Anyonic correlation functions in Chern-Simons matter theories

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Using a novel relation between the parity-even and -odd parts of a correlator, we show that the three-point function of conserved or weakly broken currents in three-dimensional conformal field theory (CFT) can be obtained from just the free fermion (FF) or the free boson (FB) theory. In the special case of large N Chern-Simons matter theories, we obtain the correlator in terms of a coupling constant dependent "anyonic phase" factor. This anyonic phase factor was previously obtained in the $2 \to 2$ exact S-matrix result and is consistent with strong-weak duality. By varying the coupling constant, the CFT correlator interpolates nicely between the same in the FF and FB theories.

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I. INTRODUCTION

Three-dimensional conformal field theory (CFT) finds important applications in diverse branches of physics such as cosmology [1–5], condensed matter physics [6–8], etc. They also play an important role in the study of various dualities such as those between CFTs and higher-spin Vasiliev theories [9–13] and Aharony-Bergman-Jafferis-Maldacena duality in the context of AdS/CFT correspondence [14]. One of the important quantities to be computed in a CFT is the correlation function of various operators. While position space CFT correlation functions are quite well studied, the same in momentum space is relatively less explored. See Refs. [4,15–20] for recent progress in momentum space three-point correlator results. Although relatively recent and less explored as compared to position space, the study of momentum space CFT correlators has led to the understanding of a lot of previously unknown structures of conformal correlators such as the double copy relations [21–23]. In this paper, we make use of yet another interesting feature of momentum space CFT correlators.

Conformal correlators comprising exactly conserved currents in three dimensions generally have two parity-even and one parity-odd structures, which have been constructed explicitly in position space [24,25]. The parity-even structures can be obtained from free bosonic and free fermionic theories, whereas the parity-odd structure in general arises in an interacting theory which violates parity, such as Chern-Simons (CS) matter theories [26,27]. A direct computation of these correlators using Feynman diagrams in Chern-Simons matter theories is complicated and has been done for only a few specific correlators in specific kinematic regimes in momentum space [28–30]. Momentum space parity-even and parity-odd three-point correlators comprising arbitrary higher-spin currents were computed recently in Refs. [20,31]. In Ref. [33], helicity structures of three-point spinning correlation functions for higher-spin currents and their relation to bulk anti-de Sitter (AdS) couplings were discussed [34].

In this paper, using results from direct computation of the parity-even part of the correlator from free boson (FB) and free fermion (FF) theories and the conformal Ward identity, we relate the parity-odd part of the CFT correlator to the parity-even part from the FB or FF theory. This relation in spinor-helicity variables can be used to express the three-point function of conserved or weakly broken higher-spin currents in three-dimensional (3D) CFTs in terms of either the free bosonic or free fermionic theory answers. Remarkably, in the special case of CS matter theory at large N, we show that the full three-point correlator is given by either the FB theory or the FF theory with an appropriate anyonic phase factor which nicely interpolates between the correlator in the FB and FF theories. Explained another way, we can start with the correlator in the FF theory, multiply with an appropriate anyonic phase which gives the correlator in the CS matter theory which has a parity-odd part as well, and then we can tune that phase to go all way to the FB theory correlator. Interestingly, the same anyonic phase factor was observed in the calculation of the all loop $2 \to 2$ S-matrix in Chern-Simons matter theories [35,36], and it also appeared in the context of nonrelativistic Aharanov-Bohm scattering [37,38].

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We first introduce some necessary background details.

II. SOME BACKGROUND DETAILS

Three-point functions of conserved or weakly broken higher-spin currents [39] in a generic 3D CFT can be written as the combination of three independent structures: coming from the FB theory, the FF theory, and a parity-odd term [24,25],

$$\langle J_{s_1}J_{s_2}J_{s_3}\rangle = n_B\langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\mathrm{FB}} + n_F\langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\mathrm{FF}} + n_{\mathrm{odd}}\langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\mathrm{odd}}. \quad (1)$$

Let us emphasize here that the correlators in the FB and FF theories are parity even and are independent, whereas the parity-odd part cannot be obtained from a free theory and in general takes a complicated form as was shown in position space in Ref. [24]. However, we show working in momentum or spinor-helicity variables that all the three structures in (1) can be obtained from just the FB theory or the FF theory, and we apply this result to the special case of CS matter theories. Before doing so, let us very briefly review some of the background details.

