Interpreting B anomalies within an extended 331 gauge theory

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In light of the recent $R_{K^{(*)}}$ data on neutral current flavor anomalies in $B \to K^{(*)}\ell^+\ell^-$ decays, we reexamine their quantitative interpretation in terms of an extended 331 gauge theory framework. We achieve this by adding two extra lepton species with novel 331 charges, while ensuring that the model remains anomaly-free. In contrast to the canonical 331 models, the gauge charges of the first and second lepton families differ from each other, allowing lepton-flavor universality violation. We further expand the model by adding the neutral fermions required to provide an adequate description for small neutrino masses.

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I. INTRODUCTION

In the past decade, several measurements of b-quark decays with final leptons have shown disagreement with the overly successful Standard Model (SM). Such disagreements are collectively referred to as "flavor anomalies," and they typically feature tensions at the level of 2–3 standard deviations between experimental results and SM predictions. An interesting aspect of these anomalies lies in the fact that they all seem to point toward the presence of lepton-flavor universality (LFU) violation in the interactions mediating the processes. Last year, the measurements of rare decays $B^+ \to K^+\ell^+\ell^-$, with ℓ denoting an electron or a muon, provided further evidence for the breaking of LFU in beauty-quark decays in a single process, with a significance of 3.1 standard deviations based on 9 fb^{-1} of proton-proton collision data collected at LHCb [1].

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The accuracy of the predictions for the branching fractions of semileptonic B decays is generally higher than the one of hadronic decays, due to the reliability of perturbative techniques. Moreover, this precision can be further increased by taking ratios of processes with electrons or muons in the final state since they are affected equally by the strong force, which does not couple directly to leptons. Thus, to minimize the hadronic uncertainties, one usually introduces branching fraction ratios, which in the case of $B \to K^{(*)}\ell^+\ell^-$, can be defined as

$$R_{K^{(*)}[q^2_{\min},q^2_{\max}]} = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)_{q^2\in[q^2_{\min},q^2_{\max}]}}{\mathcal{B}(B \to K^{(*)}e^+e^-)_{q^2\in[q^2_{\min},q^2_{\max}]}}, \quad (1)$$

where $\mathcal{B}$ denotes the branching fraction for the given decay mode measured over a bin size of $[q^2_{\min}, q^2_{\max}]$. The resulting $R_{K^{(*)}}$ are measured over specific ranges for the squared dilepton invariant mass q^2.

The $B \to K^{(*)}\ell^+\ell^-$ decays are driven at the quark level by the $b \to s\ell^+\ell^-$ decay. The hadronic process involved is mediated by flavor changing neutral currents, which are forbidden at tree level in the SM. The branching fractions in the ratio $R_{K^{(*)}}$ differ only by the leptons in the final state; hence, this ratio is expected to be 1 by virtue of LFU, with small deviations induced by phase space differences and QED corrections. By comparing recent LHCb experimental values with theoretical determinations, we have [1–4]

$$R^{\rm exp}_{K^+[1.1,6.0]} = 0.846^{+0.042+0.013}_{-0.039-0.012}\,, \qquad R^{\rm th}_{K^+} = 1.00 \pm 0.01\,, \qquad 3.1\sigma,$$
$$R^{\rm exp}_{K^{*0}[0.045,1.1]} = 0.66^{+0.11}_{-0.07} \pm 0.03, \qquad R^{\rm th}_{K^{*0}[0.045,1.1]} = 0.922 \pm 0.022\,, \qquad 2.3\sigma,$$
$$R^{\rm exp}_{K^{*0}[1.1,6.0]} = 0.69^{+0.11}_{-0.07} \pm 0.05\,, \qquad R^{\rm th}_{K^{*0}[1.1,6.0]} = 1.000 \pm 0.006\,, \qquad 3.4\sigma, \tag{2}$$

where q^2 is given in GeV^2. In the experimental data, the first errors are statistical and the second ones systematic. The first result is the most precise measurement to date and consistent with the SM prediction with a p value of 0.10%. This gives evidence for the violation of lepton universality in these decays with a significance of 3.1σ. We have also listed the statistical significance of the anomalies for the other experimental results.

Recently, LHCb investigated $B^0 \to K^0_S \ell^+\ell^-$ and $B^+ \to K^{*+}\ell^+\ell^-$ decays, with ℓ being an electron or a muon. Notice that these decays involve mesons which are the isospin partners of the ones in the previously measured channels $B^+ \to K^+\ell^+\ell^-$ and $B^0 \to K^{*0}\ell^+\ell^-$. Although these decays have similar branching fractions as their isospin partners, they suffer from a reduced experimental efficiency at LHCb, due to the presence of a long-lived K^0_S or π^0 meson in the final states. The measured ratios are

$$R^{\rm exp}_{K^0_S[1.1,6.0]} = 0.66^{+0.20+0.02}_{-0.14-0.04}\ [5],$$
$$R^{\rm exp}_{K^{*+}[0.045,6.0]} = 0.70^{+0.18+0.03}_{-0.13-0.04}\ [5], \tag{3}$$

and provide $\sim 1.5\sigma$ hints of departures from the SM [5].

Recent experimental determinations of R_{K^*} have also been given by the Belle Collaboration, using the full $\Upsilon(4S)$ data sample containing 772×10^6 $B\bar{B}$ events. For the same range of dilepton invariant mass reported in (2), they find

$$R^{\rm exp}_{K^{*0}[0.045,1.1]} = 0.46^{+0.55}_{-0.27} \pm 0.13\ [6],$$
$$R^{\rm exp}_{K^{*0}[1.1,6.0]} = 1.06^{+0.63}_{-0.38} \pm 0.14\ [6],$$
$$R^{\rm exp}_{K^{*+}[0.045,6.0]} = 0.62^{+0.60}_{-0.36} \pm 0.09\ [6]. \tag{4}$$

BABAR, Belle, and LHCb have provided other prominent contributions to ratio determinations in these as well as in different channels [2,5–12].

The primary requirement for any model to explain these $b \to s$ anomalies is to have a symmetry which distinguishes between semileptonic B decays to $\mu^+\mu^-$ and to e^+e^- such that $R_{K^{(*)}}$ deviates appreciably from 1. Within the SM, this cannot be achieved as the theory is sequential, so that the e^- and μ^- carry the same gauge charges. One possible way out is to postulate a new $\mathrm{U}(1)_X$ gauge symmetry under which e^- and μ^- carry different charges [13–17].

Here we propose that the deviation from $R_{K^{(*)}} = 1$ can be achieved from a bigger nontrivial gauge symmetry, from which the standard $\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y}$ gauge group emerges as a subgroup. We do this in the framework of the so-called 331 models [18–20], which constitute one of the simplest well-motivated extensions of the SM. The name 331 follows from their extended $\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(3)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{X}$ gauge group. Several issues which remain unanswered within the SM, for instance, the origin of light-neutrino masses, typically call for larger gauge structures and/or new particles. Grand unified theories certainly play an important role in this respect, but 331 models have the advantage that they can provide scenarios where larger gauge symmetries can be probed already at the TeV scale. Moreover, so far no hard evidence in favor of conventional unification schemes has been found. Here we note that 331 models lead to a consistent theoretical structure and also a phenomenologically viable weak neutral current.[1,2] As a result, 331-based extensions have attracted a lot of interest; see, for instance, [28–32]. These models experience two stages of breaking: at a larger scale Λ_NP, the extended group is broken down to the SM gauge group, while the electroweak symmetry breaking occurs at the lower scale Λ_EW. Phenomenologically, these models feature additional heavy gauge bosons, as well as an extended Higgs sector to drive the two spontaneous symmetry breakdowns.

Left-handed fermions transform according to one of the two fundamental representations, i.e., triplets (or antitriplets) under the action of $\mathrm{SU}(3)_L$. In the simplest version of 331 theories [18,19], exactly three families emerge from the cancellation of chiral anomalies, which requires that the number of triplets matches the number of antitriplets. In contrast to the SM, where the anomaly is canceled within each generation of fermions, in these 331 models all families must be considered to fulfill the anomaly cancellation. Since quarks come in three colors, there must be three families of quarks and leptons, with leptons appearing in the same fundamental representation of the group. As a result, their couplings with gauge bosons are necessarily family independent, preventing any LFU violation in their gauge couplings.

[1]Earlier 331 models were suggested to account for the high y anomaly, which turned out to be a fake [21–24], while shedding light on mysteries such as the number of particle families.

[2]Recently, there have been some attempts to relate the $(g-2)_\mu$ anomaly with 331 models (see Refs. [25–27]).

Here we are concerned with other versions of the 331 model extending the lepton sector with additional species. This assumption allows us to choose at least one lepton family transforming differently from the others, ensuring the presence of LFU violation. The minimal choice preserving anomaly cancellation requires two additional lepton species. These versions of the 331 model have been considered in Refs. [33–38]. In the preliminary analysis [39], it was studied whether they can reproduce the anomalies observed in $b \to s\ell\ell$ processes under simple assumptions: LFU violation is dominated by neutral gauge boson exchange with no significant lepton-flavor violation of the form $b \to s\ell_1\ell_2$ or large contributions to $B_s - \bar{B}_s$ mixing. It was found that under these simple assumptions, an extended 331 model without exotic electric charges for fermions and gauge bosons can yield large contributions to (C_9^μ, C_{10}^μ) in good agreement with 2018 global fit analyses [40]. This result is rather nontrivial, given that the model is quite constrained.

