Analysis of FCNC $\Xi_{QQ} \to \Lambda_Q l^+ l^-$ decay in light-cone sum rules

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The weak decays of doubly heavy Ξ_{QQ} baryon induced by flavor changing $b \to d$ and $c \to u$ flavor changing neutral current within the light-cone sum rules are studied. The sum rules are constructed and analyzed for the corresponding transition form factors using the parallel components of the light-cone distribution amplitudes for Λ_Q baryon. Having the results for the form factors, the corresponding branching ratios for $\Xi_{QQ} \to \Lambda_Q l^+ l^-$ decays are estimated. While the branching ratio due to $b \to d$ transition is around 10^{-9}, in $c \to u$ transition case, it is found as around 10^{-13}. Hence, $\Xi_{bb} \to \Lambda_b l^+ l^-$ decay has the potential of being discovered at LHCb. However, $\Xi_{bb} \to \Lambda_b l^+ l^-$ is difficult to be measured due to the extremely small values of obtained branching ratio. Our results on the branching ratio are also compared with the results of the light-front approach.

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I. INTRODUCTION

The quark model is a very useful tool for classifying and studying hadrons' properties. Many hadrons predicted by the quark model have already been discovered. However, there are still many undiscovered states even though the quark model predicts them. For instance, even though doubly heavy baryons are predicted by the quark model, only Ξ_{cc}^{++} baryons with mass $m_{\Xi_{cc}^{++}} = (3621.40 \pm 0.72 \pm 0.27 \pm 0.14)$ MeV has been observed via $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ [1] at LHCb. This state was also confirmed in the $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$ [2] and later verified in a series of experiments [3–6].

Doubly heavy baryons are one of the subjects studied intensively in many works (see [7–9] and references therein). The process induced by the flavor changing neutral current (FCNC) at quark level induced by $b \to d/s$ and $c \to u$ transitions occurs at loop level in the standard model; hence, their decay widths are expected to be small.

In the diquark-quark picture, the quantum numbers of possible doubly heavy baryons with spin-$1/2$ and spin-$3/2$ are presented in Table I.

The FCNC induced by doubly heavy baryons processes deserves much theoretical and experimental attention. The weak decays of doubly heavy baryons induced by FCNC transitions are a significant class of decays for studying the properties of such baryons. Analyzing these channels can give useful information about the helicity structure of effective Hamiltonian and Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and also look for new physics effects.

While the semileptonic decays induced by charged current occur at tree level, the decays induced by flavor-changing neutral current take place at loop level. The leptonic part of these decays is well known, and all the QCD dynamics are encoded in the hadronic matrix elements induced by the weak current. These matrix elements are parametrized in terms of the form factors that play a crucial role in analyzing the semileptonic and nonleptonic decays. Therefore determining these form

TABLE I. Quantum numbers of ground state doubly heavy baryons.

Doubly heavy baryon	J^P	Doubly heavy baryon	J^P
Ξ_{QQ}	$\frac{1}{2}^+$	Ω_{QQ}	$\frac{1}{2}^+$
Ξ_{QQ}^*	$\frac{3}{2}^+$	Ω_{QQ}^*	$\frac{3}{2}^+$
$\Xi_{QQ'}$	$\frac{1}{2}^+$	$\Omega_{QQ'}$	$\frac{1}{2}^+$
$\Xi'_{QQ'}$	$\frac{1}{2}^+$	$\Omega'_{QQ'}$	$\frac{1}{2}^+$
$\Xi_{QQ'}^*$	$\frac{3}{2}^+$	$\Omega_{QQ'}^*$	$\frac{3}{2}^+$

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factors constitutes a central problem in studying such a class of decays.

The form factors belong to the nonperturbative domain of QCD, and some nonperturbative methods are needed for their determination. QCD sum rule is one of the powerful methods that take into account the nonperturbative effects [10]. One of the modifications of the traditional QCD sum rules method is the light-cone version [11] where operator product expansion (OPE) is performed over the twist of operators rather than the dimensions of operators.

The form factors induced by charged currents for $\Xi_{QQ'} \to \Lambda'_Q$ are estimated within light-cone sum rules [12], in QCD sum rules methods [12], and in light-front formalism [13]. A similar analysis for the $\Xi_{QQ'} \to \Sigma'_Q$ transition within the light-front quark model and light-cone QCD sum rules are investigated in [14,15,8], respectively.

FCNC processes of doubly heavy baryons in the framework of the light-front approaches are comprehensively studied in [7]. In the present work, we apply light-cone QCD sum rules to study doubly heavy baryon decays induced by FCNC transition. The paper is organized as follows. In Sec. II, we present the effective Hamiltonian responsible for $b \to s/dl^+l^-$ and $c \to ul^+l^-$ transition, and then the transition form factors are derived. The numerical results for the form factors are presented in Sec. III. The final section contains the summary and comparison of our findings with the predictions of other approaches.