The FB theory that we consider is given by

$$S = \int d^3x \partial^\mu\bar{\phi}\partial_\mu\phi, \quad (2)$$

where ϕ is a massless scalar field in the fundamental representation of $\mathrm{SU}(N_b)$. The operator spectrum of single-trace primary operators in the theory consists of a scalar primary $O = \bar{\phi}\phi$ with scaling dimension 1 and spin-s currents with scaling dimension $s+1$. One also defines a critical bosonic theory by Legendre transforming the FB theory with respect to the scalar operator O. More precisely,

$$S = \int d^3x[\partial_\mu\bar{\phi}\partial^\mu\phi + \sigma_B\bar{\phi}\phi], \quad (3)$$

where σ_B is an auxiliary field. The conformal dimension of the scalar primary operator [40] for this case is $\Delta = 2 + O(\frac{1}{N_b})$.

The FF theory that we consider is given by

$$S = \int d^3x\bar{\psi}\gamma^\mu\partial_\mu\psi, \quad (4)$$

where ψ is a massless fermion field in the fundamental representation of $\mathrm{SU}(N_f)$. The operator spectrum of single-trace primary operators in the theory consists of a scalar primary $O = \bar{\psi}\psi$ with scaling dimension 2 and is odd under parity. Other primary operators are conserved spin-s currents with scaling dimension $s+1$. Similar to the critical boson (CB) theory (3), one can also define the critical fermion (CF) theory. For details, see Ref. [41].

Another class of theories that we consider is Chern-Simons gauge field at level κ_f coupled to matter at large N. For example, the fermionic theory coupled to $SU(N_f)$ Chern-Simons gauge field has the following action:

$$S = \int d^3x\left[\bar{\psi}\gamma_\mu D^\mu\psi + i\epsilon^{\mu\nu\rho}\frac{\kappa_f}{4\pi}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right)\right]. \quad (5)$$

The scalar primary operator has conformal dimension $\Delta = 2 + O(\frac{1}{N})$ [42]. The spin-1 and spin-2 conserved currents have dimensions 2 and 3, respectively. The theory also has an infinite tower of higher-spin currents J_s with spin $s > 2$ that are weakly broken with conformal dimension $\Delta = s + 1 + O(\frac{1}{N})$ [25,43]. At large N_f and κ_f, the t'Hooft coupling is defined as

$$\lambda_f = \lim_{N_f,\kappa_f\to\infty}\frac{N_f}{k_f}. \quad (6)$$

One can also define bosonic theory coupled to $SU(N_b)$ Chern-Simons gauge field at level κ_b, CF theory coupled to CS gauge field, and CB theory coupled to CS gauge field; see Ref. [41] for details, [44]. The CS gauge theory coupled to matter at large N has a remarkable property that it shows strong-weak duality [25–28,45–47]. For example, a fermion coupled to CS gauge field in (5) is dual to CB coupled to CS gauge field. In Ref. [25], these two theories were together named quasifermion (QF) theory. The other dual pair, scalar coupled to CS gauge field and CF coupled to CS gauge field, is called quasiboson (QB) theory. In Ref. [25], three-point functions in these classes of theories were calculated. For example, in the notation of (1) for the QF theory, it was shown that [25]

$$n_F = \tilde{N}\frac{1}{1+\tilde{\lambda}^2}, \qquad n_B = \tilde{N}\frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2}, \qquad n_{\mathrm{odd}} = \tilde{N}\frac{\tilde{\lambda}}{1+\tilde{\lambda}^2}, \quad (7)$$

where for the specific case of CS gauge field coupled to fermion (5) we have

$$\tilde{N} = N_f\frac{\sin(\pi\lambda_f)}{\pi\lambda_f}, \qquad \tilde{\lambda} = \tan\left(\frac{\pi\lambda_f}{2}\right). \quad (8)$$

Having reviewed some basics, let us now move on to the calculation of correlation functions. Before turning our attention to three-point functions, let us first focus on two-point functions.

