

Feasibility of searching for the Cabibbo-favored $D^* \rightarrow \bar{K}\pi^+, \bar{K}^*\pi^+, \bar{K}\rho^+$ decays

Yueling Yang^{1,*}, Kang Li,¹ Zhenglin Li,¹ Jinshu Huang^{2,†}, Qin Chang,¹ and Junfeng Sun^{1,‡}

¹*Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang 453007, China*

²*School of Physics and Electronic Engineering, Nanyang Normal University, Nanyang 473061, China*



(Received 20 July 2022; accepted 19 August 2022; published 31 August 2022)

The current knowledge on the D^* mesons is still inadequate. Encouraged by the positive development prospects of high-luminosity and high-precision experiments, the Cabibbo-favored nonleptonic $D^* \rightarrow \bar{K}\pi^+, \bar{K}^*\pi^+, \bar{K}\rho^+$ weak decays are studied with the naive factorization approach. It is found that branching ratios of these processes can reach up to $\mathcal{O}(10^{-10})$ or more, and can be accessible at STCF, CEPC, FCC-ee, and LHCb@HL-LHC experiments in the future. It might even be possible to search for the $D^{*0} \rightarrow K^{*-}\pi^+$ and $K^-\rho^+$ decays at the running SuperKEKB experiments.

DOI: [10.1103/PhysRevD.106.036029](https://doi.org/10.1103/PhysRevD.106.036029)

I. INTRODUCTION

The need for a charm quark, as its natural incorporation in a unified description of the weak and electromagnetic interactions, was first postulated by Bjorken, Glashow, Iliopoulos, and Maiani [1,2]. The charm flavor is strictly conserved in electromagnetic and strong interactions. Although the J/ψ particle was first discovered by two vastly different experiments in 1974 [3,4], and interpreted as a bound system of the $c\bar{c}$ pair, the charm flavor is hidden away in the quantity R near the narrow resonance J/ψ peak. The observable quantity R measures the total hadron production rate relative to the electrodynamic $\mu^+\mu^-$ process in e^+e^- collisions, $R \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, determines the properties of the species of quark carrying color and fractional electric charge. The value of R with the lowest order approximation should remain approximately constant with center-of-mass energy of the e^+e^- pair annihilation, as long as no new final state fermion pair appears. From the view of R , the most striking evidence for the existence of a charm quark is the discovery of charmed mesons in positron-electron annihilation at SPEAR experiments in 1976 [5,6]. The clearly evident increase in R near the 4 GeV region in Fig. 1 has been widely taken by particle physicists to be the characteristic features of charm threshold.

The ordinary charmed mesons consist of the charmed quark plus a light nonstrange quark forming an isodoublet, $c\bar{u}$ and $c\bar{d}$, symbolized respectively by D^0 and D^+ corresponding to the 1^1S_0 state with spin-parity quantum $J^P = 0^-$ for ground pseudoscalar mesons, D^{*0} and D^{*+} corresponding to the 1^3S_1 state with spin-parity quantum $J^P = 1^-$ for ground vector mesons. In electron-positron collisions, the charmed quark pair originated from the virtual timelike photon combines with the light u and d quarks out of the vacuum with the orbit-spin $L-S$ coupling and forms the charmed mesons through strong interactions. In principle, the charmed meson pairs are equally likely to be neutral and charged. It is clearly seen from Fig. 1 that there are many broader resonances above charm threshold, such as $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and so on. These broader sibling ψ resonances presumably decay strongly to pairs of oppositely charmed mesons, $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$. The relative production cross sections of charmed mesons close to threshold were once thought to be, for example, $\sigma(D\bar{D}) : \sigma(D\bar{D}^*) : \sigma(D^*\bar{D}^*) \simeq 1 : 4 : 7$ [9–12] or other ratios [13–16] based on different theoretical calculation. The measured production ratio may differ considerably from those theoretical estimates. At energies a little far above the $D^*\bar{D}^*$ threshold, it is generally believed that the production cross sections of D^* mesons are greater than those of D mesons in e^+e^- collisions, which is clearly seen from Fig. 2.

The center-of-mass energy for $D^*\bar{D}^*$ production is larger than that for $D\bar{D}$ production. Due to the engineering design of e^+e^- accelerators and detectors, plus the actual operation energy and time, more experimental data on the pseudoscalar D mesons rather than the vector D^* mesons have been accumulated at CLEC-c and BES experiments. The properties of D mesons have been extensively and carefully studied by many experimental groups. To go

*yangyueling@htu.edu.cn

†jshuang@foxmail.com

‡sunjunfeng@htu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

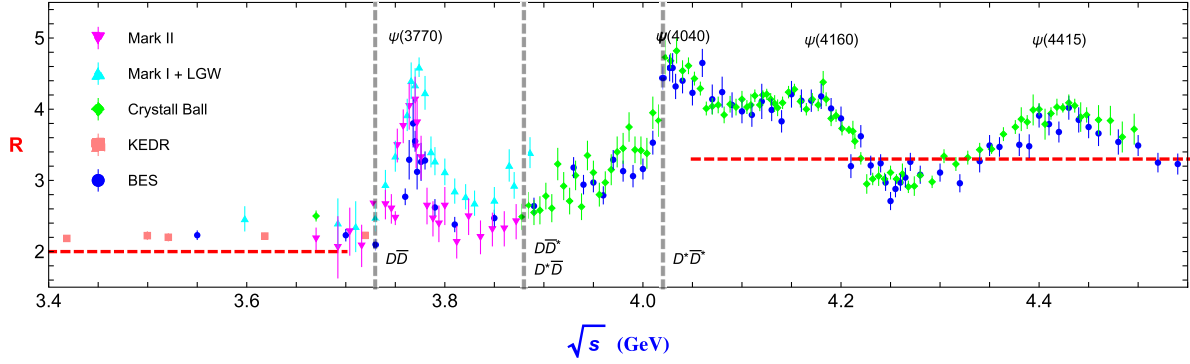


FIG. 1. R near the charm threshold regions versus the center-of-mass energy $\sqrt{s}$ of e^+e^- annihilation, corresponding to Fig. 53.3 of Ref. [7], where the experimental data are available [8]; the lower and upper horizontal dashed lines correspond to $R = 2$ and $10/3$, respectively; and the vertical dashed lines denote charm thresholds.

