

Tightening geometric and dynamical constraints on dark energy and gravity: Galaxy clustering, intrinsic alignment, and kinetic Sunyaev-Zel'dovich effect

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Conventionally, in galaxy surveys, cosmological constraints on the growth and expansion history of the Universe have been obtained from the measurements of redshift-space distortions and baryon acoustic oscillations embedded in the large-scale galaxy density field. In this paper, we study how well one can improve the cosmological constraints from the combination of the galaxy density field with velocity and tidal fields, which are observed via the kinetic Sunyaev-Zel'dovich (kSZ) and galaxy intrinsic alignment (IA) effects, respectively. For illustration, we consider the deep galaxy survey by Subaru Prime Focus Spectrograph, whose survey footprint perfectly overlaps with the imaging survey of the Hyper Suprime-Cam and the CMB-S4 experiment. We find that adding the kSZ and IA effects significantly improves cosmological constraints, particularly when we adopt the nonflat cold dark matter model which allows both time variation of the dark energy equation-of-state and deviation of the gravity law from general relativity. Under this model, we achieve 31% improvement for the growth index γ and >35% improvement for other parameters except for the curvature parameter, compared to the case of the conventional galaxy-clustering-only analysis. As another example, we also consider the wide Galaxy survey by the *Euclid* satellite, in which shapes of galaxies are noisier but the survey volume is much larger. We demonstrate that when the above model is adopted, the clustering analysis combined with kSZ and IA from the deep survey can achieve tighter cosmological constraints than the clustering-only analysis from the wide survey.

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I. INTRODUCTION

The baryon acoustic oscillations (BAO) [1–3] and redshift-space distortions (RSD) [4–6] imprinted in large-scale galaxy distribution have been widely used as powerful tools to constrain the expansion and growth history of the Universe. Measurements of these signals enable galaxy clustering from redshift surveys to be one of the most promising probes to clarify the origin of the late-time cosmic acceleration, which could be explained by dark energy or modification of gravity [7–17]. Upcoming spectroscopic galaxy surveys, including the Subaru Prime Focus Spectrograph (PFS) [18], the Dark Energy Spectroscopic Instrument (DESI) [19], the *Euclid* space telescope [20–22], and the Nancy Grace Roman Space Telescope [23–25], aim to constrain the dark energy equation-of-state and deviation of the gravitational law from general relativity (GR) with a precision at the subpercent level.

In order to maximize the information encoded in the galaxy distribution in the large-scale structure (LSS) and to constrain cosmological parameters as tightly as possible, one needs to effectively utilize synergies between galaxy redshift surveys and other observations. In this respect, there is a growing interest of using two effects below as new probes of the LSS to improve cosmological constraints, complementary to the conventional galaxy clustering analysis. The first is the kinetic Sunyaev-Zel'dovich (kSZ) effect [26,27], which can be observed via the measurement of cluster velocities by a synergy between galaxy surveys and cosmic microwave background (CMB) experiments. Theoretical and forecast studies suggest that kSZ measurements could provide robust tests of dark energy and modified gravity theories on large scales [28–33]. The kSZ effect has been detected through the cross-correlations of CMB data with galaxy positions from various redshift surveys [34–41].

The second probe is intrinsic alignment (IA) of galaxy shapes with the surrounding large-scale matter density field. The IA was originally proposed as a source of

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systematic effects on the measurement of the cosmological gravitational lensing [42–55]. However, since the spatial correlation of IA follows the gravitational tidal field induced by the LSS, it contains valuable information and is considered as a cosmological probe complimentary to the galaxy clustering [56–65]. Ongoing and future galaxy surveys focus on observing LSS at higher redshifts, $z > 1$, at which the emission line galaxies (ELG) would be an ideal tracer of the LSS [18–20, 66–68]. Although IA has not yet been detected for ELG [50, 69–71], recent work [72] has proposed an effective estimator to determine the IA of dark-matter halos using ELG, enhancing the signal-to-noise ratio at a statistically significant level. In any case, the accurate determination of galaxy shapes is of critical importance for IA to be a powerful tool to constrain cosmology. Thus, the synergy between imaging and spectroscopic surveys is essential because the accurate galaxy shapes and positions are determined from the former and latter, respectively.

In this paper, using the Fisher matrix formalism, we simultaneously analyze the velocity and tidal fields observed by the kSZ and IA effects, respectively, together with galaxy clustering. The combination of galaxy clustering with either IA or kSZ has been studied in earlier studies [e.g., [30, 32, 62]]. This is the first joint analysis of these three probes and we want to see if cosmological constraints can be further improved by combining the combination. We emphasize that the question we want to address is not trivial at all because these probes utilize the information embedded in the same underlying matter fluctuations. Nevertheless, a key point is that these different probes suffer from different systematic effects, and can be in practice complementary to each other, thus used as a test for fundamental observational issues, such as the Hubble tension [73], if the constraining power of each probe is similar. Furthermore, analyzing the kSZ and IA simultaneously enables us to study the correlation of galaxy orientations in phase space as proposed in our recent series of work [60, 61, 74, 75]. For our forecast, we mainly consider the PFS-like deep galaxy survey [18] which overlaps with the imaging survey of the Hyper Suprime-Cam (HSC) [76, 77] and the CMB Stage-4 experiment (CMB-S4) [78]. To see how the cosmological gain by adding the IA and kSZ effects to galaxy clustering can be different for different survey geometries, we also analyze the *Euclid*-like wide galaxy survey [20, 22].

The rest of this paper is organized as follows. In Sec. II we briefly summarize geometric and dynamic quantities to be constrained. Section III presents power spectra of galaxy density, velocity and ellipticity fields and their covariance matrix. We perform a Fisher matrix analysis and present forecast constraints in Sec. IV, with some details further discussed in Sec. V. Our conclusions are given in Sec. VI. Appendix A describes the CMB prior used in this paper. In Appendix B, we present conservative forecast constraints

by restricting the analysis to large scales where linear perturbation theory is safely applied.

II. PRELIMINARIES

A. Distances

The comoving distance to a galaxy at redshift z , $\chi(z)$, is given by

$$\chi(z) = \int_0^z \frac{cdz'}{H(z')}, \quad (1)$$

with c being the speed of light. The function $H(z)$ is the Hubble parameter which describes the expansion rate of the Universe. Writing it as $H(z) = H_0 E(z)$, we define the present-day value of the Hubble parameter by $H_0 \equiv H(z=0)$, which is often characterized by the dimensionless Hubble constant, h , as $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$. Then the time-dependent function $E(z)$ is obtained from the Friedmann equation, and is expressed in terms of the (dimensionless) density parameters. In this paper, we consider the universe whose cosmic expansion is close to that in the standard cosmological model, with the dark energy having the time-varying equation of state. Allowing also the nonflat geometry, the function $E(z)$ is given by

$$E^2(z) = \Omega_m(1+z)^3 + \Omega_K(1+z)^2 + \Omega_{DE}(1+z)^{3(1+w_0+w_a)} \exp\left[-3w_a \frac{z}{1+z}\right], \quad (2)$$

where Ω_m , Ω_{DE} , and Ω_K are the present-day energy density fractions of matter, dark energy, and curvature, respectively, with $\Omega_m + \Omega_{DE} + \Omega_K = 1$. In Eq. (2), the time-varying equation-of-state parameter for dark energy, denoted by $w(z)$, is assumed to be described by a commonly used and well tested parametrization [79, 80],

$$w(z) = w_0 + w_a \frac{z}{1+z} = w_0 + w_a(1-a), \quad (3)$$

where $a = (1+z)^{-1}$ is the scale factor, and w_0 and w_a characterize the constant part and the amplitude of time variation of the dark energy equation of state, respectively (see e.g., Ref. [81], which studied how the different parametrization of $w(z)$ affects the constraining power of the deviation of a cosmological constant.)

The angular diameter distance, $D_A(z)$, is given as

$$D_A(z) = (1+z)^{-1} \frac{c}{H_0} S_K\left(\frac{\chi(z)}{c/H_0}\right), \quad (4)$$

where

$$S_K(x) = \begin{cases} \sin(\sqrt{-\Omega_K}x)/\sqrt{-\Omega_K} & \Omega_K < 0, \\ x & \Omega_K = 0, \\ \sinh(\sqrt{\Omega_K}x)/\sqrt{\Omega_K} & \Omega_K > 0. \end{cases} \quad (5)$$

Negative and positive values of Ω_K correspond to the closed and open universe, respectively. The geometric quantities, $D_A(z)$ and $H(z)$, are the key quantities we directly constrain from the measurement of the BAO imprinted in the power spectra.

B. Perturbations

Density perturbations for a given component i ($i = \{m, g\}$ for matter and galaxies, respectively) are defined by the density contrast from the mean $\bar{\rho}_i(z)$,

$$\delta_i(\mathbf{x}; z) \equiv \rho_i(\mathbf{x}; z)/\bar{\rho}_i(z) - 1. \quad (6)$$

Throughout the paper, we assume the linear relation for the galaxy bias with which the galaxy density fluctuation δ_g is related to the matter fluctuation δ_m through $\delta_g = b_g \delta_m$ [82]. Then, an important quantity to characterize the evolution of the density perturbation is the growth rate parameter, defined as

$$f(z) = -\frac{d \ln D(z)}{d \ln(1+z)} = \frac{d \ln D(a)}{d \ln a}, \quad (7)$$

where $D(z)$ is the linear growth factor of the matter perturbation, $D(z) = \delta_m(\mathbf{x}; z)/\delta_m(\mathbf{x}; 0)$. The parameter f quantifies the cosmological velocity field and the speed of structure growth, and thus is useful for testing a possible deviation of the gravity law from GR [11]. For this purpose, it is common to parametrize the f parameter as

$$f(z) = [\Omega_m(z)]^\gamma, \quad (8)$$

where $\Omega_m(z) = \Omega_m(1+z)^3/E^2(z)$ is the time-dependent matter density parameter and the index γ specifies a model of gravity, e.g., $\gamma \approx 6/11$ for the case of GR [4,83].

It is known that a class of modified gravity models exactly follows the same background evolution as in the Λ CDM model. However, the evolution of density perturbations can be different in general (see, e.g., Ref. [84] for degeneracies between the expansion and growth rates for various gravity models). Thus, it is crucial to simultaneously constrain the expansion and growth rate of the universe to distinguish between modified gravity models.

III. POWER SPECTRA AND THE FISHER MATRIX

In this paper we consider three cosmological probes observed in redshift space, i.e., density, velocity and ellipticity (tidal) fields. While nonlinearity of the density

field has been extensively studied and a precision modeling of its redshift-space power spectrum has been developed [e.g., [85–88]], the understanding of the nonlinearities of velocity and tidal fields are relatively poor. However, there are several numerical and theoretical studies discussed beyond the linear theory, among which a systematic perturbative treatment has been also exploited (See, e.g., Refs. [29,31,32] and [89–91] for the nonlinear statistics of velocity and tidal fields, respectively). It is thus expected that a reliable theoretical template of their power spectra would be soon available, and an accessible range of their templates can reach, at least, at the weakly nonlinear regime. Hence, in our analysis, we consider the weakly nonlinear scales of $k \leq 0.2 \, h \text{ Mpc}^{-1}$, as our default setup. Nevertheless, in order for a robust and conservative cosmological analysis, we do not use the shape information of the underlying matter power spectrum, which contains ample cosmological information but is more severely affected by the nonlinearities. That is, our focus in this paper is the measurements of BAO scales and RSD imprinted in the power spectra, and through the geometric and dynamical constraints on $D_A(z)$, $H(z)$ and $f(z)$, we further consider cosmological constraints on models beyond the Λ cold dark matter (Λ CDM) model. In Appendix B, we perform a more conservative forecast by restricting the analysis to large scales, $k \leq 0.1 \, h \text{ Mpc}^{-1}$, where linear perturbation theory predictions can be safely applied.

In what follows, we discuss how well one can maximize the cosmological information obtained from the BAO and RSD measurements, based on the linear theory predictions. While the linear-theory based template is no longer adequate at weakly nonlinear scales, the signal and information contained in the power spectrum can be in general maximized as long as we consider the Gaussian initial condition. In this respect, the results of our analysis presented below may be regarded as a theoretical upper bound on the cosmological information one can get. Furthermore, we assume a plane-parallel approximation for the cosmological probes [92,93], taking the z -axis to be the line-of-sight direction. While properly taking into account the wide-angle effect provides additional cosmological constraints (see, e.g., Refs. [94], [95–97] and [98] for the studies of the wide-angle effects on density, velocity and ellipticity fields, respectively), we leave the inclusion of this effect to our analysis as future work.

A. Density, velocity, and ellipticity fields

In this subsection, based on the linear theory description, we write down the explicit relation between cosmological probes observed in redshift space to the matter density field. First, the density field of galaxies in redshift space, which we denote by δ_g^S , is a direct observable in galaxy redshift surveys, and in Fourier space, it is related to the

underlying density field of matter in real space on large scales, through $\delta_g^S(\mathbf{k}; z) = K_g(\mu; z)\delta_m(\mathbf{k}; z)$. The factor K_g is the so-called linear Kaiser factor given by [5,99,100]

$$K_g(\mu; z) = b_g(z) + f(z)\mu^2, \quad (9)$$

where b_g is the galaxy bias and μ is the directional cosine between the wave vector and line-of-sight direction, $\mu = \hat{k} \cdot \hat{z}$, with a hat denoting a unit vector. Note that setting f to zero, the above equation is reduced to the Fourier counterpart of δ_g in Eq. (6).

Next, the cosmic velocity field is related to the density field through the continuity equation [101,102]. The observable through the kSZ effect is the line-of-sight component of the velocity, v_{\parallel} , and in linear theory, we have (in Fourier space) $v_{\parallel}(\mathbf{k}; z) = if(z)\mu aH\delta_m(\mathbf{k}; z)/k$. To be precise, the kSZ effect measures the temperature distortion of CMB, δT , detected at the position of foreground galaxies. It is explicitly written in Fourier space as $\delta T(\mathbf{k}; z) = (T_0\tau/c)v_{\parallel}(\mathbf{k}; z) = iK_v(\mathbf{k}; z)\delta_m(\mathbf{k}; z)$, where

$$K_v(k, \mu; z) = \frac{T_0\tau f(z)\mu aH(z)}{c k}, \quad (10)$$

with τ being the optical depth. Since the distance to tracers of the velocity field is measured by redshift, the observed velocity field is affected by RSD, similarly to the density field in redshift space. Unlike the density field, however, the RSD contribution to the redshift-space velocity field appears at higher order [29]. Thus, at leading order, the velocity field traced in redshift space coincides with that in real space in linearized theory, $v_{\parallel}^S = v_{\parallel}$. Note that the kSZ effect, which appears as secondary CMB anisotropies, is given by a line-of-sight integral of the velocity field, and thus the expression of Eq. (10) is just an approximation. We discuss the validity of this approximation in Sec. V C.

