients in the charm sector, e.g., of whether it makes sense to include light quarks into SD contributions evaluated in perturbation theory.[17] We shall simply follow the literature. It is fortunate that the SD contributions turn out to be subdominant in the SM.

The different contributions discussed above are conveniently discussed in terms of so-called effective Wilson coefficients. The latter consists of the pure Wilson coefficient $C_{7,8}(m_c)$ and matrix elements which can be rewritten in terms of $\mathcal{O}_{7,8}$, which we denote by $\delta C^{\mathrm{eff}}_{7,8}(m_c)$,

$$C^{\mathrm{eff}}_{7,8}(m_c) = C_{7,8}(m_c) + \delta C^{\mathrm{eff}}_{7,8}(m_c). \qquad \text{(C5)}$$

From a conceptual point of view, the Wilson coefficient can be divided into two further subparts,

$$C_{7,8}(m_c) = C^{(m_W)}_{7,8}(m_c) + C^{(m_b)}_{7,8}(m_c). \qquad \text{(C6)}$$

The notation above is nonstandard but hopefully useful for clarity. For the reminder of this section, we closely follow the notation of [45]. For C_8^{eff}, only $C_8^{(m_W)}(m_c) = \eta_c^{\frac{14}{25}}\eta_b^{\frac{14}{23}}C_8(m_W)$, $\eta_b = \alpha_s(m_W)/\alpha_s(m_b)$, and $\eta_c = \alpha_s(m_b)/\alpha_s(m_c)$, is known explicitly in the literature. For C_7^{eff}, all three parts are known which we shall quote, almost explicitly, below,

$$C_7^{(m_W)}(m_c) = \left[\eta_c^{\frac{16}{25}}\eta_b^{\frac{16}{23}}C_7(m_W) - \frac{16}{3}(\eta_c^{\frac{14}{25}}\eta_b^{\frac{14}{23}} - \eta_c^{\frac{16}{25}}\eta_b^{\frac{16}{23}})C_8(m_W)\right]$$
$$C_7^{(m_b)}(m_c) = -\lambda_b\sum_{i,j}C_j(m_b)X_{ji}\eta_c^{z_i}, \qquad \text{(C7)}$$

where $i = 1..8$, $j = 1..6$. Note that $C_7^{(m_W)}(m_c)$ describes the evolution directly from m_W to m_c and $C_7^{(m_b)}(m_c)$ originates from integrating out the b quark at the m_b scale and running from m_b to m_c. We hasten to add that the above expressions are given in the leading logarithm approximation. The term from the four quark matrix element is given by [32]

$$\delta C_7^{\mathrm{eff}}(m_c) = \frac{\alpha_s(m_c)}{4\pi}C_2(m_c)(\lambda_s f[(m_s/m_c)^2] + \lambda_d f[(m_d/m_c)^2]). \qquad \text{(C8)}$$

The strong phase results from the charmed meson's four momentum cutting the diagram through light quark lines. The contribution of $C_1(m_c)$ vanishes whereas the $C_{3,4,5,6}(m_c)$ have not been given but are small as they originate from SD contributions, which themselves are small. In fact, the numerical hierarchy is as follows [32]:

$$|C_7^{(m_W)}(m_c)| \approx 2\times 10^{-7} \ll |C_7^{(m_b)}(m_c)| \approx 8\times 10^{-6} \ll |\delta C_7^{\mathrm{eff}}(m_c)| = 5\times 10^{-3}. \qquad \text{(C9)}$$

The hierarchy between the first two was noted in [30] and numerically improved in [32]. The fact that matrix element dominates the Wilson coefficient was pointed out in [32]. The expression of $C_7^{(m_b)}(m_c)$ for operators other than $\mathcal{O}_{1,2}$ was given recently in Ref. [45]. As mentioned previously, we are not aware of explicit results for $C_8^{(m_b)}(m_c)$ and $\delta C_8^{\mathrm{eff}}(m_c)$ in the literature, yet they can be expected to be close to their C_7 counterparts as they differ only by color factors. Excluding cancellation effects, we would expect them to equal up to $\mathcal{O}(1/N_c)$ effects, say equal to about 30%–50%. Given the uncertainties of the estimates the approximations, $C_8^{(m_b)}(m_c) \approx C_7^{(m_b)}(m_c)$ and $\delta C_8^{\mathrm{eff}}(m_c) \approx \delta C_7^{\mathrm{eff}}(m_c)$, are good for our purposes.[18] Furthermore, with $C_8(m_c) \approx C_8^{(m_b)}(m_c) \approx C_7^{(m_b)}(m_c) \approx (-0.3+0.8i)\times 10^{-5}$, we see that the SM value is 2 to 3 orders of magnitude below the reference value $\mathrm{Im}[C_8^{\mathrm{NP}}] \approx 0.4\times 10^{-2}$.