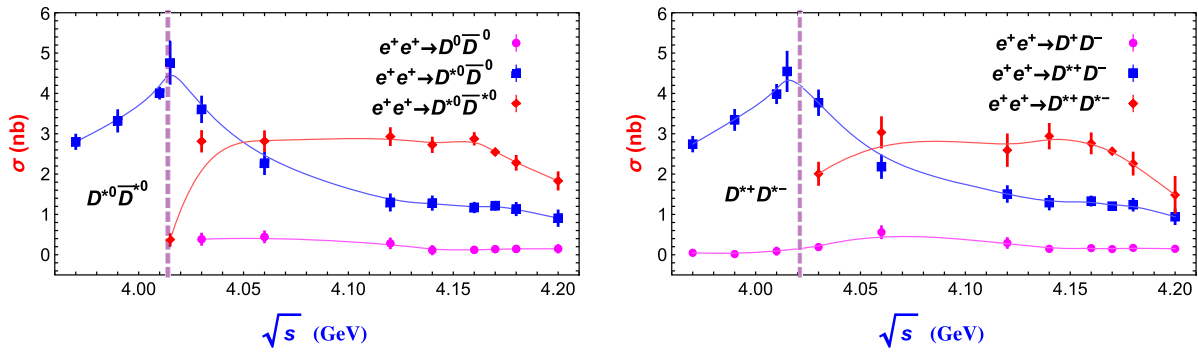


FIG. 2. Measured production cross sections for charmed meson pairs near the threshold regions versus the center-of-mass energy $\sqrt{s}$ of e^+e^- annihilation, where the experimental data are available from the CLEO-c group in Refs. [17,18]; the vertical dashed lines denote charm thresholds.

along with this, the experimental information on D^* mesons is still very limited by now [7]. The measurement of the mass of D^* mesons which was quoted but not adopted by Particle Data Group was carried out in 1977 [19,20], not updated for 45 years. In addition, only three decay modes of D^{*0} (D^{*+}) mesons have been observed [7].

The fitted D^* masses are $m_{D^{*0}} = 2006.85(5)$ MeV and $m_{D^{*+}} = 2010.26(5)$ MeV [7]. There is a hierarchical relationship among hadron mass,

$$m_{D^{*0}} - m_{D^0} > m_{\pi^0}, \quad (1)$$

$$m_{D^{*0}} - m_{D^+} < m_{\pi^-}, \quad (2)$$

$$m_{D^{*+}} - m_{D^0} > m_{\pi^+}, \quad (3)$$

$$m_{D^{*+}} - m_{D^+} > m_{\pi^0}, \quad (4)$$

which results in the $D^{*0} \rightarrow D^+ \pi^-$ decay being kinematically forbidden. The D^* mesons can decay mainly through the strong and electromagnetic interactions, and are dominated by $D^* \rightarrow D\pi$ processes. Kinematically, the $D^* \rightarrow D\pi$ decays are inhibited by very compact phase space. Dynamically, for the $D^* \rightarrow D\pi$ decays, the π emission

processes are suppressed by the phenomenological Okubo-Zweig-Iizuka rule [21–23], and the recoiled π processes are suppressed by a color match factor owing to the color singlet requirements of quark combinations. Consequently, the decay width of D^* mesons is very narrow, for example, $\Gamma_{D^{*+}} = 83.4 \pm 1.8$ keV [7]. Experimentally, the momentum of the pion in the rest frame of D^* mesons is very soft, about 40 MeV. The difficulties in signal reconstruction result in inefficiency of particle identification.

Besides, the D^* meson decay through weak interactions is also legal and allowable within the standard model (SM) of elementary particle. As it is well known, the strong and electromagnetic interactions are related only to vector currents, while the weak interactions are related to both vector and axial vector currents. Study of the D^* meson weak decays can not only improve our knowledge of the properties D^* mesons, but also test the axial vector current interactions in SM and enrich our understanding on the decay mechanism of D^* mesons. In addition, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [24,25] describing the quark mixing can be determined and overconstrained from D^* meson weak decays.

It is not hard to visualize that the occurrence probability of D^* meson weak decays is very small, insignificant when compared with that of strong and electromagnetic decays. Fortunately, the experimental data on D^* mesons are accumulating increasingly. Now, more than 5×10^7 $D^{*\pm}$ mesons have been accumulated in energy regions $\sqrt{s} \in [4.1, 4.6]$ at BESIII experiments [26]. It is promisingly expected that there will be a total of about 5×10^{10} $c\bar{c}$ pairs at the SuperKEKB [27]. Given the charm quark fragmentation fractions $f(c \rightarrow D^{*+}) \approx 25\%$ and $f(c \rightarrow D^{*0}) \approx 23\%$ [28], more than 2×10^{10} D^{*0} and $D^{*\pm}$ will be available at SuperKEKB. If it is considered optimistically that the production cross sections $\sigma(e^+e^- \rightarrow D^0\bar{D}^{*0}) \approx \sigma(e^+e^- \rightarrow D^+D^{*-}) \approx 2$ nb and $\sigma(e^+e^- \rightarrow D^{*0}\bar{D}^{*0}) \approx \sigma(e^+e^- \rightarrow D^{*+}D^{*-}) \approx 3$ nb near $\psi(4040)$ resonance, as illustrated in Fig. 2 (or Tables 1 and 2 of Ref. [26]), about 8×10^{10} D^{*0} and $D^{*\pm}$ mesons will be available at the super τ -charm factory (STCF) [29,30] with a total integrated luminosity of 10 ab^{-1} . It is highly expected that there will be a total of about 10^{12} Z^0 bosons at the Circular Electron Positron Collider (CEPC) [31] and 10^{13} Z^0 bosons at the Future Circular Collider (FCC-ee) [32]. Considering the inclusive branching ratios $\mathcal{B}(Z^0 \rightarrow D^{*\pm}X) = (11.4 \pm 1.3)\%$ [7] and the approximation $\mathcal{B}(Z^0 \rightarrow D^{*0}X/\bar{D}^{*0}X) \approx \mathcal{B}(Z^0 \rightarrow D^{*\pm}X)$, more than 10^{11} and 10^{12} D^* mesons will be available at CEPC and FCC-ee with a total integrated luminosity of 20 ab^{-1} , respectively. In addition, the inclusive production cross sections in hadron-hadron collisions are measured to be $\sigma(pp \rightarrow D^{*+}X) \approx 0.8$ mb at center-of-mass energy of $\sqrt{s} = 13$ TeV by the LHCb group [33] and $\sigma_{\text{tot}}(D^{*+}) \approx 2.1$ mb at $\sqrt{s} = 7$ TeV by the ALICE group [34], respectively. A conservative estimate is that more than 2×10^{14} D^* mesons will be available by the LHCb detector at the future high luminosity LHC (HL-LHC) hadron collider with a total integrated luminosity of 300 fb^{-1} [35]. A huge amount of experimental data provides a solid foundation and valuable opportunities for studying and understanding the properties of D^* mesons, including the search for D^* meson weak decays.