An alternative way to measure the velocity field v_{\parallel} without observing the temperature distortion is to use velocity surveys, which enable us to uniquely constrain $f(z)$ [102]. We, however, do not consider observables from peculiar velocity surveys. The main reason is that these observations are limited to the nearby universe ($z \approx 0$) while we consider joint constraints with other probes from a single observation of the LSS. Thus, throughout this paper we refer the velocity field as the temperature distortion δT .

Finally, we use the ellipticity of galaxies as a tracer of the tidal field. The two-component ellipticity of galaxies is defined as

$$\gamma_{(+,\times)}(\mathbf{x}; z) = \frac{1-q}{1+q}(\cos(2\phi_x), \sin(2\phi_x)), \quad (11)$$

where ϕ_x is the position angle of the major axis relative to the reference axis, defined on the plane normal to the line-of-sight direction, and q is the minor-to-major axis ratio of a galaxy shape. We set q to zero for simplicity [52]. As a tracer of LSS, a leading-order description of the ellipticity field is to relate $\gamma_{(+,\times)}$ linearly to the tidal gravitational field, known as the linear alignment (LA) model [46,48,60,75]. In Fourier space, this is given by

$$\gamma_{(+,\times)}(\mathbf{k}; z) = b_K(z)(k_x^2 - k_y^2, 2k_x k_y) \frac{\delta_m(\mathbf{k}; z)}{k^2}. \quad (12)$$

Just like the velocity field, the ellipticity field is not affected by RSD in linear theory [60]. We then define E-/B-modes, $\gamma_{(E,B)}$, which are the rotation-invariant decomposition of the ellipticity field [103],

$$\gamma_E(\mathbf{k}; z) + i\gamma_B(\mathbf{k}; z) = e^{-2i\phi_k}\{\gamma_{+}(\mathbf{k}; z) + i\gamma_{\times}(\mathbf{k}; z)\}, \quad (13)$$

where ϕ_k is the azimuthal angle of the wave vector projected on the celestial sphere (Note that ϕ_k has nothing to do with the directional cosine of the wave vector, and thus $\phi_k \neq \cos^{-1}\mu$). By writing $\gamma_{(E,B)}(\mathbf{k}; z) = K_{(E,B)}(\mu; z)\delta_m(\mathbf{k}; z)$, we have $K_B = 0$ and

$$K_E(\mu; z) = b_K(z)(1 - \mu^2). \quad (14)$$

In Eq. (12) or (13), the parameter b_K quantifies the response of individual galaxy shapes to the tidal field of LSS, and it is conventionally characterized by introducing the parameter A_{IA} as follows [e.g., [104,105]]:

$$b_K(z) = 0.01344A_{\text{IA}}(z)\Omega_m/D(z). \quad (15)$$

Note that the parameter A_{IA} generally depends on properties of the given galaxy population as well as redshift. The analysis of numerical simulations, however, demonstrated that for fixed galaxy/halo properties, A_{IA} is nearly redshift-independent [105]. We thus treat A_{IA} as a constant throughout this paper.

B. Linear power spectra of the three fields

As summarized in the previous subsection, the three cosmological fields, i.e., density, velocity, and ellipticity, are related to the matter field linearly through the coefficients, K_g , K_v , and K_E , respectively. Provided their explicit expressions, we can analytically compute the auto-power spectra of these fields and their cross-power spectra. There are in total six power spectra measured in redshift space, each of which exhibits anisotropies characterized by the μ dependence [60,106–108]. Writing these spectra as $P_{ij}(\mathbf{k}; z) = P_{ij}(k, \mu; z)$ with $i, j = \{g, v, E\}$, they are expressed in a concise form as

TABLE I. Statistics and their abbreviations considered for given probes. Note that when two fields, A and B , are considered, we use not only the autocorrelations (P_{AA} and P_{BB}) but also the cross-correlation, P_{AB} .

Probes	Statistics	Abbreviations	No. of parameters N_θ	Parameters $\{\theta_\alpha\}$	
				Nuisance	Geometric/Dynamical
Clustering	P_{gg}	g	4	$b\sigma_8$	$f\sigma_8, H, D_A$
kSZ	P_{vv}	v	4	τ	$f\sigma_8, H, D_A$
IA	P_{EE}	E	3	A_{IA}	H, D_A
Clustering + IA	$P_{gg} + P_{EE} + P_{gE}$	$g + E$	5	$b\sigma_8, A_{IA}$	$f\sigma_8, H, D_A$
Clustering + kSZ	$P_{gg} + P_{vv} + P_{gv}$	$g + v$	5	$b\sigma_8, \tau$	$f\sigma_8, H, D_A$
IA + kSZ	$P_{EE} + P_{vv} + P_{vE}$	$v + E$	5	A_{IA}, τ	$f\sigma_8, H, D_A$
Clustering + IA + kSZ	$P_{gg} + P_{EE} + P_{vv} + P_{gE} + P_{gv} + P_{vE}$	$g + v + E$	6	$b\sigma_8, A_{IA}, \tau$	$f\sigma_8, H, D_A$

$$P_{ij}(k, \mu; z) = K_i(k, \mu; z)K_j(k, \mu; z)P_{\text{lin}}(k; z), \quad (16)$$

where $P_{\text{lin}}(k; z)$ is the linear power spectrum of matter fluctuation in real space. The normalization of the density fluctuation is characterized by the σ_8 parameter, defined by the linear RMS density fluctuation within a sphere of radius $8 h^{-1}$ Mpc, and thus $P_{\text{lin}}(k; z) \propto \sigma_8^2(z)$. While each of the three autopower spectra, P_{gg} , P_{vv} , and P_{EE} , can be measured from each of the three individual probes, namely galaxy clustering, kSZ and IA, respectively, the cross-power spectra become measurable only when two probes are simultaneously made available.¹ Particularly, the correlation between velocity and ellipticity fields, P_{vE} , has been proposed recently by our earlier studies and it can be probed by the joint analysis of the kSZ (or peculiar velocities) and IA effects [60,61,74,75,109]. Table I summarizes all the statistics used in this paper.

To measure the power spectra, the observed galaxy positions measured with redshift and angular position need to be converted into the comoving positions by introducing a reference cosmology, with a help of Eqs. (1) and (4). An apparent mismatch between the reference and true cosmology causes a geometric distortion in the measured power spectra, which is yet another anisotropy known as the Alcock-Paczynski (AP) effect [110]. This AP effect has been extensively investigated for the galaxy power spectrum in redshift space [106,107,111,112]. The AP effect on the kSZ and IA statistics has been studied relatively recently by Refs. [32,62], respectively. In all of the six power spectra, P_{ij} , their observable counterpart P_{ij}^{obs} are related to the true ones through the relation,

$$P_{ij}^{\text{obs}}(k_\perp^{\text{fid}}, k_\parallel^{\text{fid}}; z) = \frac{H(z)}{H^{\text{fid}}(z)} \left\{ \frac{D_A^{\text{fid}}(z)}{D_A(z)} \right\}^2 P_{ij}(k_\perp, k_\parallel; z), \quad (17)$$

where k_\perp and k_\parallel are the wave number perpendicular and parallel to the line of sight, $(k_\perp, k_\parallel) = k(\sqrt{1 - \mu^2}, \mu)$. The quantities $D_A^{\text{fid}}(z)$ and $H^{\text{fid}}(z)$ are the angular diameter distance and expansion rate computed from fiducial cosmological parameters in the reference cosmology, and $k_\parallel^{\text{fid}} = k_\parallel H^{\text{fid}}(z)/H(z)$ and $k_\perp^{\text{fid}} = k_\perp D_A(z)/D_A^{\text{fid}}(z)$. The prefactor $\frac{H(z)}{H^{\text{fid}}(z)} \left\{ \frac{D_A^{\text{fid}}(z)}{D_A(z)} \right\}^2$ accounts for the difference in the cosmic volume in different cosmologies.

As formulated above, K_g , K_v , and K_E , respectively contain two (b, f) , two (τ, f) , and one (A_{IA}) parameters, and all the power spectra depend on (H, D_A) through the AP effect (see Table I). Thus, we have six parameters in total, $\theta_\alpha = (b\sigma_8, A_{IA}\sigma_8, \tau, f\sigma_8, H, D_A)$, among which the first three are nuisance parameters that we want to marginalize over. The latter three parameters carry the cosmological information which characterize the growth of structure and geometric distances, and are determined by measuring the anisotropies in the power spectra.

C. Covariance matrix

Writing all the power spectra obtained from the galaxy clustering, kSZ and IA as $P_a = (P_{gg}, P_{EE}, P_{vv}, P_{gE}, P_{gv}, P_{vE})$, we will below examine several forecast analysis with a different number of power spectra, which we denote by N_P . Specifically, depending on how many probes are simultaneously available, we consider seven possible cases with $N_P = 1, 3$ or 6 , summarized in Table I. Correspondingly, the covariance matrix Cov_{ab} becomes a $N_P \times N_P$ matrix, defined as $\text{Cov}_{ab}(k, \mu; z) = \langle P_a P_b \rangle - \langle P_a \rangle \langle P_b \rangle$, for a given wave vector, $\mathbf{k} = (k, \mu)$. The full 6×6 Gaussian covariance matrix reads

¹Note that this terminology is different from that used in past studies: while in this paper the kSZ and IA power spectra stand for only P_{vv} and P_{EE} , respectively, the past studies included the cross-power spectrum with density field, P_{gv} and P_{gE} , into kSZ and IA spectra.

$$\text{Cov}_{ab}(k, \mu; z) = \begin{bmatrix} 2\{\tilde{P}_{gg}\}^2 & 2\{P_{gE}\}^2 & 2\{P_{gv}\}^2 & 2\tilde{P}_{gg}P_{gE} & 2\tilde{P}_{gg}P_{gv} & 2P_{gv}P_{gE} \\ 2\{P_{gE}\}^2 & 2\{\tilde{P}_{EE}\}^2 & 2\{P_{vE}\}^2 & 2P_{gE}\tilde{P}_{EE} & 2P_{gE}P_{vE} & 2\tilde{P}_{EE}P_{vE} \\ 2\{P_{gv}\}^2 & 2\{P_{vE}\}^2 & 2\{\tilde{P}_{vv}\}^2 & 2P_{gv}P_{vE} & 2\tilde{P}_{vv}P_{gv} & 2P_{vE}\tilde{P}_{vv} \\ 2\tilde{P}_{gg}P_{gE} & 2P_{gE}\tilde{P}_{EE} & 2P_{gv}P_{vE} & \tilde{P}_{gg}\tilde{P}_{EE} + \{P_{gE}\}^2 & \tilde{P}_{gg}P_{vE} + P_{gE}P_{gv} & P_{gv}\tilde{P}_{EE} + P_{gE}P_{vE} \\ 2\tilde{P}_{gg}P_{gv} & 2P_{gE}P_{vE} & 2\tilde{P}_{vv}P_{gv} & \tilde{P}_{gg}P_{vE} + P_{gE}P_{gv} & \tilde{P}_{gg}\tilde{P}_{vv} + \{P_{gv}\}^2 & P_{gE}\tilde{P}_{vv} + P_{gv}P_{vE} \\ 2P_{gv}P_{gE} & 2\tilde{P}_{EE}P_{vE} & 2P_{vE}\tilde{P}_{vv} & P_{gv}\tilde{P}_{EE} + P_{gE}P_{vE} & P_{gE}\tilde{P}_{vv} + P_{gv}P_{vE} & \tilde{P}_{EE}\tilde{P}_{vv} + \{P_{vE}\}^2 \end{bmatrix}, \quad (18)$$

where $\tilde{P}_{ii} = \tilde{P}_{ii}(k, \mu; z)$ denotes an autopower spectrum ($i = \{g, v, E\}$) including the shot noise. Assuming the Poisson shot noise, we have

$$\tilde{P}_{gg} = P_{gg} + \frac{1}{n_g}, \quad (19)$$

$$\tilde{P}_{vv} = P_{vv} + (1 + R_N^2) \left(\frac{T_0 \tau}{c} \right)^2 \frac{(faH\sigma_d)^2}{n_v}, \quad (20)$$

$$\tilde{P}_{EE} = P_{EE} + \frac{\sigma_\gamma^2}{n_\gamma}, \quad (21)$$

where the quantities n_g , n_v , and n_γ are the number density of the galaxies obtained from galaxy clustering, kSZ and IA observations, respectively. Though different notations are explicitly used for these three samples, $n_g = n_v = n_\gamma$ when one considers a single galaxy population for the analysis. When one uses a single galaxy population as a tracer of the density, velocity and ellipticity fields, there should be a shot noise contribution in the cross correlations. Such a noise term, however, vanishes because $\langle v_\parallel \rangle = 0$ [32] and $\langle \gamma_E \rangle = 0$ [62].

In the shot noise terms of P_{vv} and P_{EE} , there appear factors $\sigma_d = \sqrt{\langle v_\parallel^2 \rangle}$ and $\sigma_\gamma = \sqrt{\langle \gamma_E^2 \rangle}$, which respectively represent the velocity dispersion and shape noise of galaxies, respectively. Using perturbation theory, σ_d can be evaluated as

$$\sigma_d^2 = \frac{1}{3} \int \frac{d^3 q}{(2\pi)^3} \frac{P_{\theta\theta}(q; z)}{q^2} = \frac{1}{6\pi^2} \int dq P_{\theta\theta}(q; z), \quad (22)$$

where $P_{\theta\theta}$ is the power spectrum of velocity divergence. In the limit of linear theory, we have $P_{\theta\theta} = P_{\text{lin}}$, and in the standard cosmological model, it is predicted to give $aH\sigma_{d,\text{lin}} \simeq 600D(z)$ km/s [113], and hence $faH\sigma_{d,\text{lin}} \simeq 600f(z)D(z) \approx 300$ km/s over the redshift considered in this work. Finally, the parameter R_N is the inverse signal-to-noise ratio of the kSZ temperature fluctuations [32]. The rms noise for the kSZ measurement of the CMB-S4 experiment is $\langle \delta T \rangle \sim 2\mu K$, leading to $R_N \sim 10$ [32].