2. $A_{\mathrm{CP}}(D^0 \to V\gamma)$ in the SM

In the SM, we identify three main sources contributing to the direct CP asymmetry: (a) $C_8(m_c) \approx C_8^{(m_b)}(m_c)$, (b) $\delta C_7^{\mathrm{eff}}(m_c)$, and (c) $\delta C_8^{\mathrm{eff}}(m_c)$. Right-handed operators $\mathcal{O}_{7,8}^{(\prime)}$ are negligible in the SM as Wilson coefficients as well as matrix elements are suppressed. As previously mentioned, we shall use $C_8(m_c) \approx C_7(m_c)$ for cases (a) and (c), which is good up to $1/N_c$ corrections. Note, as the leading LD amplitude is proportional to λ_d, it is only λ_s or λ_b that can contribute to the direct CP asymmetry.

[17] We are grateful to Ikaros Bigi and Ayan Paul to draw our attention to this point.

[18] Though the values $C_{7,8}^{(m_W)}(m_c)$ differ substantially for various reasons, this is of no concern as they are small.

(a) It is found that [32]

$$C_7^{(m_b)}(m_c) \approx 0.06\lambda_b \approx (0.3 - 0.8i) \times 10^{-5}, \quad \text{(C10)}$$

and assuming, as discussed above, $C_8^{(m_b)}(m_c) \approx C_7^{(m_b)}(m_c)$, we get that this contribution compares with $C_8^{\rm NP}$ in $A_{\rm CP}$ as follows:

$$\frac{0.06 \mathrm{Im}[\lambda_b]}{\mathrm{Im}[C_8^{\rm NP}]} \approx -0.2 \times 10^{-2}. \quad \text{(C11)}$$

(b) It is found that

$$\delta C_7^{\rm eff}(m_c) = (0.6 + 2.2i) \times 10^{-2}\lambda_s + c\lambda_d, \quad \text{(C12)}$$

where the imaginary part, other than λ_s, corresponds to a strong phase. The number c is of no importance for CP violation as it can be absorbed into WA, which is proportional to λ_d and much larger. The contribution $A_{\rm CP}$ compares with $C_8^{\rm NP}$ as follows:

$$\frac{\mathrm{Im}[\lambda_s]\mathrm{Im}[(0.6 + 2.2i) \times 10^{-2}]T_1(0)}{\mathrm{Im}[C_8^{\rm NP}]\mathrm{Im}[G_1(0)]} \approx -1 \times 10^{-2}, \quad \text{(C13)}$$

for reference values (5), $T_1(0) = 0.7$ and $\mathrm{Im}[G_1(0)] = -0.2$.

(c) As discussed above, we expect $\delta C_8^{\rm eff}(m_c) \approx \delta C_7^{\rm eff}(m_c)$, and this leads to a result for (c) with $\mathrm{Im}[G_1(0)]/T_1(0) \approx 2/7$ suppression factor as compared to (C13).

Summa summarum, the SM contributions is 1 order of magnitude below the values $\mathrm{Im}[C_8^{\rm NP}]$ (5) contribution and with the value in (22), we get

$$A_{\rm CP}|_{\rm SM}(D^0 \to (\rho^0, \omega)\gamma) \approx \left(-1.5\% \frac{1}{\sqrt{3}}\right)(-2 \times 10^{-2}) \approx 3\frac{1}{\sqrt{3}} \times 10^{-4}. \quad \text{(C14)}$$

We refrain from quoting a specific uncertainty. We would though think that the value catches the right order of magnitude. As possible criticisms, one could advocate, for example, the estimate $C_8^{(m_b)}(m_c) \approx C_7^{(m_b)}(m_c)$ and question the accuracy of local duality in (C12). The charged case is obtained by replacing $\mathrm{Im}[G_1^{D^0}] \to \mathrm{Im}[G_1^{D^+}]$ in (C13) and this would lead to $A_{\rm CP}|_{\rm SM}^{D^+} \approx 3.9\% \frac{1}{\sqrt{3}}(-3 \times 10^{-2}) \approx -1\frac{1}{\sqrt{3}} \times 10^{-3}$.