It is natural to want to know the probability of D^* meson weak decays, and whether the study on the D^* meson weak decays is feasible or not in the coming future. Our study [36] has tentatively shown that the purely leptonic D^{*+} decays might be measurable at e^+e^- colliders, such as SuperKEKB, STCF, CEPC and FCC-ee, and the HL-LHC hadron collider [35]. Purely leptonic D^{*+} decays are Cabibbo suppressed, while leptonic D^{*0} decays are induced by the flavor-changing neutral weak currents. In this paper, we would like to investigate the $D^* \rightarrow \bar{K}\pi^+$, $\bar{K}\rho^+$ and $\bar{K}^*\pi^+$ decays within the SM. The motivation is as follows. Dynamically, these decays are induced by W boson emission, and favored by CKM elements, and thus should have relatively larger branching ratios and higher priority to do research among D^* meson weak decays. Kinematically,

the final states are back to back, and have definitive momentum and discrete energy in the rest frame of D^* mesons. Experimentally, the energetic charged pion and kaon of final states of nonleptonic D^* weak decays are more easily captured by the sensitive detectors, when compared with $D^* \rightarrow D\pi$ decays where the soft pion identifications are very challenging, and when compared with leptonic D^{*+} decays where the final neutrinos will bring the signal reconstruction into additional complications. Hence, there would be a relatively higher reconstruction efficiency for nonleptonic D^* weak decays.

The remainder of this paper is organized as follows. Theoretical framework and phenomenological approach dealing with nonleptonic D^* weak decays are presented in Sec. II. Numerical results of branching ratios and our comments are given in Sec. III. Section IV is a brief summary. The analytic expressions of decay amplitudes are collected in Appendices A and B.

II. THEORETICAL FRAMEWORK

The effective Hamiltonian in charge of nonleptonic $D^* \rightarrow \bar{K}\pi^+$, $\bar{K}\rho^+$, and $\bar{K}^*\pi^+$ decays is written as [37]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \{C_1 O_1 + C_2 O_2\} + \text{H.c.}, \quad (5)$$

where both Fermi constant $G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [7] and Wilson coefficients $C_{1,2}$ are universal effective couplings. Wilson coefficients are computable with the perturbation theory at the mass scale of W boson. The scale dependence of Wilson coefficients can be obtained with the renormalization group equation [37]. The nature of the product of $G_F C_i$ is similar to a gauge parameter, for example, the electric charge e for electromagnetic interactions. The CKM elements correspond to different effective operators, and have been determined precisely, $|V_{cs}| = 0.987(11)$, and $|V_{ud}| = 0.97370(14)$ [7]. The explicit expressions of effective four-quark operators are written as

$$O_1 = [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta], \quad (6)$$

$$O_2 = [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\alpha], \quad (7)$$

where α and β are the color indices. Owing to only one kaon meson in final states, there is no penguin operators.

Taking the $D^{*0} \rightarrow K^-\pi^+$ decay for example, the amplitude can be written as

$$\begin{aligned} \mathcal{A}(D^{*0} \rightarrow K^-\pi^+) &= \langle K^-\pi^+ | \mathcal{H}_{\text{eff}} | D^{*0} \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \sum_{i=1}^2 C_i \langle K^-\pi^+ | O_i | D^{*0} \rangle. \end{aligned} \quad (8)$$

Clearly, the remaining part of decay amplitudes is to reasonably evaluate the hadronic matrix elements (HMEs) $\langle K^-\pi^+ | O_i | D^{*0} \rangle$, which sandwich the initial and final states together with quark operators. HMEs closely relate to transitions between quarks and hadron states, and involve short- and long-distance contributions which complicate theoretical calculations.

Phenomenologically, the naive factorization (NF) approach [38] was often used to deal with HMEs, owing to its simple and clear physics picture, and good performance for nonleptonic B and D weak decays induced by external W emission processes. Based on the color transparency hypothesis [39] that energetic final hadron states have flown far away from each other before influence of soft gluons, HMEs of four-quark operators using NF approach are divided into two HMEs of hadron currents, and the final state interactions can be dismissed for the time being. HMEs of hadron currents are conventionally parametrized by decay constants and hadron transition form factors, which can be obtained from data and nonperturbative methods. In addition, it has been shown in Ref. [40] that although the contributions from W -exchange and W -annihilation diagrams are the same order

as those from tree-emission diagrams due to the large nonperturbative contributions for charmed meson decays, the tree-emission diagrams always give the largest contributions in the total amplitudes, thereby give the correct orders of magnitude of branching fractions. In this paper, we will apply NF approach to the concerned nonleptonic D^* weak decays.

The parametrization schemes of hadron current HMEs are generally written as [41–43]

$$\langle 0 | \bar{q}_1 \gamma_\mu q_2 | P(k) \rangle = 0, \quad (9)$$

$$\langle 0 | \bar{q}_1 \gamma_\mu \gamma_5 q_2 | P(k) \rangle = i f_P k_\mu, \quad (10)$$

$$\langle 0 | \bar{q}_1 \gamma_\mu q_2 | V(k, \epsilon) \rangle = f_V m_V \epsilon_\mu, \quad (11)$$

$$\langle 0 | \bar{q}_1 \gamma_\mu \gamma_5 q_2 | V(k, \epsilon) \rangle = 0, \quad (12)$$

$$\langle P(p_2) | \bar{q} \gamma_\mu c | D^*(\epsilon_1, p_1) \rangle = \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\nu P^\alpha q^\beta \frac{V^{D^*P}(q^2)}{m_{D^*} + m_P}, \quad (13)$$

$$\begin{aligned} \langle P(p_2) | \bar{q} \gamma_\mu \gamma_5 c | D^*(\epsilon_1, p_1) \rangle &= +i2m_{D^*} \frac{\epsilon_1 \cdot q}{q^2} q_\mu A_0^{D^*P}(q^2) + i\epsilon_{1,\mu} (m_{D^*} + m_P) A_1^{D^*P}(q^2) \\ &\quad + i \frac{\epsilon_1 \cdot q}{m_{D^*} + m_P} P_\mu A_2^{D^*P}(q^2) - i2m_{D^*} \frac{\epsilon_1 \cdot q}{q^2} q_\mu A_3^{D^*P}(q^2), \end{aligned} \quad (14)$$

$$\begin{aligned} \langle V(\epsilon_2, p_2) | \bar{q} \gamma_\mu c | D^*(\epsilon_1, p_1) \rangle &= -(\epsilon_1 \cdot \epsilon_2^*) \{ P_\mu V_1^{D^*V}(q^2) - q_\mu V_2^{D^*V}(q^2) \} \\ &\quad + \frac{(\epsilon_1 \cdot q)(\epsilon_2^* \cdot q)}{m_{D^*}^2 - m_V^2} \left\{ \left[P_\mu - \frac{m_{D^*}^2 - m_V^2}{q^2} q_\mu \right] V_3^{D^*V}(q^2) + \frac{m_{D^*}^2 - m_V^2}{q^2} q_\mu V_4^{D^*V}(q^2) \right\} \\ &\quad - (\epsilon_1 \cdot q) \epsilon_{2,\mu}^* V_5^{D^*V}(q^2) + (\epsilon_2^* \cdot q) \epsilon_{1,\mu} V_6^{D^*V}(q^2), \end{aligned} \quad (15)$$

