Inaccessibility of traversable wormholes

Daniel R. Terno

School of Mathematical and Physical Sciences, Macquarie University, Sydney, New South Wales 2109, Australia

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Wormhole solutions to the equations of general relativity have some spectacular local and global properties. As these unusual features are not explicitly forbidden by known physics, wormholes are considered in various astrophysical and cosmological scenarios. The paradigmatic traversable wormhole models are described by static spherically symmetric Ellis-Morris-Thorne and Simpson-Visser metrics. We show that no dynamical solution of the semiclassical Einstein equations can have these metrics as their static limit. On the other hand, possible static limits of the dynamical solutions are not traversable. Moreover, they lead to violation of a quantum energy inequality that bounds violations of the null energy condition by quantum fields. This conclusion does not depend on specific properties of fields that may be proposed for wormhole construction. As a result, spherically symmetric wormholes cannot exist in semiclassical gravity.

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I. INTRODUCTION

Wormhole solutions in general relativity [1–3] have been known for almost as long as the Schwarzschild metric. However, their status is different. The black hole paradigm explains all current observations of dark massive ultracompact objects. Wormholes are just one of the alternative models. As such, they aim to describe astrophysical black holes without causing conceptual problems inherent in the notions of an event horizon and singularity [4–6]. Even the revival of interest in wormholes as traversable shortcuts between separated spacetime regions had science fiction as its primary motivation [7,8].

Several wormhole features, even if not forbidden by the laws of physics, are unusual enough to make their existence unlikely [8]. The null energy condition (NEC) is the weakest energy requirement that is used in general relativity. The NEC is satisfied by normal classical matter [9,10]. Quantum field theory permits its violations that are, however, constrained by quantum energy inequalities [11,12]. NEC violation is a generic and universal feature of (traversable) wormholes [13]. Creation of a wormhole implies a change in the topology of space, and using a pair of wormholes is a simple way to generate closed timelike loops, i.e., to create a time machine [1–3,8].

On the other hand, the NEC could be sufficiently violated in the early Universe to enable formation of wormholes [1,2,14]. Its violation is a necessary condition for formation of a trapped spacetime region, i.e., a physical black hole (PBH) [15], in finite time of a distant observer [9,16]. Moreover, while the phantom matter that might effect the necessary NEC violation is unphysical [17,18], asymmetric wormhole solutions may be allowed [19]. Topology changes are expected to occur in quantum gravity and are a basic component of the path integral approach to it [2,20,21]. These arguments provide additional impetus for search for astrophysical wormholes, using both electromagnetic and gravitational radiation [4–6].

The original static wormhole solutions were characterized using the embedding diagrams and explicit description of the two spatial sheets that are connected at the wormhole's throat [7,22,23]. The invariant characterization of the throat that is valid for generic wormholes identifies it as an outer marginal trapped surface subject to additional conditions [13,21]. This allows us to describe dynamical wormholes [21,24] and also to apply the self-consistent analysis of black hole horizons [25]. In the case of spherical symmetry, it allows an exhaustive description of the admissible solutions [16].

Using properties of these solutions, we find that the standard static traversable wormhole (TWH) solutions are not static limits of dynamical solutions. Moreover, the admissible static limits are not only nontraversable wormholes, but violate the quantum energy inequalities (QEIs), making their introduction in semiclassical physics self-contradictory.

This article is organized as follows. In the next section, we review the semiclassical physics of spherically symmetric horizons. Section III reviews the basic properties of the static wormhole solutions and positions the Ellis-Morris-Thorne and the Simpson-Visser metrics within a general scheme of self-consistent solutions. Section IV contains the main original results of this work and presents the dynamic wormhole solutions and their static limits.

The results, their implications, and future directions are discussed in Sec. V.

We use the $(-+++)$ signature and use the Planck units. Derivatives of a function of a single variable are marked with a prime, e.g., $r_g'(t) \equiv \mathrm{d}r_g/dt$. Derivatives with respect to the proper time τ are denoted by the dot, $\dot{R} \equiv dR/d\tau$.