Apart from providing an updated numerical analysis including the recent B-anomaly data, here we fully develop the proposal in Ref. [39] by adding the neutral fermions required for an adequate description of the neutrino mass matrix. In addition to gauge symmetries, we assume the presence of two auxiliary discreet $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries, which are needed in order to ensure an adequate pattern of fermion masses. As we describe in Sec. II in more detail, the primary purpose of the $\mathbb{Z}_3$ symmetry is to forbid direct gauge invariant couplings between SM and exotic leptons. The presence of such a coupling would imply either unacceptable large masses for SM leptons or unacceptable small masses for exotic charged leptons, both scenarios, of course, being experimentally rejected. An additional $\mathbb{Z}_2$ is further needed to generate different masses for SM and exotic fermions which carry the same gauge quantum numbers, without the need for fine-tuning.

The paper is organized as follows. In Sec. II, we sketch the model and its field representations. In Sec. III, we discuss the Yukawa interactions, including those used in the implementation of the seesaw mechanism. In Sec. IV, we comment on fermion mass generation, including neutrino masses. In Sec. V, we perform a comparison with B flavor global analyses. We find that this 331 model can generate large new physics contributions to (C_9^μ, C_{10}^μ) parameters, in agreement with new physics scenarios favored by global fits. In Sec. VI, we present our conclusions.

II. THE MODEL

Apart from gluons, any 331 model has nine vector bosons associated with each generator of the gauge group, eight W_μ^a for $\mathrm{SU}(3)_\mathrm{L}$ and one X_μ for $\mathrm{U}(1)_\mathrm{X}$. We indicate the generators of the $\mathrm{SU}(3)_\mathrm{L}$ gauge group with $\hat{T}^1 \cdots \hat{T}^8$ normalized as $\mathrm{Tr}[\hat{T}^i\hat{T}^j] = \delta^{ij}/2$, and define the $\mathrm{U}(1)_X$ generator as $\hat{T}^9 = \mathbb{1}/\sqrt{6}$, where $\mathbb{1} = \mathrm{diag}(1,1,1)$ is the identity matrix. The electric charge is defined in general as a linear combination of the diagonal generators of the group

$$\hat{Q} = a\hat{T}^3 + \beta\hat{T}^8 + X\mathbb{1}, \tag{5}$$

where the values of the proportionality constants a and β distinguish different 331 models. We have $\hat{T}^3 = 1/2\hat{\lambda}^3 = 1/2\mathrm{diag}(1,-1,0)$ and $\hat{T}^8 = 1/2\hat{\lambda}^8 = 1/(2\sqrt{3})\mathrm{diag}(1,1,-2)$, where $\hat{\lambda}^i$ are the Gell-Mann matrices. X is the quantum number associated with $\mathrm{U}(1)_X$. We set $a = 1$ to obtain isospin doublets which embed $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ into $\mathrm{SU}(3) \otimes \mathrm{U}(1)$. In order to restrict β, we demand that no new particle introduced in the model has exotic charges (i.e., different from the SM ones). This can be done by choosing the particular value

$$\beta = -1/\sqrt{3}, \tag{6}$$

which is the original assignment made in [18]. We will thus have the following definition of the electric charge operator:

$$\hat{Q} = \hat{T}^3 - \frac{1}{\sqrt{3}}\hat{T}^8 + X\mathbb{1}. \tag{7}$$

Complex gauge fields are defined by the combinations $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$, $V_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^6 \mp iW_\mu^7)$, and $Y_\mu^{0(0_*)} = \frac{1}{\sqrt{2}}(W_\mu^4 \mp iW_\mu^5)$, where the superscripts $\pm, 0$ denote the electric charges of the fields, a notation we will follow throughout this work. In general, the values of the electric charges of the V_μ and Y_μ bosons depend on the value of β. With our choice of $\beta = -\frac{1}{\sqrt{3}}$, the electric charges of all gauge bosons are fixed to either ± 1 or 0, i.e., nonanomalous values.

A. Symmetry breaking

Starting from the $\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(3)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{X}$ gauge group (with gauge couplings g_S, g, g_X), the model will undergo two spontaneous symmetry breakings (SSBs) triggered by color singlet scalar fields acquiring nonvanishing vacuum expectation values, in a way analogous to the SM. The overall pattern of SSB is the following:

$$\mathrm{SU}(3)_\mathrm{c} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{U}(1)_X \underset{\Lambda_\mathrm{NP}}{\rightarrow} \mathrm{SU}(3)_\mathrm{c} \times \mathrm{SU}(2)_\mathrm{L} \times U(1)_Y \underset{\Lambda_\mathrm{EW}}{\rightarrow} \mathrm{SU}(3)_\mathrm{c} \times \mathrm{U}(1)_\mathrm{EM}.$$

The first SSB occurs at an energy scale Λ_NP and allows us to recover the SM gauge group. The subsequent one, at energy scale $\Lambda_{\rm EW}$, reproduces the electroweak symmetry breaking (EWSB) of the SM. We assume that $\Lambda_{\rm NP} \gg \Lambda_{\rm EW}$, and introduce a small parameter $\epsilon = \Lambda_{\rm EW}/\Lambda_{\rm NP}$ characterizing the order of magnitude of the new physics (NP).

As in the SM, the Higgs fields, besides giving mass to the gauge bosons, are used to generate fermion mass terms through gauge invariant Yukawa terms. The need to build gauge invariant terms in such a way to obtain appropriate mass terms after SSB constrains possible scalar Higgs field representations. Since the fermions transform either as a 3 or as a $\bar{3}$ under $\mathrm{SU}(3)_{\rm L}$, we only have a limited number of possibilities [36] for a scalar field Φ, which at both stages can only be a triplet, a sextet, or a singlet.[3]

We assume that the breaking of the $\mathrm{SU}(3)_{\rm L}$ symmetry is accomplished through two triplets χ and $\tilde{\chi}$ and a sextet S_1. There are five gauge fields that acquire a mass of the order of $\Lambda_{\rm NP}$, whereas the remaining three gauge fields are the SM gauge bosons. At the first SSB stage, the gauge bosons acquiring mass are the charged ones $V^{\pm}$, the neutral gauge bosons $Y^{0(0_*)}$, and a massive neutral gauge boson Z' given as a combination of the two neutral gauge bosons X, W^8, which also yields the gauge boson B. Their mixing angle θ_{331} is given by

$$\begin{pmatrix} Z' \\ B \end{pmatrix} = \begin{pmatrix} \cos\theta_{331} & -\sin\theta_{331} \\ \sin\theta_{331} & \cos\theta_{331} \end{pmatrix} \begin{pmatrix} X \\ W^8 \end{pmatrix}. \tag{8}$$

The angle θ_{331} is found by singling out the Z' field in the sector of the Lagrangian including the masses of the gauge bosons, which follow from the covariant derivative in the Higgs Lagrangian. It yields

$$\sin\theta_{331} = \frac{g}{\sqrt{g^2 + \frac{g_X^2}{18}}}, \qquad \cos\theta_{331} = -\frac{\frac{g_X}{3\sqrt{2}}}{\sqrt{g^2 + \frac{g_X^2}{18}}}, \tag{9}$$

where g,g_X denote the coupling constants for $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$, respectively.

The second stage of symmetry breaking is the usual electroweak symmetry breaking to the electromagnetic subgroup, i.e., $\mathrm{SU}(3)_{\rm c} \otimes \mathrm{SU}(2)_{\rm L} \otimes \mathrm{U}(1)_{\rm Y} \to \mathrm{SU}(3)_{\rm c} \otimes \mathrm{U}(1)_{\rm EM}$. This breaking is driven by the triplets $\eta, \rho, \tilde{\eta}, \tilde{\rho}$, and the sextet S_c. After electroweak symmetry breaking, the neutral gauge bosons W^3 and B mix with each other to give the SM Z and γ bosons as follows:

$$\begin{pmatrix} Z \\ \gamma \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}, \tag{10}$$

where the mixing angle θ_W is the usual electroweak mixing angle.

Summarizing, our scalar sector is similar to that in Ref. [39], except for the addition of the triplets $\tilde{\chi}, \tilde{\eta}, \tilde{\rho}$, and the removal of the sextet S_b, for reasons that will be detailed later. The two-step spontaneous symmetry breaking ensures that all the new gauge bosons indeed get large masses through the large VEVs of the scalars breaking $\mathrm{SU}(3)_{\rm c} \otimes \mathrm{SU}(3)_{\rm L} \otimes \mathrm{U}(1)_{\rm X} \to \mathrm{SU}(3)_{\rm c} \otimes \mathrm{SU}(2)_{\rm L} \otimes \mathrm{U}(1)_{\rm Y}$. Only the SM gauge bosons get their masses in the second symmetry breaking step due to the electroweak-scale VEV carried by the scalars breaking $\mathrm{SU}(3)_{\rm c} \otimes \mathrm{SU}(2)_{\rm L} \otimes \mathrm{U}(1)_{\rm Y} \to \mathrm{SU}(3)_{\rm c} \otimes \mathrm{U}(1)_{\rm EM}$.

