Anomaly-induced vacuum effective action with torsion: Covariant solution and ambiguities

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The conformal anomaly in curved spacetime with antisymmetric torsion is reconsidered, taking into account new important details. We formulate, for the first time, the covariant solution of the anomaly-induced effective action. The covariant effective action includes local terms corresponding to total derivatives in the conformal anomaly. The contribution of massless fermions to these terms is characterized by multiplicative anomaly, coming from two different choices of doubling for the spinor operator. On the other hand, the nonlocal part of anomaly-induced action is free of ambiguities and admits a low-energy limit, similar to the effective potential in the metric-scalar theory.

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I. INTRODUCTION

General features of conformal and the induced action of gravity corresponding to anomaly represent an interesting and active subject of interest starting from the epoch when anomaly was discovered [1–4] and anomaly-induced effective action derived in two [5] and four (4D) [6,7] spacetime dimensions. The reason for this special interest is related to important applications to black hole physics [8,9], cosmology [10–12] (see also [4,13–15] for review and further references), effective approaches to quantum gravity based on the anomaly-induced action in 4D [16–18] and possible nonperturbative generalizations in the form of a- and c-theorems (see, e.g., [19–21]).

From a more general perspective, the anomaly is technically simple and elegant way for describing loop corrections in the semiclassical approach. Usually, the trace anomaly is associated with the UV limit because classical conformal symmetry is typical only in a massless theory. In other words, the anomaly-induced action is a direct generalization of the renormalization group improved classical action based on the minimal subtraction scheme of renormalization. The generalization, in this case, means that the constant rescaling of the metric (curved-space equivalent of rescaling the momenta [22–24]) is replaced by the local conformal transformation with the conformal factor depending on the coordinates. On the other hand, the anomaly-induced action can be adapted to describe the low-energy (IR) limit of a massless theory. The first examples of this sort can be found in [25,26], where the anomaly has been used to obtain the one-loop effective potential of scalar fields in curved spacetime.

Anomaly is directly related to the logarithmic divergences in the conformal massless theory [4,14]. This is true not only for a purely metric background but also for the theory with extra background fields, such as scalars, vector fields, and torsion. One can prove that the divergences satisfy the conformal Noether identity [23] and, for this reason, the anomaly is composed by the following three types of terms [3,27]:

(i) Topological term, such as the Gauss-Bonnet term in 4D or its analogs in higher even dimensions. The conformal transformation of this term greatly simplifies if supplemented by a specially chosen surface terms. This fundamental feature is the main basis of integrating anomaly, which was verified in 2D [5], in 4D [6,7] and in 6D [28]. The general proof for higher dimensions is not known and remains a conjecture [29]. Assuming this is correct, the topological term is the main source of the nonlocal structures in the anomaly-induced action.

(ii) Legitimate conformal terms (we will call them C-terms), such as the square of the Weyl tensor in 4D; three possible conformal structures in 6D [30], etc. The integration of these terms is relatively simple assuming the aforementioned conjecture. The result for the covariant induced action is nonlocal.

(iii) The total derivative terms in the divergences provide the same terms in the anomaly. These terms are

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known to be ambiguous [4] and this ambiguity equivalent to adding a finite local covariant nonconformal term to the classical action, as discussed in [31,32] for dimensional and Pauli-Villars regularizations. Strictly speaking, *(i)*- and *(iii)*-type terms are not conformal invariant. However, those are surface terms that satisfy the conformal Noether identity. For this reason, we shall call them N-terms.

Verifying the generality of the described classification may go in two different ways, namely either increasing the dimension or trying to enrich space-time geometry. The first approach is extremely difficult for practical realization (see, e.g., [28]). At the same time, the second possibility may be related to introducing torsion or nonmetricity of the spacetime. The present work reports on the verification of the scheme described above for the theory with torsion. Namely, we explore the uniqueness of the topological term, the possibility to construct covariant forms for anomaly-induced action with torsion, and the ambiguities in the local covariant terms, responsible for the total derivatives in the anomaly.

The case of the anomaly with torsion has been explored in several works [33,34], including for the different realizations of conformal symmetry [35,36]. However, the covariant (nonlocal and local) form of the anomaly-induced effective action of gravity with torsion was never formulated, and the first purpose of the present work is to fill this gap. We shall see that the integration of all terms of an anomaly in the theory with torsion can be done in a very standard way, with one important addition. As we are working with fermions, there is always a possibility to have an ambiguity related to different ways of doubling the Dirac operator. Previously, it was shown that, in case of massive fermions, this ambiguity leads to the nonlocal multiplicative anomaly [37], something one cannot consistently achieve [38,39] using ζ-regularization [40] owing to the presence of renormalization μ-dependence in the local terms in the effective action. In what follows we show that, in the theory with torsion, the multiplicative anomaly is possible even in the *massless* case, i.e., in the local terms coming from the integration of total derivative terms in the trace anomaly. Since these terms are not renormalized, this new version of multiplicative anomaly avoids the μ-dependence and the arguments of [38,39].

The paper is organized as follows. In Sec. II we briefly review the notations for gravity with torsion and define the actions of free matter fields with conformal symmetry. Section III describes the calculation of one-loop divergences in the fermion case, generalizing the previous works on the subject [33–36]. Since one of our main concerns is ambiguity in the anomaly, we perform the calculation in two different ways and meet the difference which leads to the local multiplicative anomaly in the effective action. In Sec. IV the anomaly is used to find the covariant (nonlocal and local) solutions for the anomaly-induced vacuum effective action. Section V describes the low-energy limit in the effective action in the metric-torsion theory, constructed in analogy to the effective potential of a scalar field [26]. Finally, in Sec. VI we draw our conclusions.

II. CONFORMAL FIELDS WITH TORSION

In what follows we shall give only a brief list of necessary formulas about gravity with torsion and conformal matter fields. The path integrals over these fields require renormalization of vacuum action and produce trace (conformal) anomaly. A more detailed review can be found, e.g., in [36,41]. In the last reference, the notations are the same as here.

The affine connection without torsion is the Christoffel symbol (Levi-Civita connection),

$$\Gamma^{\alpha}_{\beta\gamma} = \left\{{}^{\alpha}_{\beta\gamma}\right\} = \frac{1}{2} g^{\alpha\lambda}(\partial_\beta g_{\lambda\gamma} + \partial_\gamma g_{\lambda\beta} - \partial_\lambda g_{\beta\gamma}). \qquad (1)$$

The corresponding covariant derivative satisfies the metricity condition $\nabla_\lambda g_{\alpha\beta} = 0$ and is free of torsion, i.e., $\Gamma^{\tau}{}_{\alpha\beta} = \Gamma^{\tau}{}_{\beta\alpha}$. In what follows, we do not consider the theories with nonmetricity but include nonzero torsion, that is making geometry more extensive and, in particular, links it to the spin of matter fields [41].