II. THE SETUP

Properties of TWHs are usually investigated by first designing a metric with the desired properties. This is possible if the classical notions and semiclassical and/or modified Einstein equations are applicable. Then the Einstein equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \equiv 8\pi\langle \hat{T}_{\mu\nu}\rangle_\omega \tag{1}$$

are reverse engineered to determine the energy-momentum tensor (EMT) that is the source of this geometry [2,26]. Here the Einstein tensor $G_{\mu\nu} \coloneqq R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}$, where $R_{\mu\nu}$ and $\mathcal{R}$ are the Ricci tensor and Ricci scalar, respectively, is equated with the effective EMT. The latter is an expectation value of the renormalized EMT operator plus all additional curvature terms. They appear, e.g., as a result of the renormalization procedure or are derived from the Lagrangian of an effective field theory of gravity [27–29]. We do not make any assumptions about the nature of the fields or of the state ω.

A general four-dimensional spherically symmetric metric in Schwarzschild coordinates is given by [30,31]

$$ds^2 = -e^{2h(t,r)}f(t,r)dt^2 + f(t,r)^{-1}dr^2 + r^2 d\Omega_2. \tag{2}$$

These coordinates provide geometrically preferred foliations with respect to Kodama time, which is derived from a natural divergence-free vector field [31,32]. Invariantly defined Misner-Sharp mass [31,33] $C(t,r)/2$ allows us to write

$$f(t,r) \coloneqq 1 - C/r \coloneqq \partial_\mu r \partial^\mu r. \tag{3}$$

In asymptotically flat spacetimes, the coordinate t is the physical time of a distant static observer (Bob). However, our results do not depend on this interpretation.

Using the advanced null coordinate v the metric takes the form

$$ds^2 = -e^{2h_+(v,r)}f(v,r)dv^2 + 2e^{h_+(v,r)}dvdr + r^2 d\Omega_2, \tag{4}$$

where $f = 1 - C_+(v,r)/r$ and the invariance of the Misner-Sharp mass ensures $C_+(v,r) \equiv C(t(v,r),r)$. The coordinates are related via

$$dt = e^{-h}(e^{h_+}dv - f^{-1}dr). \tag{5}$$

In all foliations that respect spherical symmetry [34], components of the apparent horizon (a 3D boundary of the trapped region) [9,16,31] coincide with the roots of

$$f(t,r) = 0. \tag{6}$$

The Schwarzschild radius $r_g(t)$, being the largest root, corresponds to the outer apparent horizon [16,31]. For the wormhole solutions $r_g(t)$ is the only root on each sheet of the radial coordinate. It corresponds to the throat of the wormhole, and the circumferential radius is restricted to the range $r_g \leq r < \infty$. Hence, constructing solutions of the Einstein equations that satisfy Eq. (6) is the first step in developing wormhole solutions.

The self-consistent approach to black holes jointly identifies the forms of the EMT and of the metric functions h and C in the vicinity of the apparent horizon [16,25]. It is adopted here to generate the wormhole solutions. Two requirements [16,25] allow us to describe all potential geometries in the vicinity of r_g. First, Eq. (6) is required to have a solution for $t_S \leq t \leq t_* < \infty$ for some finite t_S. This allows for formation and a possible closure of the wormhole. It is important to note that, despite the Misner-Sharp mass invariance, having solutions of $f(v,r) = 0$ or $f(v,u) = 0$, where v and u are the advanced and the retarded null coordinates, respectively, does not imply that a TWH forms at the finite time of Bob.

