

Gravitational particle production and the validity of effective descriptions in loop quantum cosmology

G. S. Vicente^{1,*}, Rudnei O. Ramos^{2,†} and L. L. Graef^{3,‡}

¹*Faculdade de Tecnologia, Universidade do Estado do Rio de Janeiro, 27537-000 Resende, RJ, Brazil*

²*Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro, 20550-013 Rio de Janeiro, RJ, Brazil*

³*Instituto de Física, Universidade Federal Fluminense, 24210-346 Niterói, RJ, Brazil*



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The effective approach in loop quantum cosmology (LQC) has provided means to obtain predictions for observable quantities in LQC models. While an effective dynamics in LQC has been extensively considered in different scenarios, a robust demonstration of the validity of effective descriptions for the perturbative level still requires further attention. The consistency of the description adopted in most approaches requires the assumption of a test field approximation, which is limited to the cases in which the backreaction of the particles gravitationally produced can be safely neglected. Within the framework of LQC, some of the main approaches to quantize the linear perturbations are the dressed metric, the hybrid approaches and the closed or deformed algebra approach. Here, we analyze the consistency of the test field assumption in these frameworks by computing the energy density stored in the particles gravitationally produced compared to the background energy density. This analysis ultimately provides us with a consistency test of the effective descriptions of LQC.

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I. INTRODUCTION

In the early 1980s the inflationary scenario brought new perspectives for connection between fundamental physics with experiment. Inflation was the first paradigm to make concrete predictions for the structure of the large-scale Universe based on causal physics [1]. Many decades later, with the improvement on experiments aiming to accurately measure the cosmic radiation background (CMB), several inflationary scenarios still show good agreement with data [2,3]. However, as it is well known, many inflationary scenarios require the fields to be very homogeneous initially or start with fine-tuned initial conditions. This leaves inflation at a crossroads, since general relativity (GR) inevitably implies an initial singularity, where it is not clear how one should impose the initial conditions. Inflation is very sensitive to Planck-scale physics [4]. The assumption adopted in inflationary cosmology that “the spacetime can be treated classically” is clearly questionable. The well-known successes of inflation motivate the community to search for means to past complete this cosmological scenario with a more fundamental and consistent quantum gravity theory in the ultraviolet (UV) scale.

Cosmological spacetimes have the advantage of simplicity for being highly symmetric, since homogeneity reduces to a finite number, the infinite number of degrees that one would have otherwise. This favors the development of spacetime quantization schemes. In particular, it is worth mentioning the recent progresses on the quantization of cosmological spacetimes using the approach of loop quantum gravity (LQG), a nonperturbative quantum gravity theory which has opened new avenues to explore Planck-scale physics. The reduced version of LQG is loop quantum cosmology (LQC) [5–15], an approach which uses the symmetries considered in cosmology. Besides allowing for the construction of nonsingular early Universe models, the increasing progress on the analysis of cosmological fluctuations in LQC has bridged quantum gravity with cosmological observations [16].

In the framework of LQC, the background evolution can be divided into two classes: the kinetic-dominated bounce and the potential-dominated bounce. A bounce which is dominated by potential energy would either fail in producing sufficient slow-roll inflation or lead to a too large amount of expansion [17]. In the latter case, all the new physics is washed out, and no signal from the quantum regime is present. In the case of kinetic energy initial domination, the early evolution can always be divided into three phases after the contraction: the bouncing phase (with equation of state $\omega = 1$), the transition phase ($-1 < \omega < 1$) and inflation ($\omega \simeq -1$). The presence of these three stages is

*gustavo@fat.uerj.br

†rudnei@uerj.br

‡leilagraef@id.uff.br

universal in the kinetic-dominated case and does not depend on the form of the inflaton potential.

Unlike the general evolution of the background (zero modes), the linear perturbations depend on the methods used to quantize them. Within the framework of LQC, there are mainly four different approaches: the dressed metric [18,19], the closed or deformed algebra [20,21], the hybrid [22] and separated universe approach [23,24].

In the dressed metric approach, the perturbative degrees of freedom are quantized using the Fock quantization procedure while the background metric is quantized by the loop method. The quantum dynamics of the perturbations can be described by a quantum field evolving in a dressed background in the cases in which the energy density of the perturbations are small compared to the Planck energy. In the dressed metric approach, and also in the other approaches considered in this work, if a potential is added to the dominant field, the dynamics becomes too complicated and in this case an effective field theory needs to be adopted.

In the hybrid quantization approach, the background and the perturbed degrees of freedom are also treated differently, since a LQG-like quantization of the background is performed along with a Fock-like quantization of the perturbations [25–30]. As a result of the nonhomogeneous degrees of freedom being not loop but Fock quantized, the kinematic Hilbert space is a tensor product of the individual Hilbert space for each sector, that is, $\mathcal{H}_{\text{kin}} = \mathcal{H}_{\text{kin}}^{\text{grav}} \otimes \mathcal{H}_{\text{kin}}^{\text{matt}} \otimes \mathcal{F}$ [31]. While the background geometry is loop quantized, the zero mode of the scalar field is quantized in the standard Schrödinger representation, and the nonhomogeneous perturbations are Fock quantized. Similar to the dressed metric approach, for sharply peaked semiclassical background states, there exists an effective description of the quantum dynamics, which greatly simplifies the dynamical equations.¹ Despite the hybrid approach also providing effective perturbative equations that are similar to the dressed approach, sharing several similarities with the latter, they incorporate differently quantum gravity corrections. This leads to a few differences [33]. In Ref. [31] a robust evidence was provided of differences in predictions between dressed and hybrid approaches due to the respective underlying constructions in the context of the modified LQC-I model (mLQC-I) [34,35]. Although it has been shown that, in the effective description, these approaches do not lead to significant differences in the observable parts of the spectrum, the behavior of the nonobservable part of the CMB spectrum deserves investigation. As shown in Ref. [36], for the case of the dressed metric approach, the behavior of the nonobservable range of mode frequencies would imply, if it were not for invalidating the effective

description of the approach, in a preinflationary phase dominated by radiation, which would delay and shorten the inflationary phase.

Another important framework is the closed or deformed algebra approach [37,38], which considers effective constraints coming from the quantum corrections. In this approach the effective constraint algebra must be closed after the quantum corrections are considered. In this approach one encounters the problem that the signature of spacetime effectively changes from Lorentzian to Euclidean at high curvatures [38,39]. The transition point between Lorentzian and Euclidean spacetime implies a “state of silence,” characterized by a vanishing speed of sound, which can be interpreted as due to a decoupling of different space points [40]. This effect comes from the necessity to have a closed or deformed algebra of quantum corrected effective constraints when including holonomy corrections from loop quantum gravity.

Finally, it is important to also mention the separate universe approach [23,24]. This approach considers a spacetime with small perturbations discretized in a lattice. At each cell, which is considered to be homogeneous and noninteracting, loop quantization is applied. The dynamics of cosmological perturbations can be approximated by effective equations whenever we have a sharply peaked wave function in each cell. We also do not consider this approach since its results are only applicable to infrared modes.

An important open question in all of the above approaches concerns the choice of initial conditions for the evolution of the nonhomogeneities. Usually, in the traditional scenario, the initial data are set at the beginning of inflation or during the slow-roll phase. However, the situation is very different in LQC, since it is not clear where to set the initial conditions and whether a vacuum state can be defined at those points. This is one of the most important conceptual questions to be understood in such scenarios. As shown in Refs. [38,41–48], in addition to the differences at the perturbative level, different choices of initial conditions can severely affect the duration of inflation in LQC.

While in the dressed metric and hybrid approaches the initial conditions are set either in the contracting phase or in the bounce, in the closed or deformed algebra approach the initial conditions must be set at the silent point, soon after the bounce. This state is the beginning of the Lorentzian phase in this approach and, in some sense, the beginning of time [40]. As discussed in Ref. [38], this is the only known initial condition that can lead to a spectrum compatible with the observations in this approach. On the other hand, in the dressed metric and hybrid approaches, both the usual Bunch-Davis vacuum and adiabaticlike initial conditions for the perturbations at the bounce, or at the contracting phase, have been considered.

As mentioned before, most of the approaches in LQC rely on effective descriptions to provide more treatable equations for the nonhomogeneous sector. For such an

¹For a different treatment on the hybrid approach, which does not consider effective background equations, see, for example, Ref. [32].

effective description to be valid, the backreaction of the modes gravitationally produced must be negligible. However, according to the Parker gravitational particle production (GPP) mechanism [49,50], a test scalar field χ , evolving from a prebounce Minkowski vacuum state to a postbounce different Minkowski vacuum state, will develop a final state containing χ particles, similarly to linear cosmological perturbations on the cosmological background. This effect is usually negligible in inflationary phases, but in a bounce phase the situation can be different [51–58]. In the dressed metric approach, the gravitational particle production was calculated in Ref. [36]. In that work it was shown that the relativistic particles gravitationally produced during the bounce would come to dominate the energy density of the Universe before inflation, which invalidates the test field approximation required in this approach. Motivated by this result, we revisit the particle production in the dressed metric approach and we also investigate whether the test field approximation is valid in the closed or deformed algebra and in the hybrid approaches. We proceed by comparing the energy density stored in particles gravitationally produced with the background energy density in each approach.

This paper is organized as follows. In Sec. II, we describe the background dynamics of the LQC model with a kinetic-dominated bounce. In Sec. III, we present the dynamics of the perturbative modes in the dressed metric, hybrid and deformed or closed algebra approaches and introduce the GPP mechanism. In Sec. IV, we present the results for the energy density of the particles produced in each case. Finally, our concluding remarks are presented in Sec. V.

II. BACKGROUND MODEL

We consider LQC as the quantum background scenario, which provides GR in the classical regime and quantum corrected GR equations in the Planck regime. The scale factor a arises from the definition of a fiducial fixed cubic cell, with a volume described by $v = \mathcal{V}_0 a^3 m_{\text{Pl}}^2 / (2\pi\gamma)$, \mathcal{V}_0 being the comoving volume of a cell in LQC and γ the Barbero-Immirzi parameter whose value we are going to consider to be $\gamma \simeq 0.2375$, according to black hole entropy calculations [59]. Above G is the Newtonian constant of gravitation and the Planck mass is $m_{\text{Pl}} \equiv 1/\sqrt{G} = 1.22 \times 10^{19}$ GeV. The variable b denotes the conjugate momentum to v and it is given by $b = -4\pi\gamma P_{(a)} / (3a^2 \mathcal{V}_0 m_{\text{Pl}}^2)$, where $P_{(a)}$ is the momentum conjugate to the scale factor.

