Possible assignments of the scalar $K_0^*(1950)$ and $K_0^*(2130)$ within the $^3P_0$ model

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We have evaluated the strong decays of the $K_0^*(1950)$ and $K_0^*(2130)$ within the $^3P_0$ model by employing the meson wave functions from the relativized quark model. By comparing with the experimental measurements, the $K_0^*(2130)$ could be assigned as $K_0^*(3^3P_0)$, while the $K_0^*(1950)$ is difficult to be explained as the $q\bar{q}$ meson. We also predict that the $K_0^*(2^3P_0)$ state has a mass of about 1811 MeV and a width of about 656 MeV, while the $K_0^*(4^3P_0)$ state has a mass of about 2404 MeV and a width of about 180 MeV.

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I. INTRODUCTION

According to the theory of quantum chromodynamics, in addition to the conventional $q\bar{q}$ mesons, the so-called exotic states are also permitted, such as tetraquarks, molecules, glueballs, and hybrids [1–3]. Although many exotic states have been observed experimentally, such as $X(3872)$, $Z_c(3900)$, $Z_c(4025)$, P_c, and T_{cc}, it is still difficult to distinguish between the exotic states with conventional quantum numbers and the ordinary $q\bar{q}$ and qqq hadrons.

One puzzle in hadron spectra is the scalar mesons, since there are too many states to be accommodated within the quark model without difficulty [4]. For example, the $K_0^*(700)$ state (also known as κ), together with its multiple partners $a_0(980)$, $f_0(500)(\sigma)$, and $f_0(980)$, does not fit well into the predictions of the quark model, since the observed mass ordering of these lowest scalar states is $m_\sigma < m_\kappa < m_{a_0,f_0}$ [5], while in the conventional quark model, by a naive counting of the quark mass, the mass ordering of the scalar $q\bar{q}$ nonet should be $m_\sigma \sim m_{a_0} < m_\kappa < m_{f_0}$. These scalar states below 1 GeV are generally believed not to be $q\bar{q}$ states [2,3,6–9].

Within the naive quark model, it is natural to assume that the $a_0(1450)$, $K_0^*(1430)$, $f_0(1710)$, and $f_0(1370)$ are the 1^3P_0 members of the SU(3) flavor nonet [5]. The isovector scalar mesons $a_0(2020)/a_0(1950)$ are suggested to be the good candidates of the $a_0(3^3P_0)$ in our previous work [10]. However, the assignments of the excited scalar K_0^* states are still unclear. Up to now, above the $K_0^*(1430)$ mass, only two scalar states $K_0^*(1950)$ and $K_0^*(2130)$ are reported [5,11]. Their masses and widths are listed in Table I.

Since $K_0^*(1950)$ was first reported in the $K\pi$ invariant mass distribution of the $K^-p \to K^-\pi^+n$ reaction by the Large Aperture Solenoid Spectrometer in 1988 [12], it is difficult to interpret its properties within the quark model. The $K_0^*(1950)$ mass is close to the $K_0^*(2^3P_0)$ mass of about 1890 MeV predicted by the Godfrey-Isgur (GI) quark model [13]. However, it is expected that the $K_0^*(2^3P_0)$ with a mass of 1850 MeV has a width of about 450 MeV within the $^3P_0$ decay model [14], larger than the $K_0^*(1950)$ width. In addition, Ref. [15] recently analyzed the kaon family within the modified GI (MGI) model involving the color screening effect and predicted the mass and width of the $K_0^*(2^3P_0)$ to be $M = 1829$ and $\Gamma = 1000$ MeV, respectively, both of which disfavor the assignment of $K_0^*(1950)$ as the candidate

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TABLE I. The masses and widths of the excited scalar $K_0^*(1950)$ and $K_0^*(2130)$ (in MeV).

State	Mass	Width	Reference
$K_0^*(1950)$	$1945 \pm 10 \pm 20$	$201 \pm 34 \pm 79$	[12]
	$1979 \pm 26 \pm 3$	$144 \pm 44 \pm 21$	[11]
$K_0^*(2130)$	$2128 \pm 31 \pm 9$	$95 \pm 42 \pm 76$	[11]

of the $K_0^*(2^3P_0)$ state. Recently, the *BABAR* Collaboration also reported the signal of $K_0^*(1950)$ in the process $\eta_c \to \eta' K^+ K^-$ [11] and presented two solutions. One solution gives $M = 1942 \pm 22 \pm 5$ and $\Gamma = 80 \pm 32 \pm 20$ MeV with the significance of 3.3σ, and the other solution gives $M = 1979 \pm 26 \pm 3$ MeV and $\Gamma = 144 \pm 44 \pm 21$ MeV with the significance of 4.3σ by adding the $K_0^*(2130)$ state. Since *BABAR* has shown that the hypothesis including the $K_0^*(2130)$ state gives an overall improvement of the measurements, we take the $K_0^*(1950)$ properties of the second solution, as tabulated in Table I.