B. Matter content

In the previous section, we have discussed the gauge structure and symmetry breaking pattern; here we focus on the matter content of our 331 model, looking into the details of the charge assignment. This 331 model contains three families of left-handed quarks and five families of left-handed leptons [33,34,37–39]. They all belong to the fundamental representations of $\mathrm{SU}(3)_{\rm L}$. Two generations of quarks and one of leptons behave as antitriplets, and all the others as triplets of $\mathrm{SU}(3)_{\rm L}$. This fermion content ensures at the same time the cancellation of the anomalies and allows LFU violation, but otherwise departs from the SM as little as possible. Fixing $\beta = -1/\sqrt{3}$ has ensured that both SM and new fields in the spectra all have nonexotic charges.

Using the notation $(\mathrm{SU}(3)_{\rm c}, \mathrm{SU}(3)_{\rm L}, \mathrm{U}_X(1))$ while referring to the representations of the fermions, we write for the left-handed ones

(i) three families of quarks[4]

$$q_m = \begin{pmatrix} d_m^L \\ -u_m^L \\ B_m^L \end{pmatrix} \sim (3, \bar{3}, 0), \quad m = 1, 2,$$
$$q_3 = \begin{pmatrix} u_3^L \\ d_3^L \\ T_3^L \end{pmatrix} \sim \left(3, 3, \frac{1}{3}\right), \tag{11}$$

[3]A 331 gauge singlet scalar can in principle contribute to the neutral fermion mass term; however, since it does not change our conclusions, we ignore this possibility for simplicity. Though we have a different number of triplets and sextets from in Ref. [36], their conclusions on the structure of the gauge boson mass sector do not change, since we assume the same vacuum expectation value (VEV) alignments.

[4]Note that the order in which the triplet components are arranged is a matter of choice. An alternative convention is to have the first component of quark triplets to be up-type, whereas the others are down-type. For leptons, the upper one would be charged, while the others neutral. The third component is always exotic [18].

TABLE I. Particle content of the 331 model, where in addition to the $SU(3)_c$, $SU(3)_L$, $U_X(1)$ gauge symmetries, we have listed two Abelian discrete symmetries; see text.

	Fields	$SU(3)_c \otimes SU(3)_L \otimes U(1)_X$	$\mathbb{Z}_3$	$\mathbb{Z}_2$	Fields	$SU(3)_c \otimes SU(3)_L \otimes U(1)_X$	$\mathbb{Z}_3$	$\mathbb{Z}_2$
Quarks	$q_{1,2}$	$(\mathbf{3},\bar{\mathbf{3}},\mathbf{0})$	$\mathbf{1}$	$\mathbf{1}$	q_3	$(\mathbf{3},\mathbf{3},\mathbf{1/3})$	$\mathbf{1}$	$\mathbf{1}$
	$u_{1,2,3}$	$(\mathbf{3},\mathbf{1},\mathbf{2/3})$	ω^2	$\mathbf{1}$	$d_{1,2,3}$	$(\mathbf{3},\mathbf{1},-\mathbf{1/3})$	ω	$\mathbf{1}$
	T_3	$(\mathbf{3},\mathbf{1},\mathbf{2/3})$	ω^2	$\mathbf{1}$	$B_{1,2}$	$(\mathbf{3},\mathbf{1},-\mathbf{1/3})$	ω	$\mathbf{1}$
Leptons	ℓ_1	$(\mathbf{1},\bar{\mathbf{3}},-\mathbf{2/3})$	$\mathbf{1}$	$\mathbf{1}$	$\ell_{2,3}$	$(\mathbf{1},\mathbf{3},-\mathbf{1/3})$	ω	$\mathbf{1}$
	$e_{1,2}$	$(\mathbf{1},\mathbf{1},-\mathbf{1})$	ω	$\mathbf{1}$	E_1	$(\mathbf{1},\mathbf{1},-\mathbf{1})$	ω	$-\mathbf{1}$
	L_4	$(\mathbf{1},\mathbf{3},-\mathbf{1/3})$	ω	$-\mathbf{1}$	L_5	$(\mathbf{1},\mathbf{3},\mathbf{2/3})$	ω	$-\mathbf{1}$
	$\nu^R_{1,2,3}$	$(\mathbf{1},\mathbf{1},\mathbf{0})$	$\mathbf{1}$	$\mathbf{1}$				
Scalars	χ	$(\mathbf{1},\mathbf{3},\mathbf{1/3})$	ω	$\mathbf{1}$	S_1	$(\mathbf{1},\mathbf{6},-\mathbf{2/3})$	ω^2	$\mathbf{1}$
	$\tilde{\chi}$	$(\mathbf{1},\mathbf{3},\mathbf{1/3})$	ω	$-\mathbf{1}$	$\tilde{\eta}$	$(\mathbf{1},\mathbf{3},\mathbf{1/3})$	ω	$-\mathbf{1}$
	η	$(\mathbf{1},\mathbf{3},\mathbf{1/3})$	ω	$\mathbf{1}$	ρ	$(\mathbf{1},\mathbf{3},\mathbf{2/3})$	ω^2	$\mathbf{1}$
	S_c	$(\mathbf{1},\mathbf{6},\mathbf{4/3})$	ω^2	$\mathbf{1}$	$\tilde{\rho}$	$(\mathbf{1},\mathbf{3},\mathbf{2/3})$	$\mathbf{1}$	$\mathbf{1}$

(ii) five species of leptons

$$\ell_1 = \begin{pmatrix} e_1^{-L} \\ -\nu_1^L \\ E_1^{-L} \end{pmatrix} \sim \left(1, \bar{3}, -\frac{2}{3}\right),$$

$$\ell_n = \begin{pmatrix} \nu_n^L \\ e_n^{-L} \\ N_n^{0L} \end{pmatrix} \sim \left(1, 3, -\frac{1}{3}\right), \quad n = 2, 3,$$

$$L_4 = \begin{pmatrix} \nu_4^{0L} \\ E_4^{-L} \\ N_4^{0L} \end{pmatrix} \sim \left(1, 3, -\frac{1}{3}\right),$$

$$L_5 = \begin{pmatrix} (E_4^{-R})^c \\ N_5^{0L} \\ (e_3^{-R})^c \end{pmatrix} \sim \left(1, 3, \frac{2}{3}\right). \tag{12}$$

Notice that, as in the original 331 model in [18], no positively charged leptons have been introduced in the triplets. Indeed, they would only appear in L_5, but we identify them with the charge conjugate of the right-handed components of E_4^- and e_3^-. This economical identification avoids the presence of charged exotic particles at the electroweak scale. We have labeled the SM fermions with lowercase (e_i, ν_i with $i = 1, 2, 3$), and the exotic ones with ν_4 and uppercase ($E_{1,4}, N_{2,3,4,5}$), choosing letters and/or superscripts recalling their electric charge assignments and chirality. In contrast, reference to chirality has been eliminated for simplicity when naming left-handed triplets/antitriplets as a whole: left-handed SM quarks, SM leptons, and exotic leptons are indicated with $q_{1,2,3}$, $\ell_{1,2,3}$, and $L_{4,5}$, respectively. Capital letters have been used for the last two triplets because they include only exotic fermions.

The right-handed components of charged fermions are defined as singlets of $SU(3)_L$; the SM ones are labeled as $u_{1,2,3}$, $d_{1,2,3}$, and $e_{1,2}$ with lowercase, and the exotic ones $B_{1,2}$, T_3, and E_1 with uppercase, without any chirality or charge superscript. Altogether, we have the following list of right-handed fermions:

(i) The quark fields

$$\begin{aligned} d_{1,2,3} &\sim (3, 1, -1/3), \\ B_m &\sim (3, 1, -1/3), \quad m = 1, 2, \\ u_{1,2,3} &\sim (3, 1, 2/3), \\ T_3 &\sim (3, 1, 2/3). \end{aligned} \tag{13}$$

(ii) The charged lepton fields

$$\begin{aligned} e_{1,2} &\sim (1, 1, -1), \\ E_1 &\sim (1, 1, -1). \end{aligned} \tag{14}$$

As already mentioned, the right-handed parts of e_3^- and E_4^- are included in the $SU(3)_L$ lepton triplet L_5.

(iii) The neutral lepton fields[5]

$$\nu^R_{1,2,3} \sim (1, 1, 0). \tag{15}$$

We do not include right-handed partners for the neutral lepton fields $N^{0L}_{2,3,4,5}$ and ν_4^{0L}, which get Majorana mass terms.