The quantum Friedmann equation, obtained by solving the effective LQC equations, reads [45]

$$\frac{1}{9} \left(\frac{\dot{v}}{v} \right)^2 \equiv H^2 = \frac{\sin^2(2\lambda b)}{4\gamma^2 \lambda^2} = \frac{8\pi}{3m_{\text{Pl}}^2} \rho \left(1 - \frac{\rho}{\rho_c} \right), \quad (2.1)$$

where $\lambda = (48\pi^2 \gamma^2 / m_{\text{Pl}}^4)^{1/4}$ and b lies in the interval $(0, \pi/\lambda)$. Above, ρ accounts for the energy density, $\rho_c = 3m_{\text{Pl}}^2 / (8\pi\gamma^2 \lambda^2) \simeq 0.41 m_{\text{Pl}}^4$ represents the critical density and the dot represents derivative with respect to the cosmological time. The energy density ρ is connected to the variable b through the relation $\rho = 3m_{\text{Pl}}^2 \sin^2(\lambda b) / (8\pi\gamma^2 \lambda^2)$. For an energy density much smaller than the critical density we reobtain GR as expected in the classical limit. Due to quantum effects, the singularity is not present in this framework and a bounce phase is obtained when the energy density has a value close to the critical density. After the bounce, the Universe transits into a decelerated expansion phase with a subsequent inflationary phase.

The inflaton field can be considered as behaving as a fluid with equation of state $p = \omega\rho$. The solution for the scale factor in LQC for single fluid is given by (see, e.g., Ref. [60])

$$a(t) = a_B \left[1 + \frac{\gamma_B (1 + \omega)^2}{4} \left(\frac{t}{t_{\text{Pl}}} \right)^2 \right]^{\frac{1}{3(1+\omega)}}, \quad (2.2)$$

where $\gamma_B \equiv 24\pi\rho_c / m_{\text{Pl}}^4 \simeq 30.9$ while $t_{\text{Pl}} \equiv 1/m_{\text{Pl}}$ is the Planck time. The evolution until the end of inflation can be divided mainly into a contracting phase, a bounce phase and the classical slow-roll phase.

We consider a cosmological background dominated by the inflaton field with equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \quad (2.3)$$

where $V(\phi)$ is the potential energy of the field. We consider in this setup also an extra scalar field denoted by χ , that couples to ϕ and couples gravitationally to the Standard Model particles, which behaves as a *spectator* field [15,18,61,62]. This field will be produced gravitationally in the bounce phase and it is assumed to be dynamically relevant only in the postbounce evolution, particularly in the preinflationary dynamics, where it might behave as radiation. If we consider that χ has a very small mass in comparison with the postbounce H parameter, then χ will behave as radiation in the preinflationary phase. The analysis of the GPP of a spectator scalar field has already been considered in the dressed metric approach [15,18,61,62] and in the hybrid approach [63]. The equation of motion for the scalar perturbations as well as the scalar field equations have the same form as in GR. Later we are going to further analyze the particle production associated to the field χ . Here we will follow an analysis similar to the one used in Ref. [36], which was originally applied to the dressed metric approach.

Below we summarize the results of Ref. [64], where a full computation of the background dynamics in each phase can be found. As an example, we consider in the following the chaotic model for the inflaton field. However, as we will see, the results for particle production in the dressed and

hybrid approaches do not depend on the choice of the potential for the inflaton field, as the GPP happens mainly during the kinetic energy-dominated bounce phase. However, we will see that, to obtain results for the closed algebra approach, they will explicitly involve the background dynamics after the bounce phase.

For later reference, let us briefly review below the background dynamics in LQC. We consider the chaotic quadratic inflaton potential $V(\phi) = m^2\phi^2/2$ as an example, although the overall description is not expected to change significantly for other forms of potentials.

A. Contracting phase

The scale factor in the classical contracting phase, written in terms of the conformal time η ($dt = a d\eta$), follows the expression

$$a(\eta) = \lambda_0 \eta^2, \quad \text{with} \quad \lambda_0 = \frac{a_{\text{in}} H_{\text{in}}^2}{4}, \quad (2.4)$$

where a_{in} and H_{in} are the initial values for the scale factor and for the Hubble parameter, respectively.

There are two timescales in our original system of equations. One is given by $1/m$, associated with the classical evolution of the inflaton field. The other one is $1/\sqrt{G\rho_c}$, which is associated with the quantum regime. The ratio between these two timescales is defined by the quantity Γ :

$$\Gamma = \frac{m}{\sqrt{24\pi G\rho_c}}, \quad (2.5)$$

where $\Gamma \ll 1$. Here, we assume $m = 10^{-6} m_{\text{pl}}$, as suggested by the observations. Since $\rho_c = 0.41 m_{\text{pl}}^4$, this leads to $\Gamma \sim 2 \times 10^{-7}$. By also defining

$$x(t) \equiv \frac{m\phi}{\sqrt{2\rho_c}}, \quad y(t) \equiv \frac{\dot{\phi}}{\sqrt{2\rho_c}}, \quad (2.6)$$

in the classical contracting phase, x and y can be expressed as

$$x(t) = \sqrt{\frac{\rho(t)}{\rho_c}} \sin(mt + \theta_0), \quad (2.7)$$

$$y(t) = \sqrt{\frac{\rho(t)}{\rho_c}} \cos(mt + \theta_0). \quad (2.8)$$

When $H \approx -m/3$, the term proportional to H in Eq. (2.3) becomes dominant. It can be considered as the end of the prebounce contracting phase and the start of the bouncing phase. We denote the density at the end of the contracting phase by ρ_A , which is given by $\rho_A = \Gamma^2 \rho_c$,

so that before the bounce phase starts, there are still no significant quantum effects.

B. Bounce phase

We can define the starting of the bounce phase when $\rho = \rho_A$. At this time, the quantities x and y can be written as

$$x_A = \Gamma \sin \theta_A, \quad y_A = \Gamma \cos \theta_A. \quad (2.9)$$

The inflaton field kinetic energy dominates in the bounce phase and this phase is then like stiff matter, i.e., like a fluid with equation of state $\omega \approx 1$. From Eq. (2.9), one can see that we must have $\cos \theta_A \sim 1$ due to the kinetic energy domination in this phase. In particular, with ($\omega \approx 1$) from Eq. (2.2), the scale factor reads

$$a(t) = a_B \left(1 + \gamma_B \frac{t^2}{t_{\text{pl}}^2} \right)^{\frac{1}{6}}. \quad (2.10)$$

The evolution of the inflaton field in this phase is described by

$$\phi(t) = \phi_B \pm \frac{m_{\text{pl}}}{2\sqrt{3}\pi} \operatorname{arcsinh} \left(\sqrt{\gamma_B} \frac{t}{t_{\text{pl}}} \right), \quad (2.11)$$

where the plus sign applies when $\dot{\phi} > 0$ and the minus sign for $\dot{\phi} < 0$. The inflaton amplitude at the bounce, ϕ_B , expressed in terms of the variable x_B , can be written as

$$x_B = x_A - \epsilon \Gamma \ln \left(\frac{1}{2} \Gamma \cos \theta_A \right), \quad (2.12)$$

where $\epsilon \equiv \operatorname{sgn}(\cos \theta_A)$.

C. Slow-roll phase

In the starting of the slow-roll phase, a time we denote by t_{SR} , the energy density is $\rho \ll \rho_c$ and the Universe is already classical. The time of the beginning of this phase can be determined by solving $\dot{\rho}(t_{\text{SR}}) = 0$. Having this condition we can obtain the relation $t_{\text{SR}} = t_B + f/m$, in which f can be written in terms of W (the Lambert function), which is the solution of the equation $z = W(z)e^{W(z)}$. The function f is given by

$$f \equiv \sqrt{\frac{2}{W(z)}}, \quad \text{with} \quad z = \frac{8}{\Gamma^2} \exp \left(\frac{2|x_B|}{\Gamma} \right). \quad (2.13)$$

For $\cos \theta_A \sim 1$ and $\Gamma = 2 \times 10^{-7}$ as we are considering, we have that $f \sim 0.18$.

At the time t_{SR} we have that [64]

$$x_{\text{SR}} = x_A - 2\epsilon\Gamma \ln \left(\frac{1}{2}\Gamma \sqrt{\frac{|\cos\theta_A|}{f}} \right). \quad (2.14)$$

Shortly after t_{SR} , one has that $y_{\text{SR}} \equiv -\epsilon\Gamma$ and a very small slow-roll parameter is achieved, as expected, and it is given by

$$\epsilon_H = 3 \left| \frac{\Gamma}{x_{\text{SR}}} \right|^2, \quad (2.15)$$

which for the values of Γ and $\cos\theta_A$ that we are using assumes the value $\epsilon_H \sim 0.003$.

The Hubble parameter in this phase is given by

$$H(t) = H_{\text{SR}} \left| 1 - \epsilon \frac{\Gamma}{x_{\text{SR}}} m(t - t_{\text{SR}}) \right|, \quad (2.16)$$

where $H_{\text{SR}} = \sqrt{8\pi G\rho_c/3}|x_{\text{SR}}|$ (and $a_{\text{SR}} = a_B\Gamma^{-1/3}$).

III. SOLVING FOR THE QUANTUM FIELD MODES IN LQC

We are interested in how the particle production of the spectator scalar field χ could change the LQC preinflationary phase. In this section we will describe the mechanism of GPP for each approach within the framework of LQC.

The quantum fields are described using the standard procedure for classical spacetimes, but using techniques from LQG to incorporate quantum gravity effects [62], which are suitable to treat curvature and matter densities at the Planck scale. We first introduce each approach of LQC we are considering in this paper, highlighting the equations of motion for the Fourier modes of the spectator fields. In the sequence, we introduce the general details about the Parker mechanism for these fields, which has irrelevant interactions with the other components of the Universe, except for gravity.

One very important aspect is that in order to have a solvable model, the usual procedure is to assume effective equations based on the supposition that quantum corrections due to fluctuations are small enough so that they have negligible influence on the evolution of expectation values. Including a significant backreaction would result that the evolution becomes more quantum, i.e., more dependent on how the quantum variables behave. The states can then be deformed from a Gaussian initial distribution. The backreaction results in a change on the quantum state shape, and this then affects the motion of its expectation values. This effect is important for the long-term evolution of cosmology. Therefore, in order for the usual quantization scheme to be valid and to obtain a consistent solution in the effective description, we must assure that the energy

density of perturbations is negligible compared to the background energy density [61] during the whole evolution. Either the backreaction is ignored in the effective equations, or we need to consider a complete quantum gravity theory (for a further discussion on this aspect, see for example Ref. [65]).