The mass of $K_0^*(2130)$ observed by the *BABAR* Collaboration [11] is $2128 \pm 31 \pm 9$ MeV, close to the predicted $K_0^*(3^3P_0)$ mass of 2176 MeV from the MGI model [15]. In addition, we have estimated that the $n\bar{n}(3^3P_0)$ mass is about 1.9–2.0 GeV [10], thus one can naturally expect the $K_0^*(3^3P_0)$ mass should be about 100–200 MeV larger than the $a_0(3^3P_0)$ mass. Based on its mass information, the $K_0^*(2130)$ seems a good candidate of the $K_0^*(3^3P_0)$.

The mass information alone is insufficient to identify the $K_0^*(2130)$ as the $K_0^*(3^3P_0)$ state. We shall discuss the possibility of the $K_0^*(2130)$ as the $K_0^*(3^3P_0)$ state by studying its strong decay properties.

In this work, we will investigate the possible assignment of $K_0^*(2130)$ by analyzing the strong decay behaviors within the $^3P_0$ decay model. For completeness, we also check the possibility of the $K_0^*(1950)$ as the ordinary scalar mesons, since it is natural and necessary to exhaust the possible $q\bar{q}$ descriptions of a newly observed state before restoring to the more exotic assignments. This paper is organized as follows. In Sec. II, we introduce the $^3P_0$ strong decay model used in our calculations, and the results and discussions are given in Sec. III. Finally, a summary is given in Sec. IV.

II. MODEL AND PARAMETERS

The $^3P_0$ model has been widely used to study the Okubo-Zweig-Iizuka–allowed open flavor two-body strong decays, originally introduced by Micu [16] and further developed by Le Yaouanc *et al.* [17–19]. In the $^3P_0$ model, the meson strong decay takes place by producing a quark-antiquark pair with vacuum quantum number $J^{PC} = 0^{++}$. The newly produced quark-antiquark pair, together with the $q\bar{q}$ within the initial meson, regroups into two outgoing mesons in two possible quark rearrangement ways, as shown in Fig. 1. The $^3P_0$ model has been widely applied to study strong decays of hadrons with considerable success [10,14,20–35].

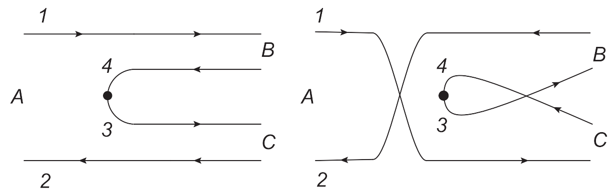

FIG. 1. Two-body decay diagrams of $A \to BC$ according to the $^3P_0$ model, where a pair of quark-antiquark is created to form the final two mesons. Left: meson B is formed by the quark in meson A combined with the created antiquark, and meson C is formed by the antiquark in meson A combined with the created quark. Right: meson B is formed by the antiquark in meson A combined with the created quark, and meson C is formed by the quark in meson A combined with the created antiquark.

Following the conventions in Refs. [20,21], the transition operator T of the decay $A \to BC$ in the $^3P_0$ model is given by

$$T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int d^3\boldsymbol{p}_3 d^3\boldsymbol{p}_4 \delta^3(\boldsymbol{p}_3 + \boldsymbol{p}_4) \times \mathcal{Y}_1^m\left(\frac{\boldsymbol{p}_3 - \boldsymbol{p}_4}{2}\right)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34} b_3^\dagger(\boldsymbol{p}_3) d_4^\dagger(\boldsymbol{p}_4), \quad (1)$$

where the γ is a dimensionless parameter corresponding to the production strength of the quark-antiquark pair $q_3\bar{q}_4$ with quantum number $J^{PC} = 0^{++}$. $\boldsymbol{p}_3$ and $\boldsymbol{p}_4$ are the momenta of the created quark q_3 and antiquark $\bar{q}_4$, respectively. $\chi_{1,-m}^{34}$, ϕ_0^{34}, and ω_0^{34} are the spin, flavor, and color wave functions of $q_3\bar{q}_4$ system, respectively. The solid harmonic polynomial $\mathcal{Y}_1^m(\boldsymbol{p}) \equiv |\boldsymbol{p}|^1 Y_1^m(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\overline{q_4}$.

