

## Identifying the $\Xi_b(6100)$ as the $P$ -wave bottom baryon of $J^P = 3/2^-$

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We study the  $\Xi_b(6100)$  using the methods of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory. Our results suggest that the  $\Xi_b(6100)$  can be well interpreted as the  $P$ -wave bottom baryon of  $J^P = 3/2^-$ , belonging to the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$  representation. It has a partner state of  $J^P = 1/2^-$ , labeled as  $\Xi_b(1/2^-)$ , whose mass and width are predicted to be  $m_{\Xi_b(1/2^-)} = 6.08_{-0.11}^{+0.13}$  GeV and  $\Gamma_{\Xi_b(1/2^-)} = 4_{-4}^{+29}$  MeV, respectively, with the mass splitting  $\Delta M = m_{\Xi_b(6100)} - m_{\Xi_b(1/2^-)} = 9 \pm 3$  MeV. We propose to search for it in the  $\Xi_c(1/2^-) \rightarrow \Xi'_b \pi$  decay channel. Our results also suggest that the  $\Lambda_b(5912)$  and  $\Lambda_b(5920)$  are their partner states with  $J^P = 1/2^-$  and  $3/2^-$ , respectively, and, moreover, the  $\Lambda_c(2595)$ ,  $\Lambda_c(2625)$ ,  $\Xi_c(2790)$ , and  $\Xi_c(2815)$  are their charmed partner states.

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### I. INTRODUCTION

A singly heavy baryon consists of a heavy quark and two light quarks, where the light quarks together with gluons circle around the nearly static heavy quark. In the infinite  $m_Q$  limit, the heavy quark is decoupled from the dynamics of the light quarks, which greatly simplifies a triquark system into a diquark system. This system is similar to the deuterium atom [1–5]. It is well known that electromagnetic interaction produces light spectroscopies with fine splitting. Similarly, strong interaction also produces hadron spectroscopies with fine splitting, and the singly heavy baryon system provides an ideal platform to study this effect [6–10].

In the past 50 years, much important experimental progress has been made in the field of singly heavy baryons. Since the lowest-lying charmed baryon  $\Lambda_c$  was reported by Fermilab in 1976 [11], a lot of singly heavy baryons have been observed in experiments [12]; e.g.,

- (i) The  $\Lambda_c(2595)$ ,  $\Lambda_c(2625)$ ,  $\Xi_c(2790)$ , and  $\Xi_c(2815)$  can be well interpreted as the  $P$ -wave charmed baryons completing two flavor  $\bar{\mathbf{3}}_F$  multiplets of

$J^P = 1/2^-$  and  $3/2^-$  [13–16]. Their masses and widths (or upper limits at 90% credibility level) were measured to be [12]

$$\Lambda_c(2595)^+: M = 2592.25 \pm 0.28 \text{ MeV},$$

$$\Gamma = 2.59 \pm 0.30 \pm 0.47 \text{ MeV}; \quad (1)$$

$$\Lambda_c(2625)^+: M = 2628.11 \pm 0.19 \text{ MeV},$$

$$\Gamma < 0.97 \text{ MeV} \quad \text{at 90\% C.L.}; \quad (2)$$

$$\Xi_c(2790)^0: M = 2793.9 \pm 0.5 \text{ MeV},$$

$$\Gamma = 10.0 \pm 0.7 \pm 0.8 \text{ MeV}; \quad (3)$$

$$\Xi_c(2815)^0: M = 2819.79 \pm 0.30 \text{ MeV},$$

$$\Gamma = 2.54 \pm 0.18 \pm 0.17 \text{ MeV}. \quad (4)$$

- (ii) In 2012, the LHCb Collaboration discovered two narrow states, the  $\Lambda_b(5912)$  and  $\Lambda_b(5920)$ , in the  $\Lambda_b \pi \pi$  mass spectrum [17]. Their masses and widths were measured to be [12]

$$\Lambda_b(5912)^0: M = 5912.19 \pm 0.17 \text{ MeV},$$

$$\Gamma < 0.25 \text{ MeV} \quad \text{at 90\% C.L.}; \quad (5)$$

$$\Lambda_b(5920)^0: M = 5920.09 \pm 0.17 \text{ MeV},$$

$$\Gamma < 0.19 \text{ MeV} \quad \text{at 90\% C.L.} \quad (6)$$

- (iii) Recently, the  $\Xi_b(6100)$  was observed by the CMS Collaboration in the  $\Xi_b^- \pi^+ \pi^-$  invariant mass

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spectrum [18]. Its mass and width were measured to be

$$\begin{aligned} \Xi_b(6100)^-: M &= 6100.3 \pm 0.2 \pm 0.1 \pm 0.6 \text{ MeV}, \\ \Gamma &< 1.9 \text{ MeV} \quad \text{at 95\% C.L.} \end{aligned} \quad (7)$$

Experimental progress on the singly heavy baryons has attracted a lot of theorists to study them. It is a challenging issue to fully understand their internal structures. Various theoretical methods and models have been applied in this field, including various quark models [19–30], the chiral perturbation theory [31,32], the chiral unitary approach [33], various molecular interpretations [34–39], the Regge trajectory [40], the  ${}^3P_0$  model [41,42], the relativistic flux tube model [43], QCD sum rules [44–48], and the lattice QCD [49–51], etc. We refer to the reviews [9,10,52–55] for detailed discussions.

In our previous works [56–65], we have studied the mass spectra and decay properties of the  $P$ -wave charmed and bottom baryons using the methods of QCD sum rules [66,67] and light-cone sum rules [68–73] within the framework of heavy quark effective theory [74–76], especially as follows.

- (i) In Refs. [56,57], we have systematically investigated the  $P$ -wave charmed and bottom baryons of the  $SU(3)$  flavor  $\mathbf{6}_F$  and studied their  $S$ - and  $D$ -wave decays into the ground-state heavy baryons and light pseudoscalar and vector mesons.
- (ii) In Ref. [62], we have investigated the  $P$ -wave charmed baryons of the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$  and studied their  $S$ -wave decays into the ground-state charmed baryons and light pseudoscalar and vector mesons.

In the present study, we will further investigate the  $P$ -wave charmed and bottom baryons of the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$  and study their  $S$ - and  $D$ -wave decays into the ground-state heavy baryons and light pseudoscalar and vector mesons. We shall concentrate on the  $\Xi_b(6100)$  recently observed by CMS [18] and study its mass and decay properties. We refer to Refs. [77–83] for relevant quark model calculations.

Previously, in Ref. [62], we found that the  $\Lambda_c(2595)$ ,  $\Lambda_c(2625)$ ,  $\Xi_c(2790)$ , and  $\Xi_c(2815)$  can be well explained as the  $P$ -wave charmed baryons of the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$ , which complete two flavor  $\bar{\mathbf{3}}_F$  multiplets of  $J^P = 1/2^-$  and  $3/2^-$ . In this study, we shall find that the  $\Lambda_b(5912)$ ,  $\Lambda_b(5920)$ , and  $\Xi_b(6100)$ , as the bottom partner states of the  $\Lambda_c(2595)$ ,  $\Lambda_c(2625)$ , and  $\Xi_c(2815)$ , respectively, can be well explained as the  $P$ -wave bottom baryons of the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$ . Hence, there is a  $P$ -wave bottom baryon still missing, which is the bottom partner state of the  $\Xi_c(2790)$ . We shall also study its mass and decay properties.

This paper is organized as follows. In Sec. II, we briefly introduce our notations and apply the QCD sum rule method to calculate the mass of  $\Xi_b(6100)$  as a  $P$ -wave bottom baryon of the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$ . The obtained parameters are further used to study its decay properties

through the light-cone sum rule method in Sec. III. In Sec. IV, we discuss the results and conclude this paper.

## II. MASS ANALYSES FROM QCD SUM RULES

A bottom baryon consists of a heavy bottom quark and two light up, down, and strange quarks. It has a rich internal structure with color, flavor, spin, and orbital degrees of freedom. Especially, the orbital excitation of a  $P$ -wave bottom baryon can be between the bottom quark and the light quarks, or it can also be inside the two light quarks. We call the former  $\lambda$ -mode excitation with  $l_\lambda = 1$  and  $l_\rho = 0$ , and the latter  $\rho$ -mode excitation with  $l_\lambda = 0$  and  $l_\rho = 1$ .

When investigating the bottom baryon, we need to pay attention to the two light up, down, and strange quarks.