II. LIGHT-CONE SUM RULES FOR THE TRANSITION FORM FACTORS

The effective Hamiltonian responsible for $b \to q$ ($q = s$ or d) is given by

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu), \tag{1}$$

where G_F is the Fermi coupling constant, V_{tb} and V_{tq} are the elements of CKM matrix elements. $\mathcal{O}_i$ corresponds to the local operators and C_i are their Wilson coefficients. Their explicit forms $\mathcal{O}_i$ and C_i can be obtained in [16]. The transition amplitude for $\Xi_{bb} \to \Lambda_b l^+l^-$ is given

$$\mathcal{M}(\Xi_{bb} \to \Lambda_b l^+l^-) = \frac{G_f \alpha_{em}}{2\sqrt{2}\pi} V_{tb} V_{td}^* \times \{[C_9^{\rm eff} \langle \Lambda_b | \bar{d} \gamma_\mu (1-\gamma_5) b | \Xi_{bb} \rangle - 2m_b C_7^{\rm eff} \langle \Lambda_b | \bar{d} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (1+\gamma_5) b | \Xi_{bb} \rangle] \bar{l} \gamma_\mu l \\ + C_{10} \langle \Lambda_b | \bar{d} \gamma_\mu (1-\gamma_5) b | \Xi_{bb} \rangle \bar{l} \gamma_\mu \gamma_5 l \}. \tag{2}$$

The transition amplitude for $\Xi_{cc} \to \Lambda_c l^+l^-$ induced by $c \to u$ transition can be obtained form Eq. (2) by replacing $V_{tb}V_{td}^*$ by $\sum_i V_{ci}V_{ui}^*$. Obviously, the numerical values of corresponding Wilson coefficients will be different than for $b \to d$ case and the values of C_9, C_7, and C_{10} for $c \to u$ transition can be found in [17–19].

The matrix element $\langle \Lambda_b(p) | \bar{d} \gamma_\mu (1-\gamma_5) b || \Xi_{bb}(p+q) \rangle$ where initial and final baryons are spin-1/2 states is parametrized in terms of the six form factors as follows:

$$\langle B_{Q'}(p) | \bar{q} \gamma_\mu (1-\gamma_5) Q | \Xi_{bb}(p+q) \rangle \\ = \bar{u}(p) \left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M} f_2 + \frac{q_\mu}{M} f_3 \right. \\ \left. - \left(\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M} g_2 + \frac{q_\mu}{M} g_3 \right) \gamma_5 \right] u(p+q), \tag{3}$$

where $q_\mu = p' - p$ and M is the mass for doubly heavy baryon.

The matrix element $\langle \Lambda_b(p) || \bar{q} i\sigma_{\mu\nu} q^\nu (1+\gamma_5) Q || \Xi_{bb}(p) \rangle$ is expressed in terms of the four form factors in the following way

$$\langle B_{Q'}(p) | \bar{q} i\sigma_{\mu\nu} q^\nu (1+\gamma_5) Q | \Xi_{bb}(p') \rangle \\ = \bar{u}(p) \left[\frac{f_1^T}{M} (\gamma_\mu q^2 - q_\mu \not{q}) + i f_2^T \sigma_{\mu\nu} q^\nu \right. \\ \left. + \frac{g_1^T}{M} (\gamma_\mu q^2 - q_\mu \not{q}) \gamma_5 + i g_2^T \sigma_{\mu\nu} q^\nu \gamma_5 \right]. \tag{4}$$

Before starting our analysis, we would like to note that the form factors f_i, g_i entering to Eq. (3) induced by $\bar{q}\gamma_\mu(1-\gamma_5)Q$ current within the same framework is studied in [8], and therefore, we will concentrate only on the calculation of f_i^T and g_i^T form factors. The calculations of the form factors are the main issue in analyzing semileptonic decays of baryons. In the present work, we employ the light-cone sum rules method (LCSR) to calculate the f_i^T and g_i^T form factors. For this goal, we consider the following correlation function

$$\Pi_\mu(p,q) = i \int d^4x e^{iqx} \langle B'_Q(p) | T\{ \bar{q} i\sigma_{\mu\alpha} q^\alpha (1+\gamma_5) Q \\ \times \bar{J}_{QQ'}(0) \} | 0 \rangle. \tag{5}$$

Here J_μ^{V-A} is the transition current, and $J_{QQ'}$ is the interpolating current for the spin-1/2 doubly heavy baryon.

For $Q = Q' = b$ or c, the interpolating currents of spin-$1/2$ are

$$J_{QQ} = \epsilon^{abc}(Q^{aT} C\gamma^{\alpha} Q^b)\gamma_\alpha \gamma_5 q^c, \tag{6}$$

where $q = u$ or d for the Ξ_{QQ} baryon. To construct the LCSR for the relevant form factors, the correlation functions should be calculated both from the hadronic and QCD sides. Then, the results of these two representations are matched according to quark-hadron duality ansatz.

The representation of the correlation function from the hadronic part is obtained by inserting a complete set of baryon states and isolating the ground state of Ξ_{bb} baryon. As a result, for the hadronic part of the correlation function Π_μ, we get

$$\begin{aligned}\Pi_\mu^{II} = \frac{\lambda}{M^2 - (p+q)^2}\bar{u}(v)\bigg\{&\left[\left(\frac{m}{M} - 1\right) f_1^T + f_2^T\right] \not{q} q_\mu \\ &+ 2m f_2^T \not{q} v_\mu + \left[\left(\frac{m}{M} + 1\right) g_1^T + g_2^T\right] \not{q}\gamma_5 q_\mu \\ &- 2m g_2^T \not{q}\gamma_5 v_\mu + \text{other structures}\bigg\},\end{aligned} \tag{7}$$

where v is the velocity of the heavy baryon, λ, and M are the residue and mass of doubly heavy baryons, and m corresponds to the mass of a single heavy baryon. To derive this equation, we used the definition of the decay constant

$$\langle B(p+q)_{Q'Q} | \bar{J}_{BB'} | 0 \rangle = \lambda \bar{u}(p+q). \tag{8}$$