The S matrix of the process $A \to BC$ is defined by

$$\langle BC|S|A\rangle = I - 2\pi i\delta(E_A - E_B - E_C)\langle BC|T|A\rangle, \quad (2)$$

where $|A\rangle$, $|B\rangle$ and $|C\rangle$are the wave functions of the mock mesons defined by Ref. [36].

The transition matrix element $\langle BC|T|A\rangle$ can be written as

$$\langle BC|T|A\rangle = \delta^3(\boldsymbol{p}_A - \boldsymbol{p}_B - \boldsymbol{p}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\boldsymbol{p}), \quad (3)$$

where $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\boldsymbol{p})$ is the helicity amplitude.

The partial wave amplitude $\mathcal{M}^{LS}(\boldsymbol{p})$ can be given by [37]

$$\mathcal{M}^{LS}(\boldsymbol{p}) = \sum_{\substack{M_{J_B},M_{J_C},\\ M_S,M_L}} \langle LM_L SM_S | J_A M_{J_A}\rangle \times \langle J_B M_{J_B} J_C M_{J_C} | SM_S\rangle \times \int d\Omega Y^*_{LM_L} \mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\boldsymbol{p}). \quad (4)$$

Various $^3P_0$ models exist in literature and typically differ in the choices of the pair-production vertex, the phase space conventions, and the meson wave functions employed. In this work, we restrict to the simplest vertex as introduced originally by Micu [16], which assumes a spatially constant pair-production strength γ, adopt the relativistic phase space, and employ the relativized quark model wave functions [13].

With the relativistic phase space, the decay width $\Gamma(A \to BC)$ can be expressed in terms of the partial wave amplitude

$$\Gamma(A \to BC) = \frac{\pi|\boldsymbol{p}|}{4M_A^2} \sum_{LS} |\mathcal{M}^{LS}(\boldsymbol{p})|^2, \quad (5)$$

where $|\boldsymbol{p}| = \sqrt{[M_A^2 - (M_B+M_C)^2][M_A^2 - (M_B-M_C)^2]}/(2M_A)$, and M_A, M_B, and M_C are the masses of the mesons A, B, and C, respectively.

III. RESULTS

In our calculations, the parameters involve the $q\bar{q}$ pair-production strength γ and the ones in the relativized quark model, as used in the work of Godfrey and Isgur [13]. The flavor wave functions for the mesons are adopted by following the conventions of Refs. [13,14] except for (1) $f_1(1285) = -0.28n\bar{n} + 0.96s\bar{s}$ and $f_1(1420) = -0.96n\bar{n} - 0.28s\bar{s}$ as Ref. [38], (2) $\eta(1295) = (n\bar{n} - s\bar{s})/\sqrt{2}$ and $\eta(1475) = (n\bar{n} + s\bar{s})/\sqrt{2}$ as Ref. [39], where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$. The masses of the final mesons are taken from the Review of Particle Physics (RPP) [5].

We take $\gamma = 0.52$ by fitting to the total width of $K_0^*(1430)$ as the 1^3P_0 state. The decay widths of $K_0^*(1430)$ as the $K_0^*(1^3P_0)$ state are listed in Table II. According to our results, the dominant decay mode of $K_0^*(1430)$ is $K\pi$, which is consistent with the experimental data [5].

TABLE II. Decay widths of $K_0^*(1430)$ as 1^3P_0 state (in MeV), and the $K_0^*(1430)$ mass is taken from RPP [5].

Channel	Mode	$\Gamma_i(1^3P_0)$
$0^+ \to 0^- + 0^-$	$K\pi$	262.53
	$K\eta$	9.42
$0^+ \to 0^- + 1^+$	πK_{1B}	< 0.1
	Total width	271.95
	Experiment	270 ± 80 [5]