- (i) Their color structure is antisymmetric ( $\bar{\mathbf{3}}_C$ ).
- (ii) Their flavor structure is either symmetric ( $\mathbf{6}_F$ ) or antisymmetric ( $\bar{\mathbf{3}}_F$ ).
- (iii) Their spin structure is either symmetric ( $s_l = 1$ ) or antisymmetric ( $s_l = 0$ ).
- (iv) Their orbital structure is either symmetric ( $\lambda$  mode) or antisymmetric ( $\rho$  mode).

Applying the Pauli principle to the two light quarks, we can categorize the  $P$ -wave bottom baryons into eight multiplets. We denote them as  $[F(\text{flavor}), j_l, s_l, \rho/\lambda]$ , where  $j_l = l_\lambda \otimes l_\rho \otimes s_l$  is the total angular momentum of the light components. Every multiplet contains one or two baryons with the total angular momenta  $j = j_l \otimes s_b = |j_l \pm 1/2|$ . Within the QCD sum rule method, we can construct their corresponding interpolating currents  $J_{j,P,F,j_l,s_l,\rho/\lambda}^{\alpha_1 \dots \alpha_{j-1/2}}$ , which couple to the bottom baryons belonging to the  $[F, j_l, s_l, \rho/\lambda]$  multiplet through

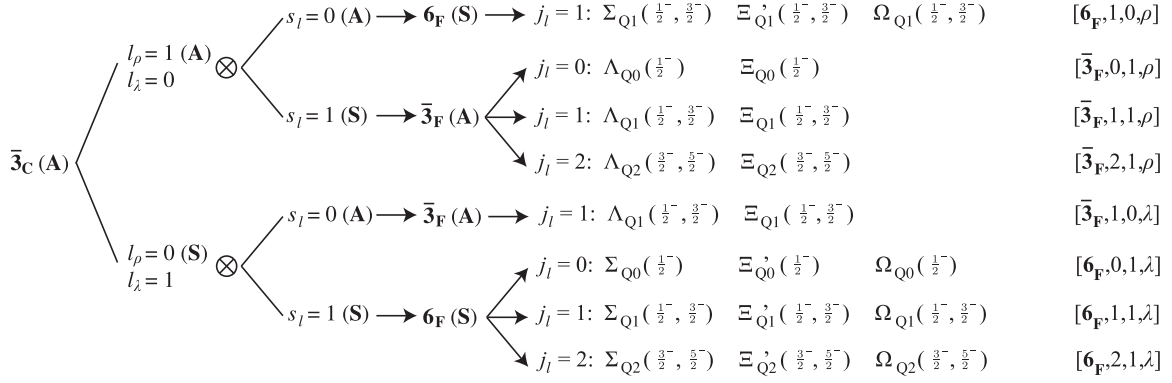
$$\langle 0 | J_{j,P,F,j_l,s_l,\rho/\lambda}^{\alpha_1 \dots \alpha_{j-1/2}} | j, P, F, j_l, s_l, \rho/\lambda \rangle = f_{F,j_l,s_l,\rho/\lambda} u^{\alpha_1 \dots \alpha_{j-1/2}}, \quad (8)$$

with  $f_{F,j_l,s_l,\rho/\lambda}$  the decay constant.

As shown in Fig. 1, there are four multiplets belonging to the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$  representation. In this paper, we study the  $\Xi_b(6100)$  as the  $P$ -wave bottom baryon of  $J^P = 3/2^-$  belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet. Accordingly, we concentrate on this doublet and use the following currents to study it [64]:

$$J_{1/2,-,\bar{\mathbf{3}}_F,1,1,\rho} = i\epsilon_{abc} ([\mathcal{D}_i^\mu q^{aT}] \mathcal{C} \gamma_i^\nu q^b - q^{aT} \mathcal{C} \gamma_i^\nu [\mathcal{D}_i^\mu q^b]) \sigma^{\mu\nu} h_v^c, \quad (9)$$

$$\begin{aligned} J_{3/2,-,\bar{\mathbf{3}}_F,1,1,\rho}^\alpha &= i\epsilon_{abc} ([\mathcal{D}_i^\mu q^{aT}] \mathcal{C} \gamma_i^\nu q^b - q^{aT} \mathcal{C} \gamma_i^\nu [\mathcal{D}_i^\mu q^b]) \\ &\times \left( g_i^{\alpha\mu} \gamma_i^\nu \gamma_5 - g_i^{\alpha\nu} \gamma_i^\mu \gamma_5 - \frac{1}{3} \gamma_i^\alpha \gamma_i^\mu \gamma_i^\nu \gamma_5 + \frac{1}{3} \gamma_i^\alpha \gamma_i^\nu \gamma_i^\mu \gamma_5 \right) h_v^c, \end{aligned} \quad (10)$$

FIG. 1.  $P$ -wave heavy baryons belonging to the  $SU(3)$  flavor  $\bar{\mathbf{3}}_F$  representation.

where  $a\dots c$  are color indices,  $\mathcal{C}$  is the charge-conjugation operator,  $\mathcal{D}^\mu = \mathcal{D}^\mu - v \cdot \mathcal{D} v^\mu$ ,  $\gamma_\nu^\nu = \gamma^\nu - v \not{v}^\nu$ , and  $g_t^{\rho\beta} = g^{\rho\beta} - v^\rho v^\beta$ . The covariant derivative operator has been explicitly added to these currents, and we also refer to Ref. [84], where we use this covariant derivative operator to construct the  $D$ -wave fully strange tetraquark currents with the exotic quantum number  $J^{PC} = 4^{+-}$ .

We employ QCD sum rules to investigate these two currents and calculate the mass of  $\Xi_b(6100)$  through

$$m_{\Xi_b} = m_b + \bar{\Lambda}_{\Xi_b} + \delta m_{\Xi_b}, \quad (11)$$

$$\delta m_{\Xi_b} = -\frac{1}{4m_b} (K_{\Xi_b} + d_{j,j_l} C_{\text{mag}} \Sigma_{\Xi_b}), \quad (12)$$

where  $m_b$  is the bottom quark mass,  $\bar{\Lambda}_{\Xi_b}$  is the sum rule result at the leading order, and  $\delta m_{\Xi_b}$  is the result at the  $\mathcal{O}(1/m_Q)$  order.

The obtained results are summarized in Table I, where we use three criteria to restrict the two free parameters: the threshold value  $\omega_c$  and the Borel mass  $T$ . The first criterion requires that the higher-order corrections be less than 30%, the second criterion requires that the pole contribution be larger than 20%, and the third criterion requires a weak

dependence of the mass  $m_{\Xi_b}$  on these two free parameters. We refer to Refs. [63,64] for detailed discussions, where all the QCD sum rule calculations have been done for the  $P$ -wave charmed and bottom baryons. In the calculations, we work at the renormalization scale 2 GeV and use the following QCD parameters [12,85–91]:

$$\begin{aligned} \langle \bar{q}q \rangle &= -(0.24 \pm 0.01 \text{ GeV})^3, \\ \langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\ \langle g_s \bar{q}\sigma Gq \rangle &= M_0^2 \times \langle \bar{q}q \rangle, \\ \langle g_s \bar{s}\sigma Gs \rangle &= M_0^2 \times \langle \bar{s}s \rangle, \\ M_0^2 &= 0.8 \text{ GeV}^2, \\ \langle g_s^2 GG \rangle &= (0.48 \pm 0.14) \text{ GeV}^4, \\ m_b &= 4.66 \pm 0.03 \text{ GeV}, \\ m_s &= 95_{-3}^{+9} \text{ MeV}. \end{aligned} \quad (13)$$

There are considerable uncertainties in our results for the absolute value of the mass. This allows us to fine-tune some of the parameters in order to get a better description of the  $\Xi_b(6100)$ . However, the mass difference within the same doublet is produced quite well with much less uncertainty.

TABLE I. Parameters of the  $P$ -wave bottom baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets, calculated using the method of QCD sum rules within the framework of heavy quark effective theory. Decay constants in the last column satisfy  $f_{\Xi_b^0} = f_{\Xi_b^-}$ .