$$\begin{aligned} \langle V(\epsilon_2, p_2) | \bar{q} \gamma_\mu \gamma_5 c | D^*(\epsilon_1, p_1) \rangle &= -i\epsilon_{\mu\nu\alpha\beta} \epsilon_1^\alpha \epsilon_2^{*\beta} \left\{ \left[P^\nu - \frac{m_{D^*}^2 - m_V^2}{q^2} q^\nu \right] A_1^{D^*V}(q^2) + \frac{m_{D^*}^2 - m_V^2}{q^2} q^\nu A_2^{D^*V}(q^2) \right\} \\ &\quad - \frac{i\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{m_{D^*}^2 - m_V^2} \{ (\epsilon_2^* \cdot q) \epsilon_1^\nu A_3^{D^*V}(q^2) - (\epsilon_1 \cdot q) \epsilon_2^{*\nu} A_4^{D^*V}(q^2) \}, \end{aligned} \quad (16)$$

where m_P and f_P are the mass and decay constant of pseudoscalar P meson, respectively; m_V , f_V , and ϵ_V are the mass, decay constant, and polarization vector of vector V meson, respectively; $A_i^{D^*h}$ and $V_i^{D^*h}$ are mesonic transition form factors; the momentum $P = p_1 + p_2$ and $q = p_1 - p_2$. There are some relationships among form factors:

$$A_0^{D^*P}(0) = A_3^{D^*P}(0), \quad (18)$$

$$A_1^{D^*V}(0) = A_2^{D^*V}(0), \quad (19)$$

$$V_3^{D^*V}(0) = V_4^{D^*V}(0). \quad (20)$$

$$\begin{aligned} (m_{D^*} + m_P) A_1^{D^*P}(q^2) + (m_{D^*} - m_P) A_2^{D^*P}(q^2) \\ = 2m_{D^*} A_3^{D^*P}(q^2), \end{aligned} \quad (17)$$

The numerical values of form factors have been comprehensively calculated with the light front quark model in Ref. [43], and are listed in Table I.

TABLE I. Numerical values of decay constants [7,36] and form factors at the pole $q^2 = 0$ [43].

$f_\pi = 130.2 \pm 1.2$ MeV	$f_K = 155.7 \pm 0.3$ MeV	$f_\rho = 207.7 \pm 1.6$ MeV	$f_{K^*} = 202.5_{-6.7}^{+6.5}$ MeV
$V_1^{D^*\rho} = 0.65$	$V_2^{D^*\rho} = 0.51$	$V_3^{D^*\rho} = 0.29$	$V_5^{D^*\rho} = 1.42$
$V_6^{D^*\rho} = 0.68$	$A_1^{D^*\rho} = 0.59$	$A_3^{D^*\rho} = 0.22$	$A_4^{D^*\rho} = 0.24$
$V_1^{D^*K^*} = 0.74$	$V_2^{D^*K^*} = 0.43$	$V_3^{D^*K^*} = 0.26$	$V_5^{D^*K^*} = 1.50$
$V_6^{D^*K^*} = 0.78$	$A_1^{D^*K^*} = 0.69$	$A_3^{D^*K^*} = 0.15$	$A_4^{D^*K^*} = 0.22$
$V^{D^*\pi} = 0.92$	$A_0^{D^*\pi} = 0.68$	$A_1^{D^*\pi} = 0.74$	$A_2^{D^*\pi} = 0.61$
$V^{D^*K} = 1.04$	$A_0^{D^*K} = 0.78$	$A_1^{D^*K} = 0.85$	$A_2^{D^*K} = 0.68$

III. BRANCHING RATIO

The branching ratio of two-body nonleptonic D^* decays is defined as

$$\mathcal{B}r = \frac{G_F^2}{48\pi} |V_{cs}|^2 |V_{ud}|^2 \frac{p_{\text{cm}}}{m_{D^*}^2 \Gamma_{D^*}} \sum_{s_i} \mathcal{M} \mathcal{M}^\dagger, \quad (21)$$

where p_{cm} is the center-of-mass momentum of final states in the rest frame of the D^* meson, and Γ_{D^*} is the decay width of the D^* meson. $\mathcal{M}$ denotes the decay amplitude. For $D^* \rightarrow \bar{K}\pi^+$ decays, there is only the p -wave amplitude. For $D^* \rightarrow \bar{K}^*\pi^+$ and $\bar{K}^*\rho^+$ decays, there are s -, p - and d -wave amplitudes. The analytic expressions of decay amplitudes with NF approach are collected in Appendices A and B.

The decay width of the D^{*+} meson has been experimentally determined to be $\Gamma_{D^{*+}} = 83.4 \pm 1.8$ keV [7], while there only was an upper limit for the decay width of the D^{*0} meson, $\Gamma_{D^{*0}} < 2.1$ MeV [7]. It is widely assumed that there should be a relation between decay widths for the p -wave strong decay $D^* \rightarrow D\pi^0$ [44–46],

$$\frac{\Gamma(D^{*0} \rightarrow D^0\pi^0)}{\Gamma(D^{*+} \rightarrow D^+\pi^0)} = \frac{p_{\text{cm}D^0\pi^0}^3 m_{D^{*+}}^2}{p_{\text{cm}D^+\pi^0}^3 m_{D^{*0}}^2}. \quad (22)$$

So it is expected that the decay width of D^{*0} ,

$$\begin{aligned} \Gamma_{D^{*0}} &= \Gamma_{D^{*+}} \frac{\mathcal{B}r(D^{*+} \rightarrow D^+\pi^0)}{\mathcal{B}r(D^{*0} \rightarrow D^0\pi^0)} \frac{p_{\text{cm}D^0\pi^0}^3 m_{D^{*+}}^2}{p_{\text{cm}D^+\pi^0}^3 m_{D^{*0}}^2} \\ &= 55.9_{-5.4}^{+5.9} \text{ keV}, \end{aligned} \quad (23)$$

where the comprehensive uncertainties are conservative estimates, and come from data error of $\Gamma_{D^{*+}}$, branching ratios and mesonic mass. The full decay width $\Gamma_{D^{*0}}$ in Eq. (23) is well consistent with theoretical prediction of Ref. [46], and will be used in our calculation to estimate branching ratios for D^{*0} decays.