Now let us turn our attention to the calculation of the correlation function from the QCD side. To achieve this, we used the operator product expansion (OPE) carried over twists of nonlocal operators and involves the single heavy baryon distribution amplitudes. Before presenting the details of the calculation of the correlation function from the QCD side, a few words about the distribution amplitudes (DAs) of Λ_Q baryon is in order. The light-cone distribution amplitudes for single-heavy baryons are obtained in [20,21] by using the sum rules method at the heavy quark mass limit. In this study, we used the DAs of Λ_Q obtained in [20]. The DAs of the antitriplet Λ_Q baryon are defined with the help of four matrix elements of nonlocal operators:

$$\begin{aligned}\frac{1}{v_+}\langle 0|[q_1(t_1)\mathcal{C}\gamma_5 \not{n} q_2(t_2)]Q_\gamma|H_b^{j=0}\rangle &= \psi_2(t_1,t_2) f^{(1)}_{H_b^{j=0}} u_\gamma, \\ \frac{i}{2}\langle 0|[q_1(t_1)\mathcal{C}\gamma_5 \sigma_{\bar{n}n} q_2(t_2)]Q_\gamma|H_b^{j=0}\rangle &= \psi_3^\sigma(t_1,t_2) f^{(2)}_{H_b^{j=0}} u_\gamma, \\ \langle 0|[q_1(t_1)\mathcal{C}\gamma_5 q_2(t_2)]Q_\gamma|H_b^{j=0}\rangle &= \psi_3^s(t_1,t_2) f^{(2)}_{H_b^{j=0}} u_\gamma, \\ v_+\langle 0|[q_1(t_1)\mathcal{C}\gamma_5 \not{\bar{n}} q_2(t_2)]Q_\gamma|H_b^{j=0}\rangle &= \psi_4(t_1,t_2) f^{(1)}_{H_b^{j=0}} u_\gamma,\end{aligned} \tag{9}$$

where n and $\bar{n}$ are two light-cone vectors, $\bar{v}^\mu = \frac{1}{2}(\frac{n^\mu}{v_+ - v_+\bar{n}^\mu})$, and t_i is the difference between the ith light quark and origin along the direction of n. The space coordinates of light quarks is $t_i n^\mu$. The four velocity of a single baryon is defined as $v^\mu = \frac{1}{2}(\frac{n^\mu}{v_+} + v_+\bar{n}^\mu)$. In our analysis, we will work in the rest frame of single heavy baryon, i.e., $v_+ = 1$. In addition $\psi_2, \psi_3^\sigma, \psi_3^s$, and ψ_4 denote the DAs with twists 2, 3, and 4, respectively. In present work, to determine the transition form factors, we only use the parallel DAs for Λ_Q baryon.

The following matrix element $\epsilon^{abc}\langle \Lambda_Q(v)|\bar{q}^a_{1\alpha}(t_1) \bar{q}^b_{2\beta}(t_2)\bar{Q}^c_\gamma(0)|0\rangle$ in the calculation of the correlation function. This matrix element is determined in terms of the heavy baryon distribution amplitudes as follows

$$\begin{aligned}\epsilon^{abc}\langle \Lambda_Q(v)|\bar{q}^a_{1\alpha}(t_1)\bar{q}^b_{2\beta}(t_2)\bar{Q}^c_\gamma(0)|0\rangle = {}&\frac{1}{8}\psi_2^*(t_1,t_2) f^{(1)}\bar{u}_\gamma(C^{-1}\gamma_5 \not{\bar{n}})_{\alpha\beta} - \frac{1}{8}\psi^*_{3\sigma}(t_1,t_2) f^{(2)}\bar{u}_\gamma(C^{-1}\gamma_5 i\sigma^{\mu\nu})_{\alpha\beta}\bar{n}_\mu n_\nu \\ &+ \frac{1}{4}\psi^*_{3s}(t_1,t_2) f^{(2)}\bar{u}_\gamma(C^{-1}\gamma_5)_{\alpha\beta} + \frac{1}{8}\psi_4^*(t_1,t_2) f^{(1)}\bar{u}_\gamma(C^{-1}\gamma_5 \not{n}\pi)_{\alpha\beta}.\end{aligned} \tag{10}$$

The DAs $\psi(t_1, t_2)$ is defined as

$$\psi(t_1,t_2) = \int_0^\infty d\omega\omega \int_0^1 du e^{-i\bar{u}\omega v(x_2 - x_1)} e^{-iwvx_1}\psi(\omega, u), \tag{11}$$

where ω is the total diquark momentum. Using the expressions of the interpolating current for Ξ_{QQ} baryon as well as the transition current, the correlation function can be written as

$$\begin{aligned}\Pi_\mu(p,q) = -\frac{i}{4}\int d^4x \int_0^\infty d\omega\omega \int_0^1 du e^{i(q+\bar{u}\omega v)x} \\ \times \left\{\bar{u}\sum_{i=1}^4 a_i(\gamma_\nu C)^T S_Q^T(x)\mathcal{T}_\mu^T(C^{-1}\gamma_5\Gamma_i)^T\gamma_\nu\gamma_5\right\},\end{aligned} \tag{12}$$

$$\begin{aligned}
&\mathcal{T}_\mu = i\sigma_{\mu\rho}q^\rho(1+\gamma_5), \\
&a_1 = f^{(1)}\psi_2, \qquad \Gamma_1 = \bar{\not{n}}, \\
&a_2 = -f^{(2)}\psi_{3\sigma}, \qquad \Gamma_2 = i\sigma_{\alpha\beta}\bar{n}^\alpha n^\beta, \\
&a_3 = 2f^{(2)}\psi_{3s}, \qquad \Gamma_3 = 1, \\
&a_4 = -f^{(1)}\psi_4, \qquad \Gamma_4 = \not{n},
\end{aligned} \tag{13}$$

and $S_Q(x)$ is the free heavy quark propagator. $\bar{v}$ as well as the light-cone vectors n, $\bar{n}$ are defined as

$$\begin{aligned}
n_\mu &= \frac{1}{vx}v_\mu, \\
\bar{n}_\mu &= 2v_\mu - n_\mu, \\
\bar{v}_\mu &= n_\mu - v_\mu.
\end{aligned} \tag{14}$$