Multiplets	B	$\omega_c$ (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	Baryon ( $j^P$ )	Mass (GeV)	Difference (MeV)	Decay constant (GeV <sup>4</sup> )
$[\bar{\mathbf{3}}_F, 1, 1, \rho]$	$\Lambda_b$	1.63	$0.27 < T < 0.31$	$1.20 \pm 0.10$	$\Lambda_b(1/2^-)$	$5.92_{-0.10}^{+0.13}$	$10 \pm 4$	$0.060 \pm 0.014$ ( $\Lambda_b^0(1/2^-)$ )
					$\Lambda_b(3/2^-)$	$5.93_{-0.10}^{+0.13}$		$0.028 \pm 0.007$ ( $\Lambda_b^0(3/2^-)$ )
	$\Xi_b$	1.83	$0.27 < T < 0.32$	$1.37 \pm 0.10$	$\Xi_b(1/2^-)$	$6.08_{-0.11}^{+0.13}$	$9 \pm 3$	$0.083 \pm 0.020$ ( $\Xi_b^-(1/2^-)$ )
					$\Xi_b(3/2^-)$	$6.09_{-0.11}^{+0.12}$		$0.039 \pm 0.009$ ( $\Xi_b^-(3/2^-)$ )
$[\bar{\mathbf{3}}_F, 1, 0, \lambda]$	$\Lambda_b$	1.70	$0.32 < T < 0.33$	$1.13 \pm 0.10$	$\Lambda_b(1/2^-)$	$5.91_{-0.11}^{+0.11}$	$6 \pm 2$	$0.031 \pm 0.007$ ( $\Lambda_b^0(1/2^-)$ )
					$\Lambda_b(3/2^-)$	$5.92_{-0.11}^{+0.11}$		$0.015 \pm 0.003$ ( $\Lambda_b^0(3/2^-)$ )
	$\Xi_b$	1.83	$0.33 < T < 0.35$	$1.28 \pm 0.09$	$\Xi_b(1/2^-)$	$6.09_{-0.10}^{+0.10}$	$5 \pm 2$	$0.044 \pm 0.009$ ( $\Xi_b^-(1/2^-)$ )
					$\Xi_b(3/2^-)$	$6.10_{-0.10}^{+0.10}$		$0.021 \pm 0.004$ ( $\Xi_b^-(3/2^-)$ )

Moreover, the decay constant  $f_{F,j_i,s_i,\rho/\lambda}$  is an important input parameter, which will be used to study the decay properties of  $\Xi_b(6100)$  in the next section.

Besides the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet, we have also investigated the  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublet through the following interpolating currents:

$$J_{1/2,-,\bar{\mathbf{3}}_F,1,0,\lambda} = i\epsilon_{abc}([\mathcal{D}_i^\mu q^{aT}]C\gamma_5 q^b + q^{aT}C\gamma_5[\mathcal{D}_i^\mu q^b]) \times \gamma_i^\mu \gamma_5 h_v^c, \quad (14)$$

$$J_{3/2,-,\bar{\mathbf{3}}_F,1,0,\lambda}^\alpha = i\epsilon_{abc}([\mathcal{D}_i^\mu q^{aT}]C\gamma_5 q^b + q^{aT}C\gamma_5[\mathcal{D}_i^\mu q^b]) \times \left(g_i^{\alpha\mu} - \frac{1}{3}\gamma_i^\alpha \gamma_i^\mu\right) h_v^c. \quad (15)$$

The obtained results are also summarized in Table I. We find that the  $\rho$ -mode doublet  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  is lower than the  $\lambda$ -mode doublet  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ . This behavior is consistent with our previous QCD sum rule results for their

corresponding doublets of the  $SU(3)$  flavor  $\mathbf{6}_F$  [56–58], but in contrast to the quark model expectation [92,93]. However, this is possible simply because the mass difference between different multiplets have considerable uncertainties within our QCD sum rule framework, so our results for the mass spectra cannot distinguish the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets. Therefore, we move on to investigate their decay properties in the next section.

### III. DECAY ANALYSES FROM LIGHT-CONE SUM RULES

In this section, we study the decay properties of  $\Xi_b(6100)$  as the  $P$ -wave bottom baryon of  $J^P = 3/2^-$  belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet. We shall apply the method of light-cone sum rules to investigate all the bottom baryons from this doublet and study their  $S$ - and  $D$ -wave decays into the ground-state bottom baryons and light pseudoscalar and vector mesons, including

$$(a1) \Gamma[\Lambda_b[1/2^-] \rightarrow \Lambda_b + \pi] = \Gamma[\Lambda_b^0[1/2^-] \rightarrow \Lambda_b^0 + \pi^0], \quad (16)$$

$$(a2) \Gamma[\Lambda_b[1/2^-] \rightarrow \Sigma_b + \pi] = 3 \times \Gamma[\Lambda_b^0[1/2^-] \rightarrow \Sigma_b^+ + \pi^- \rightarrow \Lambda_b^0 + \pi^+ + \pi^-], \quad (17)$$

$$(a3) \Gamma[\Lambda_b[1/2^-] \rightarrow \Sigma_b^* + \pi \rightarrow \Lambda_b + \pi + \pi] = 3 \times \Gamma[\Lambda_b^0[1/2^-] \rightarrow \Sigma_b^{*+} + \pi^- \rightarrow \Lambda_b^0 + \pi^+ + \pi^-], \quad (18)$$

$$(a4) \Gamma[\Lambda_b[1/2^-] \rightarrow \Lambda_b + \rho \rightarrow \Lambda_b + \pi + \pi] = \Gamma[\Lambda_b^0[1/2^-] \rightarrow \Lambda_b^0 + \pi^+ + \pi^-], \quad (19)$$

$$(a5) \Gamma[\Lambda_b[1/2^-] \rightarrow \Sigma_b + \rho \rightarrow \Sigma_b + \pi + \pi] = 3 \times \Gamma[\Lambda_b^0[1/2^-] \rightarrow \Sigma_b^0 + \pi^+ + \pi^-], \quad (20)$$

$$(a6) \Gamma[\Lambda_b[1/2^-] \rightarrow \Sigma_b^* + \rho \rightarrow \Sigma_b^* + \pi + \pi] = 3 \times \Gamma[\Lambda_b^0[1/2^-] \rightarrow \Sigma_b^{*0} + \pi^+ + \pi^-], \quad (21)$$

$$(b1) \Gamma[\Lambda_b[3/2^-] \rightarrow \Lambda_b + \pi] = \Gamma[\Lambda_b^0[3/2^-] \rightarrow \Lambda_b^0 + \pi^0], \quad (22)$$

$$(b2) \Gamma[\Lambda_b[3/2^-] \rightarrow \Sigma_b + \pi \rightarrow \Lambda_b + \pi + \pi] = 3 \times \Gamma[\Lambda_b^0[3/2^-] \rightarrow \Sigma_b^+ + \pi^- \rightarrow \Lambda_b^0 + \pi^+ + \pi^-], \quad (23)$$

$$(b3) \Gamma[\Lambda_b[3/2^-] \rightarrow \Sigma_b^* + \pi \rightarrow \Lambda_b + \pi + \pi] = 3 \times \Gamma[\Lambda_b^0[3/2^-] \rightarrow \Sigma_b^{*+} + \pi^- \rightarrow \Lambda_b^0 + \pi^+ + \pi^-], \quad (24)$$

$$(b4) \Gamma[\Lambda_b[3/2^-] \rightarrow \Lambda_b + \rho \rightarrow \Lambda_b + \pi + \pi] = \Gamma[\Lambda_b^0[3/2^-] \rightarrow \Lambda_b^0 + \pi^+ + \pi^-], \quad (25)$$

$$(b5) \Gamma[\Lambda_b[3/2^-] \rightarrow \Sigma_b + \rho \rightarrow \Sigma_b + \pi + \pi] = 3 \times \Gamma[\Lambda_b^0[3/2^-] \rightarrow \Sigma_b^0 + \pi^+ + \pi^-], \quad (26)$$

$$(b6) \Gamma[\Lambda_b[3/2^-] \rightarrow \Sigma_b^* + \rho \rightarrow \Sigma_b^* + \pi + \pi] = 3 \times \Gamma[\Lambda_b^0[3/2^-] \rightarrow \Sigma_b^{*0} + \pi^+ + \pi^-], \quad (27)$$

$$(c1) \Gamma[\Xi_b[1/2^-] \rightarrow \Lambda_b + \bar{K}] = \Gamma[\Xi_b^-[1/2^-] \rightarrow \Lambda_b^0 + K^-], \quad (28)$$

$$(c2) \Gamma[\Xi_b[1/2^-] \rightarrow \Xi_b + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Xi_b^0 + \pi^-], \quad (29)$$

$$(c3) \Gamma[\Xi_b[1/2^-] \rightarrow \Sigma_b + \bar{K}] = 3 \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Sigma_b^0 + K^-], \quad (30)$$

$$(c4) \Gamma[\Xi_b[1/2^-] \rightarrow \Xi_b' + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Xi_b'^0 + \pi^-], \quad (31)$$

$$(c5) \Gamma[\Xi_b[1/2^-] \rightarrow \Sigma_b^* + K] = 3 \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Sigma_b^{*0} + \bar{K}^-], \quad (32)$$