Our numerical calculation shows that branching ratios for nonleptonic D^* decays are

$$\mathcal{B}r(D^{*+} \rightarrow \bar{K}^0\pi^+) \approx 1.6 \times 10^{-10}, \quad (24)$$

$$\mathcal{B}r(D^{*+} \rightarrow \bar{K}^{*0}\pi^+) \approx 4.4 \times 10^{-10}, \quad (25)$$

$$\mathcal{B}r(D^{*+} \rightarrow \bar{K}^0\rho^+) \approx 8.3 \times 10^{-10}, \quad (26)$$

$$\mathcal{B}r(D^{*0} \rightarrow K^-\pi^+) \approx 7.3 \times 10^{-10}, \quad (27)$$

$$\mathcal{B}r(D^{*0} \rightarrow K^{*-}\pi^+) \approx 2.0 \times 10^{-9}, \quad (28)$$

$$\mathcal{B}r(D^{*0} \rightarrow K^-\rho^+) \approx 2.9 \times 10^{-9}. \quad (29)$$

Our comments on branching ratios are as follows.

- (1) The light valence quarks of the D^{*0} and D^{*+} meson are usually call spectator quarks, and they do not take part in the weak interaction directly, when W annihilation and W exchange contributions are not considered with the NF approach. The partial width for D^{*0} and D^{*+} meson decays into the similar isospin final states should in principle be approximately equal. In fact, D^{*+} meson decays are dynamically induced by both external and internal W emission interactions, and the interference between these two contributions is destructive owing to a combination of Wilson coefficients. This is the main reason for the hierarchical relationship, $\mathcal{B}r(D^{*+} \rightarrow \bar{K}^0\pi^+) < \mathcal{B}r(D^{*0} \rightarrow K^-\pi^+)$, $\mathcal{B}r(D^{*+} \rightarrow \bar{K}^{*0}\pi^+) < \mathcal{B}r(D^{*0} \rightarrow K^{*-}\pi^+)$, and $\mathcal{B}r(D^{*+} \rightarrow \bar{K}^0\rho^+) < \mathcal{B}r(D^{*0} \rightarrow K^-\rho^+)$. Similar hierarchical phenomena have also been observed experimentally in the Cabibbo-favored pseudoscalar D^0 and D^+ meson decays, for example, $\mathcal{B}r(D^+ \rightarrow K_S^0\pi^+) = 1.562(31)\%$ and $\mathcal{B}r(D^0 \rightarrow K^-\pi^+) = 3.950(31)\%$ [7].
- (2) There are three partial wave amplitudes for $D^* \rightarrow PV$ decays, while only the p -wave amplitude contributes to $D^* \rightarrow PP$ decays. Hence, there is a hierarchical relationship among branching ratios, $\mathcal{B}r(D^{*+} \rightarrow PP) < \mathcal{B}r(D^{*+} \rightarrow PV)$ and $\mathcal{B}r(D^{*0} \rightarrow PP) < \mathcal{B}r(D^{*0} \rightarrow PV)$. In addition, because of decay constants $f_\rho > f_\pi$, branching ratio for emission ρ^+ is generally larger than that for emission π^+ , for either D^{*0} or D^{*+} decays. Similar phenomena have also been seen in pseudoscalar D meson decays, for example, $\mathcal{B}r(D^0 \rightarrow K^-\rho^+) = (11.3 \pm 0.77)\%$ and

TABLE II. The potential event numbers of $D^* \rightarrow \bar{K}\pi^+$, $\bar{K}^*\pi^+$, $\bar{K}\rho^+$ decays at experiments, where the D^* meson data available has been estimated in Sec. I.

Experiment	SuperKEKB	STCF	CEPC	FCC-ee	LHCb@HL-LHC
N_{D^*}	2×10^{10}	8×10^{10}	10^{11}	10^{12}	2×10^{14}
$N_{D^{*+} \rightarrow \bar{K}^0 \pi^+}$	3	13	16	160	3.2×10^4
$N_{D^{*+} \rightarrow \bar{K}^{*0} \pi^+}$	9	35	44	440	8.8×10^4
$N_{D^{*+} \rightarrow \bar{K}^0 \rho^+}$	17	66	83	830	1.66×10^5
$N_{D^{*0} \rightarrow K^- \pi^+}$	14	58	72	720	1.44×10^5
$N_{D^{*0} \rightarrow K^{*-} \pi^+}$	40	160	200	2000	4.0×10^5
$N_{D^{*0} \rightarrow K^- \rho^+}$	57	230	287	2870	5.74×10^5
$N_{D^{*0} \rightarrow K^- \pi^+ \pi^0}$	100	400	500	5000	1.0×10^6

$\mathcal{B}r(D^0 \rightarrow K^{*-} \pi^+) = (2.31_{-0.20}^{+0.40})\%$ determined from $D^0 \rightarrow K^- \pi^+ \pi^0$ decays [7].

- (3) As it is well known the vector ρ and K^* mesons are resonances, and they will decay promptly into pseudoscalar mesons via the strong interactions, with branching ratios $\mathcal{B}r(\rho \rightarrow \pi\pi) \sim 100\%$ and $\mathcal{B}r(K^* \rightarrow K\pi) \sim 100\%$ [7]. The vector ρ (K^*) meson in $D^{*0} \rightarrow K^- \rho^+$ ($K^{*-} \pi^+$) decay should be reconstructed by the final pseudoscalar mesons. Besides, both $D^{*0} \rightarrow K^- \rho^+$ and $D^{*0} \rightarrow K^{*-} \pi^+$ decays contribute to $D^{*0} \rightarrow K^- \pi^+ \pi^0$. The branching ratio of the three-body $D^{*0} \rightarrow K^- \pi^+ \pi^0$ decay can be approximately the sum of the branching ratios of these two two-body $D^{*0} \rightarrow PV$ modes whose interference effect happens in a small region of the Dalitz plot. A similar case can be seen by the comparison between the branching ratio $\mathcal{B}r(D^0 \rightarrow K^- \pi^+ \pi^0) = (14.4 \pm 0.5)\%$ and the sum of partial branching ratios $\mathcal{B}r(D^0 \rightarrow K^- \rho^+) = (11.3 \pm 0.7)\%$ and $\mathcal{B}r(D^0 \rightarrow K^{*-} \pi^+) = (2.31_{-0.20}^{+0.40})\%$ [7]. Therefore, it may be an educated guess that the three-body $D^{*0} \rightarrow K^- \pi^+ \pi^0$ decay will have a relatively larger branching ratio, $\mathcal{B}r(D^{*0} \rightarrow K^- \pi^+ \pi^0) \sim 5.0 \times 10^{-9}$, and can be more easily investigated in experiments, compared to the two-body $D^{*0} \rightarrow K^- \rho^+$ and $K^{*-} \pi^+$ decays.
- (4) Theoretical predictions on branching ratios are easily influenced by a number of factors, including final state interactions and other contributions. For example, studies [40,47] using the flavor topology diagrammatic approach have shown that for Cabibbo-favored D meson decays, the W exchange and annihilation contributions should deserve due attention, although the external W emission contributions always give the largest contributions in the total amplitudes. We would like to point out that what we want to know is whether it is feasible to investigate the nonleptonic D^* weak decays at the future experiments, so the magnitude order estimation rather than precise calculation on branching ratio