Taking Fourier transformation of heavy quark propagator, we get

$$\Pi_\mu(p,q) = \frac{1}{4}\int d^4x \int d\omega\omega \int du \int \frac{d^4k}{(2\pi)^4} e^{i(q+\bar{u}wv-k)x} \times \left[\bar{u}\sum_{i=1}^{4} a_i\gamma_\nu \frac{\not{k}-m_Q}{k^2-m_Q^2} C\mathcal{T}_\mu^T\Gamma_i^T\gamma_5^T C^{-1}\gamma_\nu\gamma_5\right]. \tag{15}$$

After performing integration over x, one can obtain the explicit expression for the correlation function from the QCD side as follows:

$$\Pi_\mu^{\rm QCD}[(p+q)^2, q^2] = \int du \int dw \sum_{n=1}^{2}\left\{\frac{\rho_n^{(1)}(u,w)}{(\Delta-m_Q^2)^n}\not{q}q_\mu + \frac{\rho_n^{(2)}(u,w)}{(\Delta-m_Q^2)^n}\not{q}v_\mu + \frac{\rho_n^{(3)}(u,w)}{(\Delta-m_Q^2)^n}\not{q}\gamma_5 q_\mu + \frac{\rho_n^{(4)}(u,w)}{(\Delta-m_Q^2)^n}\not{q}\gamma_5 v_\mu + \text{other structures}\right\}, \tag{16}$$

where $\Delta = (q+\bar{u}wv)^2$ and $\bar{u} = 1-u$. The explicit expressions of $\rho_n^{(i)}(u,w)$ are presented in Appendix A. By choosing the coefficients of the structures $\not{q}q_\mu$, $\not{q}v_\mu$, $\not{q}\gamma_5 q_\mu$, and $\not{q}\gamma_5 v_\mu$ in both sides of the correlation function and performing Borel transformation on variable $-(p+q)^2$, we get the following sum rules for the form factors f_1^T, f_2^T, g_1^T, and g_2^T:

$$\begin{aligned}
\lambda\left[f_1^T\left(\frac{m}{M}-1\right) + f_2^T\right]e^{-\frac{M^2}{M_B^2}} &= \Pi_1^B, \\
2\lambda m f_2^T e^{-\frac{M^2}{M_B^2}} &= \Pi_2^B, \\
-\lambda\left[g_1^T\left(\frac{m}{M}+1\right) + g_2^T\right]e^{-\frac{M^2}{M_B^2}} &= \Pi_3^B, \\
-2m\lambda g_2^T e^{-\frac{M^2}{M_B^2}} &= \Pi_4^B,
\end{aligned} \tag{17}$$

where Π_i^B is the Borel transformed coefficients of structures mentioned above from the QCD side, and m_B is the Borel mass parameter. The Borel transformation is performed with the help of the master formula

$$\int dw \frac{\rho_n(u,w)}{(\Delta-m_Q^2)^n} = \sum_{n=1}^{\infty}\left\{(-1)^n\int_0^{w_0} dw e^{(-s(w,q^2))/M_B^2}\frac{1}{(n-1)!(M_B^2)^{n-1}}\frac{\rho_n(u,w)}{(\bar{u}w/m)^n} - \left[\frac{(-1)^{n-1}}{(n-1)!}e^{(-s(w,q^2))/M_B^2}\sum_{j=1}^{n-1}\frac{1}{(M_B^2)^{n-j-1}}\frac{1}{s'}\left(\frac{\mathrm{d}}{\mathrm{d}w}\frac{1}{s'}\right)^{j-1}\frac{\rho_n(u,w)}{(\bar{u}w/m)^n}\right]_{w=w_0}\right\}, \tag{18}$$

where $s(u,\omega) = \frac{m_Q^2 - \bar{u}\omega(\bar{u}\omega - m) - (1-\frac{\bar{u}\omega}{m})q^2}{\frac{\bar{u}\omega}{m}}$, and $s' = \frac{ds}{d\omega}$ and ω_0 are the solutions of the $s_{\rm th} = s$ and $s = 4m_Q^2$ equations, respectively.