Second, the throat at $r = r_g$ is a regular two surface in a sense that the curvature scalars that are constructed from polynomials of components of the Riemann tensor are finite. Apart from a minimal compliance with the cosmic censorship conjecture, it is also part of the requirements that ensure traversability of the wormhole [2,7]. We use two quantities that can be obtained directly from EMT components,

$$\tilde{\mathrm{T}} \coloneqq T^\mu{}_\mu, \qquad \tilde{\mathfrak{T}} = T_{\mu\nu}T^{\mu\nu}. \tag{7}$$

The Einstein equations relate them to the curvature scalars as $\tilde{\mathrm{T}} \equiv -\mathcal{R}/8\pi$ and $\tilde{\mathfrak{T}} \equiv R^{\mu\nu}R_{\mu\nu}/64\pi^2$. For the spherically symmetric solutions, finite values of these scalars as $r \to r_g$ ensure that all independent scalar invariants are finite [16].

It is convenient to introduce the effective EMT components

$$\tau_t \coloneqq e^{-2h}T_{tt}, \qquad \tau^r \coloneqq T^{rr}, \qquad \tau_t{}^r \coloneqq e^{-h}T_t{}^r. \tag{8}$$

Then the Einstein equations for the components G_{tt}, G_t^r, and G^{rr} are

$$\partial_r C = 8\pi r^2 \tau_t / f, \tag{9}$$

$$\partial_t C = 8\pi r^2 e^h \tau_t{}^r, \tag{10}$$

$$\partial_r h = 4\pi r(\tau_t + \tau^r)/f^2. \tag{11}$$

To ensure the finite values of the curvature scalars, it is sufficient to work with

$$\mathrm{T} \coloneqq (\tau^r - \tau_t)/f,$$
$$\mathfrak{T} \coloneqq ((\tau^r)^2 + (\tau_t)^2 - 2({\tau_t}^r)^2)/f^2, \tag{12}$$

where the contribution of $T^\theta{}_\theta \equiv T^\phi{}_\phi$ is disregarded, and then to verify that the resulting metric functions do not introduce further divergences [16]. Thus, the three effective EMT components either diverge, converge to finite limits, or converge to zero in such a way that the above combinations are finite. One option is the scaling

$$\tau_t \sim f^{k_E}, \qquad \tau^r \sim f^{k_P}, \qquad {\tau_t}^r \sim f^{k_\Phi}, \tag{13}$$

for some powers $k_a > 1$, $a = E, P, \Phi$. Another involves convergence or divergence with the same $k \leq 1$. For PBHs, only solutions with $k = 0$, 1 are relevant.

Solutions of the $k = 0$ class satisfy

$$\tau_t \to \tau^r \to -\Upsilon^2(t), \qquad {\tau_t}^r \to \pm\Upsilon^2(t), \tag{14}$$

as $r \to r_g$. The negative sign of τ_t and thus of τ^r is necessary to obtain the real-valued solutions of Eqs. (9)–(11). The leading terms of the metric functions near the outer apparent horizon are

$$C = r_g - 4\sqrt{\pi} r_g^{3/2} \Upsilon \sqrt{x} + \mathcal{O}(x), \tag{15}$$

$$h = -\frac{1}{2}\ln\frac{x}{\xi} + \mathcal{O}(\sqrt{x}), \tag{16}$$

where $\xi(t)$ is determined by the choice of time variable, and the higher-order terms are matched with the higher-order terms in the EMT expansion. Equation (10) must then hold identically. Both sides contain terms that diverge as $1/\sqrt{x}$, and their matching results in the consistency condition

$$r_g'/\sqrt{\xi} = 4\epsilon_\pm \sqrt{\pi r_g}\Upsilon, \tag{17}$$

where $\epsilon_\pm = \pm 1$ corresponds to the expansion and contraction of the Schwarzschild sphere, respectively.

Geometry near and across the Schwarzschild sphere is conveniently expressed [16,25] in (v, r) coordinates for $r_g' < 0$ and in (u, r) coordinates (where u is the retarded null coordinate) for $r_g' > 0$. The extended solutions describe an evaporating PBH ($r_g' < 0$) and an expanding white hole ($r_g' > 0$). Vaidya metrics [with $M'(v) < 0$ and $M'(u) > 0$, respectively] are examples of such objects belonging to the $k = 0$ class.