may be enough. It is generally believed that the NF approach can give a reasonable and correct magnitude order estimation on branching ratio for non-leptonic heavy flavored meson decays arising from the external W emission weak interactions. In this sense, the magnitude order of branching ratios in Eqs. (24)–(29) seems to be reliable. The potential event numbers of the concerned $D^* \rightarrow PP$, PV decays are listed in Table II. It is clear that the Cabibbo-favored $D^* \rightarrow \bar{K}\pi^+$, $\bar{K}^*\pi^+$, $\bar{K}\rho^+$ decays can be measurable at the future STCF, CEPC, FCC-ee and LHCb@HL-LHC experiments. The $D^{*+} \rightarrow K^{*-} \pi^+$ and $K^- \rho^+$ decays can also be investigated at SuperKEKB experiments.

IV. SUMMARY

Now, more than 45 years after the discovery of the D^* mesons, our knowledge and understanding of the nature of the D^* mesons is far from enough, and needs to be substantially improved. One of the major reasons that excessively hindered experimental investigation on D^* mesons is that data are too scarce. We should thank the high-luminosity particle physics experiments for offering us a huge amount of D^* meson data and a tempting opportunity to explore the wanted D^* meson in the future. Compared with the dominant $D^* \rightarrow D\pi$ decays which are subject to kinematical factors, one advantage of nonleptonic D^* weak decays is that the final pion and kaon mesons are energetic and easily detectable by the sensitive high-resolution detectors. In addition, study of nonleptonic D^* weak decays is scientifically significant, and provide us with a new venue for testing SM. In this paper, the Cabibbo-favored two-body nonleptonic $D^* \rightarrow PP$, PV decays were studied by using the NF approach within SM. It is found that branching ratios for $D^* \rightarrow \bar{K}\pi^+$, $\bar{K}^*\pi^+$, $\bar{K}\rho^+$ decays can reach up to $\mathcal{O}(10^{-10})$ or more, and can be accessible at STCF, CEPC, FCC-ee and LHCb@HL-LHC experiments, which indicate that study of these weak interaction processes is experimentally feasible and practicable in the future.

ACKNOWLEDGMENTS

The work is supported by the National Natural Science Foundation of China (Grants No. 11705047, No. U1632109, and No. 11875122) and Natural Science Foundation of Henan Province (Grant No. 222300420479), and the Excellent Youth Foundation of Henan Province (Grant No. 212300410010). We would like to acknowledge the useful help and valuable discussion from Professor Haibo Li (IHEP@CAS), Professor Shuangshi Fang (IHEP@CAS), Professor Frank Porter (Caltech), Professor Antimo Palano (INFN), Professor Chengping Shen (Fudan University), Dr. Ping Xiao (Fudan University), Dr. Qingping Ji (Henan Normal University), Dr. Huijing Li (Henan Normal University), and Ms. Liting Wang (Henan Normal University), and positive comments and constructive suggestions from referees.

APPENDIX A: AMPLITUDES FOR $V \rightarrow P_1 + P_2$ DECAYS

With the conventions of Eqs. (9), (10), (13), and (14), the general expression of HMEs with NF approach for $V \rightarrow P_1 + P_2$ transition can be written as

$$\mathcal{M} = \langle P_2 | V_\mu - A_\mu | 0 \rangle \langle P_1 | V^\mu - A^\mu | V \rangle = \mathcal{M}_p (\epsilon_V \cdot p_{P_2}), \quad (\text{A1})$$

$$\mathcal{M}_p = 2 f_{P_2} m_V A_0^{VP_1}(0), \quad (\text{A2})$$

$$\sum_{s_V} \mathcal{M} \mathcal{M}^\dagger = |\mathcal{M}_p|^2 p_{\text{cm}}^2, \quad (\text{A3})$$

$$p_{\text{cm}} = \lambda^{1/2}(m_V^2, m_{P_1}^2, m_{P_2}^2)/2m_V, \quad (\text{A4})$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca. \quad (\text{A5})$$

With the NF approach, there is only one amplitude for the neutral D^{*0} meson decay in question, which corresponds to external W emission. There are two amplitudes for the charged $D^{*\pm}$ meson decay in question, which correspond to external and internal W emissions. The partial wave amplitudes for $D^{*0} \rightarrow K^- \pi^+$ decay are written as

$$\mathcal{M}_p = 2m_{D^*} a_1 f_\pi A_0^{D^*K}, \quad (\text{A6})$$

and for $D^{*+} \rightarrow \bar{K}^0 \pi^+$ decay,

$$\mathcal{M}_p = 2m_{D^*} \{a_1 f_\pi A_0^{D^*K} + a_2 f_K A_0^{D^*\pi}\}, \quad (\text{A7})$$

where the coefficients a_1 and a_2 correspond to external and internal W emission, respectively; and they are defined as

$$a_1 = C_1 + C_2/N_c, \quad (\text{A8})$$

$$a_2 = C_2 + C_1/N_c. \quad (\text{A9})$$

In practice, it is generally believed that coefficients $a_{1,2}$ are also influenced by nonfactorizable contributions and final state interactions. In many phenomenological studies on charmed meson weak decays, such as Refs. [38,40,48–54], coefficients $a_1 \approx 1.2$ and $a_2 \approx -0.5$ are often used for charmed meson decays by including comprehensive contributions.