III. TWO-POINT FUNCTIONS

In this section, we consider two-point functions of spinning operators in spinor-helicity variables. In general, the two-point function of conserved currents can have parity-even and parity-odd contributions

$$\langle J_s J_s\rangle = c_s^{\rm even}\langle J_s J_s\rangle_{\rm even} + c_s^{\rm odd}\langle J_s J_s\rangle_{\rm odd}. \tag{9}$$

For simplicity, let us first consider the two-point function of spin-1 current

$$\langle JJ\rangle = c_1^{\rm even}\langle JJ\rangle_{\rm even} + c_1^{\rm odd}\langle JJ\rangle_{\rm odd}. \tag{10}$$

In spinor-helicity variables, we get [48]

$$\langle J^-(k_1)J^-(-k_1)\rangle = (c_1^{\rm even} + ic_1^{\rm odd})\frac{\langle 12\rangle^2}{16\pi k_1}. \tag{11}$$

We introduce $c_1^{\rm even} + ic_1^{\rm odd} = |c_J|e^{i\pi\theta}$ to express the above as

$$\langle J^-(k_1)J^-(-k_1)\rangle = |c_J|e^{i\pi\theta}\frac{\langle 12\rangle^2}{16\pi k_1}. \tag{12}$$

The other nonzero helicity component $\langle J^+J^+\rangle$ can be obtained by a simple complex conjugation of the above result.

Let us now consider the special case of CS gauge field coupled to fermion (5). For this case, we have

$$\langle JJ\rangle_{\rm F+CS} = \frac{N\sin\pi\lambda_f}{16\pi\lambda_f}\langle JJ\rangle_{\rm even} + i\frac{N(\cos\pi\lambda_f - 1)}{16\pi\lambda_f}\langle JJ\rangle_{\rm odd}. \tag{13}$$

Let us note that the parity-odd contribution $\langle JJ\rangle_{\rm odd}$ is a contact term. As was argued in Refs. [28,29], contact terms are scheme dependent and can be shifted up to an integer using appropriate counter-terms. In this case, the contact term corresponds to $\frac{i\kappa_f}{4\pi}\int \mathcal{A}\wedge d\mathcal{A}$, where κ is an integer. Using this, one can shift away the following term from (13) [49]:

$$-N\frac{i}{16\pi\lambda_f}\langle JJ\rangle_{\rm odd}. \tag{14}$$

This gives

$$\langle JJ\rangle_{\rm F+CS} = \frac{N\sin\pi\lambda_f}{16\pi\lambda_f}\langle JJ\rangle_{\rm even} + i\frac{N\cos\pi\lambda_f}{16\pi\lambda_f}\langle JJ\rangle_{\rm odd}. \tag{15}$$

In spinor-helicity variables, this leads to the following nonzero components [50]:

$$\langle J^-J^-\rangle_{\rm F+CS} = -\frac{iNe^{-i\pi\lambda_f}\langle 12\rangle^2}{32\pi\lambda_f k_1}. \tag{16}$$

The above result readily generalizes to two-point functions of arbitrary spin-s conserved currents J_s:

$$\langle J_s^- J_s^-\rangle_{\rm F+CS} = -\frac{iNe^{-i\pi\lambda_f}\langle 12\rangle^{2s}}{32\pi\lambda_f k_1}. \tag{17}$$

We note that the coefficient is independent of the spin-s of the operator, i.e., $c_{s_1} = c_{s_2}$, where c_{s_i} is the two-point function coefficient. This follows as a result of higher-spin symmetry. Let us note the presence of the factor $e^{-i\pi\lambda_f}$ in (17), which we term as an anyonic phase factor. A similar result holds for boson coupled to CS gauge field as well, for which case we instead have $e^{-i\pi\lambda_b}$.