Torsion tensor is defined as the difference between the two affine connections which are not assumed symmetric,

$$T^{\alpha}{}_{\cdot\beta\gamma} = \tilde{\Gamma}^{\alpha}_{\beta\gamma} - \tilde{\Gamma}^{\alpha}_{\gamma\beta}. \qquad (2)$$

It is useful to present torsion tensor as a sum of the irreducible components [42] (see also [36]),

$$T_{\alpha\beta\mu} = \frac{1}{3}(T_\beta g_{\alpha\mu} - T_\mu g_{\alpha\beta}) - \frac{1}{6}\varepsilon_{\alpha\beta\mu\nu}S^\nu + q_{\alpha\beta\mu}, \qquad (3)$$

namely the vector $T_\beta = T^{\alpha}{}_{\cdot\beta\alpha}$, axial vector $S^\nu = \epsilon^{\alpha\beta\mu\nu}T_{\alpha\beta\mu}$, and the remaining tensor $q^{\alpha}{}_{\cdot\beta\gamma}$, satisfying the conditions $q^{\alpha}{}_{\cdot\beta\alpha} = 0$ and $\epsilon^{\alpha\beta\mu\nu}q_{\alpha\beta\mu} = 0$.

One can derive the generalizations of the Riemann tensor, Ricci tensor and scalar curvature for the connection with torsion, e.g.,

$$\tilde{R} = R - 2\nabla_\alpha T^\alpha - \frac{4}{3}T_\alpha T^\alpha + \frac{1}{2}q_{\alpha\beta\gamma}q^{\alpha\beta\gamma} + \frac{1}{24}S^\alpha S_\alpha. \qquad (4)$$

Since our purpose is to consider the quantum theory of matter fields, regarding both metric and torsion as external fields, the parametrization of these background fields is mostly irrelevant. Thus, in what follows, we shall use the Riemannian version of the curvature tensors.

Since the interaction of torsion with the gauge fields is forbidden by the gauge invariance [41], we shall assume that gauge vectors decouple from torsion at the classical level. The same symmetry protects the theory from these interactions at the quantum level [36,43]. Thus, we need to consider only scalar and fermion fields, as described below.

A. Scalar field

The action of the real nonminimal scalar field ϕ in curved spacetime with torsion has the form,

$$S_0 = \frac{1}{2}\int d^4x\sqrt{-g}\{g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \xi_i P_i\phi^2 - m^2\phi^2\}, \qquad (5)$$

where the nonminimal parameters ξ_i correspond to the structures,

$$P_1 = R, \qquad P_2 = \nabla_\alpha T^\alpha, \qquad P_3 = T_\alpha T^\alpha,$$
$$P_4 = q_{\alpha\beta\tau}q^{\alpha\beta\tau}, \qquad P_5 = S_\alpha S^\alpha, \qquad (6)$$

repeating the ones of (4) but with arbitrary coefficients ξ_i. It is known [36,42,43] that in the curved-space theory with fermions, scalars and Yukawa coupling, arbitrary nonminimal parameters ξ_1 and ξ_5, of the interaction of scalar field(s) with R and S_α, are needed to provide renormalizable semiclassical theory, as will be explained below. In this article, we will mainly restrict the consideration by a purely antisymmetric torsion. The reasons are that *(i)* this is the most relevant part of the matter-torsion interaction, in particular linking spin to geometry [41]; *(ii)* more general cases are not expected to bring new details, concerning the aforementioned aspects of the anomaly and also compared to the previous analysis in [36]. Thus, we shall assume that $T_\mu = 0$ and $q_{\alpha\beta\tau} = 0$, such that only the axial vector component in (3) is present.

Action (5) is invariant under general coordinate transformations. On top of that, the massless model with $\xi = 1/6$ is invariant under the transformation called local conformal symmetry,

$$g_{\mu\nu} = e^{2\sigma}\bar{g}_{\mu\nu}, \qquad S_\alpha = \bar{S}_\alpha, \qquad \phi = e^{-\sigma}\bar{\phi}, \qquad (7)$$

where $\sigma = \sigma(x)$. Note that the value of ξ_5 does not affect conformal invariance. Also, if we include T_μ component, there are three types of the conformal transformations with torsion. This subject was discussed in detail in [35,43], and we will not repeat it here.

B. Massless Dirac field

The interaction of Dirac spinor field with torsion is described by the parity-preserving action,

$$S = \int d^4x\sqrt{-g}\bar{\psi}\{i\gamma^\mu(\nabla_\mu - i\eta\gamma^5 S_\mu - i\eta_2 T_\mu) - m\}\psi, \qquad (8)$$

with the nonminimal parameters η and η_2. It is important that the minimal coupling of fermion with torsion correspond to $\eta = 1/8$ and $\eta_2 = 0$ [41]. This feature explains the difference between interaction of torsion with S_μ and T_μ. Starting from the minimal actions, there is no T_μ and $q_{\alpha\beta\mu}$, and the corresponding interactions never emerge in the divergences. On the contrary, S_μ is always present in both classical theory and in the divergences. As a result, one has to renormalize the parameters η and also ξ_5, in case the theory includes Yukawa interactions between scalars and fermions. Thus, one cannot have a renormalizable theory based on the minimal coupling to the external torsion.

In the massless case, the theory (8) possesses three different symmetries. One of those is the usual Abelian gauge symmetry related to T_μ. In fact, one can trade $\eta_2 T_\mu$ by eA_μ and, in the part of the gauge symmetry, reduce the problem to the usual gauge field. Since we are interested in the $T_\mu = 0$ case, the corresponding transformation will not be considered. Another symmetry is

$$\bar{\psi} \to \bar{\psi}e^{i\eta\alpha\gamma^5}, \qquad \psi \to e^{i\eta\alpha\gamma^5}\psi, \qquad S_\mu \to S_\mu + \partial_\mu\alpha, \qquad (9)$$

where $\alpha = \alpha(x)$ is a scalar transformation parameter. Finally, there is a conformal transformation of the spinor field, supplementing the one of (7),

$$g_{\mu\nu} = e^{2\sigma}\bar{g}_{\mu\nu}, \qquad S_\alpha = \bar{S}_\alpha, \qquad \bar{\psi} = \bar{\psi}e^{-\frac{3}{2}\sigma}, \qquad \psi = \psi e^{-\frac{3}{2}\sigma}. \qquad (10)$$

Let us mention that the values of torsion nonminimal parameters, η and ξ_5, do not affect the conformal invariance.