Details of $k = 1$ solutions are given in Sec. IV and Appendix A. Both $k = 0$ and $k = 1$ solutions satisfy

$$\lim_{r \to r_g} e^h f = |r_g'|, \tag{18}$$

which ensures a finite infall time also according to a distant Bob [16,35].

The limiting form of the (tr) block of a $k = 1$ EMT as $r \to r_g$ is

$$T^a{}_b \approx \begin{pmatrix} \Upsilon^2/f & -\epsilon_\pm e^{-h}\Upsilon^2/f^2 \\ \epsilon_\pm e^h \Upsilon^2 & -\Upsilon^2/f \end{pmatrix}, \tag{19}$$

where $a, b = t, r$. According to a static (outside) observer, the local energy density, pressure, and flux diverge as $r \to r_g$.

In addition to the usual list of requirements that make a wormhole traversable, experience with PBHs indicates another necessary feature: absence of strong firewalls, i.e., of divergent negative energy density and/or pressure and flux in the frame of a traveling observer (Alice) [16,36]. These firewalls may occur even if the curvature scalars are finite. While they indicate that the apparent horizon of a PBH is a surface of intermediate curvature singularity, a sufficiently strong firewall leads to a divergent integrated energy density, violating the QEIs. In particular, along a timelike geodesic γ with a tangent four-vector u^μ_{A}, the local energy density is

$$\rho_{\mathrm{A}} \coloneqq \langle \hat{T}^{\mathrm{ren}}_{\mu\nu} \rangle_\omega u^\mu_{\mathrm{A}} u^\nu_{\mathrm{A}}, \tag{20}$$

where the expectation of the renormalized EMT is evaluated on an arbitrary Hadamard state [12,27] ω. The total integrated energy is obtained along by integration along the timelike trajectory γ of the energy density that is smeared by a sampling function $\wp$ with a compact support. It can be taken to be $\wp \cong 1$ for an arbitrarily large fraction of the domain with $\wp > 0$. Then,

$$\int_\gamma d\tau \wp^2(\tau)\rho(\tau) \geq -B(\gamma, \mathcal{R}, \wp), \tag{21}$$

where $B > 0$ is a bounded function that depends on the trajectory, Ricci scalar, and sampling function [37]. Violation of this bound by a particular solution indicates its impossibility in semiclassical gravity. While integrated energy densities in the case of the PBH firewalls are finite, we will see that some potential wormhole solutions lead to violation of this QEI.

We illustrate the issue by considering $k = 0$ solutions. For an incoming Alice, the energy density in her frame remains finite. It is $\rho_{\mathrm{A}} \propto r_g'/\dot{R}$ at the apparent horizon [16]. On the other hand, for an outgoing observer, outside a PBH,

$$\rho_{\mathrm{A}} = \frac{\dot{R}^2}{4\pi r_g X} + \mathcal{O}(1/\sqrt{X}), \tag{22}$$

where $X \coloneqq R(\tau) - r_g(T(\tau 0)$. The energy density is obtained by using the expression for the EMT of $k = 0$ solutions [Eq. (19)] and the normalization of the four velocity in the form

$$\dot{T} = \frac{\sqrt{\dot{R}^2 + F}}{e^H F} \approx \frac{|\dot{R}|}{|r_g'|}, \tag{23}$$

where $F = f(T,R)$ and $H = h(T,R)$. For a PBH, this divergence is largely a curious observation (observers that may exit the so-called quantum ergosphere cannot have $\dot{R} > 0$ at the apparent horizon [16,36]). However, if this metric describes the neighborhood of a wormhole throat for $r \gtrsim r_g$, then an exiting Alice will have $\dot{R} > 0$ at the throat, and such a divergence contradicts traversability.