3. $A_{\rm CP}(D^0 \to V\gamma)$ via $\mathrm{Im}[C_7^{\rm NP}]$ and a strong LD phase

In Ref. [17], the idea was put forward that $C_8(m_{\rm NP})$ mixes into $C_7(m_c)$, e.g., Eq. (C7) for the SM evolution. More precisely, depending on the model and the scale of NP, $M_{\rm NP}$, it was put forward [17] that this leads to comparable values.[19] An important point is that $C_7(m_c)$ hardly affects $D^0 \to \pi\pi/KK$ because of α suppression and is therefore not constrained by the latter. Following [17], we shall assume only SM degrees of freedom below the scale $M_{\rm NP} = 1$ TeV and that the NP part of the Wilson coefficients is much larger than the SM part. Amending the notation of (C7) to include the running of six quarks above the top threshold, one gets

$$\begin{aligned} C_8^{(1\,\mathrm{TeV})}(m_c) &\approx 0.42 C_8(1\,\mathrm{TeV}), \\ C_7^{(1\,\mathrm{TeV})}(m_c) &\approx 0.37 C_7(1\,\mathrm{TeV}) - 0.26 C_8(1\,\mathrm{TeV}) \\ &\approx 0.37 C_7(1\,\mathrm{TeV}) - 0.62 C_8(m_c), \end{aligned}$$

and the analogous equations for the $\mathcal{O}'_{7,8}$ operators. Equation (C15) exposes the dependence of $C_7(m_c)$ on the scale $M_{\rm NP}$ and $C_7^{(\prime)}(M_{\rm NP})$. We shall somewhat arbitrarily choose the value $\mathrm{Im}[C_7^{(\prime)\rm NP}(m_c)] \approx -0.5 \mathrm{Im}[C_8^{(\prime)\rm NP}(m_c)]$ as a reference values. This follows the model dependent assumption $|\mathrm{Im}[C_7^{(\prime)}(1\,\mathrm{TeV})]| \ll |\mathrm{Im}[C_8^{(\prime)}(1\,\mathrm{TeV})]|$ in [17].

Since the $\mathcal{O}_7$ matrix element itself, as opposed to $\delta C_7^{\rm eff}$, does not carry a strong phase and the LD strong phase vanishes at leading order in the chiral limit, as discussed in Appendix A 3, we did not include this effect in our results [(22) and (23)]. In fact, we estimated that the phases could be around $|\delta_{L,R}| \approx 10°$, and we shall investigate how the CP asymmetry changes. It is then useful to rewrite the g_L amplitude as in (B3) with the replacement,

$$[C_8(2G_1(0)) + C_7(2T_1(0))] \to [2\mathrm{Im}[C_8](\underbrace{G_1(0) - 0.5T_1(0)}_{F_1})]. \quad \text{(C15)}$$

For $T_1(0) = 0.7$ and $G_1^{D^0}(0) \approx -0.2 - 0.2i \approx 0.3e^{-i135°}$ [28], one gets $F_1 \approx -0.55 - 0.2i = 0.7e^{-i160°}$. Thus, a correction of the LD phase $\delta_{L,R} = \pm 10°$ leads to a strong phase difference between the two amplitudes in the range of 10° to 30°, which corresponds to a rescaling of the CP asymmetry by factors $\sin(10°)/\sin(20°) \approx 0.5$

[19] Note that our normalization of $\mathcal{O}_7$ differs from [17] by a factor of Q_u, which translates into $Q_u C_7 = C_7^{IK}$, where IK stands for the authors of [17].

and $\sin(30°)/\sin(20°) \approx 1.5$, respectively. Thus, in conclusion, one cannot exclude the possibility that the phases conspire to cancel a significant part, or even an order of magnitude, of the effect. A lot of things have to go wrong for this to happen though. As discussed in Sec. A 3, an $\mathcal{O}(\alpha_s)$ computation would presumably give an indication of the sign of the LD phase as well as its size and would allow us to make firmer statements.