III. NUMERICAL ANALYSIS

In this section, first, we perform a numerical analysis of the transition form factors. Then using the results for the form factors, we estimate the decay width of $\Xi_{QQ} \to \Lambda_Q l^+ l^-$ transitions. The main parameters entering the sum rules are the mass of the heavy quarks, the mass of the doubly heavy baryon and its residue, and the decay constants $f^{(1)}$ and $f^{(2)}$ of Λ_Q baryon. For the masses of c and b quarks, we used their values in $\overline{MS}$ scheme, i.e., $\bar{m}_c(\bar{m}_c) = 1.275 \pm 0.025$ GeV and $\bar{m}_b(\bar{m}_b) = 4.18 \pm 0.03$ GeV [22], respectively. Moreover, the mass and decay constant λ of doubly heavy baryons are [8]

$$M_{\Xi_{cc}} = 3.621 \text{ GeV}, \qquad \lambda_{\Xi_{cc}} = 0.109 \pm 0.020,$$
$$M_{\Xi_{bb}} = 10.143 \text{ GeV}, \qquad \lambda_{\Xi_{bb}} = 0.190 \pm 0.052.$$

The mass values of the Λ_c and Λ_b baryons are taken as [22] $m_{\Lambda_c} = 2.286$ GeV and $m_{\Lambda_b} = 5.620$ GeV. For the decay constants of Λ_c and Λ_b baryons, we used $f^{(1)} = f^{(2)} = (2.8 \pm 0.2) \times 10^{-2}$ GeV^3 [23].

The main nonperturbative input parameters of any light-cone sum rules are the light-cone DAs of corresponding baryons. Although the DAs of Λ_c have not been known yet, thanks to the heavy quark limit, they are supposed to have the same form as Λ_b. Hence, we can use the same form of DAs presented in [20,21] for both Λ_b and Λ_c baryons.

$$\begin{aligned}
\psi_2(\omega,u) &= \frac{15}{2}A^{-1}\omega^2\bar{u}u\int_{\omega/2}^{s_0} dse^{-s/y}(s-\omega/2),\\
\psi_4(\omega,u) &= 5A^{-1}\int_{\omega/2}^{s_0} dse^{-s/y}(s-\omega/2)^3,\\
\psi_{3s}(\omega,u) &= \frac{15}{4}A^{-1}\omega\int_{\omega/2}^{s_0} dse^{-s/y}(s-\omega/2)^2,\\
\psi_{3\sigma}(\omega,u) &= \frac{15}{4}A^{-1}\omega(2u-1)\int_{\omega/2}^{s_0} dse^{-s/y}(s-\omega/2)^2, \qquad (19)
\end{aligned}$$

where

$$A = \int_0^{s_0} dss^5 e^{-s/y}, \qquad (20)$$

and y and s_0 are the Borel parameter and the continuum threshold introduced by QCD sum rules in [20,21]. While y is varied in the interval 0.4 GeV $< y <$ 0.8 GeV a fixed value $s_0 = 1.2$ GeV is used. Note that the DAs are only nonvanishing in the region $0 < \omega < 2s_0$ [21].

In addition to these input parameters, the sum rules for the form factors contain two auxiliary parameters, continuum thresholds $s_{\rm th}$ and the Borel mass parameters. The choice of the working regions of M_B^2 regions is based on the standard criteria; namely, both power corrections and continuum contributions should be sufficiently suppressed.

TABLE II. The working regions of Borel mass parameter M_B^2 and the continuum threshold $s_{\rm th}$.

Channels	$M_B^2(\text{GeV}^2)$	$s_{\rm th}(\text{GeV}^2)$
$\Xi_{cc} \to \Lambda_c ll$	6 ± 1	16 ± 1
$\Xi_{bb} \to \Lambda_b ll$	14 ± 1	114 ± 2

The upper limit of M_B^2 is determined from the condition that the contributions of continuum and excited states are less than 30% of the total LCSR result, i.e., $\frac{\Pi(M_B^2,s_{\rm th})}{\Pi(M_B^2,\infty)} \lesssim 0.3$. The lower limit of M_B^2 is obtained by requiring the convergence of operator product expansion near the light cone for the correlation in the deep Euclidean domain, i.e., the contributions of the higher twist amplitudes constitute maximum 10% of the leading twist contribution, $\frac{\Pi(M_B^2,\text{twist}-4\text{ DAs})}{\Pi(M_B^2,\text{twist}-2\text{ DAs})} \lesssim 0.1$. Here $\Pi(M_B^2,\text{twist}-4\text{ DAs})$ and $\Pi(M_B^2,\text{twist}-2\text{ DAs})$ correspond to the higher twist-4 and leading twist-2 DA contributions, respectively. Both conditions are simultaneously satisfied in the regions depicted in Table II. The value of $s_{\rm th}$ is chosen in such a way that the differentiated mass sum rules reproduce 10% accuracy of the corresponding baryon. Considering these criteria, we obtain the working region for $s_{\rm th}$ that is presented in Table II.

The light-cone sum rules are reliable in the region where q^2 is not too large, and our calculation indicates that LCSR predictions are reliable up to $q^2 \lesssim 0.5$ GeV^2 for Λ_c and $q^2 \lesssim 10$ GeV^2 for Λ_b baryon, respectively. In order to study the stability of the form factors with respect to the variation of M_B^2 and $s_{\rm th}$, we present the dependency of the form factors on Borel mass square at the fixed values of $s_{\rm th}$ in Fig. 1 for $\Xi_{bb} \to \Lambda_b$ transition at $q^2 = 0$. Note that the choice of this point is due to the fact that LCSR works well at this point. From this figure, we see that all the form factors exhibit good stability with respect to the variation of M_B^2 in its working region. And, we deduce the values of the form factors at $q^2 = 0$ presented in Table III. Performing the similar analysis for the form factors responsible for $\Xi_{cc} \to \Lambda_c$ transition and at $q^2 = 0$ point, their values are also presented in Table III.