Regarding the evolution of the spectator field, we work directly in terms of its Fourier k modes χ_k . The Fourier expansion of the field χ in terms of the k modes, in conformal time, reads

$$\chi(\mathbf{x}, \eta) = \int \frac{d^3k}{(2\pi)^{3/2}} [\chi_k(\eta) a_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} + \chi_k^*(\eta) a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{x}}], \quad (3.1)$$

where $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are the annihilation and creation operators, respectively, that satisfy the canonical commutation relation. In the following, we introduce the dynamics for the Fourier modes of the spectator field χ_k in each LQC approach that we will be considering in this paper. This will later provide us with means to verify the test field supposition $\rho_{\text{pert}}/\rho_{\text{bg}} \ll 1$.

A. Dressed metric approach

The effective equation of motion for the spectator field k modes in the dressed metric approach reads [66]

$$\chi_k''(\eta) + \left[k^2 - \frac{a''(\eta)}{a(\eta)} + U_d(\eta) \right] \chi_k(\eta) = 0, \quad (3.2)$$

where a''/a is given by [67]

$$\frac{a''}{a} = \frac{4\pi}{3m_{\text{Pl}}^2} a^2 \left[\rho \left(1 + 2 \frac{\rho}{\rho_c} \right) - 3p \left(1 - 2 \frac{\rho}{\rho_c} \right) \right], \quad (3.3)$$

where p is the pressure density, prime here indicates derivative with respect to conformal time and $U_d(\eta)$ in Eq. (3.2) is given by $U_d(\eta) = a^2(f^2 V(\phi) + 2fV_{,\phi}(\phi) + V_{,\phi\phi}(\phi))$ is the effective potential, with $f \equiv \sqrt{2\pi G}(\phi'/a)/\sqrt{\rho}$. In this approach, $U(\eta)$, a and η refer to the background state quantum expectation values, $\Psi_0(a, \phi)$. In the case of background states sharply peaked, as often considered, we can approximate the dressed effective quantities by their peaked values $U_d(\eta)$, a and η .

The effective potential U_d can be shown [41] to be negligible during the whole bounce and transition phases. Therefore the equation of motion in these regimes can be written as

$$\chi_k''(\eta) + \left[k^2 - \frac{a''(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0, \quad (3.4)$$

where $-a''(\eta)/a(\eta)$ corresponds to an effective square mass term for the modes.

From the scale factor [Eq. (2.10)], we can define the characteristic momentum scale at the bounce, $k_B = \sqrt{a''(t)/a(t)}|_{t=t_B} = \sqrt{\gamma_B/3}a_B m_{\text{pl}}$. The quantity k_B plays the role of the characteristic energy scale at the bounce in the dressed approach. It is also important to define the quantity $\lambda = \sqrt{a(t)/a''(t)}$ which is the characteristic scale that plays a role similar to the comoving Hubble radius. As is well known, the modes with $k \gg k_B$ are oscillatory since they are inside the effective radius. On the other hand, modes with $k \approx k_B$ are inside the effective radius during the contracting phase and then exiting λ during the bounce phase. After that they enter again the effective radius in the transition phase.

B. Hybrid approach

The effective equation of motion for the spectator field modes in the hybrid approach is given by [22,67]

$$\chi_k''(\eta) + \left[k^2 - \frac{4\pi}{3m_{\text{pl}}^2} a^2(\rho - 3p) + \mathcal{U}_h(\eta) \right] \chi_k(\eta) = 0, \quad (3.5)$$

where the variables are the same ones described in the dressed metric approach, but now $\mathcal{U}_h(\eta) = a^2(V_{,\phi\phi} + 48\pi GV + 6a'\phi'V_{,\phi}/(a^3\rho) - 48\pi GV^2/\rho)$. In the bounce phase, where the gravitational particle production is more relevant, and up to the transition phase, the kinetic energy of the scalar field dominates the energy content of the Universe. In these regimes, we can neglect the contribution of $\mathcal{U}_h(\eta)$ such that Eq. (3.5) becomes

$$\chi_k'' + \left[k^2 - \frac{4\pi}{3m_{\text{pl}}^2} a^2(\rho - 3p) \right] \chi_k = 0, \quad (3.6)$$

with $-4\pi a^2(\rho - 3p)/(3m_{\text{pl}}^2)$ corresponding to the effective square mass term for the modes in the hybrid case. Also, in the transition phase, the energy density drops down to about $10^{-12}\rho_c$ [68], a''/a in Eq. (3.3) reduces to $4\pi a^2(\rho - 3p)/(3m_{\text{pl}}^2)$ and we recover the standard expression of the Fourier modes, with the effective mass term as in Eq. (3.6).

Analogously to the dressed metric approach, we can also define a characteristic momentum scale in the hybrid approach, which reads $k_H = k_B/\sqrt{3}$. The modes behave similarly to the dressed metric approach, but now with respect to the characteristic momentum k_H , which is subtly different from k_B .

We can now draw a parallel between the hybrid and the dressed metric approaches regarding the impact of its quantization strategies in the evolution equations for the modes [67]. There are two main differences in the evolution equations. Firstly, the effective potential $U(\eta)$ is different in each approach. However, since in the scenarios considered here these potentials can always be neglected in the relevant moments for GPP, it does not affect our results. Secondly,

and most important, the effective mass term is different throughout the evolution in each approach.

These differences are due to the proper treatment of the phase space of the perturbed Friedmann-Lemaître-Robertson-Walker cosmologies in each formalism. In the hybrid approach the standard procedure is to treat the whole phase space as a symplectic manifold. The effective mass term is thus expressed in terms of canonical variables, and the expectation value of the operator which represents such a canonical expression is then evaluated by using the effective dynamics in LQC. On the other hand, in the dressed metric case, there is no such global canonical symplectic structure on the truncated phase space and therefore the effective mass is afterward evaluated on the LQC effective solutions. In Ref. [67], for example, the difference between the dressed metric and hybrid approaches is explained in detail.

Despite the differences between the two approaches, the procedure for obtaining the Bogoliubov coefficients in both approaches, which is relevant for the GPP, is basically the same. In both approaches one can realize that the equation of motion for χ_k is analogous to a Schrödinger-type equation having an effective mass term in Eq. (3.4) and in Eq. (3.6) which acts as a potential, behaving effectively as a barrier during the phase of the bounce. This potential, $\mathcal{V}(\eta) \equiv -m_{\text{eff}}^2(\eta)$ in each case, can be, during the bounce phase, approximated by a Pöschl-Teller potential,

$$\mathcal{V}_{\text{PT}}(\eta) = \mathcal{V}_0 \cosh^{-2}[\alpha(\eta - \eta_B)], \quad (3.7)$$

for which we know the analytical solution. In the latter equation, \mathcal{V}_0 is the effective potential's height while $-2\mathcal{V}_0\alpha^2$ is the curvature of the potential at the maximum point. In the dressed metric approach, the height \mathcal{V}_0 can be obtained from the expression of a''/a , being equal to $\mathcal{V}_0 = k_B = \alpha^2/6$, while for the hybrid, the scale k_B is replaced by k_H . Hereafter, we use the notation $k_{B/H}$ when we want to refer to the characteristic scale at the bounce in the dressed (k_B) and hybrid approaches (k_H), respectively.

The solution for χ_k , in the dressed and hybrid approaches, can be written in the form of the standard hypergeometric equation's solution, given by [41]

$$\begin{aligned} \chi_k(\eta) &= a_k x^{ik/2\alpha} (1-x)^{-ik/2\alpha} \\ &\times {}_2F_1(a_1 - a_3 + 1, a_2 - a_3 + 1, 2 - a_3, x) \\ &+ b_k [x(1-x)]_2^{-ik/2\alpha} F_1(a_1, a_2, a_3, x), \end{aligned} \quad (3.8)$$

where $x \equiv x(\eta) = \{1 + \exp[-2\alpha(\eta - \eta_B)]\}^{-1}$,

$$a_1 \equiv \frac{1}{2} \left(1 + \sqrt{1 + \frac{32\pi\rho_c}{3\alpha^2}} \right) - \frac{ik}{\alpha}, \quad (3.9)$$

$$a_2 \equiv \frac{1}{2} \left(1 - \sqrt{1 + \frac{32\pi\rho_c}{3\alpha^2}} \right) - \frac{ik}{\alpha}, \quad (3.10)$$

$$a_3 \equiv 1 - \frac{ik}{\alpha}, \quad (3.11)$$

and a_k and b_k are integration constants to be determined by the initial conditions.

C. Closed or deformed algebra approach

Within the framework of the closed or deformed algebra approach, the effective equation of motion for the modes in Fourier space is given by [20,21,69]

$$\chi_k''(\eta) + W_{k,\text{eff}}^2(\eta)\chi_k(\eta) = 0, \quad (3.12)$$

where

$$W_{k,\text{eff}}^2(\eta) = \Omega(\eta)k^2 - \frac{z''(\eta)}{z(\eta)}, \quad (3.13)$$

$$\Omega \equiv 1 - \frac{2\rho}{\rho_c}. \quad (3.14)$$

When $W_{k,\text{eff}}^2 < 0$, the modes are outside the Hubble horizon and are decaying or growing modes, whereas for $W_{k,\text{eff}}^2 > 0$ the modes are inside the Hubble horizon and are oscillatory.

In this approach, there is no effective potential $U(\eta)$ like in the dressed metric and hybrid ones. However, we need to be careful with the factor Ω , which change signs at $\rho = \rho_c/2$. The instant $t = t_S$ when $\rho = \rho_c/2$, i.e., $\Omega = 0$, is the so-called silent point. At t_S all the space points are uncorrelated [40]. Depending on the signature of Ω , there can be two different regions: the Euclidean region ($\rho_c/2 < \rho < \rho_c$) and the Lorentzian region ($\rho < \rho_c/2$). In order to avoid difficulties in the calculations in the Euclidean regime [70], it is usual to consider the modes only in the Lorentzian region, which means that $t \geq t_S$.

D. Gravitational particle production

Let us now consider the evolution of the Fourier modes χ_k of the (here assumed) massless spectator scalar field χ , whose equation of motion represent a set of uncoupled oscillators with a frequency which varies in time. Due to the time variable frequency, we can define a different vacuum for each instant η . The effect of GPP was introduced by Parker [49,50], who developed an understanding about the conditions for the definition of a particle number $n(\eta)$ which is time dependent, which are (i) its vacuum expectation value varies sufficiently slowly with time as the Universe expansion rate is enough slow and (ii) the period of expansion must occur between the limit of two (“Minkowskian”) vacuum states. In the following we summarized the mathematical treatment of GPP.