[1] R. Aaij *et al.* (LHCb Collaboration), Evidence for *CP* Violation in Time-Integrated $D^0 \to h^- h^+$ Decay Rates, Phys. Rev. Lett. **108**, 111602 (2012).

[2] T. Aaltonen *et al.* (CDF Collaboration), Measurement of CP–violating asymmetries in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ decays at CDF, Phys. Rev. D **85**, 012009 (2012).

[3] R. Aaij *et al.* (LHCb Collaboration), Observation of *CP* Violation in Charm Decays, Phys. Rev. Lett. **122**, 211803 (2019).

[4] Y. S. Amhis *et al.* (HFLAV Collaboration), Averages of b-hadron, c-hadron, and τ-lepton properties as of 2018, Eur. Phys. J. C **81**, 226 (2021).

[5] M. T. Hansen and S. R. Sharpe, Multiple-channel generalization of Lellouch-Luscher formula, Phys. Rev. D **86**, 016007 (2012).

[6] I. I. Bigi, A. Paul, and S. Recksiegel, Conclusions from CDF results on *CP* violation in $D^0 \to \pi^+\pi^-, K^+K^-$ and future tasks, J. High Energy Phys. 06 (2011) 089.

[7] A. N. Rozanov and M. I. Vysotsky, *CP* violation in D-meson decays and the fourth generation, arXiv:1111.6949.

[8] H.-Y. Cheng and C.-W. Chiang, Direct *CP* violation in two-body hadronic charmed meson decays, Phys. Rev. D **85**, 034036 (2012); **85**, 079903(E) (2012).

[9] H.-n. Li, C.-D. Lu, and F.-S. Yu, Branching ratios and direct *CP* asymmetries in $D \to PP$ decays, Phys. Rev. D **86**, 036012 (2012).

[10] J. Brod, A. L. Kagan, and J. Zupan, Size of direct *CP* violation in singly Cabibbo-suppressed D decays, Phys. Rev. D **86**, 014023 (2012).

[11] B. Bhattacharya, M. Gronau, and J. L. Rosner, *CP* asymmetries in singly-Cabibbo-suppressed *D* decays to two pseudoscalar mesons, Phys. Rev. D **85**, 054014 (2012).

[12] T. Feldmann, S. Nandi, and A. Soni, Repercussions of flavour symmetry breaking on *CP* violation in D-meson decays, J. High Energy Phys. 06 (2012) 007.

[13] E. Franco, S. Mishima, and L. Silvestrini, The Standard Model confronts *CP* violation in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$, J. High Energy Phys. 05 (2012) 140.

[14] M. Golden and B. Grinstein, Enhanced *CP* violations in hadronic charm decays, Phys. Lett. B **222**, 501 (1989).

[15] G. Isidori, J. F. Kamenik, Z. Ligeti, and G. Perez, Implications of the LHCb evidence for charm *CP* violation, Phys. Lett. B **711**, 46 (2012).

[16] G. F. Giudice, G. Isidori, and P. Paradisi, Direct *CP* violation in charm and flavor mixing beyond the SM, J. High Energy Phys. 04 (2012) 060.

[17] G. Isidori and J. F. Kamenik, Shedding Light on *CP* Violation in the Charm System via D to V Gamma Decays, Phys. Rev. Lett. **109**, 171801 (2012).

[18] Y. Grossman, A. L. Kagan, and J. Zupan, Testing for new physics in singly Cabibbo suppressed D decays, Phys. Rev. D **85**, 114036 (2012).

[19] Y. Grossman, A. L. Kagan, and Y. Nir, New physics and *CP* violation in singly Cabibbo suppressed D decays, Phys. Rev. D **75**, 036008 (2007).

[20] G. Hiller, Y. Hochberg, and Y. Nir, Supersymmetric ΔA_{CP}, Phys. Rev. D **85**, 116008 (2012).

[21] S. de Boer and G. Hiller, Rare radiative charm decays within the standard model and beyond, J. High Energy Phys. 08 (2017) 091.

[22] C. Delaunay, J. F. Kamenik, G. Perez, and L. Randall, Charming *CP* violation and dipole operators from RS flavor anarchy, J. High Energy Phys. 01 (2013) 027.

[23] B. Keren-Zur, P. Lodone, M. Nardecchia, D. Pappadopulo, R. Rattazzi, and L. Vecchi, On partial compositeness and the *CP* asymmetry in charm decays, Nucl. Phys. **B867**, 394 (2013).