$$(c6) \Gamma[\Xi_b[1/2^-] \rightarrow \Xi_b^* + \pi \rightarrow \Xi_b + \pi + \pi] = \frac{9}{2} \times \Gamma[\Xi_b^{*-}[1/2^-] \rightarrow \Xi_b^{*0} + \pi^- \rightarrow \Xi_b^0 + \pi^0 + \pi^-], \quad (33)$$

$$(c7) \Gamma[\Xi_b[1/2^-] \rightarrow \Lambda_b + \bar{K}^* \rightarrow \Lambda_b + \bar{K} + \pi] = 3 \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Lambda_b^0 + K^- + \pi^0], \quad (34)$$

$$(c8) \Gamma[\Xi_b[1/2^-] \rightarrow \Xi_b + \rho \rightarrow \Xi_b + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Xi_b^0 + \pi^0 + \pi^-], \quad (35)$$

$$(c9) \Gamma[\Xi_b[1/2^-] \rightarrow \Sigma_b^* + \bar{K}^* \rightarrow \Sigma_b^{*0} + \bar{K} + \pi] = 9 \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Sigma_b^{*0} + K^- + \pi^0], \quad (36)$$

$$(c10) \Gamma[\Xi_b[1/2^-] \rightarrow \Xi_b^* + \rho \rightarrow \Xi_b^* + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[1/2^-] \rightarrow \Xi_b^{*0} + \pi^0 + \pi^-]. \quad (37)$$

$$(d1) \Gamma[\Xi_b[3/2^-] \rightarrow \Lambda_b + \bar{K}] = \Gamma[\Xi_b^-[3/2^-] \rightarrow \Lambda_b^0 + K^-], \quad (38)$$

$$(d2) \Gamma[\Xi_b[3/2^-] \rightarrow \Xi_b + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Xi_b^0 + \pi^-], \quad (39)$$

$$(d3) \Gamma[\Xi_b[3/2^-] \rightarrow \Sigma_b + \bar{K}] = 3 \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Sigma_b^0 + K^-], \quad (40)$$

$$(d4) \Gamma[\Xi_b[3/2^-] \rightarrow \Xi_b' + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Xi_b'^0 + \pi^-], \quad (41)$$

$$(d5) \Gamma[\Xi_b[3/2^-] \rightarrow \Sigma_b^* + \bar{K}] = 3 \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Sigma_b^{*0} + K^-], \quad (42)$$

$$(d6) \Gamma[\Xi_b[3/2^-] \rightarrow \Xi_b^* + \pi \rightarrow \Xi_b + \pi + \pi] = \frac{9}{2} \times \Gamma[\Xi_b^{*-}[3/2^-] \rightarrow \Xi_b^{*0} + \pi^- \rightarrow \Xi_b^0 + \pi^0 + \pi^-], \quad (43)$$

$$(d7) \Gamma[\Xi_b[3/2^-] \rightarrow \Lambda_b + \bar{K}^* \rightarrow \Lambda_b + \bar{K} + \pi] = 3 \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Lambda_b^0 + K^- + \pi^0], \quad (44)$$

$$(d8) \Gamma[\Xi_b[3/2^-] \rightarrow \Xi_b + \rho \rightarrow \Xi_b + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Xi_b^0 + \pi^0 + \pi^-], \quad (45)$$

$$(d9) \Gamma[\Xi_b[3/2^-] \rightarrow \Sigma_b^* + \bar{K}^* \rightarrow \Sigma_b^{*0} + \bar{K} + \pi] = 9 \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Sigma_b^{*0} + K^- + \pi^0], \quad (46)$$

$$(d10) \Gamma[\Xi_b[3/2^-] \rightarrow \Xi_b^* + \rho \rightarrow \Xi_b^* + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_b^-[3/2^-] \rightarrow \Xi_b^{*0} + \pi^0 + \pi^-]. \quad (47)$$

We shall calculate their partial decay widths through the following Lagrangians:

$$\mathcal{L}_{X_b(1/2^-) \rightarrow Y_b(1/2^+) \mathcal{P}}^S = g\bar{X}_b(1/2^-)Y_b(1/2^+)\mathcal{P}, \quad (48)$$

$$\mathcal{L}_{X_b(3/2^-) \rightarrow Y_b(3/2^+) \mathcal{P}}^S = g\bar{X}_{b\mu}(3/2^-)Y_b^\mu(3/2^+)\mathcal{P}, \quad (49)$$

$$\mathcal{L}_{X_b(1/2^-) \rightarrow Y_b(1/2^+) V}^S = g\bar{X}_b(1/2^-)\gamma_\mu\gamma_5 Y_b(1/2^+)V^\mu, \quad (50)$$

$$\mathcal{L}_{X_b(1/2^-) \rightarrow Y_b(3/2^+) V}^S = g\bar{X}_b(1/2^-)Y_b^\mu(3/2^+)V_\mu, \quad (51)$$

$$\mathcal{L}_{X_b(3/2^-) \rightarrow Y_b(1/2^+) V}^S = g\bar{X}_b^\mu(3/2^-)Y_b(1/2^+)V_\mu, \quad (52)$$

$$\mathcal{L}_{X_b(3/2^-) \rightarrow Y_b(3/2^+) V}^S = g\bar{X}_b^\nu(3/2^-)\gamma_\mu\gamma_5 Y_{b\nu}(3/2^+)V^\mu. \quad (53)$$

$$\mathcal{L}_{X_b(1/2^-) \rightarrow Y_b(3/2^+) \mathcal{P}}^D = g\bar{X}_b(1/2^-)\gamma_\mu\gamma_5 Y_{b\nu}(3/2^+)\partial^\mu\partial^\nu\mathcal{P}, \quad (54)$$

$$\mathcal{L}_{X_b(3/2^-) \rightarrow Y_b(1/2^+) \mathcal{P}}^D = g\bar{X}_{b\mu}(3/2^-)\gamma_\nu\gamma_5 Y_b(1/2^+)\partial^\mu\partial^\nu\mathcal{P}, \quad (55)$$

$$\mathcal{L}_{X_b(3/2^-) \rightarrow Y_b(3/2^+) \mathcal{P}}^D = g\bar{X}_{b\mu}(3/2^-)Y_{b\nu}(3/2^+)\partial^\mu\partial^\nu\mathcal{P}. \quad (56)$$

In the above expression, the superscripts  $S$  and  $D$  denote the  $S$ - and  $D$ -wave decays, respectively; the fields  $X_b^{(\mu)}$ ,  $Y_b^{(\mu)}$ ,  $\mathcal{P}$ , and  $V^\mu$  denote the  $P$ -wave bottom baryons, ground-state bottom baryons, light pseudoscalar mesons, and light vector mesons, respectively.

We use the  $\Xi_b^-(3/2^-)$  from the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet as an example and study its  $S$ -wave decays into the ground-state bottom baryon  $\Xi_b^{*0}$  and the light pseudoscalar meson  $\pi^-$ . We note that the two-body decay is insufficient due to the  $\Xi_b^* \pi$  threshold being close to the mass of  $\Xi_b(6100)$ , so the sequential three-body decay process  $\Xi_b(3/2^-) \rightarrow \Xi_b^*(3/2^+) \pi \rightarrow \Xi_b(1/2^+) \pi \pi$  is crucial for understanding the decay behavior of  $\Xi_b(6100)$ . To do this, we investigate the three-point correlation function:

$$\begin{aligned} \Pi^\alpha(\omega, \omega') &= \int d^4x e^{-ik \cdot x} \langle 0 | J_{3/2, -\Xi_b^-, 1, 1, \rho}^\alpha(0) \bar{J}_{\Xi_b^{*0}}(x) | \pi^-(q) \rangle \\ &= \frac{1 + \not{v}}{2} G_{\Xi_b^-[3/2^-] \rightarrow \Xi_b^{*0} \pi^-}^\alpha(\omega, \omega'), \end{aligned} \quad (57)$$

where  $k' = k + q$ ,  $\omega = v \cdot k$ , and  $\omega' = v \cdot k'$ .

At the hadron level, we write  $G_{\Xi_b^-[3/2^-] \rightarrow \Xi_b^{*0} \pi^-}^{\alpha\beta}$  as

$$\begin{aligned} G_{\Xi_b^-[3/2^-] \rightarrow \Xi_b^{*0} \pi^-}^{\alpha\beta}(\omega, \omega') &= g_{\Xi_b^-[3/2^-] \rightarrow \Xi_b^{*0} \pi^-} \frac{g^{\alpha\beta} f_{\Xi_b^-[3/2^-]} f_{\Xi_b^{*0}}}{(\bar{\Lambda}_{\Xi_b^-[3/2^-]} - \omega')(\bar{\Lambda}_{\Xi_b^{*0}} - \omega)} + \dots, \end{aligned} \quad (58)$$

where  $\dots$  contains other possible amplitudes.