The rest of this work is devoted to the anomaly in the local conformal symmetry (7), (10) in the vacuum part of the one-loop effective action $\Gamma^{(1)}(g, S)$.

According to the general proof [23], if the classical actions of quantum matter fields have local conformal symmetry (7) and (10), the divergent part $\Gamma^{(1)}_{\rm div}(g, S)$ satisfies the corresponding Noether identity,

$$-\frac{2}{\sqrt{-g}}g_{\alpha\beta}\frac{\delta\Gamma^{(1)}_{\rm div}(g, S)}{\delta g_{\alpha\beta}} = \Phi(g, S). \qquad (11)$$

Here $\Phi(g, S)$ is a covariant finite expression, that is also local owing to the Weinberg's theorem. We note that Eq. (11) does not include the variational derivative with respect to S_μ because, according to (7), its conformal weight is zero.

The possible vacuum divergences obey the mentioned symmetries. The Riemannian terms include the square of the Weyl tensor in 4D,

$$C^2 = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 2R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{3}R^2, \qquad (12)$$

the integrand of the Gauss-Bonnet topological term,

$$E_4 = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2, \qquad (13)$$

and the surface term $\Box R$. The important difference between these terms is that, in 4D, the integral of C^2 is invariant,

$$\int d^4x\sqrt{-g}C^2 = \int d^4x\sqrt{-\bar{g}}\bar{C}^2. \tag{14}$$

We shall call the actions satisfying this condition $\mathcal{C}$-terms. On the contrary, the integrals $\int d^4x\sqrt{-g}E_4$ and $\int d^4x\sqrt{-g}\Box R$ do not possess this property and, strictly speaking, are not conformal invariant. At the same time, both terms satisfy the conformal Noether identity. All such actions, that are not really conformal, but obey the rule,

$$-\frac{2}{\sqrt{-g}}g_{\alpha\beta}\frac{\delta S(g,S}{\delta g_{\alpha\beta}} = 0, \tag{15}$$

will be called $\mathcal{N}$-terms.

Besides the mentioned metric-dependent integrals, there are torsion-dependent $\mathcal{C}$- and $\mathcal{N}$-terms. In the first g roup, there are two new candidates [44],

$$S^4 = (S^2)^2 = (S_\mu S^\mu)^2 \quad \text{and} \quad S^2_{\mu\nu} = g^{\mu\alpha}g^{\nu\beta}S_{\mu\nu}S_{\alpha\beta}, \tag{16}$$

where $S_{\mu\nu} = \nabla_\mu S_\nu - \nabla_\nu S_\mu$. One can ask the following questions: *(i)* Whether there are torsion-dependent analogs of (or alternatives to) the Riemannian topological term E_4? *(ii)* Are there torsion-dependent $\mathcal{N}$-terms, including total derivatives, similar to $\Box R$, and *(iii)* Whether the renormalization of these torsion-dependent $\mathcal{N}$-terms has ambiguities (see [4,31]) which are present in the cases of $\Box R$ and, also, for a nonzero background scalar field, in the $\Box\phi^2$-term [26,32]. We shall address these questions by making direct calculations of divergences, anomaly and anomaly-induced action of gravity with torsion.

III. DERIVATION OF ONE-LOOP DIVERGENCES

For the sake of generality, we perform calculations for massive versions of scalar and spinor fields and for an arbitrary ξ_1. We can set masses to zero and $\xi_1 = 1/6$ at the end.

Let us start by quoting the known result for the scalar field [36],

$$\Gamma^{(1)}_{\text{div,scal}} = -\frac{\mu^{n-4}}{\varepsilon}\int d^nx\sqrt{-g} \times\left[\frac{1}{120}C^2 - \frac{1}{360}E_4 + \frac{1}{180}\Box R + \frac{1}{6}\Box P + \frac{1}{2}P^2\right], \tag{17}$$

where

$$P = \left(\frac{1}{6} - \xi_1\right)R - \xi_5 S^2 + m^2. \tag{18}$$

In the massless limit and assuming $\xi_1 = 1/6$, we meet the first $\mathcal{C}$-term from (16) and the $\mathcal{N}$-terms i.e., E_4, $\Box R$, and $\Box S^2$.

Consider the fermion contribution. In this part, we go into the detail of the calculation, regardless they can be partially found in [36]. Our purpose is to evaluate $-i\,\mathrm{Tr}\,\ln\hat{H}$, where

$$\hat{H} = i\gamma^\mu(\nabla_\mu - i\eta\gamma^5 S_\mu) - m. \tag{19}$$

In order to perform this calculation, we have to multiply (19) by a conjugate operator, such that the product belongs to the standard class of minimal operators $\hat{F} = \hat{\Box} + 2\hat{h}^\alpha\nabla_\alpha + \hat{\Pi}$, admitting application of the Schwinger-DeWitt technique [45,46]. It seems that there should be many possible choices for such a conjugate operator, but there are two constraints. First of all, the contribution of the conjugate operator should be calculable. And, on the other hand, we have to respect the chiral symmetry (9), as otherwise the result may be wrong. The last means the structure $\gamma^\mu(\nabla_\mu - i\eta\gamma^5 S_\mu)$ has to be part of the conjugate operator or, alternatively, the conjugate operator must be S_μ-independent. In what follows, we consider both these options and explore the difference.

A. First calculation of fermion contributions

As a first option, consider the conjugate operator of the form,

$$\hat{H}_1 = i\gamma^\nu(\nabla_\nu - i\eta\gamma^5 S_\nu) + m. \tag{20}$$

It is known that the change of the sign of the mass does not change the result (see, e.g., [47]), such that $\mathrm{Tr}\,\ln\hat{H} = \mathrm{Tr}\,\ln\hat{H}_1$ and we can use the relations,

$$\begin{aligned} -i\,\mathrm{Tr}\,\ln\hat{H} &= -\frac{i}{2}\mathrm{Tr}\,\ln(\hat{H}\hat{H}_1) \\ &= -\frac{i}{2}\mathrm{Tr}\,\ln(\hat{\Box} + 2\hat{h}_1^\alpha\nabla_\alpha + \hat{\Pi}_1). \end{aligned} \tag{21}$$

After some algebra, we get

$$\begin{aligned} \hat{h}_1^\alpha &= \frac{i}{2}\gamma^5(\gamma^\lambda\gamma^\alpha - \gamma^\alpha\gamma^\lambda)S_\lambda, \\ \hat{\Pi}_1 &= m^2 - \frac{1}{4}R + S^2 - i\gamma^5(\nabla_\alpha S^\alpha) - \frac{i}{2}\gamma^5\gamma^\alpha\gamma^\beta S_{\alpha\beta}. \end{aligned} \tag{22}$$

In these and subsequent formulas we made a rescaling of the external torsion field $\eta S_\mu \to S_\mu$, making formulas more compact.