In addition to the masses, the strong decay properties are also crucial to identify the assignments of the $K_0^*(1950)$ and $K_0^*(2130)$. Taking into account that the observed mass of $K_0^*(1950)$ is between the predicted masses of $K_0^*(2^3P_0)$ and $K_0^*(3^3P_0)$, we calculate the decay properties of $K_0^*(1950)$, regarded as $K_0^*(2^3P_0)$ and $K_0^*(3^3P_0)$, respectively, as shown in Table III, where the averaged mass $M_{K_0^*(1950)} = 1962$ MeV is used as the $K_0^*(1950)$ mass. The total widths of the $K_0^*(2^3P_0)$ and $K_0^*(3^3P_0)$ states with the initial mass of 1962 MeV are expected to be 1384 and 74 MeV, respectively. Thus, the $K_0^*(1950)$ cannot be explained as the $K_0^*(2^3P_0)$ state. Although the calculated width of $K_0^*(3^3P_0)$ with the initial mass 1962 MeV is close to the low limit of the experimental data of $\Gamma = 201 \pm 34 \pm 79$ [12] and $144 \pm 44 \pm 21$ MeV [11], the $K_0^*(1950)$ still seems unlikely to be the candidate of the $K_0^*(3^3P_0)$ if one takes into account the fact that the $K_0^*(1950)$ mass is lower than the estimated mass of $K_0^*(3^3P_0)$. By comparing with the $K_0^*(1950)$ width $\Gamma = 201 \pm 34 \pm 79$ [12] and $144 \pm 44 \pm 21$ MeV [11], it is hard to assign the $K_0^*(1950)$ as the ordinary scalar $q\bar{q}$ meson. In a word, the more precise measurement of $K_0^*(1950)$ mass and width should be helpful to shed light on its nature.

TABLE III. Decay widths of $K_0^*(1950)$ as 2^3P_0 and 3^3P_0 states (in MeV). The averaged mass $M_{K_0^*(1950)} = 1962$ MeV of $K_0^*(1950)$ is used as the initial mass.

Channel	Mode	$\Gamma_i(2^3P_0)$	$\Gamma_i(3^3P_0)$
$0^+ \to 0^- + 0^-$	πK	80.76	2.02
	$K\eta$	1.84	< 0.01
	$K\eta'$	13.62	1.30
	$\pi(1300)K$	165.53	4.80
	$K\eta(1295)$	140.72	2.51
	$\pi K(1460)$	86.88	0.07
$0^+ \to 0^- + 1^+$	$Ka_1(1260)$	62.49	0.09
	$Kb_1(1235)$	114.93	10.06
	$h_1(1415)K$	21.75	4.84
	$h_1(1170)K$	5.75	< 0.01
	$Kf_1(1420)$	4.92	0.05
	$Kf_1(1285)$	5.23	< 0.01
	πK_{1A}	139.93	0.84
	πK_{1B}	24.66	1.71
	ηK_{1B}	64.07	7.10
$0^+ \to 1^- + 1^-$	$K^*\rho$	226.78	19.40
	$K^*\phi(1020)$	10.77	0.07
	$K^*\omega$	213.74	18.75
$0^+ \to 0^- + 2^-$	$\pi K_2(1770)$	0.08	0.08
	Total width	1384.41	73.68
	Experiment	$201 \pm 34 \pm 79$ [12]	
		$144 \pm 44 \pm 21$ [11]	

TABLE IV. Decay widths of $K_0^*(2130)$ regarded as $K_0^*(3^3P_0)$ states (in MeV). The $K_0^*(2130)$ mass is taken to be 2128 MeV.

Channel	Mode	$\Gamma_i(3^3P_0)$	Mode	$\Gamma_i(3^3P_0)$
$0^+ \to 0^- + 0^-$	πK	8.58	$K\eta(1295)$	7.38
	$K\eta$	0.07	$\pi K(1460)$	8.83
	$K\eta'$	0.21	$K\eta(1475)$	< 0.01
	$\pi(1300)K$	4.35	$K(1460)\eta$	0.87
$0^+ \to 0^- + 1^+$	$Ka_1(1260)$	3.58	$Kf_1(1285)$	0.16
	$Kb_1(1235)$	1.63	πK_{1A}	2.01
	$h_1(1415)K$	12.41	πK_{1B}	0.06
	$h_1(1170)K$	0.84	ηK_{1B}	2.67
	$Kf_1(1420)$	0.04	ηK_{1A}	3.26
$0^+ \to 1^- + 1^-$	$K^*\rho$	14.27	$K^*\omega$	14.97
	$K^*\phi(1020)$	1.42		
$0^+ \to 1^+ + 1^-$	$K^*a_1(1260)$	1.20	ρK_{1B}	2.56
	$K^*b_1(1235)$	0.15	ωK_{1B}	2.87
	$K^*h_1(1170)$	2.33		
$0^+ \to 0^- + 2^-$	$\pi K_2(1820)$	1.28	$\eta_2(1645)K$	0.05
	$\pi K_2(1770)$	7.22		
Total width		105.33		
Experiment		$95 \pm 42 \pm 76$ [11]		

As we discussed in the Introduction, the observed $K_0^*(2130)$ mass is close to the predicted mass of $K_0^*(3^3P_0)$ by the MGI model [15], we present the decay widths of $K_0^*(2130)$ as the $K_0^*(3^3P_0)$ with the $M_{K_0^*(2130)} = 2128$ MeV in Table IV, and total width is expected to be about 105 MeV, which is in good agreement with the *BABAR* data of $95 \pm 42 \pm 76$ MeV [11]. The dependence of the total width of $K_0^*(3^3P_0)$ on the initial state mass is shown in Fig. 2, where we assume that the wave function of the initial meson is not significantly affected by its mass varying in the experimental uncertainties. From Fig. 2 one can find that the total width of $K_0^*(3^3P_0)$ is also consistent with the experimental data $\Gamma = 95 \pm 42 \pm 76$ MeV [11]. Thus, the $K_0^*(2130)$ state could be the good candidate of the $K_0^*(3^3P_0)$ state.