Although we have only discussed the case with fermion coupled to gauge field or boson coupled to gauge field, it easily generalizes to the critical theories in QF and QB theories. Even though the two-point function is trivial, it sets the stage for a discussion on three-point functions. Turning our attention to three-point functions, we show that the full three-point function in QF theory can be obtained by appropriately multiplying the same anyonic phase factor to the three-point function in the FF or FB theory. The term "anyonic phase" will also become much more transparent. In contrast to two-point functions, for three-point functions, the parity-odd term is not a contact term and in general takes a complicated form in position space.

IV. THREE-POINT FUNCTIONS

In three-dimensional CFTs, we can split three-point functions into homogeneous **h** and nonhomogeneous **nh** pieces. This is based on the action of the special conformal generator on a generic three-point correlator. This is given by

$$\tilde{K}^\kappa\left\langle \frac{J_{s_1}}{k_1^{s_1-1}}\frac{J_{s_2}}{k_2^{s_2-1}}\frac{J_{s_3}}{k_3^{s_3-1}}\right\rangle = \text{transverse Ward identity terms}. \tag{18}$$

The terms that arise from the transverse Ward identities are contact terms, which can be expressed in terms of two-point functions.

The general solution of the above differential equation is given by the sum of homogeneous and nonhomogeneous solutions,

$$\langle J_{s_1}J_{s_2}J_{s_3}\rangle = \langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\bf h} + \langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\bf nh}, \tag{19}$$

where $\langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\bf h}$ solves

$$\tilde{K}^\kappa\left\langle \frac{J_{s_1}}{k_1^{s_1-1}}\frac{J_{s_2}}{k_2^{s_2-1}}\frac{J_{s_3}}{k_3^{s_3-1}}\right\rangle_{\bf h} = 0 \tag{20}$$

and $\langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\bf nh}$ is a solution of

$$\tilde{K}^{\kappa}\left\langle \frac{J_{s_1}}{k_1^{s_1-1}}\frac{J_{s_2}}{k_2^{s_2-1}}\frac{J_{s_3}}{k_3^{s_3-1}}\right\rangle_{\mathbf{nh}} = \text{transverse Ward identity terms.} \tag{21}$$

Under the action of the special conformal generator in spinor-helicity variables, the nonhomogeneous piece contributes to the Ward-Takahashi (WT) identity, whereas the homogeneous piece goes to zero. This implies that the nonhomogeneous piece is proportional to the two-point function coefficient. See Ref. [20] for a detailed discussion.

One can check that $\langle J_{s_1}J_{s_2}J_{s_3}\rangle$ in FB and FF theories satisfy the same WT identity [51,52], which implies their nonhomogeneous contribution should be the same [53]. The difference in the values of the two correlators should then arise from the difference in their homogeneous terms. From several explicit examples, one can show that the homogeneous terms differ only up to a sign; i.e., the homogeneous contribution is always uniquely determined up to theory dependent coefficient for a given correlator. Thus, consistent with the WT identity, one has

$$\begin{aligned}\langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\rm FB} &= \langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\mathbf{nh}} + \langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\mathbf{h}}\\ \langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\rm FF} &= \langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\mathbf{nh}} - \langle J_{s_1}J_{s_2}J_{s_3}\rangle_{\mathbf{h}},\end{aligned} \tag{22}$$

which can also be shown to be consistent with the representations of correlators in terms of conformal invariants in position space in Ref. [54]. For a detailed discussion, see Ref. [55]. Let us emphasize here that the homogeneous and nonhomogeneous pieces that appear in free bosonic and free fermionic theory are the same [56].

Let us take an illustrative example. Let us consider the three-point function of the stress-tensor $\langle TTT\rangle$ in a generic CFT. Using (1) and (22), we get

$$\begin{aligned}\langle TTT\rangle = (n_B+n_F)\langle TTT\rangle_{\mathbf{nh}} + (n_B-n_F)\langle TTT\rangle_{\mathbf{h}}\\ + n_{\rm odd}\langle TTT\rangle_{\rm odd}.\end{aligned} \tag{23}$$