The elements of the Schwinger-DeWitt technique are

$$\begin{aligned} \hat{P}_1 &= \hat{\Pi}_1 + \frac{\hat{1}}{6}R - \nabla_\alpha\hat{h}_1^\alpha - \hat{h}_{1\alpha}\hat{h}_1^\alpha \\ &= m^2 - \frac{1}{12}R - 2S^2 - i\gamma^5(\nabla_\alpha S^\alpha) \end{aligned} \tag{23}$$

and

$$\hat{\mathcal{S}}_{1,\alpha\beta} = [\nabla_\beta,\nabla_\alpha] + \nabla_\beta \hat{h}_{1\alpha} - \nabla_\alpha \hat{h}_{1\beta} + \hat{h}_{1\beta}\hat{h}_{1\alpha} - \hat{h}_{1\alpha}\hat{h}_{1\beta}$$
$$= -\frac{1}{4}R_{\alpha\beta\lambda\tau}\gamma^\lambda\gamma^\tau - S^2(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha) - 2S^\lambda(S_\alpha\gamma_\beta - S_\beta\gamma_\alpha)\gamma_\lambda$$
$$+ \frac{i}{2}\gamma^5[(\nabla_\beta S^\lambda)(\gamma_\lambda\gamma_\alpha - \gamma_\alpha\gamma_\lambda) - (\nabla_\alpha S^\lambda)(\gamma_\lambda\gamma_\beta - \gamma_\beta\gamma_\lambda)]. \tag{24}$$

The general expression for the one-loop divergences is [45]

$$\Gamma^{(1)}_{\rm div} = -\frac{\mu^{n-4}}{\varepsilon}\int d^n x\sqrt{-g}\,{\rm tr}\bigg[\frac{\hat{1}}{180}(R^2_{\mu\nu\alpha\beta} - R^2_{\alpha\beta} + \Box R) + \frac{1}{2}\hat{P}^2 + \frac{1}{12}\hat{\mathcal{S}}^2_{\mu\nu} + \frac{1}{6}\Box\hat{P}\bigg], \tag{25}$$

where the trace and sign correspond to bosonic fields and for the fermions the sign should be inverted. In the present case, $\hat{P}$ and $\hat{\mathcal{S}}_{\mu\nu}$ are defined by (23) and (24). The calculation is pretty much standard, but we quote simple relation for (16),

$$\frac{1}{2}S^2_{\mu\nu} = (\nabla_\mu S_\nu)^2 - (\nabla_\mu S^\mu)^2 + R_{\mu\nu}S^\mu S^\nu + 2\nabla_\nu(S^\nu\nabla_\mu S^\mu - S^\mu\nabla_\mu S^\nu), \tag{26}$$

which proves useful, also, for integrating anomaly. Here $(\nabla_\mu S_\nu)^2 = (\nabla_\mu S_\nu)(\nabla^\mu S^\nu)$.

Finally, for the divergences we obtain the expression (with recovered η),

$$\Gamma^{(1)}_{\rm div,fer,1} = -\frac{\mu^{n-4}}{\varepsilon}\int d^n x\sqrt{-g}\bigg[\frac{m^2}{3}R + 8m^2\eta^2 S^2 - 2m^4 + \frac{1}{20}C^2 - \frac{11}{360}E_4 + \frac{1}{30}\Box R - \frac{2}{3}\eta^2 S^2_{\mu\nu} + \frac{4}{3}\eta^2\Box S^2 - \frac{4}{3}\eta^2\nabla_\beta(S^\alpha\nabla_\alpha S^\beta - S^\beta\nabla_\alpha S^\alpha)\bigg]. \tag{27}$$

There are several remarkable aspects in this formula. In the massless theory and in the limit $n \to 4$, the integrand is conformal invariant, that is, composed by the $\mathcal{C}$-type and $\mathcal{N}$-type invariants [36]. In the expression for divergences, one can identify two torsion-dependent total derivatives. On top of this, there is also the surface conformal term $-\frac{1}{3}\eta\nabla_\beta B^\beta$ in the integrand, that depends on the vector field,

$$B^\nu = R^\nu{}_{\cdot\mu\tau\lambda}\varepsilon^{\tau\lambda\alpha\mu}S_\alpha = C^\nu{}_{\cdot\mu\tau\lambda}\varepsilon^{\tau\lambda\alpha\mu}S_\alpha. \tag{28}$$

This term is not included in (27) because B^ν can be shown to vanish as a result of the first Bianchi identity for a Riemann and Weyl tensors.

B. Second calculation of fermion contribution

The second scheme of doubling the fermion operator (19) uses the torsion-independent conjugate operator,

$$\hat{H}_2 = i\gamma^\nu\nabla_\nu + m. \tag{29}$$

In this case, one has to use the formula (21) for the torsion-independent terms, which are certainly the same as in (27). However, for the S_μ-dependent terms, Tr ln $\hat{H}_2$ is irrelevant, and we have to use the modified rule,

$$-i\,{\rm Tr}\ln\hat{H} = -i\,{\rm Tr}\ln(\hat{H}\hat{H}_2) = -i\,{\rm Tr}\ln(\hat{\Box} + 2\hat{h}^\alpha_2\nabla_\alpha + \hat{\Pi}_2). \tag{30}$$

The elements of the operator, in this case, are

$$\hat{h}^\alpha_2 = \frac{i}{2}\gamma^5\gamma^\lambda\gamma^\alpha S_\lambda,$$
$$\hat{\Pi}_2 = m^2 - \frac{1}{4}R + m\gamma^5\gamma^\lambda S_\lambda. \tag{31}$$