III. STANDARD STATIC TRAVERSABLE WORMHOLES

In horizonless spacetimes, it is often convenient to represent a spherically symmetric metric as

$$ds^2 = -e^{2\Psi}dt^2 + f(t,r)^{-1}dr^2 + r^2 d\Omega_2, \tag{24}$$

where $\Psi \equiv h + \frac{1}{2}\ln f$. Requiring $g_{tt}(t, r_g)$ to be finite while $f \to 0$ implies that, in addition to Eq. (11), the equation

$$\partial_r h \approx -\frac{\partial_r f}{2f} \approx -\frac{\lambda}{2x} \tag{25}$$

holds as well, where we kept only the divergent terms. The final expression above is a leading term in the expansion in powers of $x \coloneqq r - r_g$ of $f \approx \phi(t)x^\lambda$, where $\phi(t)$ is some function.

After substituting Eq. (25) in Eq. (11) and using Eq. (9) we obtain a local algebraic relation between the metric functions mass and the EMT component. If $\Psi = 0$, then the relation $\tau^r = -Cf/(8\pi r^3)$ holds exactly. On the other hand, the expansion

$$\tau^r = -(8\pi r_g^2)^{-1} f + \ldots \tag{26}$$

near the Schwarzschild radius is valid for a general finite Φ.

For static wormholes, the physical distance allows to introduce the coordinate l, $-\infty < l(r) < \infty$, that describes both sides of the bridge via $dl = \pm dr/\sqrt{f}$. The regular metric functions have series expansions,

$$C = b_0 + b_1 x + b_2 x^2 + \ldots, \quad \Psi = \phi_1 x + \phi_2 x^2 + \ldots, \tag{27}$$

where $x \coloneqq r - b_0$, and we absorbed the constant term in Ψ by redefining the time. The leading coefficient b_1 must satisfy $b_1 \leq 1$ to make the throat at $r_g \equiv b_0$ a marginally trapped surface. Then,

$$f(r) = \frac{(1-b_1)x}{b_0} + \mathcal{O}(x^2), \tag{28}$$

and

$$h = -\frac{1}{2}\ln\frac{x}{\xi} + \left(\phi_1 + \frac{1 - b_1 + b_0 b_2}{2b_0(1-b_1)}\right)x + \ldots, \tag{29}$$

where

$$\xi \equiv \frac{b_0}{1-b_1}. \tag{30}$$

This form allows for a more convenient comparison with PBH solutions. At the throat,

$$\rho(r_g) = \frac{b_1}{8\pi b_0^2}, \qquad p(r_g) = -\frac{1}{8\pi b_0^2}. \tag{31}$$

Expansion of τ_t, τ^r, and $\tau_t{}^r$ in terms of x shows that for $b_1 \neq 0$ these solutions belong to the $k = 1$ class.

The standard Ellis-Morris-Thorne metric [7,22] corresponds to $\Psi = 0$ and

$$C = b_0^2/r, \tag{32}$$

$$h = -\frac{1}{2}\ln 2x/b_0 + \mathcal{O}(x), \tag{33}$$

with the throat at $r_g = b_0$, and thus

$$b_1 = -1, \qquad b_2 = 1/b_0. \tag{34}$$

The Simpson-Visser metric [38] interpolates between the Schwarzschild black hole and TWHs,

$$ds^2 = -\left(1 - \frac{2m}{\sqrt{\eta^2 + a^2}}\right)dt^2 + \frac{d\eta^2}{1 - \frac{2m}{\sqrt{\eta^2+a^2}}} + (\eta^2 + a^2)d\Omega_2, \tag{35}$$

where a is a parameter. The Misner-Sharp mass is $C = 2m + a^2(r - 2m)/r$, and for $a \geq 2m$ the throat is located at $r_g = a$.

Expanding the metric functions near the throat, we find for $a > 2m$,

$$C = a + \frac{4m - a}{a}x + \mathcal{O}(x^2), \tag{36}$$

$$h = -\frac{1}{2}\ln\frac{2(a-2m)}{a}x + \mathcal{O}(x), \tag{37}$$

and for $a = 2m$ (the one-way wormhole [38]),

$$C = r + \frac{2}{a}x^2 + \mathcal{O}(x^3), \tag{38}$$

$$h = -\ln\frac{\sqrt{2}x}{a} + \mathcal{O}(x). \tag{39}$$

Both Ellis-Morris-Thorne and Simpson-Visser metrics belong to the $k = 1$ class.