The Hamiltonian for $\chi_k(\eta)$ can be written as follows:

$$H(\eta) = \int d^3k (2E_k \hat{a}_k^\dagger \hat{a}_k + F_{\vec{k}} \hat{a}_{\vec{k}} \hat{a}_{-\vec{k}} + F_{\vec{k}}^* \hat{a}_{\vec{k}}^\dagger \hat{a}_{-\vec{k}}^\dagger), \quad (3.15)$$

where

$$F_k(\eta) = \frac{1}{2}(\chi_k'(\eta))^2 + \frac{\omega_k^2}{2}(\chi_k(\eta))^2, \quad (3.16)$$

$$E_k(\eta) = \frac{1}{2}|\chi_k'(\eta)|^2 + \frac{\omega_k^2}{2}|\chi_k(\eta)|^2, \quad (3.17)$$

and $\omega_k(\eta)$ is the frequency in the approximation of a massless χ field. The diagonalized Hamiltonian is obtained by performing the Bogoliubov transformation shown in the following equation:

$$\hat{b}_{\vec{k}} = \alpha_k(\eta)\hat{a}_{\vec{k}} + \beta_k^*(\eta)\hat{a}_{-\vec{k}}^\dagger, \quad (3.18)$$

where $\beta_k(\eta)$ and $\alpha_k(\eta)$ satisfy the normalization constraint given by $|\alpha_k(\eta)|^2 - |\beta_k(\eta)|^2 = 1$. The diagonalized Hamiltonian can be written as

$$H(\eta) = \int d^3k \omega_k b_k^\dagger b_{\vec{k}}. \quad (3.19)$$

Equation (3.17) can then be written as follows:

$$E_k(\eta) = \omega_k \left[\frac{1}{2} + |\beta_k(\eta)|^2 \right]. \quad (3.20)$$

We can define the vacuum states $|0_{(a)}\rangle$ and $|0_{(b)}\rangle$ in a way that $a_{\vec{k}}|0_{(a)}\rangle = b_{\vec{k}}|0_{(b)}\rangle = 0$. One can compute the number operator $\hat{N}_{\vec{k}}^{(b)} = b_{\vec{k}}^\dagger b_{\vec{k}}$ expectation value in the vacuum $|0_{(a)}\rangle$, as given by

$$n_k(\eta) = \langle 0_{(a)} | \hat{N}_{\vec{k}}^{(b)} | 0_{(a)} \rangle = |\beta_k(\eta)|^2. \quad (3.21)$$

The quantity $|\beta_k(\eta)|^2$ is interpreted as the particle number per mode. The initial Minkowski vacuum states $\chi_k^{(i)}$ can be related to the ones at a later time $\chi_k^{(f)}$ through the Bogoliubov coefficients:

$$\chi_k^{(f)}(\eta) = \alpha_k \chi_k^{(i)}(\eta) + \beta_k \chi_k^{(i)*}(\eta). \quad (3.22)$$

When there are zero produced particles, $\beta_k = 0$ and the normalization constraint implies $\alpha_k = 1$ and $\chi_k^{(i)} = \chi_k^{(f)}$.

With the above expressions we can compute $n_p(\eta)$, the total particle number density, and $\rho_p(\eta)$, the energy density gravitationally produced. The particle number density, which is integrated over all modes, is given by

$$\begin{aligned} n_p(\eta) &= \frac{1}{a^3(\eta)L^3} \left(\frac{L}{2\pi} \right)^3 \int_0^\infty d^3k n_k(\eta) \\ &= \frac{1}{2\pi^2 a^3(\eta)} \int_0^\infty dk k^2 |\beta_k(\eta)|^2, \end{aligned} \quad (3.23)$$

whereas $\rho_p(\eta)$, the energy density associated to the produced particles, is given by

$$\rho_p(\eta) = \frac{1}{2\pi^2 a^4(\eta)} \int_0^\infty dk k^2 \omega_k |\beta_k(\eta)|^2. \quad (3.24)$$

Equations (3.23) and (3.24) give us the particle number density and the energy density of the produced particles, respectively.

It is important to mention that Eq. (3.24) gives the net energy density produced between two Minkowskian vacuum states. On the other hand, we can also obtain the energy density of produced particles due to the GPP effect from the expectation value of the test field's energy-momentum tensor at a time η , that corresponds to [71]

$$\rho_p^{\text{EM}}(\eta) = \frac{1}{4\pi^2 a^4(\eta)} \int_0^\infty dk k^2 [|\chi'_k(\eta)|^2 + \omega_k^2 |\chi_k(\eta)|^2]. \quad (3.25)$$

The Minkowskian initial condition set at the contracting phase is called the Bunch-Davies (BD) vacuum [72]. Alternatively, it is also possible to impose an initial condition at the bounce, which is the fourth-order adiabatic vacuum state [73]. However, it is important to note that the quantum contributions computed with a Bunch-Davies vacuum initial condition and also with fourth-order adiabatic vacuum are the same just in the case of modes with $k \geq k_B$ ($k \geq k_H$) in the dressed metric (hybrid approach). For any other modes, the fourth-order adiabatic vacuum state at the bounce may not be applicable [41], while the Bunch-Davies vacuum can still be considered in the contracting phase.

We can also mention other two types of initial conditions, which are the nonoscillating vacuum [33,74,75] and the silent point vacuum [40]. The former relates to a method for minimizing the oscillations in the resulting power spectrum of perturbations, which can be considered in dressed and hybrid approaches, whereas the latter is particular to the closed or deformed algebra approach and it is necessary for its consistency.

Explicitly numerically solving for the mode equations given above is computationally intensive. This is particularly true in the regimes with rapidly oscillating high-momentum modes. We need to also handle the UV divergences that appear and then the particle production energy density needs to be renormalized appropriately. A typical approach is to use a Wentzel-Kramers-Brillouin (WKB) approximation for the modes to tackle these problems [76]. But even so, there are issues with both how to fix the upper limit for the momentum integrals and further issues in the infrared, which also demands to consider a lower limit for the momentum integrals when computing the total energy density due to GPP. This is also not free from ambiguities. It is important, thus, to have a

computation as analytical as possible and in such ways one can overcome the above mentioned issues but still having a reliable computation for the GPP. In the following section, we give approximate analytical solutions for the equations of motion for the scalar modes in each approach and from which we can estimate the GPP in appropriate ways.

IV. RESULTS

We present approximated analytical results for GPP in the dressed metric, hybrid and closed or deformed algebra approaches of LQC. To obtain these results, instead of fully computing the real-time backreaction of the produced particles in the background, we estimate the energy density associated to those after the bounce and then compare the result with the background energy density. As we are going to see, these estimates are already sufficient for a qualitative analysis. Later, we compare how the energy density of the produced particles redshifts with the scale factor in comparison with the behavior of the dominant background energy content. These results will help to gauge the validity of each of those approaches in LQC. We compute the energy density of gravitationally produced particles in each approach, which consists in computing the corresponding analytical expression for β_k .

A. Dressed metric approach

From the results of Ref. [41], by matching the analytical solutions for the bounce phase within the Pöschl-Teller potential approximation, the transition and slow-roll phase, the Bogoliubov coefficients can be obtained and they are given by

$$\alpha_k = \sqrt{2k} \left[a_k \frac{\Gamma(2-a_3)\Gamma(a_1+a_2-a_3)}{\Gamma(a_1-a_3+1)\Gamma(a_2-a_3+1)} + b_k \frac{\Gamma(a_3)\Gamma(a_1+a_2-a_3)}{\Gamma(a_1)\Gamma(a_2)} \right] e^{ik\eta_B}, \quad (4.1)$$

$$\beta_k = \sqrt{2k} \left[a_k \frac{\Gamma(2-a_3)\Gamma(a_3-a_1-a_2)}{\Gamma(1-a_1)\Gamma(1-a_2)} + b_k \frac{\Gamma(a_3)\Gamma(a_3-a_1-a_2)}{\Gamma(a_3-a_1)\Gamma(a_3-a_2)} \right] e^{-ik\eta_B}, \quad (4.2)$$

where the pairs (α_k, β_k) and (a_k, b_k) are arbitrary constants at the bounce phase and slow-roll inflation solution for the k modes, respectively, and η_B is the conformal time at the bounce. The Bogoliubov coefficients are determined when we impose initial conditions, i.e., choose a_k and b_k .

Assuming the absence of particles at the onset of inflation, one would impose that $\alpha_k = 1$ and $\beta_k = 0$. However the value of α_k and β_k must be obtained starting from vacuum initial conditions in the previous phases. Two different types of initial conditions were already considered in the literature [41], which are the aforementioned BD

vacuum in the contracting phase [77] and the fourth-order adiabatic vacuum at the bounce [62], which are, respectively, given by

$$\chi_k^{(\text{BD})}(\eta) = \frac{1}{2k} e^{-ik\eta}, \quad (4.3)$$

$$\chi_k^{(\text{WKB})}(\eta) = \frac{1}{2k} \left[1 - \frac{1}{4} \frac{k_B^2}{k^2} - \frac{29}{32} \frac{k_B^4}{k^4} + \mathcal{O}\left(\frac{k_B}{k}\right)^6 \right], \quad (4.4)$$

where $k_B = \sqrt{\gamma_B/3} a_B m_{\text{Pl}}$ is the energy scale in the bounce in the dressed approach. The ‘‘WKB’’ in Eq. (4.4) refers to the WKB approximation, used to obtain this result. These initial conditions lead to the same results for GPP in the case considered here and as computed explicitly in Ref. [36]. Setting the previous initial conditions, it then follows from Eq. (4.2) that

$$|\beta_k|^2 = \frac{1}{2} \left[1 + \cos\left(\frac{\pi}{\sqrt{3}}\right) \right] \text{csch}^2\left(\frac{\pi k}{\sqrt{6}k_B}\right). \quad (4.5)$$

Above $|\beta_k|^2$ corresponds to the number of particles per mode k that were produced, namely n_k . Using this quantity, in Ref. [36] the energy density of particles produced with and without backreaction was obtained. However, unlike the procedure used in that reference, here we are going to consider the contribution from all the modes for the density of produced particles, not only the modes that in the preinflationary phase exit and then reenter the effective horizon. Note also that using all the modes is typically the procedure adopted in studies of GPP in general [76]. The produced modes are effectively considered as particles after they reenter the horizon. Therefore, the energy density stored in the produced particles is then given by

$$\rho_p(\eta) = \frac{1}{2\pi^2 a^4(\eta)} \int_0^\infty dk k^2 n_k(\eta) \omega_k. \quad (4.6)$$