APPENDIX B: AMPLITUDES FOR $V_1 \rightarrow V_2 + P$ DECAYS

With the conventions of Eqs. (9)–(16), the general expression of HMEs with NF approach for $V_1 \rightarrow V_2 + P$ transition can be written as

$$\begin{aligned} \mathcal{M} &= \langle V_2 | V_\mu - A_\mu | 0 \rangle \langle P | V_\mu - A_\mu | V_1 \rangle \\ &= \mathcal{M}_s (\epsilon_{V_1} \cdot \epsilon_{V_2}^*) + \frac{\mathcal{M}_d}{m_{V_1} m_{V_2}} (\epsilon_{V_1} \cdot p_{V_2}) (\epsilon_{V_2}^* \cdot p_{V_1}) \\ &\quad + \frac{\mathcal{M}_p}{m_{V_1} m_{V_2}} \epsilon_{\mu\nu\alpha\beta} \epsilon_{V_1}^\mu \epsilon_{V_2}^{*\nu} p_{V_1}^\alpha p_{V_2}^\beta, \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} \mathcal{M}' &= \langle P | V_\mu - A_\mu | 0 \rangle \langle V_2 | V_\mu - A_\mu | V_1 \rangle \\ &= \mathcal{M}'_s (\epsilon_{V_1} \cdot \epsilon_{V_2}^*) + \frac{\mathcal{M}'_d}{m_{V_1} m_{V_2}} (\epsilon_{V_1} \cdot p_{V_2}) (\epsilon_{V_2}^* \cdot p_{V_1}) \\ &\quad + \frac{\mathcal{M}'_p}{m_{V_1} m_{V_2}} \epsilon_{\mu\nu\alpha\beta} \epsilon_{V_1}^\mu \epsilon_{V_2}^{*\nu} p_{V_1}^\alpha p_{V_2}^\beta, \end{aligned} \quad (\text{B2})$$

$$\mathcal{M}_s = -i f_{V_2} m_{V_2} (m_{V_1} + m_P) A_1^{V_1 P}(0), \quad (\text{B3})$$

$$\mathcal{M}_d = -i f_{V_2} m_{V_1} m_{V_2} \frac{2m_{V_2}}{m_{V_1} + m_P} A_2^{V_1 P}(0), \quad (\text{B4})$$

$$\mathcal{M}_p = -f_{V_2} m_{V_1} m_{V_2} \frac{2m_{V_2}}{m_{V_1} + m_P} V^{V_1 P}(0), \quad (\text{B5})$$

$$\mathcal{M}'_s = -i f_P (m_{V_1}^2 - m_{V_2}^2) V_1^{V_1 V_2}(0), \quad (\text{B6})$$

$$\mathcal{M}'_d = -i f_P m_{V_1} m_{V_2} \{V_4^{V_1 V_2}(0) - V_5^{V_1 V_2}(0) + V_6^{V_1 V_2}(0)\}, \quad (\text{B7})$$

$$\mathcal{M}'_p = -2 f_P m_{V_1} m_{V_2} A_1^{V_1 V_2}(0), \quad (\text{B8})$$

$$\begin{aligned} \sum_{s_{V_1}} \sum_{s_{V_2}} \mathcal{M} \mathcal{M}^\dagger &= |\mathcal{M}_s|^2 (x^2 + 2) + 2\mathcal{R}(\mathcal{M}_s \mathcal{M}_d^*) x (x^2 - 1) \\ &\quad + |\mathcal{M}_d|^2 (x^2 - 1)^2 + 2|\mathcal{M}_p|^2 (x^2 - 1), \end{aligned} \quad (\text{B9})$$

$$\sum_{s_{V_1}} \sum_{s_{V_2}} \mathcal{M}' \mathcal{M}'^\dagger = |\mathcal{M}'_s|^2 (x^2 + 2) + 2\mathcal{R}(\mathcal{M}'_s \mathcal{M}'_d^*) x (x^2 - 1) + |\mathcal{M}'_d|^2 (x^2 - 1)^2 + 2|\mathcal{M}'_p|^2 (x^2 - 1), \quad (\text{B10})$$

$$x = \frac{p_{V_1} \cdot p_{V_2}}{m_{V_1} m_{V_2}} = \frac{m_{V_1}^2 + m_{V_2}^2 - m_P^2}{2m_{V_1} m_{V_2}}. \quad (\text{B11})$$

For $D^{*0} \rightarrow K^{*-} \pi^+$ decay, the partial wave amplitudes are written as

$$\mathcal{M}'_s = -if_\pi a_1 (m_{D^*}^2 - m_{K^*}^2) V_1^{D^* K^*}, \quad (\text{B12})$$

$$\mathcal{M}'_d = -if_\pi a_1 m_{D^*} m_{K^*} \{V_4^{D^* K^*} - V_5^{D^* K^*} + V_6^{D^* K^*}\}, \quad (\text{B13})$$

$$\mathcal{M}'_p = -2f_\pi a_1 m_{D^*} m_{K^*} A_1^{D^* K^*}. \quad (\text{B14})$$

For $D^{*0} \rightarrow K^- \rho^+$ decay, the partial wave amplitudes are written as

$$\mathcal{M}_s = -if_\rho a_1 m_\rho (m_{D^*} + m_K) A_1^{D^* K}, \quad (\text{B15})$$

$$\mathcal{M}_d = -if_\rho a_1 m_{D^*} m_\rho \frac{2m_\rho}{m_{D^*} + m_K} A_2^{D^* K}, \quad (\text{B16})$$

$$\mathcal{M}_p = -f_\rho a_1 m_{D^*} m_\rho \frac{2m_\rho}{m_{D^*} + m_K} V^{D^* K}. \quad (\text{B17})$$

For $D^{*+} \rightarrow \bar{K}^{*0} \pi^+$ decay, each of the partial wave amplitudes can be divided into two parts,

$$\mathcal{M}_i = \mathcal{M}_i^{(1)} + \mathcal{M}_i^{(2)}, \quad \text{for } i = s, p, d. \quad (\text{B18})$$

$$\mathcal{M}_s^{(2)} = -if_{K^*} a_2 m_{K^*} (m_{D^*} + m_\pi) A_1^{D^* \pi}, \quad (\text{B19})$$

$$\mathcal{M}_d^{(2)} = -if_{K^*} a_2 m_{D^*} m_{K^*} \frac{2m_{K^*}}{m_{D^*} + m_\pi} A_2^{D^* \pi}, \quad (\text{B20})$$

$$\mathcal{M}_p^{(2)} = -f_{K^*} a_2 m_{D^*} m_{K^*} \frac{2m_{K^*}}{m_{D^*} + m_\pi} V^{D^* \pi}, \quad (\text{B21})$$

and expressions of $\mathcal{M}_i^{(1)}$ are the same as those of $\mathcal{M}'_{s,p,d}$ for $D^{*0} \rightarrow K^{*-} \pi^+$ decay in Eqs. (B12)–(B14).

For $D^{*+} \rightarrow \bar{K}^{*0} \rho^+$ decay, each of the partial wave amplitudes can also be divided into two parts similar to Eq. (B18),

$$\mathcal{M}_s^{(2)} = -if_K a_2 (m_{D^*}^2 - m_\rho^2) V_1^{D^* \rho}, \quad (\text{B22})$$

$$\mathcal{M}_d^{(2)} = -if_K a_2 m_{D^*} m_\rho \{V_4^{D^* \rho} - V_5^{D^* \rho} + V_6^{D^* \rho}\}, \quad (\text{B23})$$

$$\mathcal{M}_p^{(2)} = -2f_K a_2 m_{D^*} m_\rho A_1^{D^* \rho}, \quad (\text{B24})$$

and expressions of $\mathcal{M}_i^{(1)}$ are the same as those of $\mathcal{M}_{s,p,d}$ for $D^{*0} \rightarrow K^- \rho^+$ decay in Eqs. (B15)–(B17).