Note that considering only the Gaussian contribution of the covariance matrix [Eq. (18)] may underestimate the statistical errors. Particularly, the kSZ effect generally suffers from a correlated non-Gaussian noise due to the residual foreground contamination, e.g., cosmic infrared background and thermal SZ effect [see e.g., [37,114]]. Though our focus is on relatively large scales and we adopt the Gaussian covariance, such non-Gaussian contributions need to be taken into account for a more realistic forecast study.

D. Fisher matrix formalism

To quantify the constraining power for the dynamical and geometric parameters above and cosmological parameters, we use the Fisher matrix formalism. Although forecast studies with the Fisher matrix have been widely performed in cosmology, there is a limited number of relevant works that consider the kSZ and IA observations to constrain cosmology, specifically through the RSD and AP effect. One is the paper by Sugiyama, Okumura, and Spergel [32], who discussed a benefit of using kSZ observations. Another paper is Taruya and Okumura [62], who demonstrated that combining galaxy clustering with IA observations is beneficial and improves geometric and dynamical constraints. The present paper complements these two previous works, and further put forward the forecast study by combining all three probes.

Given a set of parameters to be estimated, $\{\theta_\alpha\}$, where $\alpha = 1, \dots, N_\theta$, and provided a set of observed power spectra $\{P_a\}$, the Fisher matrix is evaluated with

$$F_{\alpha\beta} = \frac{V_s}{4\pi^2} \int_{k_{\min}}^{k_{\max}} dk k^2 \int_{-1}^1 d\mu \times \sum_{a,b=1}^{N_p} \frac{\partial P_a(k, \mu)}{\partial \theta_\alpha} [\text{Cov}^{-1}]_{ab} \frac{\partial P_b(k, \mu)}{\partial \theta_\beta}, \quad (23)$$

where V_s is the comoving survey volume for a given redshift range, $z_{\min} \leq z \leq z_{\max}$, and k_{\min} and k_{\max} are respectively the minimum and maximum wave numbers used for cosmological data analysis, the former of which is specified with the survey volume by $k_{\min} = 2\pi/V_s^{1/3}$.

Note that for the analysis using a single probe ($N_p = 1$), namely when we consider either of P_{gg} , P_{EE} or P_{vv} , the covariance matrix Cov_{ab} is reduced to the power spectrum squared [see Eq. (18)].

Provided the Fisher matrix, the expected errors on the parameters of interest, marginalizing over other parameters, are computed by inverting the Fisher matrix and constructing the $\bar{N}_\theta \times \bar{N}_\theta$ submatrix $\bar{\mathbf{F}}$; for example, when one wants to evaluate the two-dimensional error contours for a specific pair of parameters, $\vartheta_A \in \theta_\alpha$ ($A = 1, 2$), the 2×2 submatrix is constructed with $C_{AB} \equiv [\bar{\mathbf{F}}]_{AB}^{-1}$ ($A, B = 1, 2$). Also, the one-dimensional marginalized error on a parameter ϑ_A is obtained from $\sigma_A^2 \equiv [\bar{\mathbf{F}}]_{AA}^{-1}$ (see, e.g., Ref. [107] for details).

Although our original Fisher matrix is given for the parameters $\{\theta_\alpha\}$ determined from the AP effect and RSD, the model-independent geometric and dynamical constraints are translated into specific cosmological model constraints by projecting the matrix into a new parameter space of interest,

$$S_{nm} = \sum_{\alpha, \beta}^{N_\theta} \frac{\partial \theta_\alpha}{\partial q_n} F_{\alpha\beta} \frac{\partial \theta_\beta}{\partial q_m}, \quad (24)$$

where $\{q_n\}$ is the set of parameters in the new parameter space ($n = 1, \dots, N_q$), i.e., nonflat $w_0 w_a \gamma$ CDM model and others in our case (see Sec. IV B), and \mathbf{S} is thus a $N_q \times N_q$ matrix. Once again, the uncertainties of the parameters can be obtained by taking the submatrix, e.g., $C_{AB} \equiv [\bar{\mathbf{S}}]_{AB}^{-1}$, $\sigma_A^2 \equiv [\bar{\mathbf{S}}]_{AA}^{-1}$, etc.

For a further discussion on the performance of the constraining power on multiple parameters, we compute the Figure-of-merit (FoM) defined by

$$\text{FoM} = \{\det(\bar{\mathbf{F}})\}^{1/\bar{N}_\theta}, \quad \text{FoM} = \{\det(\bar{\mathbf{S}})\}^{1/\bar{N}_q}, \quad (25)$$

where quantities with the bar, $\bar{\mathbf{F}}$ and $\bar{\mathbf{S}}$, denote $\bar{N}_\theta \times \bar{N}_\theta$ and $\bar{N}_q \times \bar{N}_q$ submatrices of \mathbf{F} and \mathbf{S} ($\bar{N}_\theta < N_\theta$, $\bar{N}_q < N_q$),

respectively, constructed through the inversion described above. In the definition provided in Ref. [115], $\bar{N}_\theta = \bar{N}_q = 2$ and the obtained FoM describes the inverse of the area of the error contour in the marginalized parameter plane for two parameters. Here, the FoM is defined for an arbitrary number of parameters, and the obtained value corresponds to a mean radius of the \bar{N}_θ (or \bar{N}_q) dimensional volume of the errors.

IV. RESULTS

In this section, we present geometric and dynamical constraints on cosmological parameters based on the Fisher matrix analysis of galaxy clustering, IA and kSZ effects. In Fig. 1, we summarize the steps of the analysis of this section graphically, motivated by Fig. 2 of Ref. [107].

A. Setup

To jointly analyze the galaxy clustering, IA and kSZ, we need to use data from galaxy surveys and CMB experiments: positions and shapes of galaxies are respectively used to quantify clustering and IA from a galaxy survey, while the velocity field is inferred by observing the CMB temperature distortion at the angular position of each galaxy.

As we mentioned in Sec. I, there are a number of planned spectroscopic galaxy surveys aiming at constraining cosmology with a high precision. These surveys are generally categorized into the two types; (narrow but) deep surveys and (shallow but) wide surveys. In the Fisher matrix analysis below, we consider the Subaru PFS and *Euclid* as examples of deep and wide surveys, respectively, both of which target emission line galaxies (ELG) as a tracer of the LSS. Tables II and III show the redshift range, survey volume, and number density and bias of the ELG samples for the PFS [18] and *Euclid* [22], respectively. Reference [72] has proposed an estimator to directly detect IA of host halos using the observation of the ELGs. In the forecast analysis presented below, we consider that the power spectra related to the IA are

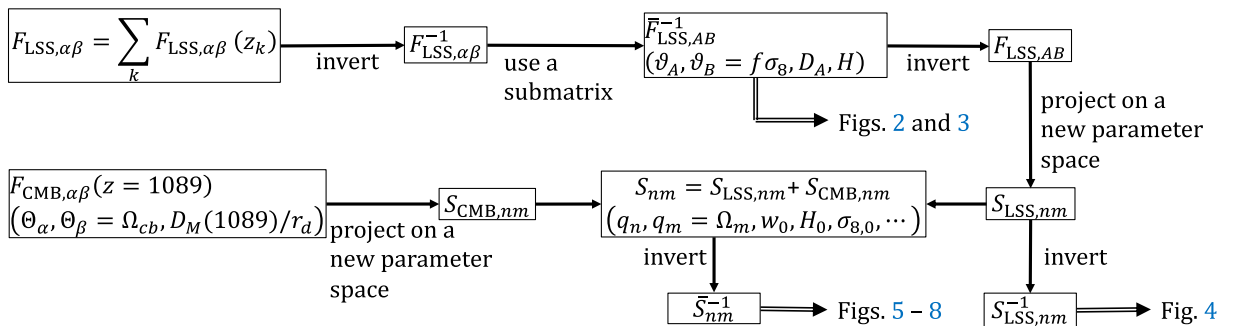


FIG. 1. Flowchart of our Fisher matrix analysis from dynamical and geometric constraints to cosmological parameter constraints. The Fisher matrices of the LSS probes, namely galaxy clustering, IA and kSZ, are given for each redshift bin z_k at the upper left. The Fisher matrix from the CMB prior is given at the lower left.

TABLE II. Expected volume, number density and bias of emission line galaxies for given redshift ranges, $z_{\min} \leq z \leq z_{\max}$ of the deep (PFS-like) survey, taken from Ref. [18].

Redshift		Volume V_s	$10^4 n$	Bias
z_{\min}	z_{\max}	($h^{-3} \text{ Gpc}^3$)	($h^3 \text{ Mpc}^{-3}$)	b_g
0.6	0.8	0.59	1.9	1.18
0.8	1.0	0.79	6.0	1.26
1.0	1.2	0.96	5.8	1.34
1.2	1.4	1.09	7.8	1.42
1.4	1.6	1.19	5.5	1.50
1.6	2.0	2.58	3.1	1.62
2.0	2.4	2.71	2.7	1.78

TABLE III. Same as Table II but for the wide (*Euclid*-like) survey, taken from Ref. [22].

Redshift		Volume V_s	$10^4 n$	Bias
z_{\min}	z_{\max}	($h^{-3} \text{ Gpc}^3$)	($h^3 \text{ Mpc}^{-3}$)	b_g
0.9	1.1	7.94	6.86	1.46
1.1	1.3	9.15	5.58	1.61
1.3	1.5	10.05	4.21	1.75
1.5	1.8	16.22	2.61	1.90

measured with this estimator.² Following the result of Ref. [72], we set the fiducial value of the IA amplitude to $A_{\text{IA}} = 18$, assuming its redshift independence. The PFS galaxy sample provides high-quality shape information thanks to the imaging survey of the HSC [76,77], and we thus set the shape noise, σ_γ , to $\sigma_\gamma = 0.2$ for the deep survey [116]. For the wide survey, following Ref. [22], we set it to $\sigma_\gamma = 0.3$. We will discuss the effect of changing the fiducial values of A_{IA} and σ_γ in Sec. V.

Similarly to the forecast study of the kSZ effect in Ref. [32], we consider CMB-S4 [78] as a CMB experiment for the expected observation of the kSZ effect. While the angular area of the PFS is completely overlapped with that of the CMB-S4, the half of the *Euclid* area is covered by the CMB-S4 [117]. Thus, when considering the statistics related to the kSZ effect, namely P_{vv} , P_{gv} , and P_{vE} , in the wide survey, the elements of the covariance matrix for these statistics are multiplied by two. Furthermore, the values of k_{\min} for these terms become larger by the factor of $2^{1/3}$. We choose $R_N = 10$ as our fiducial choice, following Ref. [32]. For the velocity dispersion, we use the linear theory value as a fiducial value, $\sigma_v = \sigma_{v,\text{lin}}$. The combination of $(1 + R_N^2)\sigma_v^2$ contributes to the shot noise of the

kSZ power spectrum. We will test the effect of these choices in Sec. V.

In the following analysis, we assume the spatially flat Λ CDM model as our fiducial model [118]; $\Omega_m = 1 - \Omega_{\text{DE}} = 0.315$, $\Omega_K = 0$, $w_0 = -1$, $w_a = 0$, $H_0 = 67.3 \text{ [km/s/Mpc]}$ and the present-day value of σ_8 , $\sigma_{8,0} \equiv \sigma_8(z=0)$, to be $\sigma_{8,0} = 0.8309$. For computation of the linear power spectrum in Eq. (16), $P_{\text{lin}}(k; z)$, we use the publicly-available CAMB code [119]. When we consider the model which allows deviation of the structure growth from GR prediction, we set the fiducial value of γ in Eq. (8) to be consistent with GR, $\gamma = 0.545$.

Finally, the maximum wave number of the power spectra used for the cosmological analysis with the Fisher matrix is set to $k_{\max} = 0.2 \text{ h Mpc}^{-1}$. While forecast results with this choice, presented below as our main results, give tight geometrical and dynamical constraints, we also consider in Appendix B a conservative choice of $k_{\max} = 0.1 \text{ h Mpc}^{-1}$, and discuss its impact on the parameter constraints.

B. Geometric and dynamical constraints

Let us first look at model-independent dynamical and geometric constraints, namely the constraints on $f(z)\sigma_8(z)$, $D_A(z)$ and $H(z)$, expected from the upcoming Subaru PFS survey. From the original Fisher matrix which includes these parameters in addition to nuisance parameters, as summarized in Table I, we obtain the marginalized constraints as described in Sec. III D.

The left panel of Fig. 2 shows the two-dimensional 1σ error contours on $f\sigma_8$, D_A and H normalized by their fiducial values, which are obtained individually from galaxy clustering (P_{gg}), kSZ (P_{vv}) and IA (P_{EE}). Since the PFS is a deep survey and has seven redshift bins at $0.6 < z < 2.4$ (see Table II), we here plot the result for the central redshift bin, $1.2 < z < 1.4$, where the number density of galaxies is the largest. Note that the left panel of Fig. 2 does not consider any cross correlation between different probes, namely P_{gE} , P_{gv} , and P_{vE} (see Table I). As clearly shown in the figure, using either P_{EE} or P_{vv} cannot constrain the growth rate. This is because the intrinsic galaxy shapes themselves are insensitive to RSD in linear theory and the kSZ only constrains the combination of $f\sigma_8$ and τ without imposing any prior on τ . Nevertheless, each single measurement of kSZ and IA can give meaningful constraints on D_A and H . Then, including the cross correlation, the combination of the two probes, namely P_{vv} and P_{EE} as well as P_{vE} , improves the constraint on (D_A, H) , depicted as the blue contour.

Interestingly, the constraining power on D_A and H , when combining kSZ and IA, can become tighter, and for the one-dimensional marginalized error, the precision on each parameter achieves a few percent level. Although the galaxy clustering still outperforms the kSZ and IA observations, systematic effects in each probe come to play differently (e.g., galaxy bias, shape noises, and optical

²Even though we use elliptical galaxies as a tracer of the tidal field as in the conventional analysis, we can present a similar analysis based on luminous red galaxy samples from, i.e., DESI, and the main results below will not change qualitatively (e.g., [62]).