The representation assignments for the fermions and scalars are summarized in Table I, where one also sees the presence of two auxiliary discrete symmetries $\mathbb{Z}_2$ and $\mathbb{Z}_3$. The latter is the discrete Abelian cyclic group of order 3. It has three elements, and a convenient representation is obtained by using the cube roots of unity. These are given by $1, \omega, \omega^2$ where $\omega = \exp[\frac{2\pi i}{3}]$ with $\omega^3 = 1$. Note that $\omega^{-1} = \omega^2$ and that $\omega^{3n} = 1$ if n is an integer. This cyclic nature further implies that $\omega^n = \omega^{n-3}$, so that $\omega^4 = \omega^3 \times \omega = \omega$, $\omega^5 = \omega^3 \times \omega^2 = \omega^2$, and so on. These extra

[5]Compared with the fermion content of Ref. [39], we have three extra neutral two-component fermions $\nu^R_{1,2,3}$ to implement neutrino mass generation *à la* seesaw.

symmetries are needed in order to ensure an adequate pattern of fermion masses. In the absence of the $\mathbb{Z}_3$ symmetry, the unwanted invariant mass term $\bar{\ell}_1(L_5)^c$ would be present. On the other hand, since the $\mathrm{SU}(3)_\mathrm{c}$, $\mathrm{SU}(3)_\mathrm{L}$, $\mathrm{U}_X(1)$ gauge charge as well as the $\mathbb{Z}_3$ charges of the SM fermion triplets $\ell_{2,3}$ and of the exotic triplet L_4 are the same, these symmetries cannot distinguish between the SM and the exotic fermions inside the L_4 triplet. To prevent having similar masses for the exotic and SM fermions, we make a distinction between them by means of an additional $\mathbb{Z}_2$ symmetry, as shown in Table I.

III. YUKAWA INTERACTIONS

Before discussing the details of the fermion masses, we summarize the Higgs scalar representations what will drive the breaking of $\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(3)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{X}$ in the Yukawa sector [36,39]. There are two stages of symmetry breaking: at the high 331 scale and the EW scale. The VEVs of a generic field ψ are denoted by $\langle\psi\rangle$.

A. 331 Breaking

This is the first SSB stage, which is accomplished by the $\mathrm{SU}(3)_\mathrm{L}$ scalar sextet S_1 and triplets $\chi, \tilde{\chi}$ with $(\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2)$ charges and nonzero VEVs as follows:

$$\langle S_1\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \langle (S_1)_{33}\rangle \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(-\frac{2}{3}, \omega^2, 1\right),$$
$$\langle \chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ \langle \chi_3\rangle \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(-\frac{1}{3}, \omega, 1\right),$$
$$\langle \tilde{\chi}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ \langle \tilde{\chi}\rangle \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(-\frac{1}{3}, \omega, -1\right). \tag{16}$$

The $\mathbb{Z}_3 \otimes \mathbb{Z}_2$ and gauge symmetry invariant Yukawa terms that can be built with the sextet are

$$\bar{\ell}_a S_1 (\ell_b)^c, \qquad a, b = 2, 3,$$
$$\bar{L}_4 S_1 (L_4)^c. \tag{17}$$

These terms lead to Majorana masses for the exotic neutral leptons $N^0_{2,3,4}$.

The $\mathbb{Z}_3 \otimes \mathbb{Z}_2$ and gauge symmetry invariant Yukawa terms that can be built with the triplets are as follows:

(i) The up- and down-quark mass terms

$$\bar{q}_m \chi^* D, \quad m = 1, 2,$$
$$\bar{q}_3 \chi U, \tag{18}$$

where D represents any right-handed $d_{1,2,3}$ or $B_{1,2}$, while U represents any right-handed $u_{1,2,3}$ or T_3. After SSB, they contribute to mix charged SM and exotic quarks, and give Dirac mass to $B_{1,2}$ and T_3.

(ii) The equivalent terms in the lepton sector

$$\bar{\ell}_1 \chi^* e_1,$$
$$\bar{\ell}_1 \chi^* e_2,$$
$$\bar{\ell}_1 \tilde{\chi}^* E_1. \tag{19}$$

Here, one sees how the scalar triplet $\tilde{\chi}$, odd under the $\mathbb{Z}_2$ symmetry, allows a coupling between E_1 with ℓ_1, providing a Dirac mass term for E_1.

(iii) We also have the antisymmetric combination of $\mathrm{SU}(3)_\mathrm{L}$ triplets or antitriplets, i.e.,

$$\epsilon_{ijk} \chi^{*i} \bar{L}_4^j (L_5)^{ck},$$
$$\epsilon_{ijk} \tilde{\chi}^{*i} \bar{\ell}_m^j (L_5)^{ck}, \quad m = 2, 3, \tag{20}$$

where the i, j, $k = 1, 2, 3$ indices refer to $\mathrm{SU}(3)_\mathrm{L}$. The first term includes mixing between N^0_5 and ν^{0L}_4 and allows a mass term for E_4.

Summarizing, all the exotic charged and neutral fermions, except for N^5_0 and ν^{0L}_4, have Yukawa couplings with scalars which get large VEVs corresponding to the first stage of spontaneous symmetry breaking. The new N^5_0 and ν^{0L}_4 fields also need to get large masses, at least in the GeV range, which can arise as discussed in the following sections.

B. Electroweak breaking

Turning now to electroweak symmetry breaking, the corresponding VEVs of the scalar fields are given as

$$\langle S_c\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \langle (S_c)_{22}\rangle & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(\frac{4}{3}, \omega^2, 1\right),$$
$$\langle \eta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \langle \eta_1\rangle \\ 0 \\ \langle \eta_3\rangle \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(-\frac{1}{3}, \omega, 1\right),$$
$$\langle \tilde{\eta}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \langle \tilde{\eta}_1\rangle \\ 0 \\ \langle \tilde{\eta}_3\rangle \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(-\frac{1}{3}, \omega, -1\right),$$
$$\langle \rho\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \langle \rho_2\rangle \\ 0 \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(\frac{2}{3}, \omega^2, 1\right),$$
$$\langle \tilde{\rho}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \langle \tilde{\rho}_2\rangle \\ 0 \end{pmatrix}, \quad (\mathrm{U}(1)_\mathrm{X}, \mathbb{Z}_3, \mathbb{Z}_2) = \left(\frac{2}{3}, 1, 1\right). \tag{21}$$

The neutral component of L_5 gets mass through invariant terms built with sextet, i.e.,

$$\bar{L}_5 S_c (L_5)^c. \tag{22}$$

This Yukawa term gives a diagonal mass term for the neutral N_5^0. Note that since S_c gets the VEV in its 22-component, a large value of $\langle (S_c)_{22} \rangle$ will change the ρ parameter from its canonical SM value. Therefore, the VEV of the S_c field needs to be small, less than 2 GeV or so. Thus, the dominant contribution to N_5^0 field's mass does not come from the above term but rather through its coupling with other fields (see Table V), a fact that we have also checked numerically.

For the triplets, the relevant Yukawa terms for quarks and leptons are the following:

(i) For quarks,

$$\begin{aligned} &\bar{q}_m \eta^* D, \\ &\bar{q}_3 \eta U, \\ &\bar{q}_3 \rho D, \\ &\bar{q}_m \rho^* U, \end{aligned} \tag{23}$$

where D represents any right-handed $d_{1,2,3}$ or $B_{1,2}$, and U represents any right-handed $u_{1,2,3}$, or T_3 and $m = 1, 2$.

(ii) For leptons,

$$\begin{aligned} &\bar{\ell}_1 \eta^* e_{1,2}, \\ &\bar{\ell}_1 \tilde{\eta}^* E_1, \\ &\bar{\ell}_m \tilde{\rho} e_{1,2}, \quad m = 2, 3, \\ &\bar{L}_4 \tilde{\rho} E_1, \\ &\epsilon_{ijk} \tilde{\eta}^{*i} \bar{\ell}_m^j (L_5)^{ck}, \quad m = 2, 3, \\ &\epsilon_{ijk} \eta^{*i} \bar{L}_4^j (L_5)^{ck}, \end{aligned} \tag{24}$$

where the i, j, $k = 1, 2, 3$ indices refer to $\mathrm{SU(3)_L}$. All these terms provide mass to charged leptons. The last two terms also provide mixing among neutral exotic states as well as mixing among SM and exotic ones. However, since the η and $\tilde{\eta}$ VEVs are of electroweak level, none of these terms lead to unacceptable large masses for any SM particles, a fact that can be seen from the explicit forms of charged and neutral lepton mass matrices given in Tables IV and V, respectively. We have also numerically cross-checked this fact.

Actually, another Higgs sextet S_b would be allowed by the symmetries of the model, with the VEV as

$$\langle S_b \rangle = \begin{pmatrix} \langle (S_b)_{11} \rangle & 0 & \langle (S_b)_{13} \rangle \\ 0 & 0 & 0 \\ \langle (S_b)_{13} \rangle & 0 & \langle (S_b)_{33} \rangle \end{pmatrix},$$

$$(U(1)_X, \mathbb{Z}_3, \mathbb{Z}_2) = \left(-\frac{2}{3}, \omega^2, 1\right),$$

leading to the $\mathrm{U}(1)_X \otimes \mathbb{Z}_3 \otimes \mathbb{Z}_2$ Majorana mass terms

$$\begin{aligned} &\bar{\ell}_n S_b (\ell_m)^c, \qquad n, m = 2, 3, \\ &\bar{L}_4 S_b (L_4)^c. \end{aligned}$$

The first of these terms gives rise to diagonal mass terms for left-handed neutrinos of the order of the EW scale. Therefore, in order to get the observed tiny neutrino masses through a seesaw mechanism, we exclude the S_b sextet from the particle content.