We then obtain that

$$\rho_p(\eta) = \frac{1 + \cos\left(\frac{\pi}{\sqrt{3}}\right)}{4\pi^2 a^4(\eta)} \int_0^\infty dk k^3 \text{csch}^2\left(\frac{\pi k}{\sqrt{6}k_B}\right), \quad (4.7)$$

where we used $n_k \equiv |\beta_k|^2$, where $|\beta_k|^2$ is given by Eq. (4.5) and $\omega_k \sim k$ in the case of relativistic particles. By performing the integration, Eq. (4.7) gives

$$\rho_p(\eta) \simeq 12.5 \times 10^{-3} \frac{k_B^4}{a^4(\eta)} \simeq 1.3 \frac{m_{\text{Pl}}^4}{a^4(\eta)}. \quad (4.8)$$

We must remember that the background energy evolves dynamically as stiff matter (since the kinetic energy of the inflaton dominates in the bounce), with $\rho_{\text{bg}} = \rho_c a^{-6}$. However the gravitationally produced particles evolve like relativistic matter, with $\rho_p \propto a^{-4}$. In a previous work [36],

it was estimated that, in order to remain subdominant before and at the beginning of inflation, the energy density on the produced particles must satisfy the condition $\rho(t_s) < 2 \times 10^{-5} m_{\text{Pl}}^4$. The time t_s , when $a''/a = 0$, is when the maximum of GPP happens. This value was estimated by considering an equal amount of energy density in radiation and in the inflaton potential energy at the beginning of inflation and then receding this radiation density backward in time by multiplying it to a^4 until the time $t = t_s$. By assuming $a_B = 1$, at the time $t_s = 0.3 t_{\text{Pl}}$, the scale factor is found to be $a(t_s) \approx 1.248$. Thus, from Eq. (4.8) we can estimate that $\rho_p(t_s) \approx 0.54 m_{\text{Pl}}^4$. Therefore, we confirm that, in the context of the dressed metric approach, the GPP density for relativistic χ particles will eventually dominate the dynamics, which is inconsistent with the premise that backreaction must be small for the dressed metric approach to be valid and as far as the production of massless spectator scalar particles are concerned.

B. Hybrid approach

The analytic form of the solutions and the matching conditions in the hybrid approach are analogous to the previous case (see Ref. [68] for more details). It is straightforward to obtain the Bogoliubov coefficients α_k and β_k by matching the solutions in the bounce phase, transition phase and slow-roll phase, whose procedure follows similarly to that done for the dressed metric approach.

Considering the BD vacuum as the initial condition [43] the Bogoliubov coefficients then reads now

$$|\beta_k|^2 = \frac{1}{2} \left[1 + \cos\left(\frac{\sqrt{5}\pi}{\sqrt{3}}\right) \right] \text{csch}^2\left(\frac{\pi k}{\sqrt{6}k_H}\right), \quad (4.9)$$

where, as already defined earlier, $k_H = k_B/\sqrt{3}$ is the energy scale at the bounce in the hybrid approach. The quantum effects in both dressed metric and hybrid approaches effective equations are qualitatively the same [68] but exhibit two quantitative differences. These differences are the characteristic energy scales k_B and k_H and the numerical factor before the hyperbolic function in Eqs. (4.5) and (4.9). In addition, in the hybrid case we have a positive time-dependent effective mass as one approaches the bounce, while in the dressed metric case the time-dependent effective mass is negative when approaching the bounce and around it [67].

Quantitatively, by comparing Eq. (4.5) with Eq. (4.9), we notice that the only differences will be a factor of $1/\sqrt{3}$ from the characteristic scale k_H as compared to k_B , in addition to the factor of $\sqrt{5}$ in the cosine argument. Then, the corresponding expression to Eq. (4.7) in the hybrid approach is simply

$$\rho_p(\eta) = \frac{1 + \cos\left(\frac{\sqrt{5}\pi}{\sqrt{3}}\right)}{4\pi^2 a^4(\eta)} \int_0^\infty dk k^3 \text{csch}^2\left(\frac{\pi k}{\sqrt{6}k_H}\right). \quad (4.10)$$

Again, the integral is ultraviolet dominated and Eq. (4.10) gives

$$\rho_p(\eta) \simeq 6.5 \times 10^{-3} \frac{k_H^4}{a^4(\eta)} \simeq 7.5 \times 10^{-2} \frac{m_{\text{Pl}}^4}{a^4(\eta)}. \quad (4.11)$$

If we estimate the energy density stored in the particles produced only for the modes that exit and reenter the effective horizon λ during the preinflationary phase, we obtain the value $\rho_p(\eta) = 10^{-3} m_{\text{Pl}}^4 / a^4(\eta)$. Equation (4.11) is only defined for $t \gtrsim t_s$, when the modes are well inside the horizon to be considered as particles and which is the moment when ρ_p assumes its highest value. From Eq. (4.11), we obtain that $\rho(t_s) \approx 3.1 \times 10^{-2} m_{\text{Pl}}^4$ for the density of particles produced at $t = t_s$, which is approximately the instant that GPP ceases. As discussed previously, in order to remain subdominant until the start of inflation, the density in the produced particles must satisfy the condition $\rho_s < 2 \times 10^{-5} m_{\text{Pl}}^4$. Despite having shown through Eq. (4.11) that ρ_s is smaller than the corresponding quantity in the dressed metric approach, we can see that the condition required for the produced particles not to dominate the background energy density is still *not* satisfied in the hybrid approach as well. Analogously to what happens in the dressed approach, this invalidates the effective description usually considered in these approaches as far as the production of massless spectator scalar particles are again involved.

Therefore, despite the difference in the maximum density of produced particles, the conclusions for the dressed and hybrid approaches will be basically the same. In Ref. [74] (and more recently in Ref. [75]), another proposal was suggested to select the initial vacuum state. It suggests to select the initial conditions for each mode in order to minimize the time variation of the spectator field amplitude since the bounce until the starting of the inflationary phase. As shown in Ref. [33], such “nonoscillating” initial conditions lead to a primordial power spectrum without the large oscillations. However, one can check that the number density of particles produced in this case, given by the quantity $|\beta_k|^2$, will not change considerably in our framework, since they get rid of the oscillations by avoiding the fast oscillating term which we have already averaged out in our analysis above. Therefore, one should not expect that this method would prevent excessive particle production. This motivates us to move further to the investigation of the particle production in another framework. In the next section we are going to consider the closed or deformed algebra approach, which consists in another way of treating the perturbations, possibly leading to different results.

C. Closed or deformed algebra approach

Here we follow the same approach considered in the previous cases to obtain the parameter β_k in the closed

algebra approach. However, this case is rather more involved than the previous ones. This is due to the fact that the description of the propagation of the modes in the transition from the Lorentzian to the Euclidean phase is not so rigorous [40] due to the presence of the silent point. This is called the *signature change problem* [64,70,78,79]. The solution to this problem can be imposing initial conditions for the modes at the silent point ($t = t_s$) in the Lorentzian phase after the bounce, where the signature changes from Euclidean to Lorentzian. In the silent point all points become uncorrelated, since the space-dependent term in the equation of motion for the modes drops out and the two-point function in this surface becomes zero. Therefore, after the silent point, in this approach, the modes $\chi_k(\eta)$ obey Eq. (3.12). In particular, in the bounce and transition phases, using the analytical approximations given by Eqs. (2.10) and (2.11), from Eq. (3.14) we obtain that

$$\Omega(\eta) = \frac{\tau^2 - 1}{\tau^2 + 1}, \quad (4.12)$$

where $\tau = t/\tau_B$ [38].

We can obtain leading-order approximate solutions for the mode functions of Eq. (3.12) using the uniform asymptotic approximation method [80,81]. The complete evaluation of the solutions was presented in Ref. [38]. Here, we summarize the main steps for completeness. First, by appropriately changing variables, the mode equation can be expressed in the form

$$\frac{d^2 \chi_k(y)}{dy^2} = [g(y) + q(y)] \chi_k(y), \quad (4.13)$$

where $y = -k\eta$ and the functions g and q are defined as

$$g(y) = \frac{1 - \tau^2}{1 + \tau^2} + \frac{\gamma_B}{4k^2\tau^2} + \frac{\gamma_B(18 + 21\tau^2 - \tau^4)}{9k^2\tau^2(\tau^2 + 1)^{5/3}}, \quad (4.14)$$

$$q(y) = -\frac{\gamma_B}{4k^2\tau^2}. \quad (4.15)$$

The analytical solution of Eq. (4.13) is found to be given by [38]

$$\chi_k(t) = \left(\frac{\xi}{g}\right)^{1/4} [a_k \text{Ai}(\xi) + b_k \text{Bi}(\xi)], \quad (4.16)$$

where $\text{Ai}(\xi)$ and $\text{Bi}(\xi)$ are the Airy functions of the first and second kind, respectively, and the parameter ξ is related to the function g through

$$\xi = \begin{cases} \left(-\frac{3k}{2} \int_{t_+}^t \frac{\sqrt{g}}{a} dt\right)^{2/3}, & t < t_+, \\ -\left(\frac{3k}{2} \int_{t_+}^t \frac{\sqrt{-g}}{a} dt\right)^{2/3}, & t > t_+, \end{cases} \quad (4.17)$$

t_+ being the turning point [where $g(y) = 0$]. Also, the Wronskian condition implies that

$$\chi_k \dot{\chi}_k^* - \dot{\chi}_k \chi_k^* = i/a, \quad (4.18)$$

$$a_k b_k^* - a_k^* b_k = i\pi/k. \quad (4.19)$$

The qualitative general behavior of the function $g(y)$ keeps the same for all modes, having only one turning point for any k in the bouncing and transition phases. However, the precise location of the turning point t_+ depends on the comoving wave number k .

Even though we cannot explicitly obtain the GPP in general from the above equations, we can still get a clear picture of GPP in the small- and long-wavelength approximations.