[24] T. Mannel and N. Uraltsev, Charm *CP* violation and the electric dipole moments from the charm scale, J. High Energy Phys. 03 (2013) 064.

[25] S. Fajfer and N. Košnik, Resonance catalyzed *CP* asymmetries in $D \to P\ell^+\ell^-$, Phys. Rev. D **87**, 054026 (2013).

[26] L. Cappiello, O. Cata, and G. D'Ambrosio, Standard Model prediction and new physics tests for $D^0 \to h^+h^-\ell^+\ell^-(h = \pi, K{:}\,\ell = e, \mu)$, J. High Energy Phys. 04 (2013) 135.

[27] N. Adolph, G. Hiller, and A. Tayduganov, Testing the standard model with $D_{(s)} \to K_1(\to K\pi\pi)\gamma$ decays, Phys. Rev. D **99**, 075023 (2019).

[28] M. Dimou, J. Lyon, and R. Zwicky, Exclusive chromomagnetism in heavy-to-light FCNCs, Phys. Rev. D **87**, 074008 (2013).

[29] P. A. Zyla *et al.* (Particle Data Group Collaboration), Review of particle physics, Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).

[30] G. Burdman, E. Golowich, J. L. Hewett, and S. Pakvasa, Radiative weak decays of charm mesons, Phys. Rev. D **52**, 6383 (1995).

[31] A. Abdesselam *et al.* (Belle Collaboration), Observation of $D^0 \to \rho^0\gamma$ and Search for *CP* Violation in Radiative Charm Decays, Phys. Rev. Lett. **118**, 051801 (2017).

[32] C. Greub, T. Hurth, M. Misiak, and D. Wyler, The c—> u gamma contribution to weak radiative charm decay, Phys. Lett. B **382**, 415 (1996).

[33] T. Janowski, B. Pullin, and R. Zwicky, Charged and neutral $\bar{B}_{u,d,s} \to \gamma$ form factors from light cone sum rules at NLO, J. High Energy Phys. 12 (2021) 008.

[34] A. Bharucha, D. M. Straub, and R. Zwicky, $B \to V\ell^+\ell^-$ in the Standard Model from light-cone sum rules, J. High Energy Phys. 08 (2016) 098.

[35] A. J. Buras, QCD factors a1 and a2 beyond leading logarithms versus factorization in nonleptonic heavy meson decays, Nucl. Phys. **B434**, 606 (1995).

[36] F. Muheim, Y. Xie, and R. Zwicky, Exploiting the width difference in $B_s \to \phi\gamma$, Phys. Lett. B **664**, 174 (2008).

[37] J. Gratrex and R. Zwicky, Parity doubling as a tool for right-handed current searches, J. High Energy Phys. 08 (2018) 178.

[38] D. Atwood, M. Gronau, and A. Soni, Mixing Induced *CP* Asymmetries in Radiative B Decays in and beyond the Standard Model, Phys. Rev. Lett. **79**, 185 (1997).

[39] A. Ali and V. M. Braun, Estimates of the weak annihilation contributions to the decays B—> rho + gamma and B—> omega + gamma, Phys. Lett. B **359**, 223 (1995).

[40] J. Lyon and R. Zwicky, Isospin asymmetries in $B \to (K^*, \rho)\gamma/l^+l^-$ and $B \to Kl^+l^-$ in and beyond the standard model, Phys. Rev. D **88**, 094004 (2013).

[41] A. Khodjamirian, G. Stoll, and D. Wyler, Calculation of long distance effects in exclusive weak radiative decays of B meson, Phys. Lett. B **358**, 129 (1995).

[42] G. Burdman and I. Shipsey, D^0-$\bar{D}^0$ mixing and rare charm decays, Annu. Rev. Nucl. Part. Sci. **53**, 431 (2003).

[43] P. Ball, G. W. Jones, and R. Zwicky, $B \to V\gamma$ beyond QCD factorisation, Phys. Rev. D **75**, 054004 (2007).

[44] J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, England, 2014), Vol. 2.

[45] A. Paul, I. I. Bigi, and S. Recksiegel, On $D \to X_u l^+ l^-$ within the standard model and frameworks like the littlest Higgs model with T parity, Phys. Rev. D **83**, 114006 (2011).