The elements of Schwinger-DeWitt technique are also different,

$$\hat{P}_2 = m^2 - \frac{1}{12}R - \frac{1}{2}S^2 + m\gamma^5\gamma^\alpha S_\alpha - \frac{i}{2}\gamma^5(\nabla_\alpha S^\alpha) + \frac{i}{4}\gamma^5\gamma^\alpha\gamma^\beta S_{\alpha\beta}, \tag{32}$$

$$\hat{\mathcal{S}}_{2,\alpha\beta} = -\frac{1}{4}R_{\alpha\beta\lambda\tau}\gamma^\lambda\gamma^\tau - \frac{1}{4}S^2(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha) - \frac{1}{2}S^\lambda\gamma_\lambda(S_\beta\gamma_\alpha - S_\alpha\gamma_\beta) + \frac{i}{2}\gamma^5\gamma^\lambda[\gamma_\alpha(\nabla_\beta S_\lambda) - \gamma_\beta(\nabla_\alpha S_\lambda)]. \tag{33}$$

Let us write only the S_μ-dependent divergences, which are obtained via (30) and (25),

$$\Gamma^{(1)}_{\rm div,fer,2} = -\frac{\mu^{n-4}}{\varepsilon}\int d^n x\sqrt{-g}\bigg[8m^2\eta^2 S^2 - \frac{2}{3}\eta^2 S^2_{\mu\nu} + \frac{2}{3}\eta^2\Box S^2 + \frac{2}{3}\eta^2\nabla_\beta(S^\beta\nabla_\alpha S^\alpha - S^\alpha\nabla_\alpha S^\beta)\bigg]. \tag{34}$$

Compared to (27), the nonsurface terms are the same. However, the total derivative, $\mathcal{N}$-terms, have different coefficients. This result represents the new kind of multiplicative anomaly, being qualitatively different from the previously known examples (starting from [37]) concerning the nonlocal part of the one-loop effective action. In these examples the multiplicative anomaly shows up only for the massive fields, and on the other hand, it cannot be compensated by the change of renormalization condition because the last concerns only the local terms. In the present case, the difference cannot be compensated by the change of renormalization conditions for the irrelevant surface integrals because such change given only finite differences. As we will see in the next section, the finite difference shows up in the local terms which are *not* total derivatives.

C. Action of torsion and UV logarithmic corrections

One of the important outputs of the one-loop calculations for scalars and fermions is that, in the semiclassical conformal theory with antisymmetric torsion, the classical action of torsion has the form [36,44],

$$S_{\rm tors} = \int d^4x\sqrt{-g}\bigg\{-a_1 S^4 - \frac{a_2}{4} S_{\mu\nu}^2 + b_1 \nabla_\beta (S^\alpha \nabla_\alpha S^\beta - S^\beta \nabla_\alpha S^\alpha) + b_2 \Box S^2\bigg\}, \qquad (35)$$

where $a_{1,2} > 0$ and $b_{1,2}$ are arbitrary parameters. The positiveness of a_1 and a_2 provides the tree-level potential of S_μ bounded from below (as will be discussed in the next sections) and the positiveness of energy for propagating torsion [48] (see also [36]).

It may look natural to set $a_2 = 1$ [48], that can be provided by rescaling S_μ and η. However, it is sometimes useful to keep a_2 arbitrary, as we shall see in what follows. From the viewpoint of conformal symmetry, $a_{1,2}$-structures represent $\mathcal{C}$-terms and the values of those parameters can be defined only from the measurements, which in the case of a_2 can be traded to the measurement of ηS_μ. At the same time, the coefficients of the $\mathcal{N}$-terms $b_{1,2}$ do not affect equations of motion and are artificial parameters that cannot be measured. Still, these terms are necessary for renormalizability of a semiclassical theory.

Let us evaluate loop corrections to the vacuum action (35). As a first step in this direction, we can recover the leading one-loop logarithms in the most relevant $\mathcal{C}$-terms. Using the standard considerations [46] (see also [14] for more details), we arrive at the one-loop corrected torsion sector of the theory,

$$\Gamma^{(1)}_{\rm tors} = -\int d^4x\sqrt{-g}\bigg\{S^2\bigg[a_1 + \frac{\beta_1}{2}\ln\bigg(\frac{\Box}{\mu^2}\bigg)\bigg]S^2 + \frac{1}{4}S_{\mu\nu}\bigg[a_2 + \frac{\beta_2}{2}\ln\bigg(\frac{\Box}{\mu^2}\bigg)\bigg]S^{\mu\nu}\bigg\}, \qquad (36)$$

From (17) and (27) [see also subsequent Eq. (34)], we can easily get

$$\beta_1 = -\frac{1}{2(4\pi)^2}\sum_{i=1}^{N_s}\xi_{5,i}^2,$$
$$\beta_2 = \frac{8}{3(4\pi)^2}\sum_{k=1}^{N_f}\eta_k^2. \qquad (37)$$

Here $\xi_{5,i}$ and η_k are nonminimal parameters for different species of scalar and spinor fields. According to the analysis of renormalization in interacting theories [43], these parameters may be different for different fields. Independent on this, the signs of the beta functions show that the sign of β_1 indicated the asymptotic freedom in the parameter a_1 and the sign of β_2 is positive, as it is typical for the Abelian vector models. It is worth mentioning that these signs correspond to the fermion and scalar contributions only, while the contribution of the proper field S_μ was not taken into account.

The integration of anomaly is, to a great extent, an elegant and useful way to work with formula (36) by constructing a local version of renormalization group. After deriving the covariant form of anomaly-induced action, we use the duality of the UV and IR limits in the massless theory and construct the low-energy alternative to (36).

IV. INTEGRATION OF ANOMALY WITH TORSION

Since the torsion field does not transform in (7), the derivation of anomaly has no novelties compared to the purely metric case [6,7] (see, e.g., [14] for detailed introduction). On top of that, in [35] one can find even more general consideration, with the torsion trace T_μ included. Thus, we shall simply write down the expression for the anomaly,

$$\langle T^\mu{}_\mu\rangle = -\bigg\{wC^2 + bE_4 + c\Box R - \beta_1 S^4 - \frac{1}{4}\beta_2 S_{\mu\nu}^2 + \gamma_1\nabla_\beta(S^\alpha\nabla_\alpha S^\beta - S^\beta\nabla_\alpha S^\alpha) + \gamma_2\Box S^2\bigg\}. \qquad (38)$$

The one-loop β-functions w, b and c do not depend on the presence of torsion and are given by the expressions [14,49],

$$\begin{pmatrix} w \\ b \\ c \end{pmatrix} = \frac{1}{360(4\pi)^2}\begin{pmatrix} 3N_s + 18N_f + 36N_v \\ -N_s - 11N_f - 62N_v \\ 2N_s + 12N_f - 36N_v \end{pmatrix}, \qquad (39)$$

where N_s, N_f and N_v are the numbers of scalar, spinor and gauge vector fields.