1. The small-wavelength regime

In the transition phase, $\xi(t)$ approaches to asymptotic negative infinity. In this region, the Airy functions assume their asymptotic form. Considering the previous definitions given above and together with the equation for the modes given by Eq. (4.16), we can then write the solution for the modes as

$$\begin{aligned} \chi_k(t) = \frac{1}{\sqrt{\pi(-g)^{1/4}}} \left\{ a_k \cos \left[\frac{2}{3} (-\xi)^{3/2} - \frac{\pi}{4} \right] \right. \\ \left. + b_k \sin \left[\frac{2}{3} (-\xi)^{3/2} - \frac{\pi}{4} \right] \right\}. \end{aligned} \quad (4.20)$$

After some algebra, it is possible to show that Eq. (4.20) can be put in the form

$$\chi_k = \frac{e^{-i\pi/4}}{2\sqrt{\pi}} (a_k - ib_k) e^{ik(\eta-\eta_{\text{TB}})} + (ia_k - b_k) e^{-ik(\eta-\eta_{\text{TB}})}. \quad (4.21)$$

On the other hand, when the horizon goes to negative infinity in the transition phase, the equation of motion for the modes becomes

$$\chi_k'' + k^2 \chi_k = 0, \quad (4.22)$$

whose solution is

$$\chi_k = \frac{1}{\sqrt{2k}} (\tilde{\alpha}_k e^{-ik\eta} + \tilde{\beta}_k e^{ik\eta}). \quad (4.23)$$

By comparing Eqs. (4.21) and (4.23), we can match the two sets of integration constants, which allows us to obtain the coefficients in the UV limit:

$$\alpha_k = \sqrt{\frac{k}{2\pi}} (ia_k - b_k) e^{ik\eta_{\text{TB}} - i\pi/4}, \quad (4.24)$$

$$\beta_k = \sqrt{\frac{k}{2\pi}} (a_k - ib_k) e^{-ik\eta_{\text{TB}} - i\pi/4}, \quad (4.25)$$

where

$$\eta_{\text{TB}} = \eta_f - \int_{\eta_+}^{\eta_f} \sqrt{-g(\eta)} d\eta. \quad (4.26)$$

The coefficients a_k and b_k are obtained by matching the power spectrum to the one given by GR. This is possible because, in the regime that we are interested, the equation of motion for the spectator field modes χ_k has the same behavior as the equation of motion of the inflaton field and of the curvature perturbations, which are basically due to the inflaton fluctuations. Therefore, the dynamics of the modes we are computing here must not present divergences, since those would be translated to divergences in the power spectrum. In order to define a behavior for the scalar modes that can be consistent with the observations in the closed algebra approach, we will interpret such modes analogously to the ones which will enter in the expression for the power spectrum of the model. Its possible to show that the only possible initial condition that allows compatibility of the power spectrum with the current CMB data is given by (for details, see, e.g., Ref. [38])

$$a_k = \sqrt{\frac{\pi}{2k}}, \quad b_k = -i\sqrt{\frac{\pi}{2k}}. \quad (4.27)$$

These coefficients lead to a spectrum equal to the classical GR result in the observational window (the observed modes in CMB correspond to the UV limit). By inserting these coefficients in Eq. (4.25), we obtain that

$$\alpha_k = i e^{i(k\eta_{\text{TB}} - \pi/4)}, \quad \beta_k = 0. \quad (4.28)$$

Any other initial condition implies a correction term (with respect to GR spectrum) proportional to the wave number, which leads to a divergent spectrum in the UV. We can see from the above expressions that $|\alpha_k|^2 = 1$, which is consistent with the condition $|\alpha_k|^2 - |\beta_k|^2 = 1$. This corresponds exactly to the classical case in GR. As the parameter $|\beta_k|^2 = 0$, this implies no gravitational particle production in the UV limit of this model. Therefore, in this framework, from the UV modes with such initial conditions (the only ones that do not produce divergences), we see no particle production in any scenario that provides a power spectrum consistent with the data. However, as discussed in Ref. [38], in the Planckian UV regime, new ingredients are expected to take place, as modified dispersion relations, for example, which could avoid possible divergences or even change the behavior of the modes. In the absence of a definite model for this regime, we are instead going to focus on the case of IR modes in order to check whether some considerably energy density can be gravitationally

produced in this regime. Since in the UV regime either we have no particle production or otherwise a new physics would be coming into play, the lack of information required to obtain definite results from the UV regime motivates us to rediscuss the possible initial conditions in the context of the IR modes. In the following we discuss the behavior of such modes.

2. Long-wavelength regime

In the IR regime, $k < m_{\text{Pl}}$, through the bounce and transition phases the equation for the modes is found to have the solution

$$\chi_k(\eta) = a_k z(\eta) + b_k \bar{z}(\eta) \int_{\eta_*}^{\eta_{\text{end}}} \frac{d\eta'}{z^2(\eta')} + \mathcal{O}(k^2), \quad (4.29)$$

where η_* denotes some particular reference time. It can be shown that this result leads to the following IR limit of the power spectrum [64]:

$$P^{\text{IR}} \approx \frac{k^3}{2\pi^2} \left| b_k \int_{\eta_*}^{\eta_{\text{end}}} \frac{d\eta'}{z^2(\eta')} \right|^2. \quad (4.30)$$

Here, we are going to consider initial conditions (i.e., the expressions for a_k and b_k) that lead to a spectrum in agreement with what we observe. Starting from Eq. (4.30), we can consider different approaches that set the initial conditions at the vicinity of the silent point. The choice of the vicinity of the silent point to set the initial conditions is justified in order to avoid the problems that can happen due to the change of signature close to the bounce. The calculations in the infrared regards the fact that, when the term k^2 is neglected, there can be analytical solutions of the mode function equations. Let us first write a more general parametrization to the coefficients:

$$a_k \equiv a_0 k^n, \quad b_k \equiv b_0 e^{-i\theta} k^l, \quad (4.31)$$

where θ is the relative phase between a_k and b_k . Besides, a_0 and b_0 are both positive and independent of k and have dimensions of m_{Pl}^{-2n} and m_{Pl}^{-2l} , respectively. The quantities a_k and b_k satisfy the Wronskian condition

$$a_k b_k^* - a_k^* b_k = i. \quad (4.32)$$

Considering the parametrization given by Eq. (4.31), the above condition implies

$$2a_0 b_0 \sin(\theta) = 1, \quad n + l = 0. \quad (4.33)$$

Therefore, we can see that the only initial condition that can imply a scale-invariant spectrum is

$$a_k \propto k^{3/2}, \quad b_k \propto k^{-3/2}, \quad (4.34)$$

and it implies scale invariance at any time until the end of inflation, since in the IR regime the term proportional to b_k in Eq. (4.29) will be dominant. The term b_k can be obtained by matching the solution for the modes in the contracting and the bounce phase. It is straightforward to show that $b_k = (3i/\sqrt{2})\lambda_0 k^{-3/2}$ (see, e.g., Ref. [64] for further details).

By comparing the expression given by Eq. (4.30) with the GR spectrum we can identify that

$$\begin{aligned} P^{\text{IR}} &\approx \frac{k^3}{2\pi^2} |b_k|^2 \left(\int_{\eta_*}^{\eta_{\text{end}}} \frac{d\eta'}{z^2(\eta')} \right)^2 = P_{\text{GR}} |\alpha_k + \beta_k|^2 \\ &\approx 2.2 \times 10^{-9} |\alpha_k + \beta_k|^2. \end{aligned} \quad (4.35)$$

Despite the initial conditions being related to the Bogoliubov coefficients through Eq. (4.35), it is not possible to identify which contribution corresponds specifically to β_k , which enters in the expression of the particle production. However, the above equation allows us to establish an upper limit in β_k . Taking into account the well-known property $|\alpha_k + \beta_k| \lesssim |\alpha_k| + |\beta_k|$ and remembering the normalization condition $|\alpha_k|^2 - |\beta_k|^2 = 1$, we can show that

$$|\alpha_k + \beta_k| \lesssim |\alpha_k| + |\beta_k| \lesssim 1 + 2|\beta_k|. \quad (4.36)$$

Therefore, from Eq. (4.35) we obtain the limit

$$|\beta_k|^2 \lesssim \frac{\lambda_0^2 \times 10^9}{\pi^2} \left| \int_{\eta_*}^{\eta_{\text{end}}} \frac{d\eta'}{z^2(\eta')} \right|^2, \quad (4.37)$$

where η_{end} is the value of η at the end of inflation and $z(\eta) = a\dot{\phi}/H$. In the above equation, we made use of Eq. (4.35) and that $b_k = (3i/\sqrt{2})\lambda_0 k^{-3/2}$. The integral in Eq. (4.37) depends only on the background, being independent of k . At this point it is important to stress that, while we have argued that it can be meaningless to define two-point correlation functions before the silent point, concerning the background there is in fact no problem in starting the initial conditions in the contracting phase. This was done in Ref. [38] and we will consider the same here. In this case, for the background dynamics we can consider the evolution presented in Sec. II. During slow-roll inflation, when the spectrum is computed, we have that $z(\eta)^2 = 2a^2\epsilon$, ϵ being the slow-roll parameter given by $\epsilon \simeq \dot{\phi}^2/(2H^2)$ during inflation. Therefore, from now on we can consider the approximation $z(\eta) \approx a(\eta)\sqrt{4\pi/(3m_{\text{Pl}}^2)}$. Therefore, the integration in Eq. (4.37) can be computed using Eq. (2.16). This procedure leads in particular to the result

$$I(\eta_e) \equiv \int_{\eta_*}^{\eta_{\text{end}}} \frac{d\eta'}{a^2(\eta')} = \frac{-m}{18\lambda_0} \frac{1}{|\cos\theta_A|} \ln \left(\frac{1}{2} \Gamma \sqrt{\frac{|\cos\theta_A|}{f}} \right). \quad (4.38)$$

In the above expression we can use the values $m = 10^{-6}m_{\text{pl}}$, $\Gamma = 2 \times 10^{-7}$, $f \approx 0.18$, $\lambda_0 = a_{\text{in}}H_{\text{in}}^2/4$ and $\cos\theta_A \approx 1$, as obtained and discussed in Sec. II. One must remember that Γ is the ratio between timescales as defined in Eq. (2.5), while the quantity $\Gamma \cos\theta_A$ corresponds to the value of y at the onset of the bounce phase. The quantity f is associated with the time of the beginning of the slow-roll phase, since $t_{\text{SR}} = t_B + f/m$ with f defined in Eq. (2.13). With such values, we obtain that Eq. (4.38) is estimated to be $I(\eta_e) \sim 10^{-6}/\lambda_0$. Therefore, using the expression for b_k , Eq. (4.37) becomes

$$|\beta_k|^2 \lesssim \frac{\lambda_0^2 \times 10^9 I(\eta_e)^2}{\pi^2 \cdot 16} \approx 10^{-5}. \quad (4.39)$$