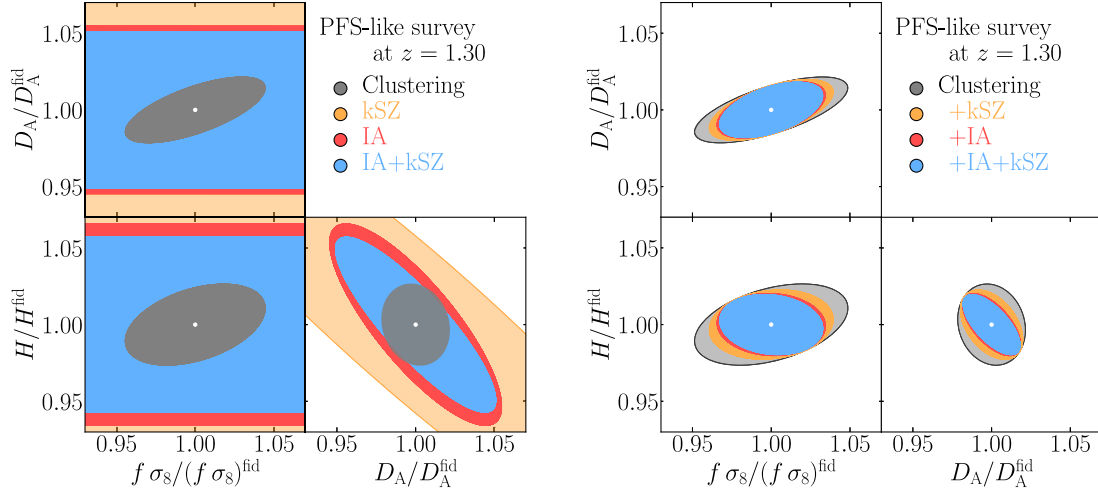


FIG. 2. Two-dimensional 1σ error contours on the geometric distances, $D_A(z)$ and $H(z)$, and the linear growth rate $f(z)\sigma_8(z)$, expected from the wide (PFS-like) survey. Since there are seven redshift bins, we here show the result for the central redshift bin, $1.2 < z < 1.4$, as an example. *Left panel*: Constraints from each of the clustering, kSZ and IA, and the combination of the latter two. *Right panel*: Similar to the left panel but joint constraints from the combination of kSZ, IA and galaxy clustering. Note that the joint constraints obtained from clustering and IA (red contours) almost overlap with those from clustering, IA and kSZ (blue contours).

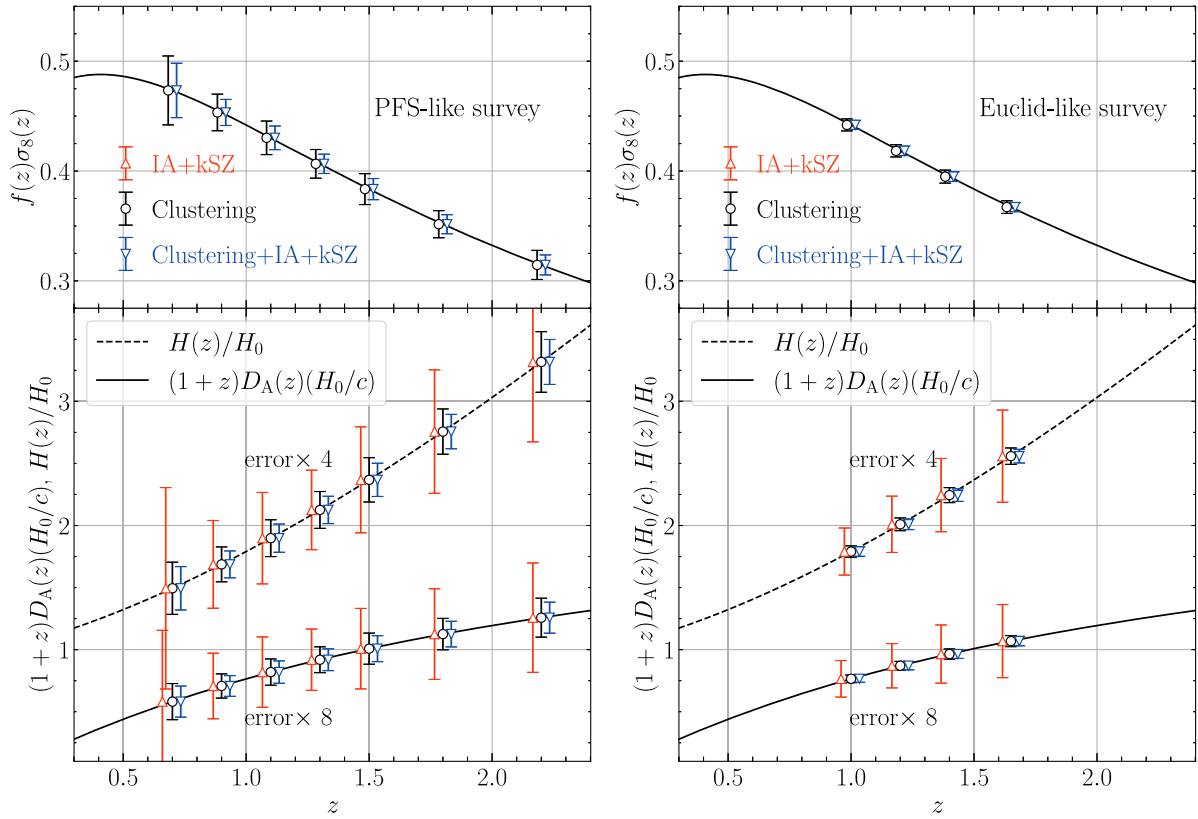


FIG. 3. *Left set*: one-dimensional marginalized errors on the growth rate $f(z)\sigma_8(z)$ (upper panel) and geometric distances (lower panel), $D_A(z)$ and $H(z)$, expected from the deep (PFS-like) survey. *Right set*: same as the left set but the result expected from wide (Euclid-like) survey. The errors on H and D_A are multiplied by 4 and 8, respectively, for illustration.

depth), and in this respect, the geometric constraints from the kSZ and IA are complementary as alternatives to those from the galaxy clustering. Thus, constraining the

geometric distances with kSZ and/or IA effects would help addressing recent systematics-related issues such as the Hubble tension.

TABLE IV. Summary of the cosmological models investigated in the forecast study.

Model	No. of free parameters N_q	Parameters $\{q_n\}$			CMB prior	Result (Fig.)
		Flat	Nonflat	MG		
w_0 flat	4	$\Omega_m, H_0, w_0, \sigma_8$	—	—	—	4
$w_0 w_a$ flat	5	$\Omega_m, H_0, w_0, w_a, \sigma_8$	—	—	Yes	5
$w_0 w_a$ nonflat	6	$\Omega_m, H_0, w_0, w_a, \sigma_8$	Ω_K	—	Yes	6
$w_0 w_a \gamma$ flat	6	$\Omega_m, H_0, w_0, w_a, \sigma_8$	—	γ	Yes	7
$w_0 w_a \gamma$ nonflat	7	$\Omega_m, H_0, w_0, w_a, \sigma_8$	Ω_K	γ	Yes	8

The right panel of Fig. 2 shows the result similar to the left panel, but the joint constraints combining kSZ and/or IA with galaxy clustering. Compared to the results from the single probe, the constraints are indeed improved, as previously demonstrated in Refs. [32] (clustering + kSZ) and [62] (clustering + IA). Here we newly show that the combination of all three probes, characterized by the six power spectra, can further tighten the constraints on both the geometric distances and growth of structure. The results imply that adding any of these power spectra can extract independent cosmological information even though they measure the same underlying matter field. The left panel of Fig. 3 summarizes the one-dimensional marginalized errors on $f\sigma_8$, D_A and H expected from the deep (PFS-like) survey, plotted as a function of z over $0.6 < z < 2.4$. Over all redshifts studied here, adding the information from kSZ and IA measurements does improve the geometric and dynamical constraints.

C. Cosmological parameter constraints

Provided the model-independent geometric and dynamical constraints estimated from the original Fisher matrix in Sec. IV B, we further discuss specific cosmological model constraints listed in Table IV. In what follows, except the w_0 flat CDM model, we add the CMB prior information to constrain cosmological parameters and follow the conventional approach adopted in the data analysis of BOSS [9,120,121], which do not use the information of the full-shape power spectra. To be precise, we introduce the following scaling parameters,

$$\alpha_{\parallel} = \frac{H(z)r_d}{H^{\text{fid}}(z)r_d^{\text{fid}}}, \quad \alpha_{\perp} = \frac{D_A(z)r_d^{\text{fid}}}{D_A^{\text{fid}}(z)r_d}, \quad (26)$$

where the quantity r_d is the sound horizon scale at the drag epoch z_d when photons and baryons are decoupled [3], given by

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (27)$$

with c_s being the sound speed in the photon-baryon fluid. We then redefine the fiducial wavenumbers k_{\parallel} and k_{\perp} , which appear in Eq. (17), as $k_{\parallel}^{\text{fid}} = k_{\parallel}/\alpha_{\parallel}$ and $k_{\perp}^{\text{fid}} = k_{\perp}\alpha_{\perp}$.

With this parametrization, the original expression for the power spectrum at Eq. (17), taking the AP effect into account, is recast as

$$P_{ij}^{\text{obs}}(k_{\perp}^{\text{fid}}, k_{\parallel}^{\text{fid}}; z) = \left(\frac{r_d^{\text{fid}}}{r_d}\right)^3 \frac{\alpha_{\parallel}}{\alpha_{\perp}^2} P_{ij}(k_{\perp}, k_{\parallel}; z), \quad (28)$$

where the prefactor $(r_d^{\text{fid}}/r_d)^3(\alpha_{\parallel}/\alpha_{\perp}^2)$ is equivalent to that in Eq. (17). Note that the dimensionless quantities r_d/D_A and Hr_d are related to the actual BAO scales measurable from galaxy surveys, i.e., angular separation and redshift width of the acoustic scales. In this respect, with the form given in Eq. (28), we are assuming that the main contribution to the AP effect comes from the BAO. As discussed in Ref. [121], the uncertainty on the r_d measurement from the *Planck* experiment is only at the level of ~ 0.2 per cent [118] and fixing r_d in Eq. (28) has a negligible effect on our cosmological parameter estimation. Based on this argument, we approximately set the prefactor $(r_d^{\text{fid}}/r_d)^3$ to unity for the Fisher matrix analysis below.

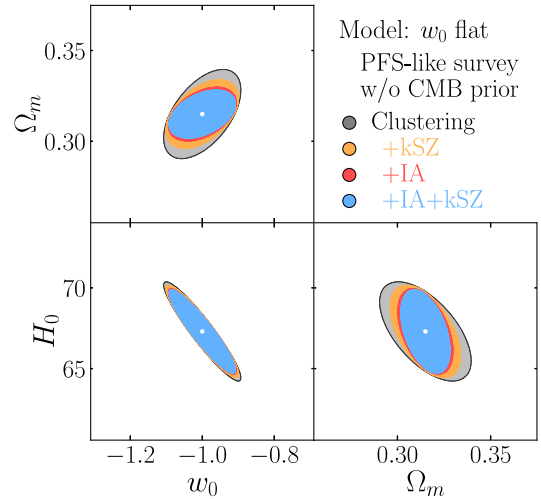


FIG. 4. Cosmological constraints on the w_0 flat model expected from the deep (PFS-like) survey without relying on the CMB prior. In each panel, contours show the 1σ confidence levels, with the amplitude parameter today, $\sigma_{8,0} = \sigma_8(0)$, marginalized over. Note that the joint constraints obtained from clustering and IA (red contours) are almost entirely behind those from clustering, IA and kSZ (blue contours).

TABLE V. Fractional marginalized errors on cosmological parameters, $\sigma/\theta^{\text{fid}}$, for the four specific models. The CMB prior information is added for all the results here. Since the fiducial values of w_a and Ω_K are zero, we show the absolute errors, σ . Since the absolute errors on Ω_K are small, we show the errors multiplied by 100.

Model	$\sigma/\theta^{\text{fid}}$	Deep (PFS-like) survey				Wide (<i>Euclid</i> -like) survey			
		Clustering only	+kSZ	+IA	+IA + kSZ	Clustering only	+kSZ	+IA	+IA + kSZ
w_0, w_a Flat	Ω_m	0.0850	0.0813	0.0765	0.0746	0.0590	0.0578	0.0559	0.0555
	w_0	0.230	0.224	0.213	0.208	0.164	0.163	0.157	0.156
	w_a	0.638	0.613	0.584	0.569	0.455	0.450	0.438	0.434
	H_0	0.0383	0.0363	0.0338	0.0329	0.0257	0.0251	0.0241	0.0239
w_0, w_a Nonflat	Ω_m	0.0877	0.0841	0.0788	0.0767	0.0592	0.0580	0.0561	0.0556
	w_0	0.240	0.232	0.220	0.214	0.164	0.163	0.157	0.156
	w_a	0.665	0.634	0.600	0.583	0.457	0.452	0.440	0.437
	H_0	0.0415	0.0397	0.0370	0.0360	0.0262	0.0256	0.0247	0.0244
w_0, w_a, γ Flat	$100\Omega_K$	0.231	0.229	0.223	0.222	0.162	0.161	0.161	0.161
	Ω_m	0.1459	0.1244	0.1075	0.0990	0.1004	0.0955	0.0840	0.0804
	w_0	0.415	0.359	0.314	0.288	0.288	0.272	0.245	0.234
	w_a	1.036	0.907	0.804	0.743	0.761	0.715	0.655	0.625
w_0, w_a, γ Nonflat	H_0	0.0691	0.0585	0.0501	0.0458	0.0452	0.0429	0.0374	0.0358
	γ	0.271	0.238	0.217	0.202	0.193	0.182	0.169	0.161
	Ω_m	0.1679	0.1380	0.1162	0.1055	0.1114	0.1039	0.0884	0.0836
	w_0	0.484	0.400	0.340	0.307	0.330	0.304	0.264	0.248
w_0, w_a, γ Nonflat	w_a	1.202	1.004	0.865	0.786	0.841	0.774	0.686	0.646
	H_0	0.0833	0.0682	0.0571	0.0516	0.0548	0.0510	0.0430	0.0404
	$100\Omega_K$	0.258	0.245	0.234	0.230	0.194	0.189	0.182	0.179
	γ	0.304	0.256	0.228	0.210	0.231	0.212	0.191	0.179

Now, the model-independent parameters in our original Fisher matrix, combining all three probes, become $\theta_\alpha = (b\sigma_8, A_{\text{IA}}\sigma_8, \tau, f\sigma_8, \alpha_\parallel, \alpha_\perp)$, and the marginalized constraints on $\vartheta_A = (f\sigma_8, \alpha_\perp, \alpha_\parallel)$ are evaluated for each z -slice by constructing the 3×3 submatrix $\bar{F}_{\text{LSS}}(z_k)$. Summing up these submatrices over all the redshift bins, i.e., $\bar{F}_{\text{LSS}} = \sum_k \bar{F}_{\text{LSS}}(z_k)$, we project it into a new parameter space to test the model-dependent cosmological parameters q_n through Eq. (24). The most general model considered in our analysis is the $w_0 w_a \gamma$ nonflat model, with $q_n = (\Omega_m, w_0, w_a, H_0, \Omega_K, \gamma, \sigma_{8,0})$. All the cosmological models we consider in this paper are summarized in Table IV.