The parity-odd part of the correlator is homogeneous. The nonhomogeneous contribution has been shown to be a contact term [20,55,57]. In spinor-helicity variables, it can be checked that the parity-odd part of the correlator and the homogeneous even contribution to the correlator are proportional [20,55,57]. For example, for the three-point function of the stress tensor, one has

$$\begin{aligned}\langle T^-T^-T^-\rangle_{\rm odd} &\propto i\langle T^-T^-T^-\rangle_{\mathbf{h}}\\ \langle T^+T^+T^+\rangle_{\rm odd} &\propto -i\langle T^+T^+T^+\rangle_{\mathbf{h}},\end{aligned} \tag{24}$$

and the remaining helicity components are zero. The normalization that we have chosen to work with is particularly suitable for discussion on CS matter theories and has been carefully fixed by demanding consistency with higher-spin equations. It gives

$$\langle T^-T^-T^-\rangle_{\rm odd} = 2i\langle T^-T^-T^-\rangle_{\mathbf{h}}. \tag{25}$$

The other nonzero helicity component for the parity-odd part is $\langle T^+T^+T^+\rangle_{\rm odd}$ [58], which is obtained by complex conjugating (25). Using (25) in (23), we obtain

$$\begin{aligned}\langle T^-T^-T^-\rangle &= (n_B+n_F)\langle T^-T^-T^-\rangle_{\mathbf{nh}}\\ &\quad + (n_B-n_F+2in_{\rm odd})\langle T^-T^-T^-\rangle_{\mathbf{h}}\\ &= (n_B+n_F)(\langle T^-T^-T^-\rangle_{\mathbf{nh}} - \gamma_T e^{-i\pi\theta}\langle T^-T^-T^-\rangle_{\mathbf{h}})\\ &= c_T^{\rm even}(\langle T^-T^-T^-\rangle_{\mathbf{nh}} - \gamma_T e^{-i\pi\theta}\langle T^-T^-T^-\rangle_{\mathbf{h}}),\end{aligned} \tag{26}$$

where we have defined $\gamma_T e^{i\pi\theta} = \frac{n_F-n_B}{n_B+n_F} - 2i\frac{n_{\rm odd}}{n_B+n_F}$. Using the Ward-Takahashi identity, it can be shown that [59] $c_T^{\rm even} = n_B + n_F$ where $c_T^{\rm even}$ is given by

$$\langle T(k_1)T(k_2)\rangle_{\rm even} = c_T^{\rm even}(z_1\cdot z_2)^2 k_1^3 \tag{27}$$

as in (9). The positive helicity components can be obtained by a complex conjugation. The mixed helicity components of the correlator only contains the nonhomogeneous piece which is exactly the same as the free theory correlator up to two-point function coefficients [60]. Let us note that in (26), when we take $\gamma_T e^{-i\pi\theta} = 1$, i.e., $\gamma_T = 1$ and $\theta = 0$, we get the FF theory and when $\gamma_T e^{-i\pi\theta} = -1$, i.e., when $\gamma_T = 1$ and $\theta = \pi$, we get the FB theory consistent with (22). In both cases, we just have parity-even contribution. For any other value of θ, we get the parity-odd term as well. It is interesting to note that (26) is valid for any generic CFT and is written entirely in terms of **nh** and **h** pieces which can be obtained from either the FB or the FF theory.

This result takes a particularly interesting form for CS matter theory. It is easy to show using (7) and (8) that for the QF theory $\gamma_s = 1$ and $\theta = \lambda_f$. When we plug this in (26), we obtain

$$\langle T^-T^-T^-\rangle_{\rm QF} = c_T(\langle T^-T^-T^-\rangle_{\mathbf{nh}} - e^{-i\pi\lambda_f}\langle T^-T^-T^-\rangle_{\mathbf{h}}). \tag{28}$$

We observe that in (28) the homogeneous piece of the correlator gets the anyonic phase which interpolates between the free fermion theory ($\lambda_f \to 0$) and the free boson theory ($\lambda_f \to 1$), which is precisely the identification that we did in (22). Under the strong-weak duality [26–28,47] $\lambda_f \to \lambda_b - sign(\lambda_b)$, $c_T \to c_T$, we have

$$\langle T^-T^-T^-\rangle = c_T(\langle T^-T^-T^-\rangle_{\mathbf{nh}} + e^{-i\pi\lambda_b}\langle T^-T^-T^-\rangle_{\mathbf{h}}), \tag{29}$$

which is consistent with (22) as $\lambda_b \to 0$ as well as the conjectured strong-weak duality in CS matter theories [61].