To extend the obtained results of the form factors to the physical region $4m_l^2 \leq q^2 \leq (M^2 - m_2)^2$, some parametrization is needed. For this goal, we look for a parametrization so that in the region where LCSR is reliable, the result of this parametrization and predictions of LCSR coincide. Numerical results show that the best parametrization of the form factors that satisfy the above criteria can be represented as a double pole parametrization

$$F_i(q^2) = \frac{F_i(0)}{1 - \frac{q^2}{m_i^2} + \alpha_i(\frac{q^2}{m_i^2})^2}. \qquad (21)$$

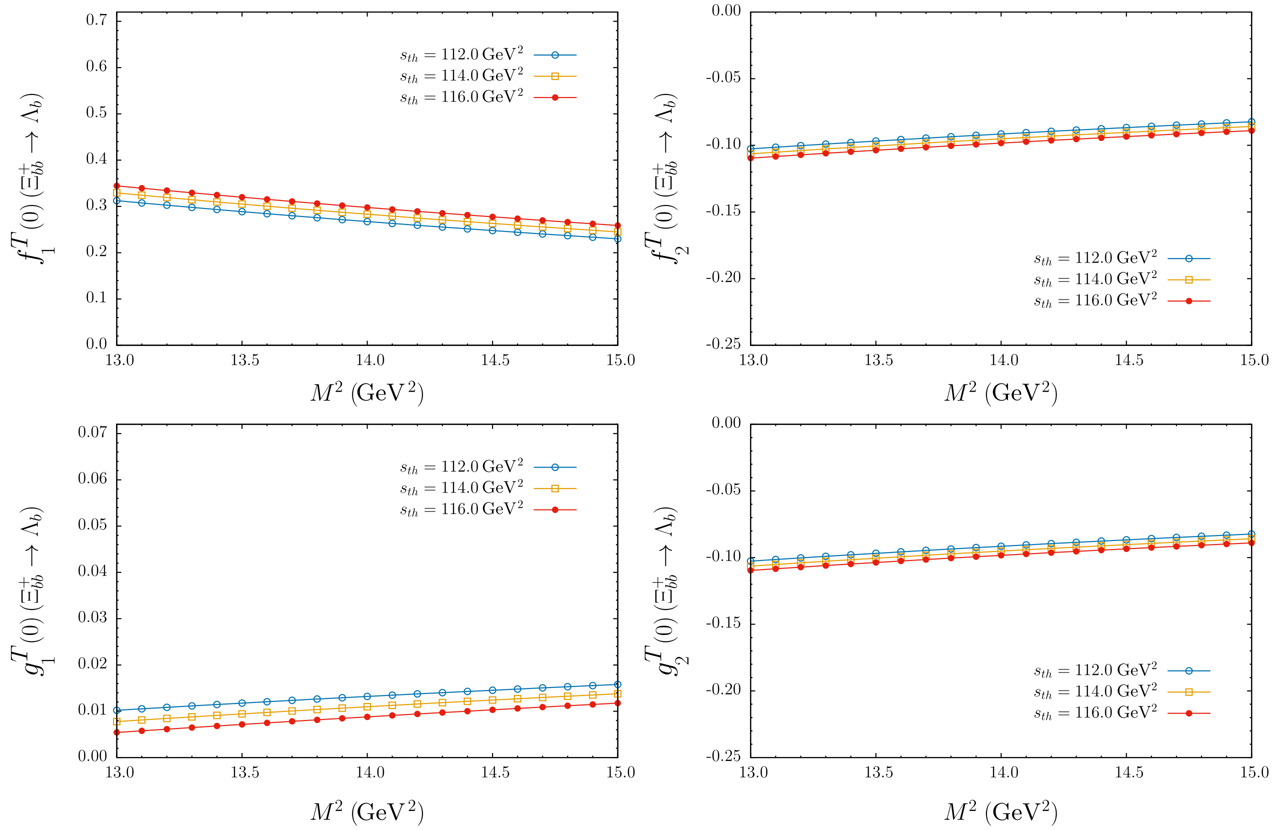

FIG. 1. The variation of form factors (at $q^2 = 0$) with respect to the Borel mass parameter at the fixed values of $s_{\rm th}$ is shown.

The obtained fitting parameters α_i and m_i are collected in Table III.

Having the form of the form factors, our final goal is the calculation of the branching ratios of $\Xi_{cc}^{++} \to \Lambda_c l^+ l^-$ and $\Xi_{bb}^{++} \to \Lambda_b l^+ l^-$ transition. After straightforward calculations of the decay width for $\Xi_{bb}^{++} \to \Lambda_b l^+ l^-$, we find

$$\frac{d\Gamma}{ds} = \frac{G_F^2 \alpha_{em}^2 M}{4096\pi^5} |V_{tb} V_{td}^*|^2 v \sqrt{\lambda(1,r,s)} \left[T_1(s) + \frac{1}{3} T_2(s) \right], \tag{22}$$

where α_{em} is the fine structure constant, $v = \sqrt{1 - 4m_l^2/q^2}$ is the velocity of the lepton, and $\lambda(1,r,s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $r = \frac{m^2}{M^2}$, $s = \frac{q^2}{M^2}$. The expressions of the $T_1(s)$ and $T_2(s)$ are presented in Appendix B.