Let us show our main results for the deep, PFS-like survey below. Figures 4–7 and the top panel of Fig. 8 plot the expected two-dimensional constraints on pairs of model parameters for different cosmological models. Also, Table V and Fig. 9 summarize the one-dimensional marginalized constraints. We will discuss all the results in detail in the rest of this subsection. Except for Fig. 4, all the following results are obtained adding the CMB prior information, as detailed in Appendix A. Thus, the constraints are obtained from the combination of the Fisher matrices of the LSS and CMB, $S = S_{\text{LSS}} + S_{\text{CMB}}$. For all cases, the nuisance parameters characterizing the power spectrum normalization on each probe namely $b\sigma_8$, τ , and $A_{\text{IA}}\sigma_8$, are marginalized over. Comparisons of the obtained constraints with those from the wide, *Euclid*-like survey will be presented in Sec. VA.

Figure 4 shows the case for the w_0 flat model, in which we vary $q_n = (\Omega_m, w_0, H_0, \sigma_{8,0})$. Only for this model, we do not add the CMB prior and use LSS probes as our

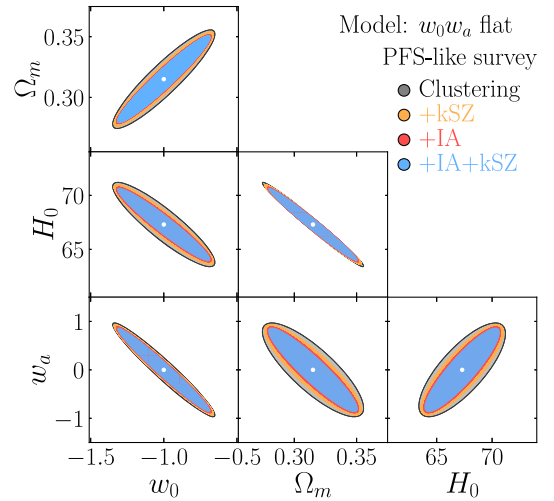
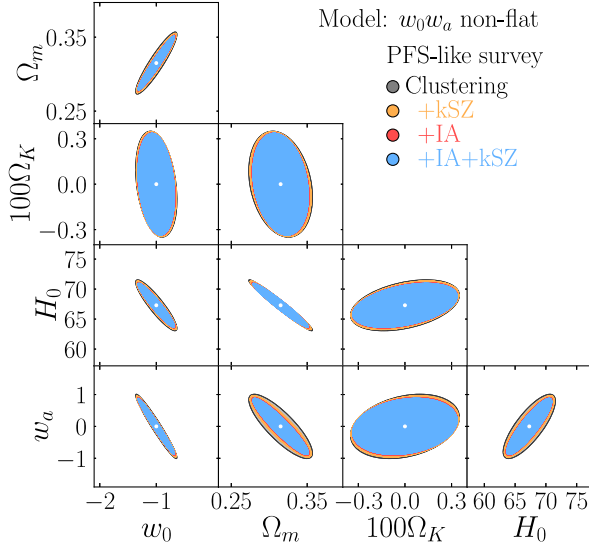
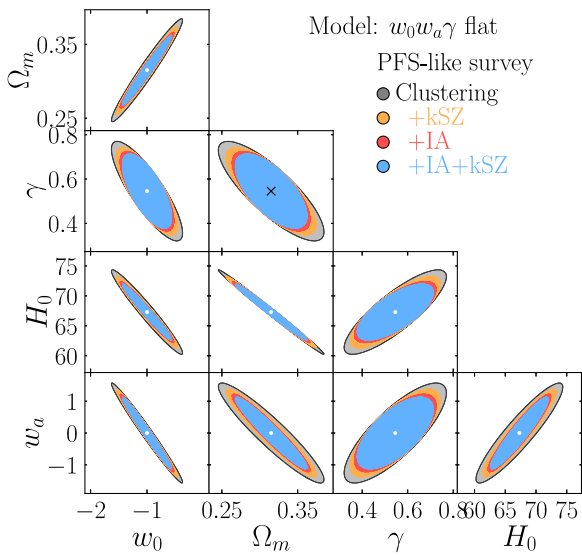
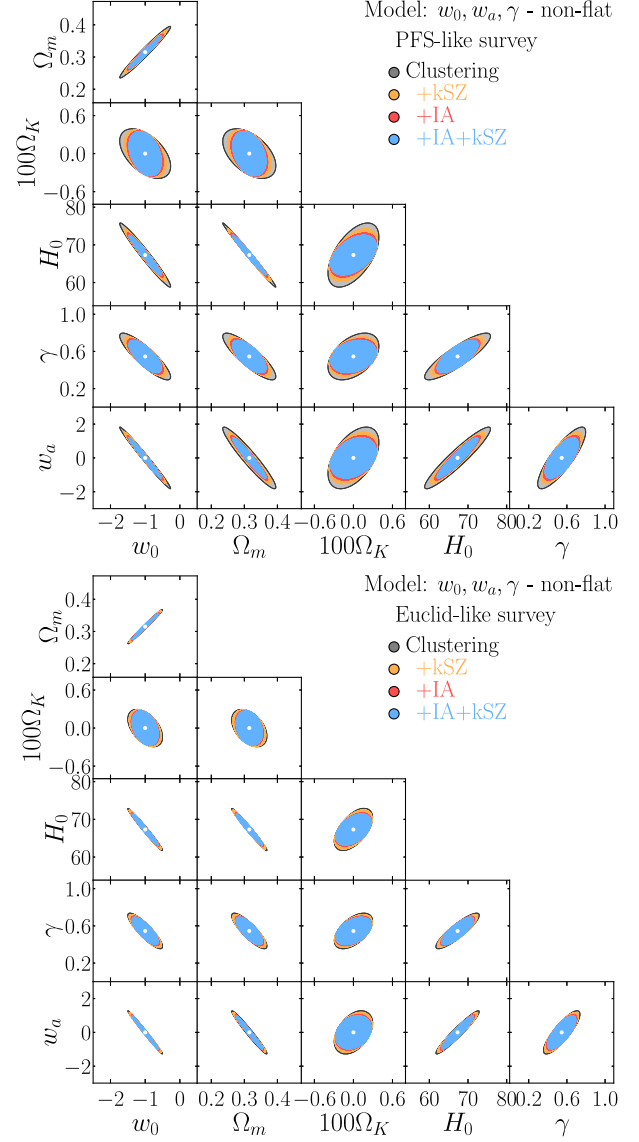


FIG. 5. Cosmological constraints on the $w_0 w_a$ flat model expected from the deep (PFS-like) survey. Unlike Fig. 4, CMB prior information is added here. In each panel, contours show the 1σ confidence levels, with the amplitude parameter today, $\sigma_{8,0} = \sigma_8(0)$, marginalized over. The joint constraints obtained from clustering and IA (red contours) are almost entirely behind those from clustering, IA and kSZ (blue contours).

FIG. 6. Same as Fig. 5 but for the w_0w_a nonflat model.

primary data set. As shown in Ref. [62], adding IA to galaxy clustering significantly improves the constraints. If the kSZ measurement is added, one can achieve a similar (but slightly weaker) improvement. Simultaneously analyzing galaxy clustering with kSZ and IA, the constraint on each cosmological parameter gets even tighter, by 15–21%, compared to the clustering-only constraints.

In Fig. 5, adding the CMB prior information, we show an extension of the parameter space by allowing the time-varying dark energy equation-of-state, which is the w_0w_a flat model described by the parameters $q_n = (\Omega_m, w_0, w_a, H_0, \sigma_{8,0})$. Here, the improvement by adding IA is not so significant compared to the former case, due mainly to a dominant contribution from the CMB prior, consistent with the result of Ref. [62]. However, combining the galaxy

FIG. 7. Same as Fig. 5 but for the $w_0w_a\gamma$ flat model.FIG. 8. *Top*: Same as Fig. 5 but for the $w_0w_a\gamma$ nonflat model from the deep (PFS-like) survey. *Bottom*: Similar to the top panel but from the wide (*Euclid*-like) survey.

clustering with both kSZ and IA measurements, we can improve the constraints further, for example, on w_a by $\sim 11\%$, as shown in Table V and Fig. 9. Figure 6 examines the case with nonzero Ω_K , by introducing another degree of freedom in the parameter space on top of the w_0w_a flat model. Note that based on the BAO experiments at high z , a best achievable precision on Ω_K , limited by the cosmic variance, has been studied in detail in Ref. [122]. In our forecast, the spatial curvature has already been tightly constrained by the CMB prior. Thus, the resulting constraints are similar with those of the model with $\Omega_K = 0$ in Fig. 5.

Now, allowing the deviation of growth of structure from the GR prediction, characterized by the parameter γ , we test and constrain both the cosmic expansion and gravity law, shown in Fig. 7 and the top panel of Fig. 8. Figure 7

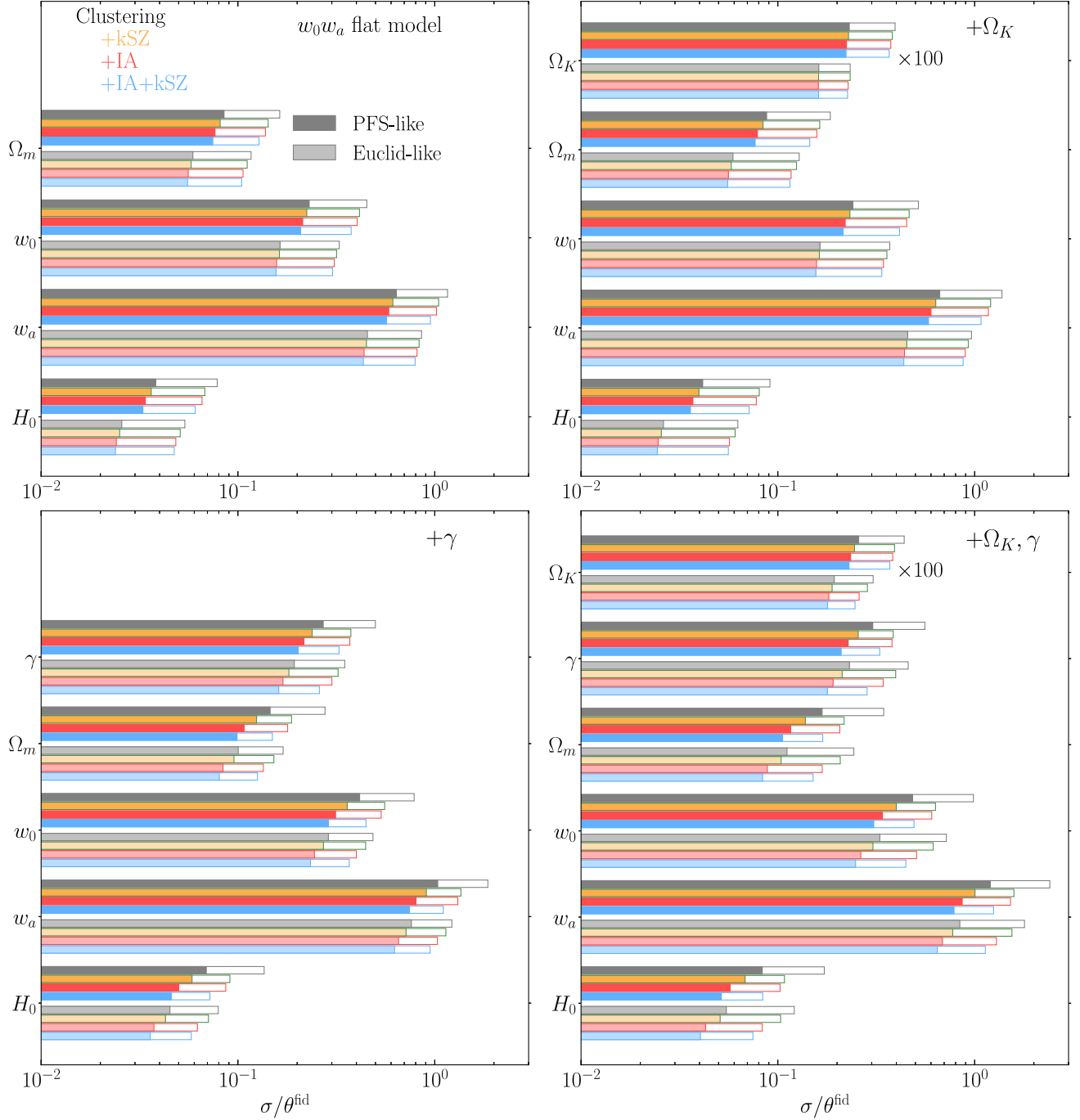


FIG. 9. Marginalized 1σ errors on cosmological parameters, relative to their corresponding fiducial values, $\sigma/\theta^{\text{fid}}$. The darkly and lightly filled bars show the errors from the deep (PFS-like) and wide (Euclid-like) surveys, respectively. The top-left and top-right panels are for the w_0w_a flat and nonflat models, respectively. Similarly, the bottom-left and bottom-right panels are for $w_0w_a\gamma$ flat and nonflat models, respectively. The CMB prior information is added for all the results here. Since the fiducial values of w_a and Ω_K are zero, we show the absolute errors, σ . Since the absolute errors on Ω_K are small, we show the errors multiplied by 100. The hollow bars are similar with the filled bars but based on the conservative analysis with the scales of $k \leq 0.1 \, h \, \text{Mpc}^{-1}$ (see Appendix B).

considers the $w_0w_a\gamma$ flat model, in which the spatial curvature is kept flat. The resulting constraints from the clustering-only analysis are generally weaker than the case of w_0w_a nonflat model despite the fact that the number of parameters remains unchanged. The main reason comes

from the newly introduced parameter γ , which can be constrained only through the measurement of the growth rate, and is strongly degenerated with Ω_m . Nevertheless, adding the information from the observations of kSZ and/or IA, the constraints get significantly tighter, and combining

all three probes, the achievable precision is improved by 25% for γ , and $\sim 30\%$ for other parameters, as shown in Table V and Fig. 9. In the top panel of Fig. 8, the significance of combining all three probes is further enhanced in $w_0w_a\gamma$ nonflat model, where we have seven parameters of $q_n = (\Omega_m, \Omega_K, w_0, w_a, H_0, \gamma, \sigma_{8,0})$. As a result, compared to the clustering-only analysis, the simultaneous analysis with the clustering, IA and kSZ further improves the constraints by 31% for γ and $> 35\%$ for others except for Ω_K (see Table V and Fig. 9).