IV. DYNAMICAL WORMHOLE SOLUTIONS AND THEIR LIMITS

There are no static $k=0$ solutions, as in this case the scalar $\tilde{\mathfrak{T}}$ cannot be finite. On the other hand, static solutions are not only possible in the class of $k=1$,

$$\tau_t \to E(t)f, \qquad \tau^r \to P(t)f, \qquad \tau_t{}^r \to \Phi(t)f, \quad (40)$$

but the two most popular static TWH metrics belong to it.

The energy density $\rho(t,r_g)=E$ and the pressure $p(t,r_g)=P$ are finite at the Schwarzschild radius. (This is also true in the proper reference frame of a static observer.) For $E<(8\pi r_g^2)^{-1}$, the resulting metric functions are

$$C = r_g(t) + 8\pi E r_g^2 x + \ldots \quad (41)$$

and

$$h = -\ln\frac{x}{\xi(t)} + \mathcal{O}(\sqrt{x}) + \ldots, \quad (42)$$

for some $\xi(t)>0$. Consistency of the Einstein equations results in

$$P = E - \frac{1}{4\pi r_g^2}, \qquad \Phi = \pm\left(\frac{1}{8\pi r_g^2} - E\right). \quad (43)$$

Equation (10) implies now

$$r_g' = 8\pi\Phi\xi r_g. \quad (44)$$

A static $k=1$ configuration can be reached from the solution where the energy density at r_g takes its maximal possible value $E=1/(8\pi r_g^2)$ [39]. While only such extreme $k=1$ solutions can describe PBHs [39], there is no such restriction on potential wormhole solutions. In the extreme limit $P=-E$, $\Phi=0$, and

$$C(t,r) = r + c_{32}(t)x^{3/2} + c_2(t)x^2 + \mathcal{O}(x^{3/2}), \quad (45)$$

for some coefficient $c_{32}(t)<0$, and

$$h = -\frac{3}{2}\ln(x/\xi) + \mathcal{O}(\sqrt{x}), \quad (46)$$

while Eq. (10) implies

$$r_g' = \pm|c_{32}|\xi^{3/2}/r_g. \quad (47)$$

The static limit is possible if as $t\to t_0$ the parameters $c_{32}(t)\to 0$ and $\xi(t)\to\xi_0$ (see Appendix A for details). In this case, the static solution has $f=|c_2|x^2/r_g+\mathcal{O}(x^{3/2})$ for some constant $c_2<0$, while h given by Eq. (46) with $\xi=\xi_0$.

As indicated by a different behavior of the function h (a prefactor $\frac{3}{2}$ instead of $\frac{1}{2}$), the resulting static metric is different from the standard TWH metrics. This can be attributed to the existence of an additional constraint: a real solution with $f=0$ and finite $\tilde{\mathrm{T}}$ and $\tilde{\mathfrak{T}}$ also has a finite nonzero g_{tt}. This extra requirement cannot be satisfied dynamically while conforming to the two basic PBH conditions.

Indeed, if $0<|g_{tt}|<\infty$, then Eq. (26) implies, since T and $\mathfrak{T}$ are finite, that the solution either belongs to the class $k=1$ or to one of the classes with $k_E>1$. In the $k=1$ case, we find $E=-P=1/(8\pi r_g^2)$, $\Phi=0$. Following through, we then arrive at Eqs. (45) and (46), contradicting Eq. (25) and thus the initial assumption. Looking at the observable quantities, we find, e.g., that for the Ellis-Morris-Thorne metric $E=+P=-1/(8\pi r_g^2)$, which is impossible in dynamical solutions.

Solutions with $k_E>1$ are possible. Consistent dynamical solutions exist for half-integer values of $k_E\geq 2$ and $k_P=k_\Phi=1$ [35]. However, in addition to the presence of a strong firewall for some of the geodesic observers, they do not have a static limit: the Ricci scalar $\mathcal{R}(t,r_g)$ diverges when $r_g'=0$ (see Appendix B for details).