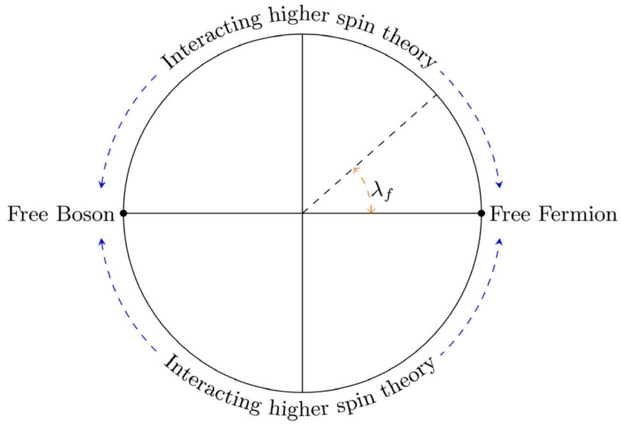

FIG. 1. The free bosonic and fermionic theories and the CS matter theories lie along the unit circle as indicated. This figure formally represents the result in (30).

The analysis of $\langle TTT\rangle$ directly extends to the three-point correlator of arbitrary spinning operators of spins s_1, s_2, s_3 and can be written as [62]

$$\langle J^-_{s_1}J^-_{s_2}J^-_{s_3}\rangle_{\mathrm{QF}} = c_s(\langle J^-_{s_1}J^-_{s_2}J^-_{s_3}\rangle_{\mathbf{nh}} - e^{-i\pi\lambda_f}\langle J^-_{s_1}J^-_{s_2}J^-_{s_3}\rangle_{\mathbf{h}}), \tag{30}$$

where c_s is the two-point function coefficient and we have used the fact that in the presence of higher-spin symmetry all two-point functions are the same, i.e., $c_{s_1} = c_{s_2}$; see the discussion below (17).

It is interesting to note that the anyonic phase factor $e^{-i\pi\lambda_f}$ that appears in (35) is exactly the same as the one that appeared in the anyonic or singlet channel of $2\to 2$ scattering amplitudes in CS matter theories [35,36]. Interestingly, in the context of scattering, this anyonic phase can also be obtained by solving the nonrelativistic Aharanov-Bohm scattering problem [37,38]. Under the duality transformation, scattering amplitudes in the boson theory coupled to CS map to those in the fermion theory coupled to CS. This is also the case for correlation functions as discussed here.

We have represented the result in (30) with a unit circle; see Fig. 1. If we start at the FF theory, with the help of the anyonic phase, we get the correlator in the CS matter theory, and it interpolates all the way to the FB theory. At the FF point, correlation functions only have the parity-even contribution, whereas for nonzero phase, it generates a parity-odd contribution as well. Changing the phase to π takes us all the way to the FB theory where there is no parity-odd contribution again. Thus, we see that higher-spin or weakly broken higher-spin theories lie on the circle.

V. DISCUSSION

In this paper, we discussed two- and three-point correlators comprising conserved and weakly broken higher-spin currents in 3D CFT. We showed that these correlators in 3D CFT are given by the free theory results dressed with an appropriate phase factor in spinor-helicity variables. This was possible using a novel relation between parity-even and parity-odd parts of correlation functions. In theories with weakly broken higher-spin symmetry such as CS matter theories, the phase factor turned out to be an anyonic phase which interpolates nicely between free theories. Given the simplicity of two-point and three-point correlation functions, it is natural to ask if for CS matter theories one can define anyonic currents whose correlation functions can be computed using Wick contraction just like in free theories. It would also be interesting to see if the anyonic structure extends to four-point functions such as $\langle TTTT\rangle$ for weakly broken higher-spin theories [63–66].