V. DISCUSSION

A. Deep vs wide surveys

So far, we have considered the PFS survey as a representative example of deep galaxy surveys. Here, we discuss how the constraining power of kSZ and IA measurements depends on types of galaxy surveys. For this purpose, we perform the forecast analysis for the *Euclid* survey as an example of wide galaxy survey. The right panel of Fig. 3 presents geometric and dynamical constraints from the *Euclid*-like survey. Though the redshift range for the *Euclid* is narrower than that for the PFS, the constraints on $f\sigma_8$, D_A , and H at each redshift bin are much tighter due to the large survey volumes (see Table III). Cosmological constraints are thus expected to be stronger as well. To see it quantitatively, let us utilize the FoM introduced in Eq. (25). Here, we marginalize over the amplitude parameter today, $\sigma_{8,0}$, via the inversion of the $N_q \times N_q$ Fisher matrix, \mathcal{S} [see Eq. (24)]. The size of the matrix $\tilde{\mathcal{S}}$ is thus $(N_q - 1) \times (N_q - 1)$. Indeed, the FoM for cosmological parameters from the wide survey is always better, roughly by a factor of two, than that from

the PFS. The comparison is shown for the four cosmological models in Fig. 10.

Constraints on each cosmological parameter is made with the projection of the Fisher matrix. The forecast results from the *Euclid* survey are summarized in the right-hand side of Table V and Fig. 9. If one uses only the information of clustering, constraints from the wide survey considered here are always tighter than those from the deep survey, by 25%–40%. Then one can improve the constraints by the joint analysis of clustering, IA and kSZ, similarly to the analysis of deep galaxy surveys. However, the improvement of the cosmological constraints are not so significant as the case of the deep survey. It is particularly prominent if we consider the model which allows the γ parameter to vary. For example, in the $w_0w_a\gamma$ flat model, while the improvement of cosmological parameters for the deep survey is 25%–34%, that for the wide survey is 17%–21%. It could be due to the fact that the γ parameter is constrained from the redshift dependence of the measured growth rate $f(z)$ at various redshifts, and thus the constraining power in the wide survey does not gain as much as that in a deep survey by combining with additional probes of kSZ and IA. As a result, if we perform a joint analysis of galaxy clustering together with kSZ and IA for a deep survey, the constraining power can be as strong as the conventional clustering-only analysis for a wide survey even though the FoM for the wide survey is twice as large. More interestingly, in the most general $w_0w_a\gamma$ nonflat model, even the deep survey with the combination of IA and clustering can have the constraining power as strong as the wide survey, as shown in Table V and Fig. 9. If one combines all the three probes in the deep survey, the constraints become stronger than the conventional clustering-only analysis in the wide survey. We also show the

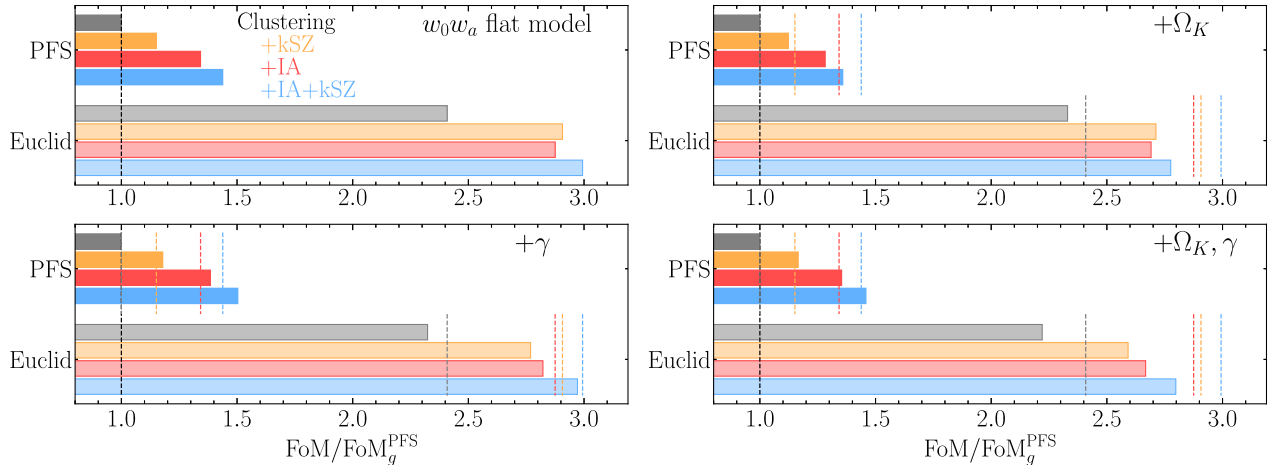


FIG. 10. FoM of cosmological impact for clustering only (gray), clustering + kSZ (yellow), clustering + IA (red) and clustering + IA + kSZ (blue). The upper-left and upper-right panels are the results for the w_0w_a flat and nonflat models, respectively. Similarly, the lower-left and lower-right panels are the results for the $w_0w_a\gamma$ flat and nonflat models, respectively. The CMB prior information is added for all the results here. In each panel, the values of FoM are normalized by that for the PFS survey with clustering-only analysis. The yellow, red and blue vertical lines indicate the FoM values obtained in the upper-left panel for comparison.

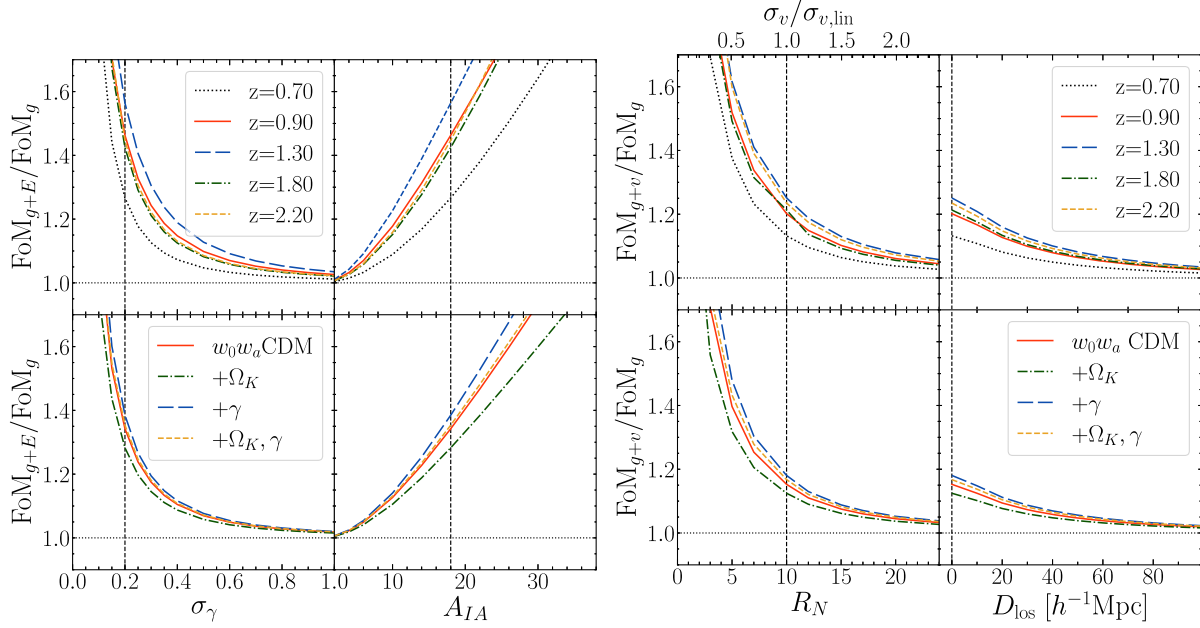


FIG. 11. Relative impact of combining IA (*left set*) and kSZ (*right set*) on the parameter constraints, defined by the ratio of figure-of-merit, $\text{FoM}_{g+E}/\text{FoM}_g$ and $\text{FoM}_{g+v}/\text{FoM}_g$, respectively. The subscripts of g , $g + E$ and $g + v$ denote the FoM expected from galaxy clustering only, the combination of clustering and IA, and that of clustering and kSZ, respectively. Upper panels show the results for geometric distances and structure growth, D_A , H , $f\sigma_8$, derived from each redshift slice of the PFS survey. Results at $z = 1.1$ and $z = 1.5$ are not shown here because they are almost equivalent to those at $z = 0.9$. Bottom panels are the results for cosmological parameters, with $\sigma_{8,0}$ marginalized over. In all cases, the vertical dashed lines indicate the default parameter setup (see Sec. IV A).

two-dimensional error contours of the cosmological parameters from the wide survey in the bottom panel of Fig. 8 which can be compared to those from the deep survey in the top panel. These results clearly demonstrate the importance of considering the IA and kSZ effects.

B. Choices of fiducial survey parameters

The results of our Fisher matrix analyses in Sec. IV and V A rely on the specific setup based on the upcoming surveys. Among several potential concerns in the actual observations, the expected amplitude and error of kSZ and IA statistics are less certain than those of galaxy clustering. Specifically, the benefit of the IA statistics largely depends on the fiducial setup of the parameters σ_γ and A_{IA} , while that of the kSZ statistics is affected by the choice of σ_v and R_N . In this subsection, we discuss the robustness of the benefit combining the IA and kSZ data set with the galaxy clustering. To elucidate this, allowing the parameters σ_γ , A_{IA} , and R_N (or σ_v) to vary, we estimate the FoM, defined by Eq. (25).

Figure 11 shows the ratio of the FoM for the combined data set of galaxy clustering and IA (or kSZ) to that for the galaxy clustering alone, $\text{FoM}_{g+E}/\text{FoM}_g$ (or $\text{FoM}_{g+v}/\text{FoM}_g$). The rightmost panels of the figure will be discussed in the next subsection. The upper panels plot the results for the geometric and dynamical constraints, i.e., D_A , H , and $f\sigma_8$ at each redshift slice. On the other hand, lower panels

show the FoM for the cosmological parameters. As seen in the upper panels, the benefit of combining kSZ and/or IA statistics increases with the number density of galaxies, e.g., $10^4 n = 1.9, 6.0, 7.8, 3.1$, and $2.7 [h^3 \text{Mpc}^{-3}]$ at $z = 0.7, 0.9, 1.3, 1.8$, and 2.2 (see Table II). Note that the results in the lower panels are obtained by adding the CMB prior information, with the fluctuation amplitude, $\sigma_{8,0}$, marginalized over. Thus, the number of cosmological parameters used to compute the FoM in Eq. (25) is $N_p = 4, 5, 5$, and 6 for the red, green, blue and yellow curves, respectively. As expected from the results in Sec. IV C, the impact of combining IA or kSZ on the improvement of cosmological parameters is more significant for the models varying γ and less significant for that varying Ω_K . Even with the suppressed amplitude of ellipticity/velocity fields or enhanced shape noise by a factor of 2, one can still expect a fruitful benefit from the combination of galaxy clustering with IA/kSZ. In particular, adopting the $w_0 w_a \gamma$ nonflat model, the improvement on each parameter reaches $\sim 20\%$, compared to the case with galaxy clustering data alone.

C. Effect of line-of-sight structures on kSZ statistics

In this paper, as in previous works [e.g., [34]], we considered that the kSZ effect is observed in a three-dimensional space, and statistical properties of the measured velocity fields are described by the three-dimensional

matter power spectrum through Eqs. (16) with (10). However, the contribution of the kSZ effect to CMB anisotropies is in general given by a line-of-sight integral of the velocity field. Thus, unless we use massive galaxy groups or clusters as a tracer of the velocity field, the measured kSZ signals would be affected by other velocity components arising predominantly from diffuse and extended sources that may not fairly trace the large-scale matter flow, hence leading to a suppression of the three-dimensional power spectra [123]. To see this effect, we approximate the impact of the line-of-sight integral by introducing a multiplicative Gaussian smoothing kernel with the typical correlation length D_{los} , $G_{\parallel}(k_{\parallel}; D_{\text{los}}) = e^{-k_{\parallel}^2 D_{\text{los}}^2/2}$. The kSZ distortion field, $\delta T(\mathbf{k})$, is then modulated as $\delta T(\mathbf{k}) \rightarrow \delta T(\mathbf{k}) G_{\parallel}(k_{\parallel}; D_{\text{los}})$. Accordingly, the power spectra that include the velocity field, $P_{gv}(\mathbf{k})$, $P_{vE}(\mathbf{k})$ and $P_{vv}(\mathbf{k})$, are modulated as $P_{gv}(\mathbf{k}) G_{\parallel}(k_{\parallel}; D_{\text{los}})$, $P_{vE}(\mathbf{k}) G_{\parallel}(k_{\parallel}; D_{\text{los}})$, and $P_{vv}(\mathbf{k}) G_{\parallel}^2(k_{\parallel}; D_{\text{los}})$, respectively. It is not trivial how the line-of-sight structure affects the velocity dispersion, σ_d^2 , which appears in the shot noise contribution [see Eq. (20)]. Although such a structure may introduce additional noise contribution to the one modeled by Eq. (22), we assume for simplicity that the velocity dispersion remains unchanged, as we have already seen the impact of the increased velocity dispersion on FoM in left panels of Fig. 11.

The rightmost panels of Fig. 11 show the ratio of the FoM for the combined data set of galaxy clustering and kSZ to that for the galaxy clustering alone, $\text{FoM}_{g+v}/\text{FoM}_g$, as a function of the smearing length, D_{los} . Note that in estimating the FoM, we consider that the damping function G is not a properly modeled factor, and for a conservative estimate, we do not take into account the AP effect of this function. The fractional gain of the FoM by adding kSZ decreases with increasing D_{los} , as expected. However, even with such a conservative setting, we can still expect 5–10% improvements at typical values of D_{los} , $D_{\text{los}} \sim 40\text{--}60 h^{-1} \text{ Mpc}$.