So far, we have seen that the standard TWH solutions are not the static limits of some allowed semiclassical solutions of the Einstein equations. We now demonstrate that the static limits of the allowed $k=1$ wormhole solutions are not traversable.

For definiteness, consider an ingoing radial trajectory of Alice, $u_{\mathrm{A}}=(\dot{T},\dot{R},0,0)$, $u_{\mathrm{A}}^2=-1$. Energy conservation on a static background determines the radial velocity via $\dot{R}=-\sqrt{\mathfrak{E}^2-f(R)}$, where $\mathfrak{E}\geq 1$ is Alice's energy per unit mass at infinity. Using the EMT that is reverse engineered from the metric of Eqs. (45) and (46) with $c_{32}=0$ and $\xi=\xi_0$, we find

$$\rho_{\mathrm{A}} = -\frac{3\mathfrak{E}^2}{8\pi r_g X} + \mathcal{O}(\sqrt{X}), \quad (48)$$

where $X(\tau)\coloneqq R(\tau)-r_g$ and $\dot{X}=\dot{R}$. We choose the sampling function $\wp=1$ in some vicinity of the throat and let $\wp\to 0$ still within the NEC-violating domain. As the trajectory passes through $X_0+r_g\to r_g$, the lhs of Eq. (21) behaves as

$$\int_\gamma \wp^2\rho_{\mathrm{A}}d\tau = \frac{3\mathfrak{E}}{8\pi r_g}\int_\gamma \frac{1+\mathcal{O}(\sqrt{X})}{X}dX \\ \propto \log X_0 \to -\infty, \quad (49)$$

where we used $\dot{R}\approx-\mathfrak{E}$ in the vicinity of the Schwarzschild radius r_g. The right-hand side of Eq. (21) is finite, and thus the QEI is violated.

Similar firewalls occur also in dynamical solutions. For example, exiting through the contracting ($r_g' < 0$) throat of the $k = 0$ wormhole with finite radial velocity $\dot{R} > 0$ leads to $\rho_A \propto 1/X$, which is stronger than weak firewalls on nongeodesic trajectories that approach the apparent horizon of a PBH [36].

V. DISCUSSION

The requirement of finite time of formation according to a distant observer not only probes the constraints that "the laws of physics place on the activities of an arbitrarily advanced civilization" [23], but investigates the local implications of the topology change. The minimal regularity requirement not only enforces compliance with the cosmological censorship, but is a part of the traversability requirement.

In spherical symmetry, these two necessary assumptions are enough to produce an exhaustive description of potential geometries. However, none of them leads in the standard static TWHs.

Impossibility of wormhole formation in finite time is different from the asymptotic nature of, say, the Schwarzschild solution. Even if the apparent horizon may never form according to Bob [16], the classical black hole geometry provides an excellent description of the exterior, while approach to it is exponentially fast. On the other hand, the defining feature of a wormhole is its being a shortcut between spacetime regions [1,3]. Without a throat where the connection happens, there is no wormhole. In the scenarios where a stationary limit can be reached, the resulting wormhole is not traversable: Alice experiences an infinite negative energy density. Moreover, similar to some dynamical configurations, this divergent negative energy violates the QEI that prescribes a finite value to the integrated NEC violation.

Various TWH solutions were designed under the assumption that the semiclassical gravity is valid. Studies of the amount of the NEC violation, necessary matter content, etc., are performed within this framework. The self-consistent analysis of the semiclassical Einstein equations does not use any information about the matter content. Its main conclusion is that none of the possible spherically symmetric wormhole solutions is a TWH. As a result, we have to accept that existence of macroscopic spherically symmetric wormholes requires not only a large-scale violation of the NEC and violation of the topology and chronology projection conjectures, but breakdown of semiclassical gravity. It is remarkable that the analysis is based on local considerations, despite the global implications of wormhole existence.