-
- [1] J. Bjorken and S. Glashow, *Phys. Lett.* **11**, 255 (1964).
[2] S. Glashow, J. Iliopoulos, and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).
[3] J. Aubert, U. Becker, P. Biggs *et al.*, *Phys. Rev. Lett.* **33**, 1404 (1974).
[4] J. Augustin, A. Boyarski, M. Breidenbach *et al.*, *Phys. Rev. Lett.* **33**, 1406 (1974).
[5] G. Goldhaber, F. Pierre, G. Abrams *et al.*, *Phys. Rev. Lett.* **37**, 255 (1976).
[6] I. Peruzzi, M. Piccolo, G. Feldman *et al.*, *Phys. Rev. Lett.* **37**, 569 (1976).
[7] R. Workman *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
[8] <https://pdg.lbl.gov/2021/hadronic-xsections>.
[9] K. Lane and E. Eichten, *Phys. Rev. Lett.* **37**, 477 (1976).
[10] A. Rújula, H. Georgi, and S. Glashow, *Phys. Rev. Lett.* **37**, 398 (1976).
[11] F. Close, *Phys. Lett.* **65B**, 55 (1976).
[12] S. Matsuda, *Phys. Lett.* **66B**, 70 (1977).
[13] J. Korner, M. Kuroda, and G. Schierholz, *Phys. Lett.* **70B**, 106 (1977).
[14] M. Suzuki and W. Wada, *Phys. Rev. D* **15**, 759 (1977).
[15] B. Humpert and R. Clark, *Phys. Rev. D* **16**, 1327 (1977).
[16] Y. Iwasaki, *Phys. Rev. D* **17**, 765 (1978).
[17] D. Cronin-Hennessy *et al.* (CLEO Collaboration), *Phys. Rev. D* **80**, 072001 (2009).
[18] X. Dong, L. Wang, and C. Yuan, *Chin. Phys. C* **42**, 043002 (2018).
[19] I. Peruzzi, M. Piccolo, G. Feldman *et al.*, *Phys. Rev. Lett.* **39**, 1301 (1977).
[20] G. Goldhaber, J. Wiss, G. Abrams *et al.*, *Phys. Lett.* **69B**, 503 (1977).
[21] S. Okubo, *Phys. Lett.* **5**, 165 (1963).
[22] G. Zweig, CERN Report No. CERN-TH-401, 1964, <https://inspirehep.net/literature/11881>; CERN Report No. CERN-TH-412, 1964, <https://inspirehep.net/literature/4674>.
[23] J. Iizuka, *Prog. Theor. Phys. Suppl.* **37-38**, 21 (1966).
[24] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963).
[25] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).

- [26] M. Ablikim *et al.* (BESIII Collaboration), *J. High Energy Phys.* **05** (2022) 155.
- [27] E. Kou *et al.*, *Prog. Theor. Exp. Phys.* **2019**, 123C01 (2019); **2020**, 029201(E) (2020).
- [28] M. Lisovyi, A. Verbytskyi, and O. Zenaiev, *Eur. Phys. J. C* **76**, 397 (2016).
- [29] X. Lyu (STCF Working Group), *Proc. Sci. BEAUTY2020* (2021) 060.
- [30] V. Anashin *et al.*, https://ctd.inp.nsk.su/wiki/images/4/47/CDR2_ScTau_en_vol1.pdf.
- [31] J. Costa *et al.*, Institute of High Energy Physics Report No. IHEP-CEPC-DR-2018-02, [arXiv:1811.10545](https://arxiv.org/abs/1811.10545).
- [32] A. Abada *et al.*, *Eur. Phys. J. C* **79**, 474 (2019).
- [33] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **03** (2016) 159; **09** (2016) 013(E); **05** (2017) 074(E).
- [34] B. Abelev *et al.* (ALICE Collaboration), *J. High Energy Phys.* **07** (2012) 191.
- [35] I. Bediaga *et al.* (LHCb Collaboration), [arXiv:1808.08865](https://arxiv.org/abs/1808.08865).
- [36] Y. Yang, Z. Li, K. Li, J. Huang, and J. Sun, *Eur. Phys. J. C* **81**, 1110 (2021).
- [37] G. Buchalla, A. Buras, and M. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).
- [38] M. Bauer, B. Stech, and M. Wirbel, *Z. Phys. C* **34**, 103 (1987).
- [39] J. Bjorken, *Nucl. Phys. B, Proc. Suppl.* **11**, 325 (1989).
- [40] H. Cheng and C. Chiang, *Phys. Rev. D* **81**, 074021 (2010).
- [41] M. Wirbel, B. Stech, and M. Bauer, *Z. Phys. C* **29**, 637 (1985).
- [42] J. Sun, L. Chen, Q. Chang, J. Huang, and Y. Yang, *Int. J. Mod. Phys. A* **30**, 1550094 (2015).
- [43] Q. Chang, L. Wang, and X. Li, *J. High Energy Phys.* **12** (2019) 102.
- [44] S. Godfrey and N. Isgur, *Phys. Rev. D* **32**, 189 (1985).
- [45] R. Thews and A. Kamal, *Phys. Rev. D* **32**, 810 (1985).
- [46] J. Rosner, *Phys. Rev. D* **88**, 034034 (2013).
- [47] C. Chiang, Z. Luo, and L. Rosner, *Phys. Rev. D* **67**, 014001 (2003).
- [48] B. Stech, *5th Moriond Workshop: Heavy Quarks* (Kim Hup Lee Printing Co. Pte. Ltd., Singapore, 1985), p. 151, ISBN 2-86332-031-9, <https://inspirehep.net/literature/213389>.
- [49] D. Fakirov and B. Stech, *Nucl. Phys.* **B133**, 315 (1978).
- [50] J. Li, M. Yang, and D. Du, *Chin. Phys. C* **27**, 665 (2003), <http://cpc.ihep.ac.cn/article/id/55f7e746-54b1-46a0-a1ab-9a8600fd309c>.
- [51] Y. Wu, M. Zhong, and Y. Zhou, *Eur. Phys. J. C* **42**, 391 (2005).
- [52] F. Yu, X. Wang, and C. Lu, *Phys. Rev. D* **84**, 074019 (2011).
- [53] H. Cheng, C. Chiang, and A. Kuo, *Phys. Rev. D* **93**, 114010 (2016).
- [54] H. Li, C. Lu, and F. Yu, *Phys. Rev. D* **86**, 036012 (2012).