As another example, let us also consider the case where the velocity dispersion including the diffuse/extended components is modeled by the line-of-sight integral just like the kSZ power spectra themselves. In such a case, the expression of the velocity dispersion in Eq. (22) is modulated as

$$\begin{aligned} \sigma_d^2 &= \int \frac{d^3 q}{(2\pi)^3} \frac{\mu^2 P_{\theta\theta}(q; z)}{q^2} G_{\parallel}^2(q\mu; D_{\text{los}}) \\ &= \frac{1}{6\pi^2} \int_0^\infty dq P_{\theta\theta}(q) \left\{ -\frac{3e^{-q^2 D_{\text{los}}^2}}{2q^2 D_{\text{los}}^2} + \frac{3\sqrt{\pi} \text{erf}(q D_{\text{los}})}{4q^3 D_{\text{los}}^3} \right\} \\ &= \frac{1}{6\pi^2} \int_0^\infty dq P_{\theta\theta}(q) \left(1 - \frac{3}{5} q^2 D_{\text{los}}^2 + \frac{3}{14} q^4 D_{\text{los}}^4 - \dots \right), \end{aligned} \quad (29)$$

where $\text{erf}(x)$ is the error function and the third equality is derived by the Taylor expansion. Adopting the estimation

of the velocity dispersion given above, we find that the fractional gain of adding the kSZ effect is almost unchanged, a few per cent, at $D_{\text{los}} = 50 h^{-1} \text{ Mpc}$, compared to the undamped case ($D_{\text{los}} = 0 h^{-1} \text{ Mpc}$).

Throughout this paper, we have considered the “homogeneous” kSZ effect, which arises when the reionization process is complete [26,27]. However, on top of that, there is a residual kSZ effect due to the “patchy” (or inhomogeneous) reionization, which arises during the process of reionization, from the proper motion of ionized bubbles around emitting sources, and it can be an additional source of the noise for the kSZ signal [124]. The contribution of the patchy kSZ effect becomes significant at small scales, $\ell \sim k\chi > 1000$ [124,125], while our analysis focuses the data only up to quasilinear scales, $k \leq 0.2 h \text{ Mpc}^{-1}$. Thus in our analysis we safely ignore this effect. However, when we perform a more aggressive analysis including higher- k modes, this patchy reionization effect needs to be properly taken into account.

D. Contribution of gravitational lensing to IA statistics

So far, we have considered the observation of IA as one of the cosmological probes ignoring the lensing effect. In principle, the shape of the galaxies, projected onto the sky, can be very sensitive to the lensing effect, and has been extensively used to detect and measure the cosmic shear signals. This implies that unless properly modeling it, the lensing effect on the E-mode ellipticity may be regarded as a potential systematics that can degrade the geometric and dynamical constraints. Nevertheless, one important point in the present analysis using the IA is that, in contrast to the conventional lensing analysis, one gets access to the cosmological information from the three-dimensional power spectrum. In this subsection, we discuss a quantitative impact of the lensing contribution on the observations of IA, particularly focusing on the three-dimensional power spectrum of E-mode ellipticity.

In the presence of the lensing effect, the observed E-mode ellipticity defined in the three-dimensional Fourier space, $\gamma_E(\mathbf{k}; z)$, is divided into two pieces, $\gamma_E = \gamma_E^{(1)} + \gamma_E^{(\text{GL})}$. Here the former is originated from the IA, and the latter represents the lens-induced ellipticity. Then the (auto) E-mode power spectrum measured at a redshift z is expressed as

$$P_{EE}(\mathbf{k}; z) = P_{EE}^{(1)}(\mathbf{k}; z) + P_{EE}^{(\text{GL})}(\mathbf{k}; z). \quad (30)$$

Note that in principle, there exists the cross talk between IA and lensing, i.e., the gravitational shear-intrinsic ellipticity correlation. However, such a cross talk becomes nonvanishing only if we take the correlation between different z -slices. Since the geometric and dynamical constraints considered in this paper are obtained from individual z -slices, the relevant

quantity to be considered is only the E-mode lensing spectrum, $P_{EE}^{(GL)}$.

Similarly, the observed density field is altered by gravitational lensing, known as the magnification effect. By denoting the observed galaxy density field as δ_{obs} , one can decompose it into the intrinsic density and the term due to magnification, $\delta_{\text{obs}} = \delta_g + \delta_\mu$. Then the cross power spectrum between galaxy density and ellipticity fields, P_{gE} , is expressed as

$$P_{gE}(\mathbf{k}; z) = P_{gE}^{(I)}(\mathbf{k}; z) + P_{\mu E}^{(GL)}(\mathbf{k}; z), \quad (31)$$

where the first term is the cross power spectrum between intrinsic density and ellipticity fields considered so far, and the second term represents the lens-induced cross-power spectrum. Again, there are also cross-talk terms, the galaxy density–lensing shear $P_{gE}^{(GL)}$ and magnification–intrinsic ellipticity $P_{\mu E}^{(I)}$ correlations. Furthermore, the lens-induced ellipticity would be correlated with the kSZ, leading to a nonzero contamination to P_{vE} . Since we consider the correlation functions in individual z -slices, these cross talks are negligible in our analysis.

Under the Limber approximation, $P_{EE}^{(GL)}$ and $P_{\mu E}^{(GL)}$ are analytically expressed as an integral of the comoving distance [e.g., [126,127]]

$$P_{EE}^{(GL)}(\mathbf{k}; z) = \left(\frac{3\Omega_m H_0^2}{2c^2} \right)^2 |W_{\parallel}(k_{\parallel})|^2 \times \int_0^\infty d\chi' \{w(\chi'; \chi(z))\}^2 \left\{ \frac{\chi(z)}{\chi'} \right\}^2 \times P_{\text{lin}}\left(\frac{\chi(z)}{\chi'} k_{\perp}; z(\chi')\right), \quad (32)$$

$$P_{\mu E}^{(GL)}(\mathbf{k}; z) = 2(\alpha_s - 1)P_{EE}^{(GL)}, \quad (33)$$

where α_s is the logarithmic slope of the cumulative galaxy luminosity function and the lensing kernel $w(\chi'; \chi)$ is given by $w(\chi'; \chi) = (\chi - \chi')\chi' / \{a(\chi')\chi\} \Theta(\chi - \chi')$ with $\Theta(x)$ being the Heaviside step function. The function $W_{\parallel}(k_{\parallel})$ is the Fourier counterpart of the survey window function along the line-of-sight direction, $W_{\parallel}(x_{\parallel})$. Equation (33) coincides with Eq. (15) of Ref. [127], ignoring the transverse survey window function W_{\perp} . Since our analysis targets spectroscopic surveys with an accurate redshift

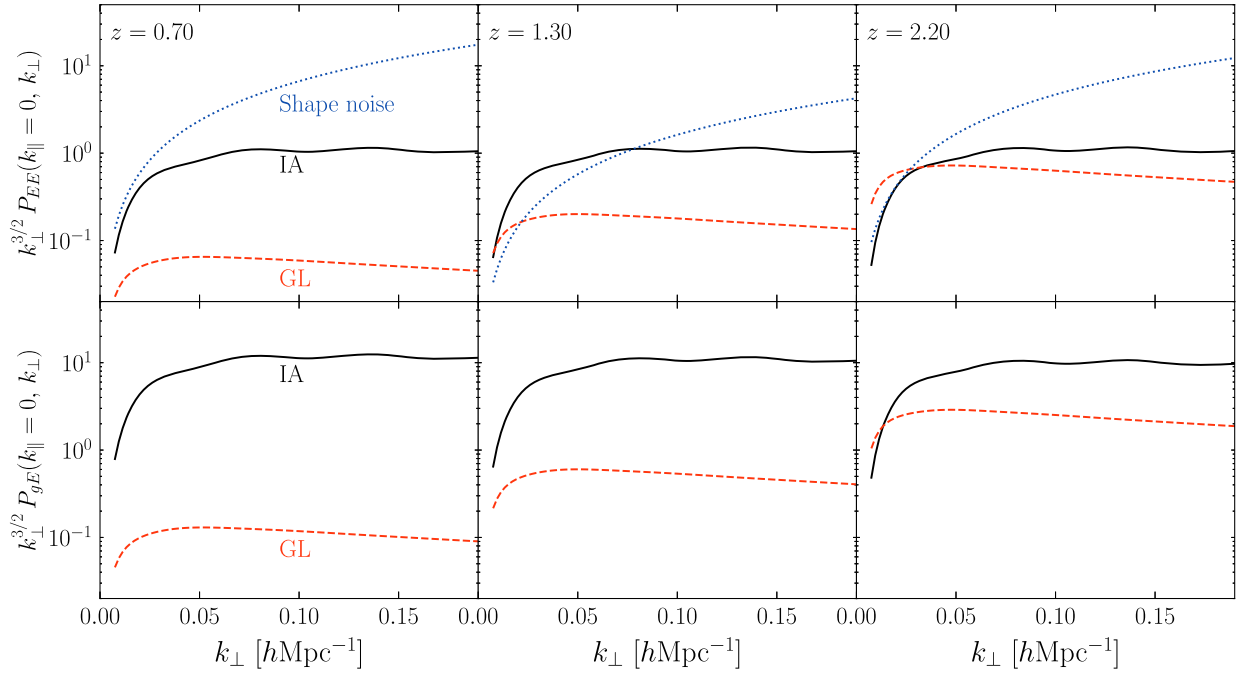


FIG. 12. Impact of lensing effects on the auto power spectra of E-mode ellipticity P_{EE} (upper row) and cross power spectra between density and E-mode ellipticity P_{gE} (lower row), at $z = 0.7$ (left), 1.30 (middle), and 2.20 (right), which are the lowest, central and highest redshift bins of the PFS survey, respectively. To highlight the most significant lensing impact, the power spectra shown here are the results with $k_{\parallel} = 0$, and the results multiplied by $k_{\perp}^{3/2}$ are plotted as function of the transverse wavenumber, k_{\perp} . The red-dashed lines represent the lensing contributions (i.e., $P_{EE}^{(GL)}$ and $P_{\mu E}^{(GL)}$ in upper and lower panels, respectively), which are computed from Eqs. (32) and (33), adopting the top-hat survey window ($P_{EE}^{(GL)}$). The redshift bin size, $\Delta z = z_{\text{max}} - z_{\text{min}}$, is $\Delta z = 0.2$ for $z = 0.7$ and 1.3, and $\Delta z = 0.4$ for $z = 2.2$. The black-solid lines are the un-lensed power spectra, $P_{EE}^{(I)}$ (upper) and $P_{gE}^{(I)}$ (lower), originated purely from the IA and clustering. In upper panels, we also show the nonvanishing noise contribution [see Eq. (21)], depicted as blue-dotted lines.

determination provided for each sample, we assume a top-hat window function,

$$W_{\parallel}(x_{\parallel}) = 1/\sqrt{L}, \quad \text{if } \bar{x} - L/2 < x_{\parallel} < \bar{x} + L/2, \quad (34)$$

and $W_{\parallel}(x_{\parallel}) = 0$ otherwise. Here L is the radial comoving size which corresponds to the redshift bin, given by $L = \chi(z_{\max}) - \chi(z_{\min}) \simeq (z_{\max} - z_{\min})c/H(z)$ (see Table II for the values of z_{\max} and z_{\min} for each redshift bin). This top-hat window leads to $|W_{\parallel}(k_{\parallel})|^2 = (4/Lk_{\parallel}^2)\{\sin(k_{\parallel}L/2)\}^2$ in Fourier space. This means that the lensing contribution becomes maximum at $k_{\parallel} \ll 1$, yielding $|W_{\parallel}|^2 \sim L$.

Figure 12 shows P_{EE} (upper row) and P_{gE} (lower row) at $z = 0.7, 1.30$, and 2.20 , which are the lowest, central and highest redshift bins of the PFS survey, respectively. The power spectra shown here are the results with $k_{\parallel} = 0$ to highlight the maximum lensing contributions, $P_{EE}^{(\text{GL})}$ [Eq. (32)] and $P_{\mu E}^{(\text{GL})}$ [Eq. (33)]. As increasing z , the amplitude of $P_{EE}^{(\text{GL})}$ depicted as red dashed lines in the upper row, gets larger. However, apart from the shape noise, the signal coming from the IA always dominates the E-mode power spectrum. Furthermore, the amplitude of $P_{EE}^{(\text{GL})}$ is always smaller than the shape noise expected from our fiducial setup of $\sigma_{\gamma} = 0.2$, depicted as blue dotted lines. On the other hand, for the power spectrum P_{gE} , the amplitude is controlled by the additional parameter α_s [Eq. (33)], which depends on magnitude and redshift of a given galaxy sample. We adopt the typical values of α_s , $\alpha_s = 2, 2.5$, and 3 for $z = 0.7, 1.30$, and 2.20 , respectively [e.g., [128]]. Due to the extra redshift dependence on α_s , the lensing contribution to $P_{\mu E}^{(\text{GL})}$ increases faster toward higher z than to $P_{EE}^{(\text{GL})}$. Nevertheless, the lensing contribution is still subdominant, and we can clearly detect the BAO signal even for the case of $k_{\parallel} = 0$.

Taking the lens-induced E-mode ellipticity and galaxy density fields to be systematic errors, we have repeated the Fisher matrix analysis, for which the lens-induced auto power spectrum of E-mode and cross power spectrum of magnification and E-mode are included in the covariance at Eq. (18). We then confirmed that the changes in the estimated errors are negligibly small. Furthermore, instead of the top-hat filter function, we have examined another filter, the Gaussian window function, given by $|W_{\parallel}(k_{\parallel})|^2 = \sqrt{4\pi\Sigma^2} \exp(-k_{\parallel}^2\Sigma^2)$ in Fourier space. If we assume $\Sigma = L/\sqrt{4\pi}$, the contribution becomes almost equivalent to the case with the top-hat window [127]. If we choose a wider window, e.g., $\Sigma = L$, the amplitude of the lens-induced power spectrum becomes $\sqrt{4\pi} \sim 3.5$ times larger. Even in that case, changes in the statistical error on each parameter are still negligible, $<1\%$, namely at most the last digits of the values quoted in Table V are modulated.

Hence, we conclude that the lensing effect on the observations of the IA gives a subdominant contribution to P_{EE} and P_{gE} as long as we consider spectroscopic surveys, and it hardly changes the cosmological constraints.

VI. CONCLUSIONS

In this paper, based on the Fisher matrix analysis, we have shown that combining IA and kSZ statistics with the conventional galaxy clustering statistics substantially improves the geometric and dynamical constraints on cosmology. As a representative of deep galaxy surveys for the forecast study, we considered the Subaru PFS, whose angular area perfectly overlaps with those from the HSC survey and the CMB-S4 experiment. We found that even without the galaxy clustering, observations of IA and kSZ enable us to constrain D_A and H , with the achievable precision down to a few percent. This demonstrated that constraining the geometric distances with kSZ and IA effects would help addressing recent systematics-related issues such as the Hubble tension.