C. Type-I seesaw mechanism in the 331 setup

To implement the type-I seesaw mechanism, we need the following terms:

$$\begin{aligned} &\bar{\ell}_m \eta \nu_a^R, \\ &\bar{L}_4 \tilde{\eta} \nu_a^R, \\ &\bar{\nu}_a^R (\nu_b^R)^c, \end{aligned} \tag{25}$$

where $m = 2$, 3 and a, $b = 1$, 2, 3. They provide Dirac and Majorana masses for the SM-like neutrinos as well as their mixing with heavy neutral fermions. The second term in (25) differs from the first, since ℓ_m is replaced by L_4. They are distinct, thanks to the $\mathbb{Z}_2$ symmetry. This ensures that the neutrinolike fermion in L_4 receives an adequately large mass because of a suitably tuned Yukawa coupling.

In addition, the following terms are also allowed by all the symmetries of the model:

$$\begin{aligned} &\bar{\ell}_m \chi \nu_a^R, \\ &\bar{L}_4 \tilde{\chi} \nu_a^R, \\ &\bar{\ell}_1 \tilde{\rho}^* \nu_a^R. \end{aligned} \tag{26}$$

As in the previous case, the first two terms in (26) are distinct due to the $\mathbb{Z}_2$ symmetry (though in this case, a single term would not be dangerous as it would only give mass to the third component of ℓ_m due to the VEV alignment of χ).

IV. FERMION MASS MATRICES

In the full Yukawa Lagrangian characterizing our model,

(i) for quarks we have

$$\begin{aligned} \mathcal{L}_Y^q = &(\bar{q}_m \chi^* Y_{mi}^d + \bar{q}_3 \rho y_{3i}^d + \bar{q}_m \eta^* j_{mi}^d) D_i \\ &+ (\bar{q}_3 \chi Y_{3j}^u + \bar{q}_m \rho^* y_{mj}^u + \bar{q}_3 \eta j_{3j}^u) U_j, \end{aligned} \tag{27}$$

where $Y^{d,u}, y^{d,u}, j^{d,u}$ represent the Yukawa couplings introduced, respectively, for χ, ρ, and η. We remind that D represents any right-handed $d_{1,2,3}$ or $B_{1,2}$, U represents any right-handed $u_{1,2,3}$, or T_3, and $m = 1, 2$.

(ii) For leptons we have

$$\begin{aligned}\mathcal{L}_Y^{\ell} = {} & (Y_{1a}\bar{\ell}_1\chi^* + f_{ma}\bar{\ell}_m\rho + y_{1a}\bar{\ell}_1\eta^*)e_a + (Y_{1E}\bar{\ell}_1\tilde{\chi}^* + y_{1E}\bar{\ell}_1\tilde{\eta}^*)E_1 + f_{4E}\bar{L}_4\tilde{\rho}E_1 \\ & + J_m\epsilon_{ijk}(\tilde{\chi}^*)^i(L_5)^{ck}\bar{\ell}_m^j + J_4\epsilon_{ijk}(\chi^*)^i(L_5)^{ck}\bar{L}_4^j + j_m\epsilon_{ijk}(\tilde{\eta}^*)^i(L_5)^{ck}\bar{\ell}_m^j \\ & + j_4\epsilon_{ijk}(\eta^*)^i(L_5)^{ck}\bar{L}_4^j + \frac{K_{mn}}{\sqrt{2}}\bar{\ell}_m S_1(\ell_n)^c + \frac{K_{44}}{\sqrt{2}}\bar{L}_4 S_1(L_4)^c + \frac{c_5}{\sqrt{2}}\bar{L}_5 S_c(L_5)^c \\ & + +(y_\eta)_{ms}\bar{\ell}_m\eta\nu_s^R + (y_{\tilde{\eta}})_{4s}\bar{L}_4\tilde{\eta}\nu_s^R + (Y_\chi)_{ms}\bar{\ell}_m\chi\nu_s^R + (Y_{\tilde{\chi}})_{4s}\bar{L}_4\tilde{\chi}\nu_s^R + (y_{\tilde{\rho}})_{1s}\bar{\ell}_1\tilde{\rho}^*\nu_s^R \\ & + \frac{M^{st}}{\sqrt{2}}\bar{\nu}_s^R(\nu_t^R)^c + \text{H.c.,}\end{aligned} \quad (28)$$

where $Y, y, K, k, f, c, J, j, M$ represent the Yukawa couplings with $m, n \in \{2,3\}$, $a, b \in \{1,2\}$, $s, t \in \{1,2,3\}$, and the $i, j, k \in \{1,2,3\}$ indices act on $\mathrm{SU(3)_L}$.

The mass matrices for the up-type ($\sqrt{2}M_{ij}^u$) and down-type ($\sqrt{2}M_{ij}^d$) quarks remain the same as before; see Tables II and III.

Turning to the lepton mass matrices, we begin with charged lepton mass matrix ($\sqrt{2}M_{ij}^e$), whose explicit form is given in Table IV.

Concerning the mass matrix of the neutral fermions ($\sqrt{2}M_{ij}^n$), it incorporates type-I seesaw mass terms. Its complete form is given in Table V. We have numerically verified that it leads to an adequate spectrum of light-neutrino masses.

V. *B* FLAVOR GLOBAL ANALYSES

These analyses are performed in the framework of the effective Hamiltonian at the *b*-mass scale, separating short-

TABLE II. Up-type quark mass matrix $\sqrt{2}M_{ij}^u$. Here, the L and R superscripts indicate the left- and right-handed fields.

Fields	u_1^R	u_2^R	u_3^R	T_3^R
$\bar{u}_1^L$	$-y_{11}^u\langle\rho_2^*\rangle$	$-y_{12}^u\langle\rho_2^*\rangle$	$-y_{13}^u\langle\rho_2^*\rangle$	$-y_{14}^u\langle\rho_2^*\rangle$
$\bar{u}_2^L$	$-y_{21}^u\langle\rho_2^*\rangle$	$-y_{22}^u\langle\rho_2^*\rangle$	$-y_{23}^u\langle\rho_2^*\rangle$	$-y_{24}^u\langle\rho_2^*\rangle$
$\bar{u}_3^L$	$j_{31}^u\langle\eta_1\rangle$	$j_{32}^u\langle\eta_1\rangle$	$j_{33}^u\langle\eta_1\rangle$	$j_{34}^u\langle\eta_1\rangle$
$\bar{T}_3^L$	$Y_{31}^u\langle\chi_3\rangle + j_{31}^u\langle\eta_3\rangle$	$Y_{32}^u\langle\chi_3\rangle + j_{32}^u\langle\eta_3\rangle$	$Y_{33}^u\langle\chi_3\rangle + j_{33}^u\langle\eta_3\rangle$	$Y_{34}^u\langle\chi_3\rangle + j_{34}^u\langle\eta_3\rangle$

TABLE III. Down-type mass matrix $\sqrt{2}M_{ij}^d$. Here, the L and R superscripts indicate the left- and right-handed fields.

Fields	d_1^R	d_2^R	d_3^R	B_1^R	B_2^R
$\bar{d}_1^L$	$j_{11}^d\langle\eta_1^*\rangle$	$j_{12}^d\langle\eta_1^*\rangle$	$j_{13}^d\langle\eta_1^*\rangle$	$j_{14}^d\langle\eta_1^*\rangle$	$j_{15}^d\langle\eta_1^*\rangle$
$\bar{d}_2^L$	$j_{21}^d\langle\eta_1^*\rangle$	$j_{22}^d\langle\eta_1^*\rangle$	$j_{23}^d\langle\eta_1^*\rangle$	$j_{24}^d\langle\eta_1^*\rangle$	$j_{25}^d\langle\eta_1^*\rangle$
$\bar{d}_3^L$	$y_{31}^d\langle\rho_2\rangle$	$y_{32}^d\langle\rho_2\rangle$	$y_{33}^d\langle\rho_2\rangle$	$y_{34}^d\langle\rho_2\rangle$	$y_{35}^d\langle\rho_2\rangle$
$\bar{B}_1^L$	$Y_{11}^d\langle\chi_3^*\rangle + j_{11}^d\langle\eta_3^*\rangle$	$Y_{12}^d\langle\chi_3^*\rangle + j_{12}^d\langle\eta_3^*\rangle$	$Y_{13}^d\langle\chi_3^*\rangle + j_{13}^d\langle\eta_3^*\rangle$	$Y_{14}^d\langle\chi_3^*\rangle + j_{14}^d\langle\eta_3^*\rangle$	$Y_{15}^d\langle\chi_3^*\rangle + j_{15}^d\langle\eta_3^*\rangle$
$\bar{B}_2^L$	$Y_{21}^d\langle\chi_3^*\rangle + j_{21}^d\langle\eta_3^*\rangle$	$Y_{22}^d\langle\chi_3^*\rangle + j_{22}^d\langle\eta_3^*\rangle$	$Y_{23}^d\langle\chi_3^*\rangle + j_{23}^d\langle\eta_3^*\rangle$	$Y_{24}^d\langle\chi_3^*\rangle + j_{24}^d\langle\eta_3^*\rangle$	$Y_{25}^d\langle\chi_3^*\rangle + j_{25}^d\langle\eta_3^*\rangle$

TABLE IV. The charged lepton mass matrix $\sqrt{2}M_{ij}^e$. Here, subscripts of the VEV-carrying scalars indicate the scalar components whose nonzero VEV comes in a given entry.