By substituting $|\beta_k|^2$ from the equation above in Eq. (4.6), we obtain the following expression for the estimated upper limit on the density of particles gravitationally produced:

$$\rho_p \lesssim \frac{10^{-5}}{2\pi^2 a^4} \int_0^{m_{\text{pl}}} dk k^3 \approx \frac{10^{-6}}{a^4} m_{\text{pl}}^4, \quad (4.40)$$

where we only have integrated modes with $k \lesssim m_{\text{pl}}$ since we are restricted to the IR limit. In the latter equation, we have considered $\omega_k(\eta) \approx \sqrt{\Omega(\eta)}k$, since the modes only contribute to the density of particles after they are well inside the effective horizon. Also, we see that the expression of ω_k has a correction factor in the closed algebra approach [see Eqs. (3.13) and (3.14)], which is given by $\Omega(\eta) = 1 - 2\rho/\rho_c$. In the silent point we have $\Omega = 0$, and after that $\Omega(\eta)$ increases until it reaches the value $\Omega = 1$, when $\rho \ll \rho_c$. Since $\Omega(\eta)$ does not depend on k , we simply consider its upper limit $\Omega = 1$ in the above equation, since we want to obtain an upper limit for the density of produced particles. The fact that Ω is zero at the silent point and then increases means that in fact the particles start being produced right after the silent point. Since the particles produced behave as radiation, its energy density will then evolve as

$$\rho_p = \rho_s a^{-4}, \quad (4.41)$$

where ρ_s is the density of particles produced right after the silent point. By comparing it with Eq. (4.40), we can see that

$$\rho_s \lesssim 10^{-6} m_{\text{pl}}^4, \quad (4.42)$$

which is right after the silent point, when the important modes are already inside the horizon. In order to know if the produced particles will not dominate the energy content of the Universe before inflation, we must compare the density of particles produced, ρ_p , with the background energy density ρ_{bg} at the beginning of inflation. Unlike the particles produced, which behave as radiation, $\rho_p = \rho_s a^{-4}$, the background energy density, on the other hand, evolves as stiff matter, $\rho_{bg} = \rho_c a^{-6}$, before inflation sets in. Therefore,

in order for the density of produced particles not to come to dominate before the onset of inflation, we must have the following condition satisfied:

$$\rho_s a^{-4} < \rho_c a^{-6}$$

and which must be satisfied in the beginning of inflation. The value of the scale factor in the beginning of inflation depends on the value of some parameters associated with the initial conditions. However, since the evolution of the background is the same in all approaches, based on previous works (see, e.g., Refs. [47,48] for example) we can estimate an amount of $4 - 5e$ -folds of preinflationary expansion (from the bounce to the beginning of inflation). In the case of four preinflationary e -folds, we have that in the onset of inflation $a_{\text{infl}} \sim 55$. We can consider that near the silent point $\rho_s = \rho_c/2 \sim \rho_c a_s^{-6}$, which leads to $a_s^{-6} \sim 1/2$ and, consequently, $a_s^6 \sim 2$. Therefore, it is a good approximation to consider $a_s \approx 1$. Considering this value, the condition for the produced particles not to come to dominate can be written as $\rho_s \lesssim 10^{-4} m_{\text{pl}}^4$. Since Eq. (4.42) shows that $\rho_s \sim 10^{-6} m_{\text{pl}}^4$, we can safely conclude that the particles gravitationally produced will not dominate the background energy density before inflation in the deformed algebra approach. This proves that the test field approximation is consistently valid in this approach with this choice of initial conditions, unlike in the hybrid and dressed approaches.

The violation of adiabaticity happens during a short time interval around the bounce phase. This coincides with the phase when the main modes exit the effective horizon. These modes will then reenter the horizon after the bounce phase, when they start behaving as actual particles. Therefore, the backreaction is expected to be not strong enough to change our conclusions. This is also corroborated by a previous analysis made in Ref. [36], which was, however, restricted to the dressed case (for related work on the backreaction of GPP in general, see, e.g., Ref. [82]).

V. CONCLUSIONS

Given the importance of the effective description of LQC in providing means to obtain the relevant cosmological quantities, it is of utmost importance to further analyze the validity of such a description. Motivated by the results obtained in a previous work [36], we extended the analysis of the backreaction from gravitational particle production to other approaches.

Firstly, for the hybrid approach, we obtain a result similar to the case of the dressed metric one, where the energy density stored in the particles produced during the bounce phase dominates the energy content of the Universe prior to inflation. Therefore, if we extend the validity of the effective description beyond the test field approximation, this would imply a preinflationary radiation-dominated phase in these scenarios. This scenario would be similar to a model including the radiation effects in LQC, as

studied in Ref. [47]. A radiation-dominated phase in the earlier stages of expansion in these scenarios tends to imply a small delay in the beginning of the inflationary phase in such models. Also, the backreaction effect leads to a state significantly different from the BD vacuum at the beginning of inflation. Indeed, radiation has been shown to be an important factor in setting initial conditions for inflation appropriately (see, e.g., Ref. [83] for a further discussion). However, since one should not expect that the dressed metric and hybrid approaches could be consistent in such a regime, this analysis actually put in check the validity of these approaches with the initial conditions considered.

On the other hand, in the case of the closed algebra approach, we obtain that the process of gravitational particle production leads to a negligible backreaction effect. The energy stored in the produced particles is very small compared to the energy density of the background all the way up to the onset of inflation. This result was obtained by considering initial conditions in the vicinity of the silent point, which is justified in order to avoid problems coming from a signature change close to the bounce. Our result corroborates the validity of the test field approximation in this framework, showing the robustness of the effective description of LQC in the closed algebra approach. Nevertheless we must point out

that this result is strictly related to the initial conditions which are chosen in such a way that guarantees the consistency of the model with CMB data. Any dynamics that could lead to significant particle production in this scenario would imply a divergent power spectrum.

In order to further confirm the analytical results obtained here, it would be important to perform a numerical analysis capable of including the backreaction effects of the particles in the background simultaneously to its production. This will be done in a future work.

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- [1] R. H. Brandenberger, Inflationary cosmology: Progress and problems, [arXiv:hep-ph/9910410](#).
 - [2] Y. Akrami *et al.* (Planck Collaboration), Planck 2018 results. X. Constraints on inflation, *Astron. Astrophys.* **641**, A10 (2020).
 - [3] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020); **652**, C4(E) (2021).
 - [4] D. Baumann, Inflation, [arXiv:0907.5424](#).
 - [5] M. Bojowald, Absence of Singularity in Loop Quantum Cosmology, *Phys. Rev. Lett.* **86**, 5227 (2001).
 - [6] A. Ashtekar, T. Pawłowski, and P. Singh, Quantum Nature of the Big Bang, *Phys. Rev. Lett.* **96**, 141301 (2006).
 - [7] A. Ashtekar, T. Pawłowski, and P. Singh, Quantum nature of the big bang: An analytical and numerical investigation. I., *Phys. Rev. D* **73**, 124038 (2006).
 - [8] A. Ashtekar, T. Pawłowski, and P. Singh, Quantum nature of the big bang: Improved dynamics, *Phys. Rev. D* **74**, 084003 (2006).
 - [9] A. Ashtekar, A. Corichi, and P. Singh, Robustness of key features of loop quantum cosmology, *Phys. Rev. D* **77**, 024046 (2008).
 - [10] P. Singh, Are loop quantum cosmos never singular?, *Classical Quantum Gravity* **26**, 125005 (2009).
 - [11] A. Ashtekar, T. Pawłowski, P. Singh, and K. Vandersloot, Loop quantum cosmology of $k = 1$ FRW models, *Phys. Rev. D* **75**, 024035 (2007).
 - [12] L. Szulc, W. Kaminski, and J. Lewandowski, Closed FRW model in loop quantum cosmology, *Classical Quantum Gravity* **24**, 2621 (2007).
 - [13] A. Ashtekar and P. Singh, Loop quantum cosmology: A status report, *Classical Quantum Gravity* **28**, 213001 (2011).
 - [14] A. Barrau, T. Cailleteau, J. Grain, and J. Mielczarek, Observational issues in loop quantum cosmology, *Classical Quantum Gravity* **31**, 053001 (2014).
 - [15] A. Ashtekar, W. Kaminski, and J. Lewandowski, Quantum field theory on a cosmological, quantum space-time, *Phys. Rev. D* **79**, 064030 (2009).
 - [16] I. Agullo and P. Singh, Loop quantum cosmology, [arXiv:1612.01236](#).
 - [17] A. Ashtekar and A. Barrau, Loop quantum cosmology: From pre-inflationary dynamics to observations, *Classical Quantum Gravity* **32**, 234001 (2015).
 - [18] I. Agullo, A. Ashtekar, and W. Nelson, A Quantum Gravity Extension of the Inflationary Scenario, *Phys. Rev. Lett.* **109**, 251301 (2012).
 - [19] M. Benetti, L. Graef, and R. O. Ramos, Observational constraints on warm inflation in loop quantum cosmology, *J. Cosmol. Astropart. Phys.* **10** (2019) 066.
 - [20] M. Bojowald, G. M. Hossain, M. Kagan, and S. Shankaranarayanan, Anomaly freedom in perturbative loop quantum gravity, *Phys. Rev. D* **78**, 063547 (2008).
 - [21] T. Cailleteau, J. Mielczarek, A. Barrau, and J. Grain, Anomaly-free scalar perturbations with holonomy corrections in loop