The beta functions $\beta_{1,2}$ are written in (37). Finally, the two functions $\gamma_{1,2}$ in (38) are ambiguous, as we have seen from the fermionic divergences (27) and (34). For these two schemes of calculation we meet, respectively,

$$\gamma_1^{(1)} = -\frac{4}{3(4\pi)^2}\sum_{k=1}^{N_f}\eta_k^2, \qquad \gamma_1^{(2)} = \frac{2}{3(4\pi)^2}\sum_{k=1}^{N_f}\eta_k^2, \qquad (40)$$

$$\gamma_2^{(1)} = \frac{4}{3(4\pi)^2}\sum_{k=1}^{N_f}\eta_k^2 - \frac{1}{6(4\pi)^2}\sum_{i=1}^{N_s}\xi_{5,k},$$
$$\gamma_2^{(2)} = \frac{2}{3(4\pi)^2}\sum_{k=1}^{N_f}\eta_k^2 - \frac{1}{6(4\pi)^2}\sum_{i=1}^{N_s}\xi_{5,k}. \tag{41}$$

Let us note that the scalar contributions to γ_2 in (41), coming from (17), also have ambiguity; however one has to perform Pauli-Villars analysis to see this. The required procedure would be a mere repetition of the one described in [26,32] for background scalars; hence we skip this part.

In the rest of this section, we describe the solution of the equation,

$$-\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta\Gamma_{\rm ind}}{\delta g_{\mu\nu}} = -\frac{1}{\sqrt{-\bar{g}}}e^{-4\sigma}\frac{\delta\Gamma_{\rm ind}}{\delta\sigma}\bigg| = \langle T^\mu{}_\mu\rangle. \tag{42}$$

The first equation here is an identity which uses σ, i.e., the conformal factor of the metric defined in (7). Also, $|$ means the procedure of replacing $(\bar{g}_{\mu\nu}, \bar{S_\mu}) \to (g_{\mu\nu}, S_\mu)$ and $\sigma \to 0$.

The 4D solution for a purely gravitational case was found in [6,7]. The generalization for a theory with torsion has been found [34,35] but only in the noncovariant formulation as a functional of $\bar{g}_{\mu\nu}$, $\bar{S}_\mu$ and σ. In what follows, we shall construct the most informative, covariant (nonlocal and local) solutions following the general scheme working for an arbitrary even dimension [28]. Thus, we need just to give a practical realization of this scheme for the theory with torsion.

The conformal invariants in (38) can be denoted in a common way as

$$Y = Y(g,S) = wC^2 - \beta_1 S^4 - \frac{1}{4}\beta_2 S_{\mu\nu}^2. \tag{43}$$

The unique topological term E_4 has the remarkable conformal property,

$$\sqrt{-g}\left(E_4 - \frac{2}{3}\Box R\right) = \sqrt{-\bar{g}}\left(\bar{E}_4 - \frac{2}{3}\bar{\Box}\,\bar{R} + 4\bar{\Delta}_4\sigma\right), \tag{44}$$

where $\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$, which obeys $\sqrt{-g}\Delta_4 = \sqrt{-\bar{g}}\bar{\Delta}_4$ [50,51].

These notations and features do not depend on the presence of torsion, and therefore, we can directly write down the nonlocal part of the solution of (42),

$$\Gamma_{\rm ind,nonloc} = \frac{b}{8}\int_x\int_y\left(E_4 - \frac{2}{3}\Box R\right)_x G(x,y)\left(E_4 - \frac{2}{3}\Box R\right)_y + \frac{1}{4}\int_x\int_y Y(x)G(x,y)\left(E_4 - \frac{2}{3}\Box R\right)_y, \tag{45}$$

where we used the notation $\int_x \equiv \int d^4x\sqrt{-g(x)}$ and the Green function of the Paneitz operator,

$$(\sqrt{-g}\Delta_4)_x G(x,y) = \delta(x,y). \tag{46}$$

Let us find a solution for the total derivative terms. For the $\Box R$ the result is well-known,

$$-\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\int_x R^2 = 12\Box R, \tag{47}$$

and for the $\Box S^2$ the answer can be easily found by direct calculation,

$$-\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\int_x RS^2 = 6\Box S^2. \tag{48}$$

Thus, the remaining problem is to integrate the γ_1 term in (38). Let us use the hypothesis that, as in all previously known cases, the solution for the total derivative should be a local covariant action. Then we have the following candidate terms:

$$\Gamma_{\rm ind,local} = \int_x\{\alpha_1(\nabla_\mu S^\mu)^2 + \alpha_2(\nabla_\mu S_\nu)^2 + \alpha_3 RS^2\}, \tag{49}$$

where the last one is already worked out in (48). We can rewrite the rhs of this formula using $\Box S^2 = 2\nabla_\nu(S_\mu\nabla^\nu S^\mu)$. It is easy to note that in (49) the dimensionally possible term $R_{\mu\nu}S^\mu S^\nu$ is missing. The reason is that the linear combination (26) gives conformal invariant functional $\int_x S_{\mu\nu}^2$, and therefore, including the mentioned term would be senseless. The application of the conformal operator to the remaining two terms gives

$$-\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\int_x(\nabla_\mu S^\mu)^2 = 4\nabla_\nu(S^\nu\nabla_\mu S^\mu),$$
$$-\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\int_x(\nabla_\nu S_\mu)^2 = 2\nabla_\nu[S^\nu\nabla_\mu S^\mu - S_\mu\nabla^\nu S^\mu - S_\mu\nabla^\mu S^\nu]. \tag{50}$$

Using (50) together with the modified version of (48), replacing the result into the linear combination of (49) and comparing to (38), we arrive at the solution for $\alpha_{1,2,3}$,

$$\alpha_1 = 0, \qquad \alpha_2 = \frac{1}{2}\gamma_1, \qquad \alpha_3 = \frac{1}{12}(\gamma_1 - 2\gamma_2). \tag{51}$$

Taking into account relations (47) and (44), the local part of the covariant induced action has the form,

$$\Gamma_{\rm ind,loc} = -\frac{3c+2b}{36}\int_x R^2 + \int_x\left\{\frac{\gamma_1}{2}(\nabla_\mu S_\nu)^2 + \frac{\gamma_1 - 2\gamma_2}{12}RS^2\right\}. \tag{52}$$

Just to complete the story, we mention that this expression may be modified by using the relations (12) and (13) in the purely metric part and (26) in the torsion-metric part.