The differential decay width for $\Xi_{cc} \to \Lambda_c l^+ l^-$ transition can easily be obtained from Eq. (22) by the following replacements:

$$\begin{aligned} m_{\Lambda_b} &\to m_{\Lambda_c}, \qquad M_{\Xi_{bb}} \to M_{\Xi_{cc}}, \\ \tau_{\Xi_{cc}} &\to \tau_{\Xi_{bb}}, \qquad V_{tb} V_{td} \to \sum V_{ci} V_{ui}^*. \end{aligned} \tag{23}$$

TABLE III. The values of the fitting parameters $F(0)$, α, and m.

Channels	F	$F(0)$	α	m_i
$\Xi_{cc} \to \Lambda_c l^+ l^-$	f_1^T	-1.25 ± 0.14	83.95 ± 6.93	4.15 ± 0.11
$\Xi_{cc} \to \Lambda_c l^+ l^-$	f_2^T	0.13 ± 0.01	0.607 ± 0.004	1.1878 ± 0.0008
$\Xi_{cc} \to \Lambda_c l^+ l^-$	g_1^T	0.51 ± 0.06	-0.0934 ± 0.0004	1.47389 ± 0.00005
$\Xi_{cc} \to \Lambda_c l^+ l^-$	g_2^T	0.13 ± 0.01	0.607 ± 0.003	1.1879 ± 0.0009
$\Xi_{bb} \to \Lambda_b l^+ l^-$	f_1^T	0.28 ± 0.03	0.786 ± 0.005	3.76 ± 0.003
$\Xi_{bb} \to \Lambda_b l^+ l^-$	f_2^T	-0.09 ± 0.01	1.891 ± 0.002	4.659 ± 0.001
$\Xi_{bb} \to \Lambda_b l^+ l^-$	g_1^T	0.01 ± 0.001	10.19 ± 5.19	2.10 ± 0.29
$\Xi_{bb} \to \Lambda_b l^+ l^-$	g_2^T	-0.09 ± 0.01	1.891 ± 0.002	4.659 ± 0.001

TABLE IV. Branching ratios for the considered decays.

Channels	Our result	[7]	[24]
$\Xi_{cc}^{++} \to \Lambda_c^+ l^+ l^-$	(1.16×10^{-13})	…	…
$\Xi_{bb}^{++} \to \Lambda_b^+ e^+ e^-$	(1.14×10^{-9})	3.63×10^{-9}	2.33×10^{-9}
$\Xi_{bb}^{++} \to \Lambda_b^+ \mu^+ \mu^-$	(4.04×10^{-10})	3.55×10^{-9}	2.24×10^{-9}
$\Xi_{bb}^{++} \to \Lambda_b^+ \tau^+ \tau^-$	(4.38×10^{-12})	9.86×10^{-10}	8.49×10^{-11}

Integrating over s in the region $\frac{4m_l^2}{M^2} \leq s \leq (1-\sqrt{r})^2$ and using the lifetimes of Ξ_{cc}^+ and Ξ_{bb} baryons $\tau_{\Xi_{cc}^+} = 45 \times 10^{-15} s$ [1] and $\tau_{\Xi_{bb}^+} = 370 \times 10^{-15} s$ [2], we get the results for the branching ratio presented in Table IV.

In this table, we also present the results obtained in the light-front approach [7,24]. From the comparison, we see that our branching ratios' results are smaller than those presented in [7,24]. The main source of the discrepancy between the predictions of the different approaches is the values of the form factors. Our final remark to this section is that our result can be improved by taking into account $\mathcal{O}(\alpha_s)$ corrections to the distribution amplitudes of Λ_Q.

IV. CONCLUSION

In the present work, we calculate the form factors of the $\Xi_{cc}^+ \to \Lambda_c^+ l^+ l^-$ and $\Xi_{bb} \to \Lambda_b^0 l^+ l^-$ induced by tensor currents within the light cone sum rules. Using the obtained results for the form factors, we estimate the corresponding branching ratios induced by neutral currents $c \to u l^+ l^-$ and $b \to d l^+ l^-$ transitions. We also compared our results with the predictions obtained in the framework of the light-front approach and found out that our predictions on the corresponding branching ratios are smaller than the ones predicted in the light-front approach. The predictions obtained for the branching ratio, especially for $\Xi_{bb} \to \Lambda_b^0 l^+ l^-$, hopefully will be inspected at LHCb; however, the branching ratio for $\Xi_{cc}^+ \to \Lambda_c^+ l^+ l^-$ is too small to be detected in the near future with current detector technologies.

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APPENDIX A: EXPLICIT EXPRESSIONS OF THE SPECTRAL DENSITIES

In this appendix, we present the explicit expressions of $\rho_n^{(i)}(u,w)$ from Eq. (16)

$$
\begin{aligned}
\rho_1^{(1)}(u,w) &= -f^{(2)} w \psi_3^{(s)}(u,w), \\
\rho_2^{(1)}(u,w) &= -\bar{u}\{2f^{(2)}[\bar{u}w - (q\cdot v)]\hat{\psi}_3^{(\sigma)}(u,w) + f^{(1)} m_Q[\hat{\psi}_4(u,w) - \hat{\psi}_2(u,w)]\}, \\
\rho_1^{(2)}(u,w) &= -2f^{(2)}\bar{u}[w^2\psi_3^{(s)}(u,w) + \hat{\psi}_3^{(\sigma)}(u,w)], \\
\rho_2^{(2)}(u,w) &= 4f^{(2)}\bar{u}^2 w(q\cdot v)\hat{\psi}_3^{(\sigma)}(u,w), \\
\rho_1^{(3)}(u,w) &= f^{(2)} w \psi_3^{(s)}(u,w), \\
\rho_2^{(3)}(u,w) &= \bar{u}\{2f^{(2)}[\bar{u}w - (q\cdot v)]\hat{\psi}_3^{(\sigma)}(u,w) - f^{(1)} m_Q[\hat{\psi}_4(u,w) - \hat{\psi}_2(u,w)]\}, \\
\rho_1^{(4)}(u,w) &= -2f^{(2)}\bar{u}[w^2\psi_3^{(s)}(u,w) + \hat{\psi}_3^{(\sigma)}(u,w)], \\
\rho_2^{(4)}(u,w) &= -4f^{(2)}\bar{u}^2 w(q\cdot v)\hat{\psi}_3^{(\sigma)}(u,w),
\end{aligned}
\tag{A1}
$$