At the quark-gluon level, we calculate  $G_{\Xi_b^-[3/2^-] \rightarrow \Xi_b^{*0} \pi^-}^\alpha$  using the method of operator product expansion:

$$\begin{aligned} G_{\Xi_b^-[3/2^-] \rightarrow \Xi_b^{*0} \pi^-}^{\alpha\beta}(\omega, \omega') &= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left( \frac{f_\pi m_s t^2 v \cdot q}{1152} \langle \bar{s}s \rangle \phi_{4;\pi}(u) - \frac{f_\pi m_s v \cdot q}{72} \langle \bar{s}s \rangle \phi_{2;\pi}(u) \right. \\ &\quad - \frac{f_\pi m_\pi^2 m_s v \cdot q}{36(m_u + m_d) \pi^2 t^2} \phi_{3;\pi}^\sigma(u) - \frac{5if_\pi m_s u t^3 (v \cdot q)^2}{13824} \langle \bar{s}s \rangle \phi_{4;\pi}(u) - \frac{5if_\pi m_s u t (v \cdot q)^2}{864} \langle \bar{s}s \rangle \phi_{2;\pi}(u) \\ &\quad + \frac{5if_\pi m_\pi^2 m_s u (v \cdot q)^2}{432(m_u + m_d) \pi^2 t} \phi_{3;\pi}^\sigma(u) - \frac{f_\pi m_s}{72 v \cdot q} \langle \bar{s}s \rangle \psi_{4;\pi}(u) - \frac{f_\pi m_\pi^2 t^2 v \cdot q}{1728(m_u + m_d)} \langle g_s \bar{s} \sigma G_s \rangle \phi_{3;\pi}^\sigma(u) \\ &\quad - \frac{f_\pi m_\pi^2 v \cdot q}{108(m_u + m_d)} \langle \bar{s}s \rangle \phi_{3;\pi}^\sigma(u) - \frac{f_\pi v \cdot q}{48 \pi^2 t^2} \phi_{4;\pi}(u) - \frac{f_\pi v \cdot q}{3 \pi t^4} \phi_{2;\pi}(u) \\ &\quad + \frac{5if_\pi m_\pi^2 u t^3 (v \cdot q)^2}{20736(m_u + m_d)} \langle g_s \bar{s} \sigma G_s \rangle \phi_{3;\pi}^\sigma(u) + \frac{5if_\pi m_\pi^2 u t (v \cdot q)^2}{1296(m_u + m_d)} \langle \bar{s}s \rangle \phi_{3;\pi}^\sigma(u) - \frac{5if_\pi u (v \cdot q)^2}{576 \pi^2 t} \phi_{4;\pi}(u) \\ &\quad \left. - \frac{5if_\pi u (v \cdot q)^2}{36 \pi^2 t^3} \phi_{2;\pi}(u) - \frac{f_\pi}{3 \pi^2 t^4 v \cdot q} \psi_{4;\pi}(u) \right) \times g^{\alpha\beta} \\ &\quad - \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2 + u\alpha_3)} e^{i\omega t(1 - \alpha_2 - u\alpha_3)} \times \frac{1}{2} \times \left( -\frac{5if_\pi \alpha_3 u^2 (v \cdot q)^2}{72 \pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u}{36 \pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) \right. \\ &\quad + \frac{f_\pi u v \cdot q}{12 \pi^2 t^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{7f_\pi u v \cdot q}{72 \pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v \cdot q}{24 \pi^2 t^2} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) \\ &\quad - \frac{5if_\pi \alpha_2 u (v \cdot q)^2}{72 \pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) - \frac{5if_\pi \alpha_3 u (v \cdot q)^2}{144 \pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) - \frac{5if_\pi \alpha_3 u (v \cdot q)^2}{144 \pi^2 t} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) \\ &\quad + \frac{5if_\pi u (v \cdot q)^2}{72 \pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi v \cdot q}{24 \pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi v \cdot q}{72 \pi^2 t^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) \\ &\quad + \frac{5f_\pi v \cdot q}{72 \pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) - \frac{5f_\pi v \cdot q}{72 \pi^2 t^2} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) - \frac{5if_\pi \alpha_2 (v \cdot q)^2}{144 \pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) \\ &\quad \left. - \frac{5if_\pi \alpha_2 (v \cdot q)^2}{144 \pi^2 t} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{5if_\pi (v \cdot q)^2}{144 \pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) + \frac{5if_\pi (v \cdot q)^2}{144 \pi^2 t} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) \right) \times g^{\alpha\beta}. \end{aligned} \quad (59)$$

Then we perform the double Borel transformation to both Eq. (59) at the hadron level and Eq. (59) at the quark-gluon level:

$$\begin{aligned}
& g_{\Xi_b^-(\frac{3}{2}^-) \rightarrow \Xi_b^*0 \pi^-} f_{\Xi_b^-(\frac{3}{2}^-)} f_{\Xi_b^*0} e^{-\frac{\bar{\Lambda}_{\Xi_b^-(\frac{3}{2}^-)}}{T_1}} e^{-\frac{\bar{\Lambda}_{\Xi_b^*0}}{T_2}} \\
& = 4 \times \left( - \int_0^{\frac{1}{2}} du_1 \frac{if_\pi}{3\pi^2} T^4 f_3 \left( \frac{\omega_c}{T} \right) \psi_{4;\pi}(u_1) - \frac{if_\pi}{3\pi^2} T^6 f_5 \left( \frac{\omega_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{2;\pi}(u_0)|_{u_0=\frac{1}{2}} - \frac{5if_\pi}{36\pi^2} T^6 f_5 \left( \frac{\omega_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{2;\pi}(u_0)|_{u_0=\frac{1}{2}} \right. \\
& \quad + \frac{if_\pi}{48\pi^2} T^4 f_3 \left( \frac{\omega_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{4;\pi}(u_0)|_{u_0=\frac{1}{2}} + \frac{if_\pi m_\pi^2 m_s}{36(m_u + m_d)\pi^2} T^4 f_3 \left( \frac{\omega_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{3;\pi}^\sigma(u_0)|_{u_0=\frac{1}{2}} \\
& \quad + \frac{5if_\pi}{576\pi^2} T^4 f_3 \left( \frac{\omega_c}{T} \right) \frac{\partial}{\partial u_0} u_0 \phi_{4;\pi}(u_0)|_{u_0=\frac{1}{2}} - \frac{5if_\pi m_\pi^2 m_s}{432(m_u + m_d)\pi^2} T^4 f_3 \left( \frac{\omega_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{3;\pi}^\sigma(u_0)|_{u_0=\frac{1}{2}} \\
& \quad - \frac{if_\pi m_s}{72} \langle \bar{s}s \rangle \int_0^{\frac{1}{2}} du_1 \psi_{4;\pi}(u_1) - \frac{if_\pi m_s}{72} \langle \bar{s}s \rangle T^2 f_1 \left( \frac{\omega_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{2;\pi}(u_0)|_{u_0=\frac{1}{2}} - \frac{if_\pi m_\pi^2}{108(m_u + m_d)} \langle \bar{s}s \rangle T^2 f_1 \left( \frac{\omega_c}{T} \right) \\
& \quad \times \frac{\partial}{\partial u_0} \phi_{3;\pi}^\sigma(u_0)|_{u_0=\frac{1}{2}} - \frac{5if_\pi m_s}{864} \langle \bar{s}s \rangle T^2 f_1 \left( \frac{\omega_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{2;\pi}(u_0)|_{u_0=\frac{1}{2}} \\
& \quad + \frac{5if_\pi m_\pi^2}{1296(m_u + m_d)} \langle \bar{s}s \rangle T^2 f_1 \left( \frac{\omega_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{3;\pi}^\sigma(u_0)|_{u_0=\frac{1}{2}} + \frac{if_\pi m_s}{1152} \langle \bar{s}s \rangle \frac{\partial}{\partial u_0} \phi_{4;\pi}(u_0)|_{u_0=\frac{1}{2}} \\
& \quad + \frac{if_\pi m_\pi^2}{1728(m_u + m_d)} \langle g_s \bar{s} G s \rangle \frac{\partial}{\partial u_0} \phi_{3;\pi}^\sigma(u_0)|_{u_0=\frac{1}{2}} + \frac{5if_\pi m_s}{13824} \langle \bar{s}s \rangle \frac{\partial^2}{\partial u_0^2} u_0 \phi_{4;\pi}(u_0)|_{u_0=\frac{1}{2}} \\
& \quad - \frac{5if_\pi m_\pi^2}{20736(m_u + m_d)} \langle g_s \bar{s} G s \rangle \frac{\partial^2}{\partial u_0^2} u_0 \phi_{3;\pi}^\sigma(u_0)|_{u_0=\frac{1}{2}} \left. - \frac{1}{2} \times \left( \frac{if_\pi}{12\pi^2} T^4 f_3 \left( \frac{\omega_c}{T} \right) \int_0^{\frac{1}{2}} d\alpha_2 \right. \right. \\
& \quad \times \int_{\frac{1}{2}-\alpha_2}^{1-\alpha_2} \left( -\frac{u_0}{3\alpha_3} \frac{\partial}{\partial \alpha_3} \Phi_{4;\pi}(\underline{\alpha}) + \frac{u_0}{\alpha_3} \frac{\partial}{\partial \alpha_3} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{7u_0}{6\alpha_3} \frac{\partial}{\partial \alpha_3} \Psi_{4;\pi}(\underline{\alpha}) + \frac{u_0}{2\alpha_3} \frac{\partial}{\partial \alpha_3} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) - \frac{1}{2\alpha_3} \frac{\partial}{\partial \alpha_3} \Phi_{4;\pi}(\underline{\alpha}) \right. \\
& \quad - \frac{1}{6\alpha_3} \frac{\partial}{\partial \alpha_3} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{5}{6\alpha_3} \frac{\partial}{\partial \alpha_3} \Psi_{4;\pi}(\underline{\alpha}) - \frac{5}{6\alpha_3} \frac{\partial}{\partial \alpha_3} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) \left. - \frac{5if_\pi}{72\pi^2} T^4 f_3 \left( \frac{\omega_c}{T} \right) \int_0^{\frac{1}{2}} d\alpha_2 \int_{\frac{1}{2}-\alpha_2}^{1-\alpha_2} d\alpha_3 \left( \frac{-u_0^2}{\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \alpha_3 \Phi_{4;\pi}(\underline{\alpha}) \right. \right. \\
& \quad - \frac{\alpha_2 u_0}{\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{u_0}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{u_0}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \alpha_3 \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{u_0}{\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{\alpha_2}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{\alpha_2}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) \\
& \quad \left. \left. + \frac{1}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\underline{\alpha}) + \frac{1}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) \right) \right). \tag{60}
\end{aligned}$$