This means, one can use the replacement $R^2 \to \frac{1}{3}R^2_{\mu\nu}$ or $R^2 \to \frac{1}{3}R^2_{\mu\nu\alpha\beta}$ in (52) and use (26) to make similar trades in the S-dependent terms.

It is worth mentioning another detail concerning fermion contributions. The local torsion-dependent terms (52) violate not only conformal (7) but also chiral symmetry (9). This symmetry breaking does not occur in the fermionic nonlocal part (45).

All in all, the general covariant solution for the anomaly-induced action is the sum of the nonlocal (45) and local (52) parts,

$$\Gamma_{\rm ind} = S_c(g,S) + \Gamma_{\rm ind,nonloc} + \Gamma_{\rm ind,loc}, \tag{53}$$

where $S_c(g,S)$ is an arbitrary conformal invariant functional which plays the role of integration constant for our main equation (42). The uncertain elements in this expression are this unknown functional and the ambiguous γ-functions in the local part $\Gamma_{\rm ind,nonloc}$ in (52). Similarly to the ambiguity in the R^2-term, these torsion-dependent local terms may be modified by adding the local nonconformal terms to the *classical* action of vacuum (35). These classical terms are not subject of renormalization and represent a new type of arbitrariness in the action, equivalent to the local multiplicative anomaly.

As usual, we can rewrite the nonlocal part of (53) in the symmetric form and get the induced action in the local covariant form with two auxiliary fields φ and ψ [52] (see also [53]),

$$\Gamma_{\rm ind} = S_c(g,S) + \Gamma_{\rm ind,loc} + \int_x \bigg\{ \frac{1}{2}\varphi\Delta_4\varphi - \frac{1}{2}\psi\Delta_4\psi + \frac{\sqrt{-b}}{2}\varphi\bigg(E_4 - \frac{2}{3}\Box R + \frac{1}{b}Y\bigg) + \frac{1}{2\sqrt{-b}}\psi Y\bigg\}, \tag{54}$$

where the local part and Y are given by (52) and (43), respectively.

The forms (53) and (54) are equivalent to the noncovariant form derived in [34]. Each of this forms has its own advantages, in particular (54) is more suitable for physical applications [9,13]. On another hand, the nonlocal form (53) is more explicit and, also, was recently shown to admit the description of the IR limit [25,26]. We shall apply this approach to the induced action with torsion (53) in the next section.

V. ANOMALY-INDUCED EFFECTIVE ACTION IN THE IR

In the recent works [54,55] it was shown that dynamical torsion may be used to construct phenomenologically successful models of dark matter (DM). On the other hand, there is a general statement that the consistency of quantum field theory of the propagating torsion requires a large torsion mass [48,56], something that can be in contradiction to the DM applications. In this respect, it looks interesting to explore the possibility of dynamical symmetry breaking in the torsion sector. In scalar field theory, this is one of the ways have a large mass in the IR and, at the same time, leave some space for the applications in the high energy physics, including to the early Universe.

In the scalar case, the analysis of symmetry breaking in initially massless theory requires the effective potential [57], that can be also derived in curved spacetime [14,44]. We shall follow [26], where the scalar potential was obtained in the IR limit of the anomaly-induced action, i.e., the scalar analog of (53). We shall concentrate only on the nonlocal part of this action because the local part is ambiguous.

Let us define the meaning of the low-energy (IR) limit in the massless conformal theory, with $\xi_1 = 1/6$. The main assumption is that torsion terms in (43) dominate over the square of the Weyl tensor. This may be a reasonable approximation in the early Universe because Weyl tensor vanishes for the homogeneous and isotropic metric and shows up only because of the metric perturbations. On the other hand, one can assume that torsion plays an important role in the formation of DM and hence should be a strong field [55]. Thus,

$$|S^4| \gg |C^2_{\mu\nu\alpha\beta} \quad |\text{and} \quad |S^2_{\mu\nu}| \gg |C^2_{\mu\nu\alpha\beta}|. \tag{55}$$

As usual in general relativity, the IR limit implies a weak gravitational field. The weak gravity can be described by a small metric perturbation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, that means, e.g., $|\Box R| \gg |R^2_{....}|$ for all curvature tensors (e.g., Weyl, Ricci tensors, and R).

In this approximation, the Green function (46) reduces to

$$G = \Delta_4^{-1} = \bigg(\Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}R^{;\mu}\nabla_\mu\bigg)^{-1} \approx \frac{1}{\Box^2}. \tag{56}$$

Thus, the nonlocal, torsion-dependent part of the effective action (45) boils down to

$$\begin{aligned}\Gamma^{\rm IR}_{\rm ind,nonloc} &= \frac{1}{6}\int_x\int_y \bigg(\beta_1 S^4 + \frac{1}{4}\beta_2 S^2_{\mu\nu}\bigg)_x \bigg(\frac{1}{\Box^2}\bigg)_{x,y} (\Box R)_y \\ &= \frac{1}{6}\int_x\int_y \bigg(\beta_1 S^4 + \frac{1}{4}\beta_2 S^2_{\mu\nu}\bigg)_x \bigg(\frac{1}{\Box}\bigg)_{x,y} R(y).\end{aligned} \tag{57}$$

On top of this expression, the IR limit of the induced effective action includes $\mathcal{O}(R^2_{...})$-terms, but those were discussed in [26], and we can refer the interested reader to this work.

In the presence of torsion, the terms $S^4\Box^{-1}R$ and $S^2_{\mu\nu}\Box^{-1}R$ have the same *global* scaling as the respective classical terms S^4 and $S^2_{\mu\nu}$; i.e., they are invariant under the transformation (7) with $\sigma \to \lambda = \text{const}$. Indeed, this is the usual feature of the nonlocal induced action, independent on extra fields and even spacetime dimension [28], but it is quite remarkable that this feature holds in the IR limit, just as in the scalar case [26].