where $\bar{u} = 1-u$. The functions $\hat{\psi}$ entering the above equation is defined as

$$
\hat{\psi}(u,\omega) = \int_0^\omega dy y \psi(u,y). \tag{A2}
$$

APPENDIX B: THE DIFFERENTIAL WIDTH FOR $\Xi_{QQ} \to \Lambda_Q l^+ l^-$ DECAY

Here, we present the differential width for $\Xi_{bb} \to \Lambda_b l^+ l^-$ decay for completeness (see [25]). As, we noted in the main body of the text, the differential decay width for this transition is given by

$$
\frac{d\Gamma}{ds} = \frac{G_F^2 \alpha_{em}^2 M}{4096\pi^5} |V_{tb} V_{td}^*|^2 v \sqrt{\lambda(1,r,s)} \left[T_1(s) + \frac{1}{3} T_2(s)\right]. \tag{B1}
$$

The functions $T_1(s)$ and $T_2(s)$ are

$$\begin{aligned}
T_1(s) = 8M^2\{&(1-2\sqrt{r}+r-s)[4m_\ell^2+M^2(1+2\sqrt{r}+r+s)]|F_1|^2\\
&-[4m_\ell^2(1-6\sqrt{r}+r-s)-M^2((1-r)^2-4\sqrt{r}s-s^2)]|F_4|^2\\
&+(1-2\sqrt{r}+r-s)[4m_\ell^2(1+\sqrt{r})^2+M^2s(1+2\sqrt{r}+r+s)]|F_2|^2\\
&+M^2s[(-1+r)^2-4\sqrt{r}s-s^2]v^2|F_4|^2+4m_\ell^2(1+2\sqrt{r}+r-s)s|F_6|^2\\
&+(1+2\sqrt{r}+r-s)[4m_\ell^2+M^2(1-2\sqrt{r}+r+s)]|G_1|^2\\
&-[4m_\ell^2(1+6\sqrt{r}+r-s)-M^2((1-r)^2+4\sqrt{r}s-s^2)]|G_4|^2\\
&+(1+2\sqrt{r}+r-s)[4m_\ell^2(1-\sqrt{r})^2+M^2s(1-2\sqrt{r}+r+s)]|G_2|^2\\
&+M^2s[(1-r)^2+4\sqrt{r}s-s^2]v^2|G_5|^2+4m_\ell^2(1-2\sqrt{r}+r-s)s|G_6|^2\\
&-4(1+\sqrt{r})(1-2\sqrt{r}+r-s)(2m_\ell^2+M^2s)\mathrm{Re}[F_1^*F_2]\\
&-4M^2(1+\sqrt{r})(1-2\sqrt{r}+r-s)sv^2\mathrm{Re}[F_4^*F_5]\\
&-8m_\ell^2(1-\sqrt{r})(1+2\sqrt{r}+r-s)\mathrm{Re}[F_4^*F_6]\\
&-4(1-\sqrt{r})(1+2\sqrt{r}+r-s)(2m_\ell^2+M^2s)\mathrm{Re}[G_1^*G_2]\\
&-4M^2(1-\sqrt{r})(1+2\sqrt{r}+r-s)sv^2\mathrm{Re}[G_4^*G_5]]\\
&+8m_\ell^2(1+\sqrt{r})(1-2\sqrt{r}+r-s)\mathrm{Re}[G_4^*G_6]\},
\end{aligned} \tag{B2}$$

$$T_2(s) = -8M^4v^2\lambda(1,r,s)[|F_1|^2+|F_4|^2+|G_1|^2+|G_4|^2-s(|F_2|^2+|F_5|^2+|G_2|^2+|G_5|^2)], \tag{B3}$$

where

$$\begin{aligned}
F_1 &= C_9 f_1 - \frac{2m_b}{M}C_7 f_1^T,\\
F_2 &= C_9 f_2 + \frac{2m_b}{q^2}M f_2^T,\\
F_3 &= C_9 f_3 - \frac{2m_b}{q^2}C_7(M-m) f_1^T,\\
G_1 &= C_9 g_1 - \frac{2m_b}{M}C_7 g_1^T,\\
G_2 &= C_9 g_2 + \frac{2m_b}{q^2}M g_2^T,\\
G_3 &= C_9 g_3 - \frac{2m_b}{q^2}C_7(M+m) g_1^T,\\
F_4 &= C_{10} f_1,\\
F_5 &= C_{10} f_2,\\
F_6 &= C_{10} f_3,\\
G_4 &= C_{10} g_1,\\
G_5 &= C_{10} g_2,\\
G_6 &= C_{10} g_3.
\end{aligned} \tag{B4}$$

The differential width for the $\Xi_{cc} \to \Lambda_c l^+ l^-$ can be obtained from the result presented above by replacements given in Eq. (23).

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