In the above expressions,  $f_n(x) \equiv 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}$ ; the parameters  $\omega$  and  $\omega'$  are transformed to be  $T_1$  and  $T_2$ , respectively; we choose  $T_1 = T_2 = 2T$  so that  $u_0 = \frac{T_1}{T_1+T_2} = \frac{1}{2}$ ; we choose  $\omega_c = 1.62$  GeV to be the average threshold value of the  $\Xi_b^-(3/2^-)$  and  $\Xi_b^*(3/2^+)$  mass sum rules; we choose  $0.268$  GeV  $< T < 0.322$  GeV to be the Borel window of the  $\Xi_b^-(3/2^-)$  mass sum rule. The light-cone distribution amplitudes contained in the above sum rule expressions can be found in Refs. [72,73,94–99].

We extract the coupling constant from Eq. (60) to be

$$\begin{aligned}
g_{\Xi_b^-(\frac{3}{2}^-) \rightarrow \Xi_b^*0 \pi^-} & = 0.08_{-0.03}^{+0.01} {}_{-0.03}^{+0.03} {}_{-0.03}^{+0.03} {}_{-0.08}^{+0.44} \text{ GeV}^{-2} \\
& = 0.08_{-0.08}^{+0.44} \text{ GeV}^{-2}, \tag{61}
\end{aligned}$$

where the uncertainties are due to the Borel mass, parameters of  $\Xi_b^*(1/2^+)$ , parameters of  $\Xi_b^-(3/2^-)$ , and various

QCD parameters given in Eqs. (13), respectively. We depict  $g_{\Xi_b^-(\frac{3}{2}^-) \rightarrow \Xi_b^*0 \pi^-}$  in Fig. 2 as a function of the Borel mass  $T$ . Its dependence on  $T$  is weak inside the Borel window  $0.268 < T < 0.322$  GeV.

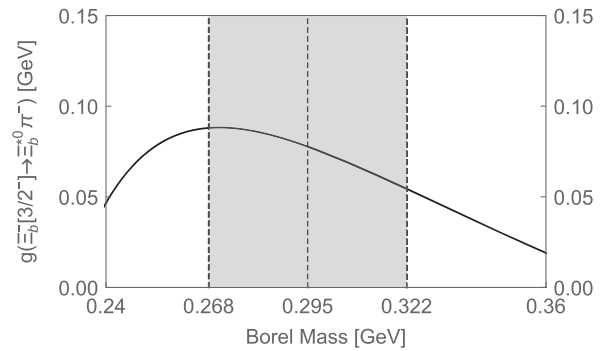


FIG. 2. The coupling constant  $g_{\Xi_b^-(\frac{3}{2}^-) \rightarrow \Xi_b^*0 \pi^-}$  as a function of the Borel mass  $T$ .

For the three-body decay process  $\Xi_b(3/2^-) \rightarrow \Xi_b^* + \pi \rightarrow \Xi_b + \pi + \pi$ , we use the formula

$$\begin{aligned}
 \Gamma(0 \rightarrow 4 + 3 \rightarrow 3 + 1 + 2) &\equiv (\Xi_b^-(3/2^-) \rightarrow \Xi_b^* + \pi \rightarrow \Xi_b + \pi + \pi) \\
 &= \frac{1}{(2\pi)^3} \frac{1}{32m_0^3} \times g_{0 \rightarrow 3+4}^2 \times g_{4 \rightarrow 2+1}^2 \times \int dm_{12} dm_{23} \frac{9}{2} \frac{1}{4} \\
 &\times \text{Tr} \left[ (\not{\epsilon}_1 + m_1) \left( g_{\beta_2 \alpha_2} - \frac{1}{3} \gamma_{\beta_2} \gamma_{\alpha_2} - \frac{P_{4,\beta_2} \gamma_{\alpha_2} - P_{4,\alpha_2} \gamma_{\beta_2}}{3m_4} - \frac{2P_{4,\beta_2} P_{4,\alpha_2}}{3m_4} \right) (\not{\epsilon}_4 + m_4) \right. \\
 &\times \left( g_{\alpha_1 \alpha_2} - \frac{1}{3} \gamma_{\alpha_2} \gamma_{\alpha_1} - \frac{P_{0,\alpha_2} \gamma_{\alpha_1} - P_{0,\alpha_1} \gamma_{\alpha_2}}{3m_0} - \frac{2P_{0,\alpha_2} P_{0,\alpha_1}}{3m_0} \right) \\
 &\times \left. (\not{\epsilon}_0 + m_0) \left( g_{\alpha_1 \beta_1} - \frac{1}{3} \gamma_{\alpha_1} \gamma_{\beta_1} - \frac{P_{4,\alpha_1} \gamma_{\beta_1} - P_{4,\beta_1} \gamma_{\alpha_1}}{3m_4} - \frac{2P_{4,\alpha_1} P_{4,\beta_1}}{3m_4} \right) (\not{\epsilon}_4 + m_4) \right] \\
 &\times \frac{P_{2,\beta_1} P_{2,\beta_2}}{|p_4^2 - m_4^2 + im_4 \Gamma_4|^2}, \tag{62}
 \end{aligned}$$

TABLE II. Decay properties of the  $P$ -wave bottom baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets, calculated using the method of light-cone sum rules within the framework of heavy quark effective theory. Possible experimental candidates are given in the last column.