This work is limited to spherically symmetric solutions. To answer the question whether existence of traversable wormholes within the confines of known physics can be ruled out on the basis of local considerations, more general configurations and, in particular, axially symmetric scenarios should be considered.

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APPENDIX A: SOME PROPERTIES OF $k=1$ SOLUTIONS

For the extreme solutions $E = 1/(8\pi r_g^2)$. Equation (43) then implies $\Phi = 0$ and $P = -E = -1/(8\pi r_g^2)$. As a result, $C(t, r)$ is given by Eq. (41), and using the next order EMT expansion leads to Eqs. (45) and (46). A potentially divergent term in expansion of the Ricci scalar,

$$\mathcal{R}_{\text{div}} = \frac{3}{2\sqrt{x}}\left(\frac{c_{32}}{r_g} - \frac{r_g r_g'}{c_{32}\xi^3}\right), \tag{A1}$$

is identically zero due to Eq. (47). However, metrics with $C - r$ starting with a higher power of x, such as $f = |c_2|x^2/r_g + \mathcal{O}(x^{3/2})$ can be only static: the Ricci scalar is finite at the apparent horizon only if $r_g' = 0$.

In the static case as $r \to r_g$,

$$T_{ab} \approx \frac{1}{8\pi r_g x}\begin{pmatrix} -c_2\xi_0^3/r_g^2 & 0 \\ 0 & 1/(c_2 x) \end{pmatrix}, \tag{A2}$$

while the leading-order expansion of the four velocity of a free-falling Alice is

$$u_A^\mu \approx \mathfrak{E}\left(\frac{r_g}{c_2\xi_0^{3/2}\sqrt{X}}, \pm 1\right), \tag{A3}$$

where $\mathfrak{E} = \text{const}$ is Alice's energy per unit mass at infinity.

APPENDIX B: SOME PROPERTIES OF $k_E > 1$ SOLUTIONS

Here we adopt the analysis of [35]. For the effective EMT components with $k_a \geq 1$, the leading terms in the Einstein equations become

$$\partial_r C \approx 8\pi r_g^2 E(t) f^{k_E - 1}, \tag{B1}$$

$$\partial_t C \approx 8\pi r_g^2 e^h \Phi(t) f^{k_\Phi}, \tag{B2}$$

$$\partial_r h \approx 4\pi r_g (E(t) f^{k_E - 2} + P(t) f^{k_P - 2}), \tag{B3}$$

for some functions $E(t)$, $P(t)$, and $\Phi(t)$ and the powers $k_E, k_\Phi, k_P \geq 1$.

The leading terms of the Misner-Sharp mass are then

$$C = r_g(t) + 8\pi E r_g^2 x^{k_E} + \dots. \tag{B4}$$

Solutions with $k_E = 1$ and $k_P > 1$ and/or $k_\Phi > 1$ lead to a divergent Ricci scalar. However, solutions with $k_E \geq \frac{3}{2}$ are consistent. In this case,

$$f = x/r_g + \dots. \tag{B5}$$

Solutions with variable $r_g(t)$ impose via Eq. (11) the logarithmic divergence of the function h, as it is necessary that $e^h \propto x^{-k_\Phi}$. It can be realized only if $k_P = 1$. Then,

$$h = 4\pi P r_g^2 \ln\frac{x}{\xi}, \qquad 4\pi P r_g^2 = -k_\Phi. \tag{B6}$$

Expressing this solution in (u, r) or (v, r) coordinates leads to $k_\Phi = 1$ and $\Phi = \pm 1/(8\pi r_g^2)$. These solutions are rather peculiar: energy density vanishes at r_g and the pressure and the flux are determined by the Schwarzschild radius. A potentially divergent term in expansion of the Ricci scalar,

$$\mathcal{R}_{\text{div}} = -\frac{1}{x}\left(\frac{1}{r_g} + \frac{r_g r_g'^2}{\xi^2}\right), \tag{B7}$$

and its leading coefficient is nonzero if $r_g' = 0$.

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