Fields	e_1^R	e_2^R	e_3^R	E_1^R	E_4^R
$\bar{e}_1^L$	$y_{11}\langle\eta_1^*\rangle$	$y_{12}\langle\eta_1^*\rangle$	0	$y_{1E}\langle\tilde{\eta}_1^*\rangle$	0
$\bar{e}_2^L$	$f_{21}\langle\rho_2\rangle$	$f_{22}\langle\rho_2\rangle$	$j_2\langle\tilde{\eta}_1^*\rangle$	0	$-(J_2\langle\tilde{\chi}_3^*\rangle + j_2\langle\tilde{\eta}_3^*\rangle)$
$\bar{e}_3^L$	$f_{31}\langle\rho_2\rangle$	$f_{33}\langle\rho_2\rangle$	$j_3\langle\tilde{\eta}_1^*\rangle$	0	$-(J_3\langle\tilde{\chi}_3^*\rangle + j_3\langle\tilde{\eta}_3^*\rangle)$
$\bar{E}_1^L$	$Y_{11}\langle\chi_3^*\rangle + y_{11}\langle\eta_3^*\rangle$	$Y_{12}\langle\chi_3^*\rangle + y_{12}\langle\eta_3^*\rangle$	0	$Y_{1E}\langle\tilde{\chi}_3^*\rangle + y_{1E}\langle\tilde{\eta}_3^*\rangle$	0
$\bar{E}_4^L$	0	0	$j_4\langle\eta_1^*\rangle$	$f_{4E}\langle\tilde{\rho}_2\rangle$	$-(J_4\langle\chi_3^*\rangle + j_4\langle\eta_3^*\rangle)$

TABLE V. The neutral lepton mass matrix $\sqrt{2}M^n_{ij}$ written so as to highlight the seesaw structure.

Fields	$(\nu_1^L)^c$	$(\nu_2^L)^c$	$(\nu_3^L)^c$	$(\nu_4^L)^c$	$(N_2^L)^c$	$(N_3^L)^c$	$(N_4^L)^c$	$(N_5^L)^c$	ν_1^R	ν_2^R	ν_3^R
$\bar{\nu}_1^L$	0	0	0	0	0	0	0	0	$-(y_{\tilde{\rho}})_{11}\langle\tilde{\rho}_2^*\rangle$	$-(y_{\tilde{\rho}})_{12}\langle\tilde{\rho}_2^*\rangle$	$-(y_{\tilde{\rho}})_{13}\langle\tilde{\rho}_2^*\rangle$
$\bar{\nu}_2^L$	0	0	0	0	0	0	0	$J_2\langle\tilde{\chi}_3^*\rangle + j_2\langle\tilde{\eta}_3^*\rangle$	$(y_\eta)_{21}\langle\eta_1\rangle$	$(y_\eta)_{22}\langle\eta_1\rangle$	$(y_\eta)_{23}\langle\eta_1\rangle$
$\bar{\nu}_3^L$	0	0	0	0	0	0	0	$J_3\langle\tilde{\chi}_3^*\rangle + j_3\langle\tilde{\eta}_3^*\rangle$	$(y_\eta)_{31}\langle\eta_1\rangle$	$(y_\eta)_{32}\langle\eta_1\rangle$	$(y_\eta)_{33}\langle\eta_1\rangle$
$\bar{\nu}_4^L$	0	0	0	0	0	0	0	$J_4\langle\chi_3^*\rangle + j_4\langle\eta_3^*\rangle$	$(y_{\tilde{\eta}})_{41}\langle\tilde{\eta}_1\rangle$	$(y_{\tilde{\eta}})_{42}\langle\tilde{\eta}_1\rangle$	$(y_{\tilde{\eta}})_{43}\langle\tilde{\eta}_1\rangle$
$\bar{N}_2^L$	0	0	0	0	$K_{22}\langle S_1\rangle$	$K_{23}\langle S_1\rangle$	0	$-j_2\langle\tilde{\eta}_1^*\rangle$	$(y_\eta)_{21}\langle\eta_3\rangle + (Y_\chi)_{21}\langle\chi_3\rangle$	$(y_\eta)_{22}\langle\eta_3\rangle + (Y_\chi)_{22}\langle\chi_3\rangle$	$(y_\eta)_{23}\langle\eta_3\rangle + (Y_\chi)_{23}\langle\chi_3\rangle$
$\bar{N}_3^L$	0	0	0	0	$K_{32}\langle S_1\rangle$	$K_{33}\langle S_1\rangle$	0	$-j_3\langle\tilde{\eta}_1^*\rangle$	$(y_\eta)_{31}\langle\eta_3\rangle + (Y_\chi)_{31}\langle\chi_3\rangle$	$(y_\eta)_{32}\langle\eta_3\rangle + (Y_\chi)_{32}\langle\chi_3\rangle$	$(y_\eta)_{33}\langle\eta_3\rangle + (Y_\chi)_{33}\langle\chi_3\rangle$
$\bar{N}_4^L$	0	0	0	0	0	0	$K_{44}\langle S_1\rangle$	$-j_4\langle\eta_1^*\rangle$	$(y_{\tilde{\eta}})_{41}\langle\tilde{\eta}_3\rangle + (Y_{\tilde{\chi}})_{41}\langle\tilde{\chi}_3\rangle$	$(y_{\tilde{\eta}})_{42}\langle\tilde{\eta}_3\rangle + (Y_{\tilde{\chi}})_{42}\langle\tilde{\chi}_3\rangle$	$(y_{\tilde{\eta}})_{43}\langle\tilde{\eta}_3\rangle + (Y_{\tilde{\chi}})_{43}\langle\tilde{\chi}_3\rangle$
$\bar{N}_5^L$	0	$J_2\langle\tilde{\chi}_3\rangle + j_2\langle\tilde{\eta}_3\rangle$	$J_3\langle\tilde{\chi}_3\rangle + j_3\langle\tilde{\eta}_3\rangle$	$J_4\langle\chi_3\rangle + j_4\langle\eta_3\rangle$	$-j_2\langle\tilde{\eta}_1\rangle$	$-j_3\langle\tilde{\eta}_1\rangle$	$-j_4\langle\eta_1\rangle$	$c_5\langle S_c\rangle$	0	0	0
$(\bar{\nu}_1^R)^c$	$-(y_{\tilde{\rho}})_{11}\langle\tilde{\rho}_2\rangle$	$(y_\eta)_{21}\langle\eta_1^*\rangle$	$(y_\eta)_{31}\langle\eta_1^*\rangle$	$(y_{\tilde{\eta}})_{41}\langle\tilde{\eta}_1^*\rangle$	$(y_\eta)_{21}\langle\eta_3^*\rangle + (Y_\chi)_{21}\langle\chi_3^*\rangle$	$(y_\eta)_{31}\langle\eta_3^*\rangle + (Y_\chi)_{31}\langle\chi_3^*\rangle$	$(y_{\tilde{\eta}})_{41}\langle\tilde{\eta}_3^*\rangle + (Y_{\tilde{\chi}})_{41}\langle\tilde{\chi}_3^*\rangle$	0	M_{11}	M_{12}	M_{13}
$(\bar{\nu}_2^R)^c$	$-(y_{\tilde{\rho}})_{12}\langle\tilde{\rho}_2\rangle$	$(y_\eta)_{22}\langle\eta_1^*\rangle$	$(y_\eta)_{32}\langle\eta_1^*\rangle$	$(y_{\tilde{\eta}})_{42}\langle\tilde{\eta}_1^*\rangle$	$(y_\eta)_{22}\langle\eta_3^*\rangle + (Y_\chi)_{22}\langle\chi_3^*\rangle$	$(y_\eta)_{32}\langle\eta_3^*\rangle + (Y_\chi)_{32}\langle\chi_3^*\rangle$	$(y_{\tilde{\eta}})_{42}\langle\tilde{\eta}_3^*\rangle + (Y_{\tilde{\chi}})_{42}\langle\tilde{\chi}_3^*\rangle$	0	M_{21}	M_{22}	M_{23}
$(\bar{\nu}_3^R)^c$	$-(y_{\tilde{\rho}})_{13}\langle\tilde{\rho}_2\rangle$	$(y_\eta)_{23}\langle\eta_1^*\rangle$	$(y_\eta)_{33}\langle\eta_1^*\rangle$	$(y_{\tilde{\eta}})_{43}\langle\tilde{\eta}_1^*\rangle$	$(y_\eta)_{23}\langle\eta_3^*\rangle + (Y_\chi)_{23}\langle\chi_3^*\rangle$	$(y_\eta)_{33}\langle\eta_3^*\rangle + (Y_\chi)_{33}\langle\chi_3^*\rangle$	$(y_{\tilde{\eta}})_{43}\langle\tilde{\eta}_3^*\rangle + (Y_{\tilde{\chi}})_{43}\langle\tilde{\chi}_3^*\rangle$	0	M_{31}	M_{32}	M_{33}

and long-distance physics in the Wilson coefficients and local operators [41,42]:

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_i C_i O_i. \tag{29}$$

The main operators of interest for this discussion are the following:

$$\begin{aligned} O_7 &= \frac{e}{16\pi^2} m_b(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}, \\ O_{7'} &= \frac{e}{16\pi^2} m_b(\bar{s}\sigma_{\mu\nu}P_L b)F^{\mu\nu}, \\ O_9^\ell &= \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\ell), \\ O_{10}^\ell &= \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma^5\ell), \\ O_{9'}^\ell &= \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\ell), \\ O_{10'}^\ell &= \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\gamma^5\ell), \end{aligned} \tag{30}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$, and the fields are understood as mass eigenstates. In the SM, only O_7, O_9^ℓ, and O_{10}^ℓ are significant, with the values of the Wilson coefficients given as $C_9^\ell \simeq 4.1$ and $C_{10}^\ell \simeq -4.3$ at the scale $\mu = m_b$. In contrast, the primed operators are m_s/m_b suppressed due to the chirality of the quarks involved.