The next step is to derive the low-energy effective action of torsion from (57). To this end, we separate the conformal factor of the metric and use the analogy with the renormalization group-based derivation of effective action [44,58]. At one loop, it is sufficient to account only for the linear in σ terms. Thus, we consider

$$g_{\mu\nu} = \bar{g}_{\mu\nu} e^{2\sigma}, \quad \sqrt{-g} S^4 = \sqrt{-\bar{g}} \bar{S}^4, \quad \sqrt{-g} S^2_{\mu\nu} = \sqrt{-\bar{g}} \bar{S}^2_{\mu\nu},$$
$$\Box^{-1} = e^{2\sigma} \bar{\Box}^{-1}, \quad R = e^{-2\sigma}[\bar{R} - 6\bar{\Box}\sigma], \tag{58}$$

where $\bar{\Box} = \bar{g}^{\mu\nu}\bar{\nabla}_\mu \bar{\nabla}_\nu = \frac{1}{\sqrt{-\bar{g}}}\partial_\mu \sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu$. In this framework, (57) becomes

$$\Gamma^{\text{IR}}_{\text{ind,nonloc}} = -\int_x \left(\beta_1 \bar{S}^4 + \frac{1}{4}\beta_2 \bar{S}^2_{\mu\nu}\right)_x \sigma(x). \tag{59}$$

This result demonstrates, as we expected, that the anomaly-induced action is a local version of the renormalization group corrected classical action (35), that means the substitution,

$$a_1 \to a_1 - \beta_1 \sigma(x), \qquad a_2 \to a_2 - \beta_2 \sigma(x). \tag{60}$$

Compared to the usual curved-space renormalization group [14,23], the constant scaling parameter λ is traded for the coordinate-dependent conformal factor of the metric σ; i.e., we arrive at the local form of renormalization group in curved space [13].

At this point, one can use (60) to recover the low-energy effective action. This requires identification of the scale parameter σ, and we have several choices because of the scaling rules,

$$S^2 = \bar{S}^2 e^{-2\sigma}, \qquad S^4 = \bar{S}^4 e^{-4\sigma}, \qquad S^2_{\mu\nu} = \bar{S}^2_{\mu\nu} e^{-4\sigma}. \tag{61}$$

For example, choosing the first option, we arrive at the identification $\sigma \to -\frac{1}{2}\ln\frac{S^2}{\mu^2}$. Then, the improvement (60) of the action (35) gives, in the torsion-dependent sector,

$$\Gamma_{\text{tors}} = -\int d^4x \sqrt{-g}\left\{\left[a_1 + \frac{\beta_1}{2}\ln\left(\frac{S^2}{\mu^2}\right)\right]S^4 + \frac{1}{4}\left[a_2 + \frac{\beta_2}{2}\ln\left(\frac{S^2}{\mu^2}\right)\right]S^2_{\mu\nu} + \cdots\right\}, \tag{62}$$

where we omitted surface terms. An obvious observation here is that (62) is not just an integral of the effective potential since there is a kinetic term $S^2_{\mu\nu}$. Thus, the result can be seen as a form of the one-loop effective action in the IR limit.

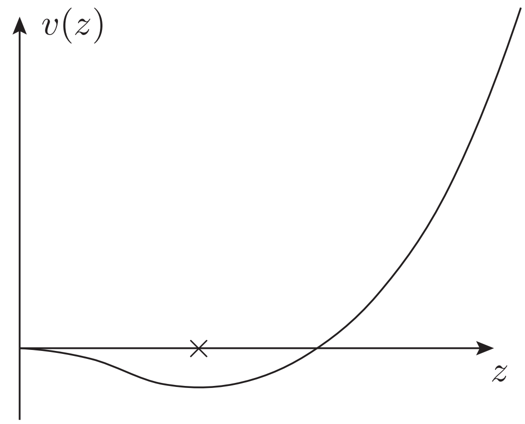

FIG. 1. Plot of $v(z)$ demonstrating the possibility of dynamical symmetry breaking for a positive β_1.

The effective potential part of (62) has the form,

$$V_{\text{eff}} = \left[a_1 + \frac{\beta_1}{2}\ln\left(\frac{S^2}{\mu^2}\right)\right]S^4, \tag{63}$$

together with the negative β_1-function (37) shows that the one-loop potential always becomes unstable for large values of S^2, where the quantum corrections start to dominate over the classical coefficient a_1. The coefficients η for fermions are experimentally bounded by very small values, at least for electrons (one can use [36] as a starting point for further references on the subject). Thus, according to (37), the strong effect of the negative β_1 may be expected only for extremely large values of S^2. Anyway, at the one-loop level the effective potential is unbounded from below.

This feature does not mean that the theory, in general, is badly defined at the quantum level because the second and higher loop contributions may restore the positive definiteness of the potential. On the other hand, assuming the change of sign of β_1 at higher loops, we can rewrite the effective potential part of (62) in terms of the dimensionless parameter $z = S^2/\mu^2$,

$$V_{\text{eff}} = \mu^4 v(z) = a_1 \mu^4 [z^2(1 + \tilde{\beta}\ln z)], \qquad \tilde{\beta} = \frac{\beta_1}{2a_1}. \tag{64}$$

The qualitative profile of the function $v(z)$ for $\beta_1 > 0$ is shown in Fig. 1. However, since the real sign of the beta function is negative, the implementation of the dynamical symmetry breaking in this theory requires further investigation and, especially, higher loops contributions to the potential.

VI. CONCLUSIONS AND DISCUSSIONS

We calculated the vacuum divergences and formulated, for the first time, the covariant version of the anomaly-induced effective action in curved spacetime with torsion. The output can be presented in the covariant nonlocal form (53) or in the local form with auxiliary scalars (54). The main novelty is the detection of the multiplicative anomaly in the total derivative part of the divergences, i.e., (27) vs (34) and the corresponding ambiguity in the local part of the induced effective action. The ambiguity cannot be removed by the change of renormalization condition and represents a new feature of massless fermionic determinants that does not take place without torsion.

Multiplicative anomaly appears in the finite local part of induced effective action, as shown in expression (52). All the terms in this action are ambiguous. The coefficient c may be modified by adding the finite $\int_x R^2$ term to the classical action or, equivalently, by the choice of the divergent Weyl-squared counterterm [31]. On the other hand, the ambiguity in the torsion-dependent terms can be compensated by adding the local nonconformal terms similar to those in (52), to the classical action (35).

Another new result of our work is the covariant expression for the low-energy limit of the anomaly-induced effective action (62). This part may be eventually useful for describing dynamical symmetry breaking in torsion theories but, independent on that, we have an interesting analogy with the scalar effective potential in the axial vector model. On the other hand, the low-energy effective action (62) by itself may serve as an evidence of breaking local conformal symmetry by quantum corrections. In this sense, it is an analog of the effective potential of a scalar field ϕ in the conformal theory, where the $\phi^4 \ln(\phi/\mu)$-term breaks the symmetry of the classical ϕ^4-type potential. It looks remarkable that we can obtain this low-energy breaking with torsion from the anomaly-induced effective action (53).

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