Multiplets	Baryon ( $j^P$ )	Mass (GeV)	Difference (MeV)	Decay channels	$S$ -wave width (MeV)	$D$ -wave width (MeV)	Total width (MeV)	Candidate	
$[\bar{\mathbf{3}}_F, 1, 1, \rho]$	$\Lambda_b(\frac{1}{2}^-)$	$5.92_{-0.10}^{+0.13}$	$10_{-4}^{+4}$	$\Lambda_b(\frac{1}{2}^-) \rightarrow \Sigma_b \pi \rightarrow \Lambda_b \pi \pi$	$4 \times 10^{-3}$	$\dots$	$4 \times 10^{-3}$	$\Lambda_b(5912)$	
				$\Lambda_b(\frac{1}{2}^-) \rightarrow \Sigma_b^* \pi \rightarrow \Lambda_b \pi \pi$	$\dots$	$2 \times 10^{-8}$			
				$\Lambda_b(\frac{1}{2}^-) \rightarrow \Lambda_b \rho \rightarrow \Lambda_b \pi \pi$	$1 \times 10^{-4}$				
	$\Lambda_b(\frac{3}{2}^-)$	$5.93_{-0.10}^{+0.13}$		$\Lambda_b(\frac{3}{2}^-) \rightarrow \Sigma_b^* \pi \rightarrow \Lambda_b \pi \pi$	$5 \times 10^{-5}$	$3 \times 10^{-8}$	$1 \times 10^{-4}$	$\Lambda_b(5920)$	
				$\Lambda_b(\frac{3}{2}^-) \rightarrow \Sigma_b \pi \rightarrow \Lambda_b \pi \pi$	$\dots$	$5 \times 10^{-7}$			
				$\Lambda_b(\frac{3}{2}^-) \rightarrow \Lambda_b \rho \rightarrow \Lambda \pi \pi$	$7 \times 10^{-5}$				
$\Xi_b(\frac{1}{2}^-)$	$6.08_{-0.11}^{+0.13}$	$9_{-3}^{+3}$	$\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b' \pi$	$3.6_{-3.6}^{+29.4}$	$\dots$	$4_{-4}^{+29}$	$\dots$		
			$\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^* \pi \rightarrow \Xi_b \pi \pi$	$\dots$	$2 \times 10^{-6}$				
			$\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b \rho \rightarrow \Xi_b \pi \pi$	$2 \times 10^{-4}$					
			$\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b \rho \rightarrow \Xi_b \pi \pi$	$2 \times 10^{-4}$					
$\Xi_b(\frac{3}{2}^-)$	$6.09_{-0.11}^{+0.12}$		$\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b' \pi$	$\dots$	$2 \times 10^{-3}$	$0.1_{-0.1}^{+3.0}$	$\Xi_b(6100)$		
			$\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b^* \pi \rightarrow \Xi_b \pi \pi$	$0.07_{-0.07}^{+2.94}$	$2 \times 10^{-5}$				
			$\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b \rho \rightarrow \Xi_b \pi \pi$	$2 \times 10^{-4}$					
			$\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b \rho \rightarrow \Xi_b \pi \pi$	$2 \times 10^{-4}$					
$[\bar{\mathbf{3}}_F, 1, 0, \lambda]$	$\Lambda_b(\frac{1}{2}^-)$	$5.91_{-0.11}^{+0.11}$	$6_{-2}^{+2}$	$\Lambda_b(\frac{1}{2}^-) \rightarrow \Sigma_b \pi \rightarrow \Lambda_b \pi \pi$	$0.92_{-0.66}^{+1.5}$	$\dots$	$0.9_{-0.7}^{+1.5}$	$\Lambda_b(5912)$	
				$\Lambda_b(\frac{1}{2}^-) \rightarrow \Sigma_b^* \pi \rightarrow \Lambda_b \pi \pi$	$\dots$	$2 \times 10^{-7}$			
				$\Lambda_b(\frac{1}{2}^-) \rightarrow \Lambda_b \rho \rightarrow \Lambda_b \pi \pi$	$3 \times 10^{-4}$				
	$\Lambda_b(\frac{3}{2}^-)$	$5.92_{-0.11}^{+0.11}$		$\Lambda_b(\frac{3}{2}^-) \rightarrow \Sigma_b^* \pi \rightarrow \Lambda_b \pi \pi$	$0.56_{-0.37}^{+0.75}$	$2 \times 10^{-7}$	$0.6_{-0.4}^{+0.8}$	$\Lambda_b(5920)$	
				$\Lambda_b(\frac{3}{2}^-) \rightarrow \Sigma_b \pi \rightarrow \Lambda_b \pi \pi$	$\dots$	$3 \times 10^{-6}$			
				$\Lambda_b(\frac{3}{2}^-) \rightarrow \Lambda_b \rho \rightarrow \Lambda \pi \pi$	$7 \times 10^{-4}$				
	$\Xi_b(\frac{1}{2}^-)$	$6.09_{-0.10}^{+0.10}$	$5_{-2}^{+2}$	$\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b' \pi$	$1200_{-850}^{+1600}$	$\dots$	$1200_{-850}^{+1600}$	$\dots$	
				$\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b^* \pi \rightarrow \Xi_b \pi \pi$	$\dots$	$1 \times 10^{-5}$			
				$\Xi_b(\frac{1}{2}^-) \rightarrow \Xi_b \rho \rightarrow \Xi_b \pi \pi$	$6 \times 10^{-4}$				
	$\Xi_b(\frac{3}{2}^-)$	$6.10_{-0.10}^{+0.10}$		$\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b' \pi$	$\dots$	$8 \times 10^{-3}$	$240_{-150}^{+290}$	$\dots$	
				$\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b^* \pi \rightarrow \Xi_b \pi \pi$	$240_{-150}^{+290}$	$8 \times 10^{-5}$			
				$\Xi_b(\frac{3}{2}^-) \rightarrow \Xi_b \rho \rightarrow \Xi_b \pi \pi$	$1 \times 10^{-3}$				



to calculate its partial decay width to be

$$\Gamma_{\Xi_b^-[3/2^-] \rightarrow \Xi_b^0 \pi^- \rightarrow \Xi_b^0 \pi^0 \pi^-}^S = 0.07_{-0.07}^{+2.94} \text{ MeV}. \quad (63)$$

Similarly, we study the other decay channels for the bottom baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets. Their partial decay widths are evaluated and summarized in Table II. Our results suggest that the  $\Xi_b(6100)$  can be well explained as the  $P$ -wave bottom baryon of  $J^P = 3/2^-$  belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet, while the  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublet is disfavored.

#### IV. SUMMARY AND DISCUSSIONS

In this paper, we study the  $\Xi_b(6100)$  as a possible  $P$ -wave bottom baryon of  $J^P = 3/2^-$  belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet. This doublet contains four  $P$ -wave bottom baryons:  $\Lambda_b(1/2^-)$ ,  $\Lambda_b(3/2^-)$ ,  $\Xi_b(1/2^-)$ , and  $\Xi_b(3/2^-)$ . We calculate their masses using the QCD sum rule method within the framework of heavy quark effective theory. We also study their  $S$ - and  $D$ -wave decays into the ground-state bottom baryons and light pseudoscalar and vector mesons through the light-cone sum rule method. The obtained results are summarized in Table II, where the masses and total widths are evaluated to be

$$\begin{aligned} M_{\Lambda_b(1/2^-)} &= 5.92_{-0.10}^{+0.13} \text{ MeV}, \\ \Gamma_{\Lambda_b(1/2^-)} &\sim 0, \\ M_{\Lambda_b(3/2^-)} &= 5.93_{-0.10}^{+0.13} \text{ MeV}, \\ \Gamma_{\Lambda_b(3/2^-)} &\sim 0, \\ M_{\Lambda_b(3/2^-)} - M_{\Lambda_b(1/2^-)} &= 10 \pm 4 \text{ MeV}, \\ M_{\Xi_b(1/2^-)} &= 6.08_{-0.11}^{+0.13} \text{ MeV}, \\ \Gamma_{\Xi_b(1/2^-)} &= 4_{-4}^{+29} \text{ MeV}, \\ M_{\Xi_b(3/2^-)} &= 6.09_{-0.11}^{+0.12} \text{ MeV}, \\ \Gamma_{\Xi_b(3/2^-)} &= 0.1_{-0.1}^{+3.0} \text{ MeV}, \\ M_{\Xi_b(3/2^-)} - M_{\Xi_b(1/2^-)} &= 9 \pm 3 \text{ MeV}. \end{aligned} \quad (64)$$

Our results suggest that the  $\Xi_b(6100)$  can be well explained as the  $P$ -wave bottom baryon of  $J^P = 3/2^-$  belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet. The  $\Lambda_b(5912)$  and  $\Lambda_b(5920)$  can be explained as its partner states with  $J^P = 1/2^-$  and  $3/2^-$ , respectively. Besides, there is a  $P$ -wave bottom baryon still missing, the  $\Xi_b(1/2^-)$ , whose mass is  $\Delta M = 9 \pm 3$  MeV smaller than the  $\Xi_b(6100)$ . We propose to search for it in the  $\Xi_c(1/2^-) \rightarrow \Xi_b^* \pi$  decay channel.