The remarkable simplicity of three-point functions when expressed in spinor-helicity variables indicates that a direct bootstrapping of correlation functions in spinor-helicity variables might give us great insights into the structure of four-point functions. See Ref. [32] for some recent progress on bootstrapping in momentum space helicity variables.

It would also be interesting to understand higher-spin equations directly in spinor-helicity variables. Because of the nontrivial relation between parity-even and parity-odd correlation functions in spinor-helicity variables, higher-spin equations in interacting theory would map to higher-spin equations in the free theory. This might also help us compute four-point functions of spinning operators.

The anyonic phase factor was previously found in $2\to 2$ scattering amplitudes. A finite N version of the phase was obtained by solving the nonrelativistic Aharanov-Bohm effect, which is given by $e^{-i\pi\frac{C_2(S)-C_2(F)-C_2(AF)}{\kappa}}$, where $C_2(R)$ represents the quadratic Casimir for the representation R. Here, S, F, and AF denote the singlet, fundamental, and antifundamental representations, respectively. For $SU(N_f)_{\kappa_f}$ CS gauge field coupled to fermion, we get $e^{-i\pi(\lambda_f-\frac{1}{N_f\kappa_f})}$, which in the limit $N,\kappa\to\infty$ gives precisely the phase in (35). It would be interesting to see if the anyonic phase observed in this paper continues to match the phase observed in scattering amplitudes at finite N.

We saw in Fig. 1 that free theories with exactly conserved currents or weakly broken higher-spin theories lie on the circle of unit radius. It would be interesting to figure out where other CFTs such as the holographic ones lie. To do this, we need to look into the conformal collider bound [67]. It would be interesting to directly formulate the conformal collider bound in spinor-helicity variables. We could also directly use previously known bounds [68–71], which indicate that generic CFTs and holographic ones lie inside the circle of radius one. The collider bound is saturated by free CFT or weakly broken higher-spin current CFTs. We will report on these exciting issues in the future.

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[59] $z_{1\mu} k_{1\nu} \langle T^{\mu\nu}(k_1) T(k_2) T(k_3)\rangle = -(z_1 \cdot k_2)\langle T(k_1 + k_2) T(k_3)\rangle + 2(z_1 \cdot z_2) k_{2\mu} z_\nu \langle T^{\mu\nu}(k_1 + k_2) T(k_3)\rangle - (z_1 \cdot k_3)\langle T(k_1 + k_3) \times T(k_2)\rangle + 2(z_1 \cdot z_3) k_{3\mu} z_{3\nu} \langle T^{\mu\nu}(k_1 + k_3) T(k_2)\rangle + (k_1 \cdot z_2) z_{1\mu} z_{2\nu} \times \langle T^{\mu\nu}(k_1 + k_2) T(k_3)\rangle + (z_1 \cdot z_2) k_{1\mu} z_{2\nu} \langle T^{\mu\nu}(k_1 + k_2) T(k_3)\rangle + (k_1 \cdot z_3) z_{1\mu} z_{3\nu} \langle T^{\mu\nu}(k_1 + k_3) T(k_2)\rangle + (z_1 \cdot z_3) k_{1\mu} z_{3\nu} \langle T^{\mu\nu}(k_1 + k_3) T(k_2)\rangle$.

[60] The homogeneous component is only nonzero for all negative or all positive helicity components; see Ref. [20].

[61] Critical boson theory coupled to CS gauge field is dual to free fermion coupled to CS. The results we have obtained make manifest the duality relation.

[62] This result takes a slightly different form for correlators with spin configuration outside the triangle inequality, $s_i > s_j + s_k$. In this situation, it takes the form $\langle J^-_{s_1} J^-_{s_2} J^-_{s_3}\rangle_{\text{QF}} = c_s(\langle J^-_{s_1} J^-_{s_2} J^-_{s_3}\rangle_{\mathbf{F+B}} - e^{-i\pi\lambda_f} \langle J^-_{s_1} J^-_{s_2} J^-_{s_3}\rangle_{\mathbf{F-B}})$, where $F \pm B$ means addition or subtraction of same correlators in the bosonic and fermionic theories.

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