For cosmological parameter estimations, a relative merit of adding kSZ and IA statistics to the galaxy clustering depends on cosmological models. We found that the improvement of combining kSZ and IA to clustering statistics is maximized if we simultaneously constrain the time-varying dark energy equation-of-state parameter $w(a) = w_0 + (1 - w_0)a$ and the growth index γ characterizing the modification of gravity in a nonflat universe ($w_0w_a\gamma$ nonflat model). In such a model, with the CMB prior information from the Planck experiment, the PFS-like deep survey is shown to improve the constraints by 31% for γ and $>35\%$ for others except the prior-dominated constraint on Ω_K .

To see the gain of adding IA and kSZ for a different survey setup, we have also performed the Fisher matrix analysis for the *Euclid*-like wide galaxy survey, whose survey area is partly overlapped by half with the CMB-S4 experiment on the sky. Due to the large volume, such a wide survey can give tighter constraints on f , D_A and H at each redshift bin. However, when considering the cosmological models which vary the growth index parameter, a deep survey is more effective than a wide survey, and can get tighter constraints. As a result, in the $w_0w_a\gamma$ nonflat model, by combining kSZ and IA measurements with the clustering measurement, cosmological constraints from the PFS-like deep survey can be tighter than those with the conventional clustering-only measurement from the *Euclid*-like wide survey. Finally, we have also discussed the potential impact of the lensing effect on the observation of IA and line-of-sight structures on the kSZ statistics, the former of which can systematically change the IA auto-power spectrum, P_{EE} [see Eq. (31)]. However, even for the deep survey considered, the lens-induced ellipticity is shown to give a negligible contribution as long as we

consider the three-dimensional power spectrum, and hence the cosmological parameter estimated from the IA data is hardly changed. For the kSZ statistics, even with a large correlation length of $D_{\text{LOS}} \sim 40\text{--}60 h^{-1} \text{ Mpc}$, the impact of the line-of-sight structures on the cosmological parameters is fairly small as long as we consider a joint analysis with the galaxy clustering.

In this paper, focusing specifically on the measurements of geometric and dynamical distortions, we have shown that the combination of both IA and kSZ with galaxy clustering is beneficial. Note, however, that the present analysis using only the BAO and RSD information is not as powerful as that using the full shape of the underlying matter power spectrum. Although one advantage in the present analysis is that the systematics arising from the nonlinearity is less severe and hence conservative, it would be highly desirable for more tighter cosmological constraints, in particular on the neutrino masses, to make use of the full shape information [129]. Indeed, the analysis with the full shape of the power spectrum has been performed in the conventional galaxy clustering analysis [10,130–133]. However, the analysis with full-shape information needs a proper nonlinear modeling, and compared to the modeling of the nonlinearities for clustering statistics, less studies have been made for velocity and ellipticity statistics [e.g., [29,90,91]]. Thus, before we extend our joint analysis of density, velocity and ellipticity fields to include the full-shape spectra, we need to develop models of nonlinear power spectra for the velocity and ellipticity fields and test them with numerical simulations. Such analytical and numerical analyses will be performed in future work.

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APPENDIX A: CMB PRIOR

In this appendix we describe the CMB prior information added to the Fisher matrix of the LSS probes (see Fig. 1). In the analysis presented in this paper, the CMB prior information is used to estimate the forecast constraints

on cosmological parameters, except for the minimal cosmological model (w_0 flat model).

First of all, our primary interest is how the geometric and dynamical constraints derived from the BAO and RSD measurements can be used to test cosmological models, with the power spectra of each LSS probe characterized by Eq. (28). For this purpose, we specifically use the information determined mainly from the CMB acoustic scales. We follow Ref. [13] and use the information on $\omega_{\text{cb}} \equiv \Omega_{\text{cb}} h^2$ and $D_{\text{M}}(1090)/r_{\text{d}}$, fixing the energy density of neutrinos ω_{ν} and baryon ω_{b} respectively to $\omega_{\nu} = 6.42 \times 10^{-4}$ and $\omega_{\text{b}} = 0.022284$, the former of which corresponds to the total mass of $\sum m_{\nu} = 0.06 \text{ [eV]}$. Here, the Ω_{cb} is the density parameter of CDM and baryons, i.e., $\Omega_{\text{cb}} = \Omega_{\text{c}} + \Omega_{\text{b}}$. The quantity $D_{\text{M}}(z) = (1+z)D_{\text{A}}(z)$ is the comoving angular diameter distance [134], and r_{d} is the sound horizon at the drag epoch, for which we use the numerically calibrated approximation,

$$r_{\text{d}} \simeq \frac{55.124 \exp[-72.3(\omega_{\nu} + 0.0006)^2]}{\omega_{\text{cb}}^{0.25351} \omega_{\text{b}}^{0.12807}} \text{ Mpc}, \quad (\text{A1})$$

with ω_{ν} and ω_{b} kept fixed to the values mentioned above. Reference [13] found that the acoustic scale information on the data vector $\Theta_{\alpha} = (\omega_{\text{cb}}, D_{\text{M}}(1090)/r_{\text{d}})$ can be described by a Gaussian likelihood with mean and covariance (see also [130]),³

$$\mu_{\Theta} = (0.1386, 94.33), \quad (\text{A2})$$

$$C_{\Theta} = \begin{pmatrix} 7.452 \times 10^{-6} & -3.605 \times 10^{-5} \\ -3.605 \times 10^{-5} & 0.004264 \end{pmatrix}. \quad (\text{A3})$$

The inverse of this error matrix is the Fisher matrix, $F_{\text{CMB}} = C_{\Theta}^{-1}$, shown in the lower left of the flowchart in Fig. 1. It is then converted to the Fisher matrix for a given cosmological model of interest, S_{CMB} , through Eq. (24). We have also tried another CMB prior used in our early study [62], based on Seo and Eisenstein [107], and confirmed that our forecast results almost remain unchanged.

APPENDIX B: FORECAST RESULTS WITH THE CONSERVATIVE CUTOFF OF $k_{\text{max}} = 0.1 h \text{ Mpc}^{-1}$

In Secs. IV and V, forecast constraints on cosmological parameters, including the geometric distances and growth of structure, were derived focusing on the upcoming deep and wide galaxy surveys, PFS and *Euclid*, respectively. In doing so, one important assumption was that the linear

³To be precise, Ref. [13] provided a Gaussian likelihood for the three parameters $\Theta_{\alpha} = (\omega_{\text{b}}, \omega_{\text{cb}}, D_{\text{M}}(1090)/r_{\text{d}})$ having the 3×3 covariance matrix. Since we consider ω_{b} to be fixed, the relevant prior information is described by the 2×2 covariance matrix given at Eq. (A3).

TABLE VI. Same as Table V but one-dimensional fractional marginalized errors on cosmological parameters, $\sigma/\theta^{\text{fid}}$, when only the data up to the linear scales $k_{\text{max}} = 0.1 \, h \text{Mpc}^{-1}$ are used.

Model	$\sigma/\theta^{\text{fid}}$	Deep survey				Wide survey			
		Clustering only	+kSZ	+IA	+IA + kSZ	Clustering only	+kSZ	+IA	+IA + kSZ
w_0, w_a Flat	Ω_m	0.163	0.142	0.138	0.128	0.117	0.111	0.106	0.105
	w_0	0.452	0.415	0.404	0.377	0.327	0.317	0.309	0.303
	w_a	1.162	1.047	1.021	0.952	0.857	0.833	0.813	0.795
	H_0	0.0785	0.0679	0.0657	0.0608	0.0538	0.0509	0.0484	0.0475
w_0, w_a Nonflat	Ω_m	0.185	0.163	0.158	0.145	0.128	0.124	0.117	0.115
	w_0	0.518	0.465	0.452	0.415	0.371	0.358	0.344	0.337
	w_a	1.377	1.206	1.175	1.077	0.964	0.928	0.896	0.874
	H_0	0.0912	0.0805	0.0777	0.0715	0.0626	0.0607	0.0568	0.0560
	$100\Omega_K$	0.394	0.382	0.375	0.368	0.233	0.233	0.227	0.226
w_0, w_a, γ Flat	Ω_m	0.277	0.187	0.179	0.150	0.170	0.152	0.135	0.126
	w_0	0.786	0.557	0.535	0.447	0.485	0.446	0.400	0.368
	w_a	1.862	1.359	1.310	1.107	1.222	1.140	1.032	0.948
	H_0	0.1360	0.0910	0.0867	0.0721	0.0795	0.0708	0.0623	0.0580
	γ	0.499	0.375	0.370	0.326	0.349	0.323	0.300	0.259
w_0, w_a, γ Nonflat	Ω_m	0.345	0.217	0.207	0.169	0.243	0.208	0.168	0.151
	w_0	0.986	0.633	0.605	0.492	0.718	0.616	0.507	0.447
	w_a	2.410	1.583	1.521	1.248	1.792	1.547	1.291	1.133
	H_0	0.1722	0.1080	0.1027	0.0838	0.1212	0.1033	0.0834	0.0747
	$100\Omega_K$	0.439	0.392	0.384	0.371	0.305	0.285	0.259	0.247
	γ	0.559	0.386	0.380	0.329	0.459	0.397	0.343	0.284

theory template for the power spectra is applicable to the weakly nonlinear scales, setting the maximum wave number to $k_{\text{max}} = 0.2 \, h \text{Mpc}^{-1}$ for all the three LSS probes. While our analysis is still conservative in the sense that we only use the geometric and dynamical information obtained from the BAO and RSD measurements, restricting the data to the linear scales of $k \leq 0.1 \, h \text{Mpc}^{-1}$ would yield a more conservative and robust forecast results, and no intricate modeling of the nonlinear systematics needs to be developed. In this appendix, repeating the Fisher matrix analysis but with $k_{\text{max}} = 0.1 \, h \text{Mpc}^{-1}$, we summarize the forecast constraints on cosmological parameters.

First we consider the deep survey. The left half of Table VI summarizes the one-dimensional marginalized errors on cosmological parameters, which are compared to results with $k_{\text{max}} = 0.2 \, h \text{Mpc}^{-1}$ listed in the left half of Table V. The results are also shown visually as the hollow bars in Fig. 9. The expected errors obtained from the clustering-only analysis with $k_{\text{max}} = 0.1 \, h \text{Mpc}^{-1}$ are roughly twice as large as those with $k_{\text{max}} = 0.2 \, h \text{Mpc}^{-1}$. Interestingly, however, the fractional gain of the cosmological power by adding the kSZ and/or IA measurements is more significant for the conservative analysis with $k_{\text{max}} = 0.1 \, h \text{Mpc}^{-1}$. For instance, in the most general model considered in this paper, namely the $w_0 w_a \gamma$ nonflat model (see Table IV), the

improvements by 48% and 41%, relative to the clustering-only analysis are respectively achieved for the constraints on w_a and γ . These are compared to the relative improvements by 35% and 31% in the cases with $k_{\text{max}} = 0.2 \, h \text{Mpc}^{-1}$.

Let us then compare the forecast results for the deep survey with those for the wide galaxy survey. As seen in the right side of Table VI (see also the hollow bars in Fig. 9), the constraining power of the clustering-only analysis from the wide survey is 25%–40% stronger than that from the deep survey. This is more or less the same as the case with the aggressive cutoff of $k_{\text{max}} = 0.2 \, h \text{Mpc}^{-1}$. However, one notable point is that the benefit of combining the IA and kSZ measurements is more significant for the deep survey than that for the wide survey. In particular, in the $w_0 w_a \gamma$ nonflat model, combining either IA or kSZ with clustering in the deep survey can beat the constraining power of the wide survey. For illustration, in Fig. 13, the expected two-dimensional error contours on the cosmological parameters are shown in the $w_0 w_a \gamma$ nonflat model. This figure is similar with Fig. 8, but here we adopt the conservative cut, $k_{\text{max}} = 0.1 \, h \text{Mpc}^{-1}$, instead of $0.2 \, h \text{Mpc}^{-1}$. Clearly, the relative impact of combining IA and kSZ is rather large for the deep survey, manifesting tighter constraints not only on the growth index but also on other parameters including the curvature parameter.

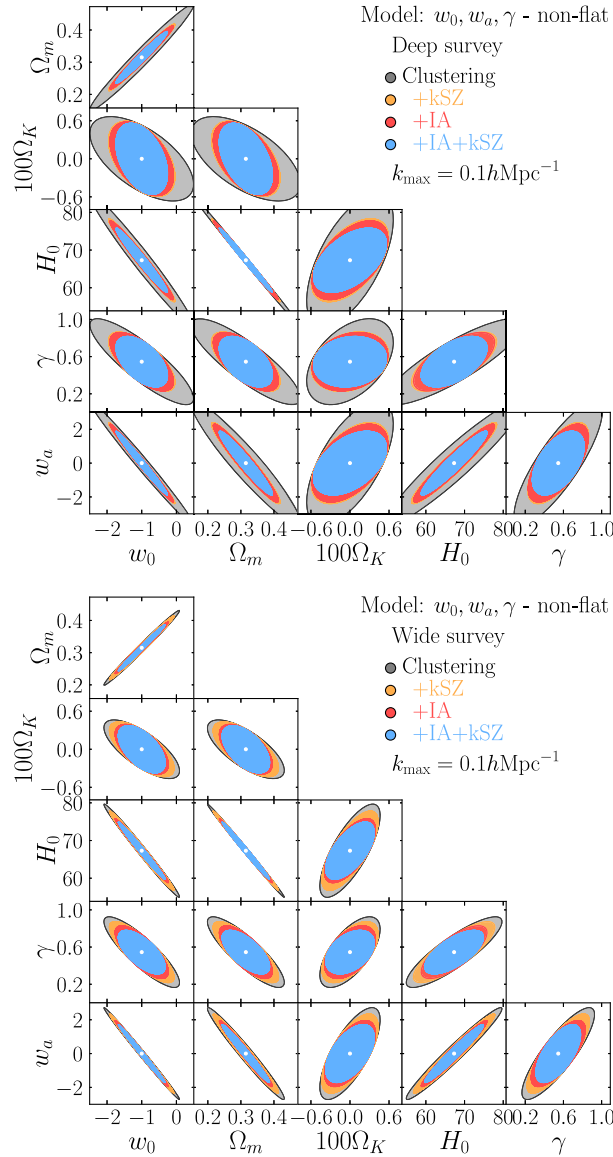


FIG. 13. Cosmological constraints on the $w_0 w_a \gamma$ nonflat model from the deep (Top) and wide (Bottom) surveys. These results are similar with Fig. 8 but using the conservative range of data of $k_{\max} = 0.1 \, h\text{Mpc}^{-1}$.

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