Besides the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet, the  $\Xi_b(6100)$  may also belong to the  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublet. In the present study, we have also investigated the latter doublet, and the obtained results are also summarized in Table II. These results can

also be used to explain the  $\Lambda_b(5912)$  and  $\Lambda_b(5920)$ , so our QCD sum rule results cannot distinguish whether they belong to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet or the  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublet. This is partly because their decays into the  $\Sigma_b^{(*)} \pi$  channels are kinematically forbidden so that their widths are limited. However, our results for the  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublet cannot be used to easily explain the  $\Xi_b(6100)$ . We would like to note that the above results are just possible explanations, and there exist many other possibilities. Especially, our QCD sum rule results for the  $\Xi_b(6100)$  seem to be not consistent with the quark model expectation [92,93], so further experimental and theoretical studies are crucially demanded to fully understand them. There exists a relevant question on how to explain the five excited  $\Omega_c$  baryons observed by LHCb [100], given that at most four of them can be explained as the  $P$ -wave excitations of the  $\lambda$  mode [56,101]. There are two possible assignments for the rest of them: either the radial  $2S$ -wave excitation or the orbital  $1P$ -wave excitation of the  $\rho$  mode. The experimental measurements on the quantum numbers of these excited  $\Omega_c$  baryons can be important and helpful to understand the  $\rho$ -mode excitations.

For completeness, we have also investigated the  $P$ -wave charmed baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets. The obtained results are summarized in the Appendix, suggesting that the  $\Lambda_c(2595)$ ,  $\Xi_c(2790)$ ,  $\Lambda_c(2625)$ , and  $\Xi_c(2815)$ , as the charmed partner states of the  $\Lambda_b(5912)$ ,  $\Lambda_b(5920)$ ,  $\Xi_b(1/2^-)$ , and  $\Xi_b(6100)$ , can be well explained as the  $P$ -wave charmed baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet.

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#### APPENDIX: $P$ -WAVE CHARMED BARYONS FROM $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ AND $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$

In this appendix, we study the  $P$ -wave charmed baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets. We apply the QCD sum rule method to study the mass spectrum, and the obtained results are summarized in Table III. We apply the light-cone sum rule method to study the decay properties, and the obtained results are summarized in Table IV. These results suggest that the  $\Lambda_c(2595)$ ,  $\Xi_c(2790)$ ,  $\Lambda_c(2625)$ , and  $\Xi_c(2815)$ , as the charmed partner states of the  $\Lambda_b(5912)$ ,  $\Lambda_b(5920)$ ,  $\Xi_b(1/2^-)$ , and  $\Xi_b(6100)$ , can be well explained as the  $P$ -wave charmed baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  doublet.

TABLE III. Parameters of the  $P$ -wave charmed baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets, calculated using the method of QCD sum rules within the framework of heavy quark effective theory.

Multiplets	B	$\omega_c$ (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	Baryon ( $j^P$ )	Mass (GeV)	Difference (MeV)	Decay constant (GeV <sup>4</sup> )
$[\bar{\mathbf{3}}_F, 1, 1, \rho]$	$\Lambda_c$	1.53	$0.27 < T < 0.29$	$1.16 \pm 0.09$	$\Lambda_b(1/2^-)$	$2.59^{+0.14}_{-0.13}$	$51^{+20}_{-18}$	$0.051 \pm 0.012$ ( $\Lambda_c^+(1/2^-)$ )
					$\Lambda_c(3/2^-)$	$2.64^{+0.13}_{-0.12}$		$0.024 \pm 0.006$ ( $\Lambda_c^+(3/2^-)$ )
	$\Xi_c$	1.78	$0.27 < T < 0.32$	$1.33 \pm 0.10$	$\Xi_c(1/2^-)$	$2.78^{+0.17}_{-0.16}$	$42^{+17}_{-15}$	$0.076 \pm 0.019$ ( $\Xi_c^0(1/2^-)$ )
					$\Xi_c(3/2^-)$	$2.82^{+0.15}_{-0.13}$		$0.036 \pm 0.009$ ( $\Xi_c^0(3/2^-)$ )
$[\bar{\mathbf{3}}_F, 1, 0, \lambda]$	$\Lambda_c$	1.43	$T = 0.30$	$0.95 \pm 0.07$	$\Lambda_b(1/2^-)$	$2.65^{+0.09}_{-0.07}$	$33^{+12}_{-12}$	$0.019 \pm 0.004$ ( $\Lambda_c^+(1/2^-)$ )
					$\Lambda_c(3/2^-)$	$2.68^{+0.09}_{-0.07}$		$0.009 \pm 0.002$ ( $\Lambda_c^+(3/2^-)$ )
	$\Xi_c$	1.68	$T = 0.33$	$1.15 \pm 0.09$	$\Xi_c(1/2^-)$	$2.91^{+0.12}_{-0.11}$	$27^{+10}_{-9}$	$0.032 \pm 0.006$ ( $\Xi_c^0(1/2^-)$ )
					$\Xi_c(3/2^-)$	$2.94^{+0.12}_{-0.11}$		$0.015 \pm 0.003$ ( $\Xi_c^0(3/2^-)$ )

 TABLE IV. Decay properties of the  $P$ -wave charmed baryons belonging to the  $[\bar{\mathbf{3}}_F, 1, 1, \rho]$  and  $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$  doublets, calculated using the method of light-cone sum rules within the framework of heavy quark effective theory. Possible experimental candidates are given in the last column.

Multiplets	Baryon ( $j^P$ )	Mass (GeV)	Difference (MeV)	Decay channels	$S$ -wave width (MeV)	$D$ -wave width (MeV)	Total width (MeV)	Candidate	
$[\bar{\mathbf{3}}_F, 1, 1, \rho]$	$\Lambda_c(\frac{1}{2}^-)$	$2.59^{+0.14}_{-0.13}$	$51^{+20}_{-18}$	$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \pi \rightarrow \Lambda_c \pi \pi$	$5.4^{+40.2}_{-4.7}$	$\dots$	$5^{+40}_{-5}$	$\Lambda_c(2595)$	
				$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$	$\dots$	$6 \times 10^{-8}$			
				$\Lambda_c(\frac{1}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	$8 \times 10^{-4}$				
	$\Lambda_c(\frac{3}{2}^-)$	$2.64^{+0.13}_{-0.12}$		$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c \pi$	$\dots$	$4 \times 10^{-3}$	$0.1^{+0.5}_{-0.1}$	$\Lambda_c(2625)$	
				$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$	$0.06^{+0.53}_{-0.06}$	$9 \times 10^{-7}$			
				$\Lambda_c(\frac{3}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	$0.01^{+0.03}_{-0.01}$				
	$\Xi_c(\frac{1}{2}^-)$	$2.78^{+0.17}_{-0.16}$	$42^{+17}_{-15}$	$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c' \pi$	$9.0^{+59.8}_{-9.0}$	$\dots$	$9^{+60}_{-9}$	$\Xi_c(2790)$	
				$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_c \pi \pi$	$\dots$	$1 \times 10^{-5}$			
				$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	$2 \times 10^{-3}$				
	$\Xi_c(\frac{3}{2}^-)$	$2.82^{+0.15}_{-0.13}$		$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_c \pi \pi$	$4.3^{+49.6}_{-4.3}$	$1 \times 10^{-4}$	$4^{+50}_{-4}$	$\Xi_c(2815)$	
				$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c' \pi$	$\dots$	$0.03$			
				$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	$0.01$				
$[\bar{\mathbf{3}}_F, 1, 0, \lambda]$	$\Lambda_c(\frac{1}{2}^-)$	$2.65^{+0.09}_{-0.07}$	$33^{+12}_{-12}$	$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \pi$	$> 560$	$\dots$	$> 560$	$\dots$	
				$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$	$\dots$	$2 \times 10^{-8}$			
				$\Lambda_c(\frac{1}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	$2 \times 10^{-3}$				
	$\Lambda_c(\frac{3}{2}^-)$	$2.68^{+0.09}_{-0.07}$		$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c \pi$	$\dots$	$0.05$	$35^{+49}_{-26}$	$\dots$	
				$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$	$35^{+49}_{-26}$	$2 \times 10^{-5}$			
				$\Lambda_c(\frac{3}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	$0.01$				
	$\Xi_c(\frac{1}{2}^-)$	$2.91^{+0.12}_{-0.11}$	$27^{+10}_{-9}$	$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c' \pi$	$> 1500$	$\dots$	$> 1500$	$\dots$	
				$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_c \pi \pi$	$\dots$	$1 \times 10^{-4}$			
				$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	$5 \times 10^{-3}$				
	$\Xi_c(\frac{3}{2}^-)$	$2.94^{+0.12}_{-0.11}$		$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_b \pi \pi$	$> 230$	$9 \times 10^{-4}$	$> 230$	$\dots$	
				$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c' \pi$	$\dots$	$0.16^{+0.23}_{-0.11}$			
				$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	$0.02$				

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