The analyses of several $b \to s\gamma$ and $b \to s\ell\ell$ observables (including angular ones) point toward a pattern of deviations consistent with a large NP short-distance contribution to C_9^μ, around 1/4 of the SM contribution; see, e.g., Refs. [40,43–45]. Scenarios with NP contributions in C_9^μ only, in (C_9^μ, C_{10}^μ) or in $(C_9^\mu, C_{9'}^\mu)$, seem particularly favored. Moreover, the LFU violating observables agree well with the absence of significant NP contributions to any electron-type Wilson coefficient C_i^e. Results of the global fit analyses seem to rule out the possibility of large contributions from other operators suppressed in the SM, in particular, scalar and pseudoscalar operators. They are constrained especially by the good agreement between the observed value for the $B_s \to \mu\mu$ branching ratio and its SM prediction, as well as by the limits on the $B \to X_s\gamma$ branching ratio.

We proceed along the lines of the phenomenological analysis of Ref. [39] to which we refer for details. We focus on the vector/axial contributions which are assumed to be the larger ones. The neutral lepton mass matrix and the neutral lepton mixing do not affect the effective Hamiltonian contributing to the process, since the relevant operators only include charged leptons. Hence, after the expansion in $\epsilon = \Lambda_{\rm EW}/\Lambda_{\rm NP}$ (NP denotes here the 331 scale), one finds that nonzero contributions at the lowest order, namely, $O(\epsilon^2)$, can only come from the neutral

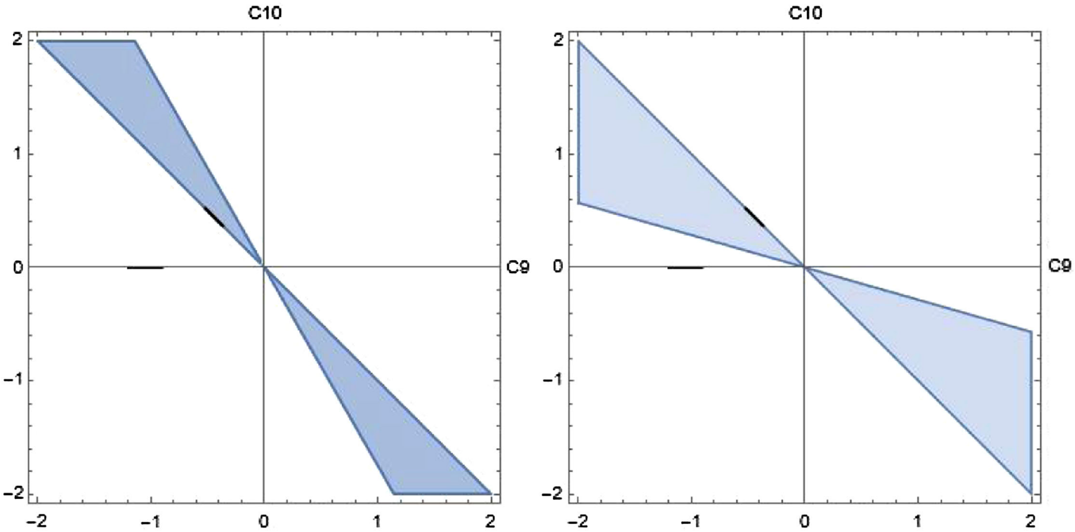

FIG. 1. Regions allowed for the Wilson coefficient C_9^μ and C_{10}^μ (abscissa and ordinate, respectively) in scenarios A (left) and B (right) described in Ref. [39]. The thick black intervals correspond to the 1σ interval for one-dimensional scenarios [46].

gauge bosons Z' and Z. The transitions mediated by the heavy gauge boson Z' are expressed in the effective Hamiltonian by the term

$$\mathcal{H}_{\rm eff} \supset \frac{g_X^2}{54\cos^2\theta_{331}} \frac{1}{M_{Z'}^2} V_{3k}^{(d)*} V_{3l}^{(d)} \frac{4\pi}{\alpha} \times \left\{ \left[-\frac{1}{2} V_{1i}^{(e)*} V_{1j}^{(e)} + \frac{1-6\cos^2\theta_{331}}{2} W_{3i}^{(e)*} W_{3j}^{(e)} + \frac{1+3\cos^2\theta_{331}}{4} \delta_{ij} \right] O_9^{klij} + \left[\frac{1}{2} V_{1i}^{(e)*} V_{1j}^{(e)} + \frac{1-6\cos^2\theta_{331}}{2} W_{3i}^{(e)*} W_{3j}^{(e)} + \frac{-1+9\cos^2\theta_{331}}{4} \delta_{ij} \right] O_{10}^{klij} \right\}, \quad (31)$$

where the indices k, l refer to the SM generations of the quark mass eigenstates (assuming $k \neq l$), while i, j refer to the SM lepton mass eigenstates (either from the same or different generations). The effective operators $O_{9,10}^{klij}$ are defined exactly as in Eq. (30), taking into account the $(\bar{q}_k q_l)(\bar{\ell}_i \ell_j)$ flavor structure. Here, $\alpha = e^2/(4\pi)$ is the fine-structure constant. The V and W matrices provide the mixing matrices arising from the diagonalization of the EWSB mass terms in the subspace of left-handed and right-handed SM fields, with the superscripts (d) and (e) referring to down-type quarks and charged leptons, respectively.

At the same lowest order, the contribution to the effective Hamiltonian given by the SM gauge boson Z can be written as

$$\mathcal{H}_{\rm eff} \supset \frac{\cos^2\theta_W(1+3\cos^2\theta_{331})}{8} \frac{g^2}{M_Z^2} \frac{4\pi}{\alpha} \sum_\lambda \hat{V}_{\lambda k}^{(d)*} \hat{V}_{\lambda l}^{(d)} \delta_{ij} \times \{(-1+9\cos^2\theta_{331}) O_9^{klij} + (1+3\cos^2\theta_{331}) O_{10}^{klij}\}, \quad (32)$$

where $\hat{V}^{(d)}$ represents the $O(\epsilon^1)$ correction to the rotation matrix $V^{(d)}$ between interaction and mass eigenstates for the left-handed down sector. Notice that at this order, the coupling is the same for all the light leptons; i.e., non-universality does not arise in the interaction with Z. LFU violating contributions arise only from the Z' contribution.

In addition to LFU violation, the model allows for lepton-flavor violation, which we assume suppressed, in agreement with experimental restrictions, and set it to zero for simplicity. These further assumptions constrain the parameter space (C_9^μ, C_{10}^μ) to two scenarios detailed in Ref. [39]. For both of them, we can compare the allowed regions with the latest data, as done in Fig. 1. In this figure, and from now on, we focus only on the non-SM contribution to the Wilson coefficients; that is, we set $C_i = C_i^{NP}$. The thick black intervals correspond to the 1σ interval for the one-dimensional scenarios from the latest data [46].

A comparison between the 2018 and 2021 intervals for $C_{9\mu}$ given by global analyses [40,46] is reported below.

(i) $C_{9\mu}$, $C_{10\mu} = 0$

$$[-1.28, -0.94] \ (2018), \quad (33)$$

$$[-1.20, -0.91] \ (2021). \quad (34)$$

(ii) $C_{9\mu} = -C_{10\mu}$

$$[-0.75, -0.49] \ (2018), \quad (35)$$

$$[-0.52, -0.37] \ (2021). \tag{36}$$

As can be seen in Table I, also with new data in both scenarios A and B we are able to account for the anomalies observed as long as we consider the $C_9^\mu = -C_{10}^\mu$ case.

In our model, the $b \to s\ell\ell$ transitions originate from the tree-level exchange of the Z and Z' gauge bosons. The former breaks the Glashow-Iliopoulos-Maiani (GIM) mechanism through the mixing between normal and exotic quarks, and depends on the Yukawa couplings. The latter involves just the unsuppressed exchange of the heavy Z' gauge boson. Both give suppressed contributions to the bsZ vertex, as can be seen in Fig. 2. To make a quantitative analysis, we must take into account phenomenological constraints on Z and Z' couplings.