- quantum cosmology, *Classical Quantum Gravity* **29**, 095010 (2012).
- [22] M. Fernandez-Mendez, G. A. Mena Marugan, and J. Olmedo, Hybrid quantization of an inflationary Universe, *Phys. Rev. D* **86**, 024003 (2012).
- [23] E. Wilson-Ewing, Separate universes in loop quantum cosmology: Framework and applications, *Int. J. Mod. Phys. D* **25**, 1642002 (2016).
- [24] E. Wilson-Ewing, Testing loop quantum cosmology, *C. R. Phys.* **18**, 207 (2017).
- [25] L. C. Gomar, M. Fernández-Méndez, G. A. M. Marugán, and J. Olmedo, Cosmological perturbations in hybrid loop quantum cosmology: Mukhanov-Sasaki variables, *Phys. Rev. D* **90**, 064015 (2014).
- [26] M. Martin-Benito, L. J. Garay, and G. A. Mena Marugan, Hybrid quantum Gowdy cosmology: Combining loop and fock quantizations, *Phys. Rev. D* **78**, 083516 (2008).
- [27] G. A. Mena Marugan and M. Martin-Benito, Hybrid quantum cosmology: Combining loop and fock quantizations, *Int. J. Mod. Phys. A* **24**, 2820 (2009).
- [28] L. J. Garay, M. Martin-Benito, and G. A. Mena Marugan, Inhomogeneous loop quantum cosmology: Hybrid quantization of the Gowdy model, *Phys. Rev. D* **82**, 044048 (2010).
- [29] M. Martin-Benito, G. A. M. Marugan, and E. Wilson-Ewing, Hybrid quantization: From Bianchi I to the Gowdy Model, *Phys. Rev. D* **82**, 084012 (2010).
- [30] D. Brizuela, G. A. D. Mena Marugan, and T. Pawłowski, Big bounce and inhomogeneities, *Classical Quantum Gravity* **27**, 052001 (2010).
- [31] B. F. Li, J. Olmedo, P. Singh, and A. Wang, Primordial scalar power spectrum from the hybrid approach in loop cosmologies, *Phys. Rev. D* **102**, 126025 (2020).
- [32] L. C. Gomar, M. Martín-Benito, and G. A. M. Marugán, Gauge-invariant perturbations in hybrid quantum cosmology, *J. Cosmol. Astropart. Phys.* **06** (2015) 045.
- [33] L. Castelló Gomar, G. A. Mena Marugán, D. Martín De Blas, and J. Olmedo, Hybrid loop quantum cosmology and predictions for the cosmic microwave background, *Phys. Rev. D* **96**, 103528 (2017).
- [34] B. F. Li, P. Singh, and A. Wang, Genericness of pre-inflationary dynamics and probability of the desired slow-roll inflation in modified loop quantum cosmologies, *Phys. Rev. D* **100**, 063513 (2019).
- [35] B. F. Li, P. Singh, and A. Wang, Qualitative dynamics and inflationary attractors in loop cosmology, *Phys. Rev. D* **98**, 066016 (2018).
- [36] L. L. Graef, R. O. Ramos, and G. S. Vicente, Gravitational particle production in loop quantum cosmology, *Phys. Rev. D* **102**, 043518 (2020).
- [37] A. Barrau, Testing different approaches to quantum gravity with cosmology: An overview, *C. R. Phys.* **18**, 189 (2017).
- [38] B. F. Li, T. Zhu, A. Wang, K. Kirsten, G. Cleaver, and Q. Sheng, Preinflationary perturbations from the closed algebra approach in loop quantum cosmology, *Phys. Rev. D* **99**, 103536 (2019).
- [39] S. Schander, A. Barrau, B. Bolliet, L. Linsefors, J. Mielczarek, and J. Grain, Primordial scalar power spectrum from the Euclidean big bounce, *Phys. Rev. D* **93**, 023531 (2016).
- [40] J. Mielczarek, L. Linsefors, and A. Barrau, Silent initial conditions for cosmological perturbations with a change of spacetime signature, *Int. J. Mod. Phys. D* **27**, 1850050 (2018).
- [41] T. Zhu, A. Wang, G. Cleaver, K. Kirsten, and Q. Sheng, Pre-inflationary universe in loop quantum cosmology, *Phys. Rev. D* **96**, 083520 (2017).
- [42] L. Linsefors and A. Barrau, Duration of inflation and conditions at the bounce as a prediction of effective isotropic loop quantum cosmology, *Phys. Rev. D* **87**, 123509 (2013).
- [43] K. Martineau, A. Barrau, and S. Schander, Detailed investigation of the duration of inflation in loop quantum cosmology for a Bianchi-I universe with different inflaton potentials and initial conditions, *Phys. Rev. D* **95**, 083507 (2017).
- [44] B. Bolliet, A. Barrau, K. Martineau, and F. Moulin, Some clarifications on the duration of inflation in loop quantum cosmology, *Classical Quantum Gravity* **34**, 145003 (2017).
- [45] A. Ashtekar and D. Sloan, Probability of inflation in loop quantum cosmology, *Gen. Relativ. Gravit.* **43**, 3619 (2011).
- [46] L. L. Graef and R. O. Ramos, Probability of warm inflation in loop quantum cosmology, *Phys. Rev. D* **98**, 023531 (2018).
- [47] L. N. Barboza, L. L. Graef, and R. O. Ramos, Warm bounce in loop quantum cosmology and the prediction for the duration of inflation, *Phys. Rev. D* **102**, 103521 (2020).
- [48] L. N. Barboza, G. L. L. W. Levy, L. L. Graef, and R. O. Ramos, Constraining the Barbero-Immirzi parameter from the duration of inflation in loop quantum cosmology, [arXiv: 2206.14881](https://arxiv.org/abs/2206.14881).
- [49] L. Parker, Particle Creation in Expanding Universes, *Phys. Rev. Lett.* **21**, 562 (1968).
- [50] L. Parker, Quantized fields and particle creation in expanding universes. 1., *Phys. Rev.* **183**, 1057 (1969).
- [51] J. Quintin, Y. F. Cai, and R. H. Brandenberger, Matter creation in a nonsingular bouncing cosmology, *Phys. Rev. D* **90**, 063507 (2014).
- [52] Y. Tavakoli and J. C. Fabris, Creation of particles in a cyclic universe driven by loop quantum cosmology, *Int. J. Mod. Phys. D* **24**, 1550062 (2015).
- [53] J. Haro and E. Elizalde, Gravitational particle production in bouncing cosmologies, *J. Cosmol. Astropart. Phys.* **10** (2015) 028.
- [54] D. C. F. Celani, N. Pinto-Neto, and S. D. P. Viteni, Particle creation in bouncing cosmologies, *Phys. Rev. D* **95**, 023523 (2017).
- [55] A. Scardua, L. F. Guimarães, N. Pinto-Neto, and G. S. Vicente, Fermion production in bouncing cosmologies, *Phys. Rev. D* **98**, 083505 (2018).
- [56] F. Zago and A. Kosowsky, Quantum particle production effects on the cosmic expansion, *Phys. Rev. D* **100**, 045023 (2019).
- [57] W. S. Hipolito-Ricaldi, R. Brandenberger, E. G. M. Ferreira, and L. L. Graef, Particle production in ekpyrotic scenarios, *J. Cosmol. Astropart. Phys.* **11** (2016) 024.
- [58] L. L. Graef, W. S. Hipolito-Ricaldi, E. G. M. Ferreira, and R. Brandenberger, Dynamics of cosmological perturbations and reheating in the anamorphic universe, *J. Cosmol. Astropart. Phys.* **04** (2017) 004.

- [59] K. A. Meissner, Black hole entropy in loop quantum gravity, *Classical Quantum Gravity* **21**, 5245 (2004).
- [60] E. Wilson-Ewing, Ekpyrotic loop quantum cosmology, *J. Cosmol. Astropart. Phys.* **08** (2013) 015.
- [61] I. Agullo, A. Ashtekar, and W. Nelson, The pre-inflationary dynamics of loop quantum cosmology: Confronting quantum gravity with observations, *Classical Quantum Gravity* **30**, 085014 (2013).
- [62] I. Agullo, A. Ashtekar, and W. Nelson, Extension of the quantum theory of cosmological perturbations to the Planck era, *Phys. Rev. D* **87**, 043507 (2013).
- [63] P. Friedrich and T. Prokopec, Scalar field dark matter in hybrid approach, *Phys. Rev. D* **96**, 083504 (2017).
- [64] B. Bolliet, J. Grain, C. Stahl, L. Linsefors, and A. Barrau, Comparison of primordial tensor power spectra from the deformed algebra and dressed metric approaches in loop quantum cosmology, *Phys. Rev. D* **91**, 084035 (2015).
- [65] M. Bojowald, Quantum nature of cosmological bounces, *Gen. Relativ. Gravit.* **40**, 2659 (2008).
- [66] T. Zhu, A. Wang, K. Kirsten, G. Cleaver, and Q. Sheng, Universal features of quantum bounce in loop quantum cosmology, *Phys. Lett. B* **773**, 196 (2017).
- [67] B. Elizaga Navascués, D. Martin de Blas, and G. A. Mena Marugán, Time-dependent mass of cosmological perturbations in the hybrid and dressed metric approaches to loop quantum cosmology, *Phys. Rev. D* **97**, 043523 (2018).
- [68] Q. Wu, T. Zhu, and A. Wang, Nonadiabatic evolution of primordial perturbations and non-Gaussianity in hybrid approach of loop quantum cosmology, *Phys. Rev. D* **98**, 103528 (2018).
- [69] T. Cailleteau, A. Barrau, J. Grain, and F. Vidotto, Consistency of holonomy-corrected scalar, vector and tensor perturbations in loop quantum cosmology, *Phys. Rev. D* **86**, 087301 (2012).
- [70] L. Linsefors, T. Cailleteau, A. Barrau, and J. Grain, Primordial tensor power spectrum in holonomy corrected Ω loop quantum cosmology, *Phys. Rev. D* **87**, 107503 (2013).
- [71] T. S. Bunch, Adiabatic regularization for scalar fields with arbitrary coupling to the scalar curvature, *J. Phys. A* **13**, 1297 (1980).
- [72] T. S. Bunch and P. C. W. Davies, Quantum field theory in de Sitter space: Renormalization by point splitting, *Proc. R. Soc. A* **360**, 117 (1978).
- [73] A. Ashtekar and D. Sloan, Loop quantum cosmology and slow roll inflation, *Phys. Lett. B* **694**, 108 (2010).
- [74] D. M. de Blas and J. Olmedo, Primordial power spectra for scalar perturbations in loop quantum cosmology, *J. Cosmol. Astropart. Phys.* **06** (2016) 029.
- [75] B. Elizaga Navascués, G. A. M. Marugán, and S. Prado, Non-oscillating power spectra in loop quantum cosmology, *Classical Quantum Gravity* **38**, 035001 (2020).
- [76] E. A. Calzetta and B. L. B. Hu, *Nonequilibrium Quantum Field Theory* (Cambridge University Press, Cambridge, England, 2008).
- [77] A. Barrau and B. Bolliet, Some conceptual issues in loop quantum cosmology, *Int. J. Mod. Phys. D* **25**, 1642008 (2016).
- [78] B. Bolliet, A. Barrau, J. Grain, and S. Schander, Observational exclusion of a consistent loop quantum cosmology scenario, *Phys. Rev. D* **93**, 124011 (2016).
- [79] J. Grain, The perturbed universe in the deformed algebra approach of loop quantum cosmology, *Int. J. Mod. Phys. D* **25**, 1642003 (2016).
- [80] T. Zhu, A. Wang, G. Cleaver, K. Kirsten, and Q. Sheng, Gravitational quantum effects on power spectra and spectral indices with higher-order corrections, *Phys. Rev. D* **90**, 063503 (2014).
- [81] T. Zhu, A. Wang, K. Kirsten, G. Cleaver, and Q. Sheng, High-order primordial perturbations with quantum gravitational effects, *Phys. Rev. D* **93**, 123525 (2016).
- [82] Y. Tavakoli, Cosmological particle production in quantum gravity, *Universe* **7**, 258 (2021).
- [83] M. Bastero-Gil, A. Berera, R. Brandenberger, I. G. Moss, R. O. Ramos, and J. G. Rosa, The role of fluctuation-dissipation dynamics in setting initial conditions for inflation, *J. Cosmol. Astropart. Phys.* **01** (2018) 002.