Restricting our discussion to the leading contributions of order $\mathcal{O}(\epsilon^2)$, the Z-exchange contribution to $B_s - \bar{B}_s$ mixing will have two such vertices, and hence, the amplitude will be suppressed by a factor $\mathcal{O}(\epsilon^4)$. On the other hand, the bs vertex is mediated by Z' at $\mathcal{O}(\epsilon^0)$, implying that in this case we have only the suppression coming from the heavy propagator that must be taken into account. The corresponding part of the effective Hamiltonian is

$$\begin{aligned}\mathcal{H}_{\text{eff}} \supset & \frac{g_X^2}{54 M_{Z'}^2 \cos^2\theta_{331}} (V_{3k}^{*(d)} V_{3l}^{(d)})^2 (\overline{D_k}\gamma^\mu D_l)(\overline{D_k}\gamma^\mu D_l) \\ = & \frac{8 G_F}{\sqrt{2}(3-\tan^2\theta_W)} \frac{M_W^2}{M_{Z'}^2} (V_{3k}^{*(d)} V_{3l}^{(d)})^2 (\overline{D_k}\gamma^\mu D_l)(\overline{D_k}\gamma^\mu D_l).\end{aligned} \tag{37}$$

Our case of interest is $k = 2$, $l = 3$. The SM contribution to the mixing reads [47]

$$\mathcal{H}_{\text{eff}}^{\text{SM}} = (V_{ts}^* V_{tb})^2 \frac{G_F^2}{4\pi^2} M_W^2 \hat{\eta}_B S\left(\frac{\overline{m_t}^2}{M_W^2}\right) (\overline{s_L}\gamma^\mu b_L)(\overline{s_L}\gamma^\mu b_L), \tag{38}$$

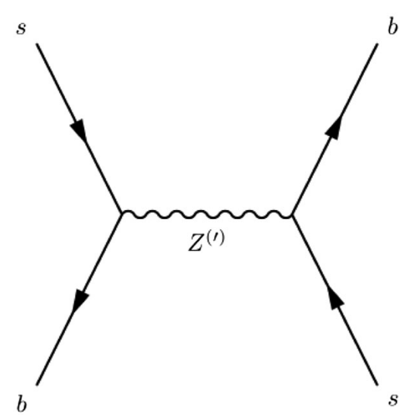

FIG. 2. Tree level contributions to $B_s - \bar{B}_s$ mixing.

where S is the Inami-Lim function, and $\overline{m_t}$ is the top quark mass defined in the $\overline{\text{MS}}$ scheme. As in Ref. [47], we take $S(\frac{\overline{m_t}^2}{M_W^2}) \simeq 2.35$ for a top mass of about 165 GeV, and $\hat{\eta}_B = 0.8393 \pm 0.0034$, which includes QCD corrections. Considering the modulus of the ratio of the NP contribution over the SM, one gets

$$\begin{aligned} r_{B_s} = & \left|\frac{C_{\text{NP}}}{C_{\text{SM}}}\right| \\ = & \frac{32\pi^2 |V_{32}^{*(d)} V_{33}^{(d)}|^2}{\sqrt{2}(3-\tan^2\theta_W)|V_{ts}^* V_{tb}|^2 G_F M_W^2 \hat{\eta}_B S} \frac{M_W^2}{M_{Z'}^2}. \end{aligned} \tag{39}$$

Here, the only variables are $d = V_{32}^{*(d)} V_{33}^{(d)}$ and $M_{Z'}^2$ or, equivalently, $M_W^2/M_{Z'}^2$. In order to get a quantitative idea of the values allowed, we perform a scan varying d in $[-1, 1]$ (since d consists of products of elements of unitary matrices). We fix the range of the other variable $M_W/M_{Z'}$ to [0, 0.1] corresponding roughly to a NP scale at least of the order of 10 times the electroweak scale, and assume that the NP contributions to the B_s mixing is at most 10% by setting $r_{B_s} \leq 0.1$. For these values, we evaluate the NP contribution to the Wilson coefficient in the one-dimensional scenario with $C_9^\mu = -C_{10}^\mu$. The allowed values found in the scan are plotted in Fig. 3.

We see that values of $C_9^\mu = -C_{10}^\mu$ can reach -0.6, in agreement with the results of global analyses of $b \to s\ell\ell$ corresponding to $r_{B_s} = 0.1$, $M_W/M_{Z'} = 0.1$, and $d \simeq -0.005$. The allowed region is limited by the fact that we have numerically taken

$$\begin{aligned} & r_{B_s} \simeq 347 \times 10^3 \times \left(\frac{M_W}{M_{Z'}}\right)^2 \times d^2 \leq 0.1, \\ & C_9^\mu \simeq 11.3 \times 10^3 \times \left(\frac{M_W}{M_{Z'}}\right)^2 \times d, \qquad |d| \leq 1. \end{aligned} \tag{40}$$

Therefore, in the simple one-dimensional scenario $C_9^\mu = -C_{10}^\mu$, the present 331 model can accommodate both $B_s - \bar{B}_s$ mixing and $b \to s\ell\ell$ data, with a NP scale

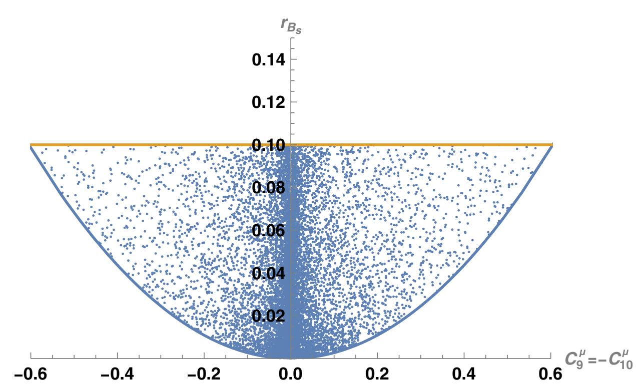

FIG. 3. Allowed points in the (C_9^μ, r_{B_s}) plane.

(and, in particular, a Z') around the TeV scale. Searches for high-mass dilepton resonances at ATLAS [48] have set higher lower limits for Z' by comparison with different 331 models [49]. As the limits on the Z' mass from direct searches gets higher, our points are pushed toward the plot edges, requiring a larger value of r_{B_s}. However, care must be used to extrapolate results from other 331 models, especially minimal ones, since different couplings and interference patterns may affect the results of the searches. The lower bounds of the Z' mass can be significantly lower than those obtained from LHC if all decay channels of Z' into new particles are included.

VI. SUMMARY AND OUTLOOK

In this paper, we have explored the possibility of explaining data on flavor anomalies for $B \to K^{(*)}$ decays within a 331 extension of the Standard Model. We have explored the possibility of having a new massive 331 Z' boson coupled in a different way to muons and electrons. We are aware of the intrinsic limitations of fiddling with gauge couplings in the absence of a dedicated family symmetry. Nevertheless, our analysis is encouraged by the previous results in Ref. [39] and motivated by recent data that tend to confirm flavor anomalies; in particular, 2021 data of LHCb achieve a 3.1σ deviation from SM predictions in the $R_{K^{(*)}}$ observable in $B^+ \to K^+\ell^+\ell^-$ decays with 9 fb^{-1} of proton-proton collision data [1].

Prompted by these new data, we have examined the viability of generalizing the scheme in Ref. [39] so as to provide a complete 331 model explaining LFU violation and generating viable neutrino masses through a type-I seesaw mechanism. We have shown the viability of a 331 gauge symmetry model setup putting together both flavor anomalies and a consistent neutrino mass spectrum. The model introduces new massive particles at mass scales allowed by current laboratory data and requires a sophisticated structure beyond the "traditional" 331 schemes. Indeed, in order to eliminate dangerous mass terms and mixings, our model employs an $\mathrm{SU}(3)_c \times \mathrm{SU}_L(3) \times \mathrm{U}(1) \times \mathbb{Z}_2 \times \mathbb{Z}_3$ symmetry. The new global discrete symmetries ensure a realistic mass hierarchy pattern for the fermions.

Within the model-independent effective approach, deviations from lepton-flavor universality in the $b \to s\ell\ell$ transitions are parametrized by new physics contributions to the Wilson coefficients. Our extended 331 model can generate such large new physics contributions to (C_9^μ, C_{10}^μ) parameters, as required by current global fits [40,46]. Trying to stick to minimality requirements, we have assumed that neutral gauge bosons give dominant contributions to the flavor violating observables without contributions to $b \to see$ or large lepton-flavor violation of the form $b \to s\ell_1\ell_2$, as suggested by experimental observations. Within a simple one-dimensional scenario with opposite contributions to C_9^μ and C_{10}^μ, we have accommodated both $B_s - \bar{B}_s$ mixing and $b \to s\ell\ell$ data, with a new physics Z' mass scale around the TeV scale. Going to different values for (C_9^μ, C_{10}^μ) would possibly extend the allowed parameter space for new physics. In order to comply with experimental limits for processes involving charged leptons, we have assumed that contributions to $b \to sl_1l_2$ as well as lepton-universality violating processes are suppressed. This has allowed us to set constraints on the fermionic mixing matrices, as discussed in Ref. [39].

In summary, we have reconciled the LFU violation data with a viable neutrino oscillation pattern in a 331 setup, a goal never achieved earlier. Our explanation for B-anomaly decays may be reformulated within alternative neutrino mass generation mechanisms such as the inverse seesaw mechanism. Likewise, the inclusion of dark matter may be implemented through a scotogenic approach.

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