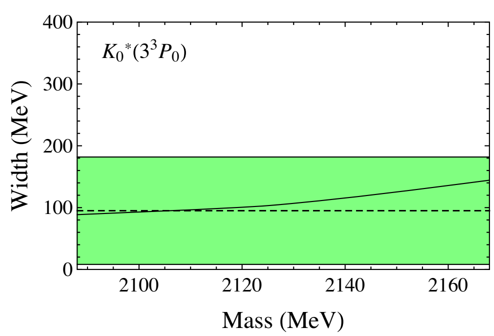

FIG. 2. The dependence of the total width of $K_0^*(3^3P_0)$ on the initial state mass. The experimental total width of the $K_0^*(2130)$ is denoted by the dashed line with a green band.

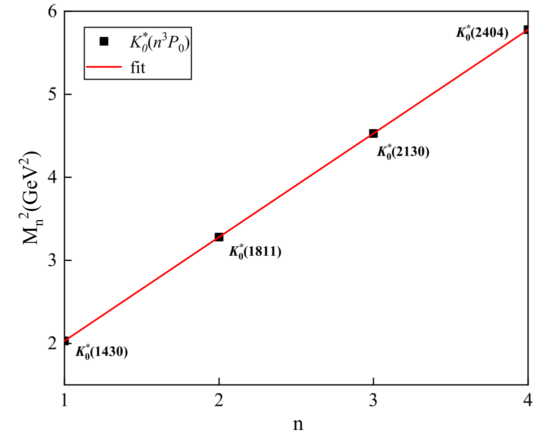

FIG. 3. The Regge trajectory of the $K_0^*(n^3P_0)$ ($n = 1, 2, 3, 4$) states, where M_n is its mass, and the red line is the fitting result.

Phenomenologically, it is suggested that the light mesons could be grouped into the following Regge trajectories [29,33,40]:

$$M_n^2 = M_0^2 + (n-1)\mu^2, \tag{6}$$

where M_0 is the lowest-lying meson mass, n is the radial quantum number, and μ^2 is the slope parameter of the corresponding trajectory. In the presence of $K_0^*(1430)$ and $K_0^*(2130)$ being the $K_0^*(1^3P_0)$ and $K_0^*(3^3P_0)$ states, we can roughly estimate the $K_0^*(2^3P_0)$ mass to be about 1811 MeV and $K_0^*(4^3P_0)$ mass to be about 2404 MeV as shown in Fig. 3.

It should be pointed out that, in our previous work [10], the mass scale for the $n\bar{n}(2^3P_0)$ nonets is expected to be 1700–1800 MeV. In addition, the mass of the $K_0^*(2^3P_0)$ state is predicted to be 1890 MeV by the GI model [13] and to be 1829 MeV by the MGI model [15]. The prediction of the $K_0^*(2^3P_0)$ mass from the Regge trajectories is consistent with these previous predictions. The strong decays of $K_0^*(2^3P_0)$ with a mass of 1811 MeV are presented in Table V. The dependence of the total width of $K_0^*(2^3P_0)$ on the initial state mass is shown in Fig. 4. When the initial

TABLE V. Decay widths of $K_0^*(2^3P_0)$ (in MeV). The initial state mass is 1811 MeV.

Channel	Mode	$\Gamma_i(2^3P_0)$	Mode	$\Gamma_i(2^3P_0)$
$0^+ \to 0^- + 0^-$	πK	57.13	$K\eta'$	0.77
	$\pi(1300)K$	96.06	$K\eta(1295)$	100.95
	$K\eta$	1.01	$\pi K(1460)$	99.91
$0^+ \to 0^- + 1^+$	$h_1(1170)K$	4.42	$Kb_1(1235)$	33.29
	$Ka_1(1260)$	16.35	πK_{1A}	64.31
	$Kf_1(1285)$	0.42	πK_{1B}	24.37
$0^+ \to 1^- + 1^-$	$K^*\rho$	81.42	$K^*\omega$	75.93
Total width		656.34		

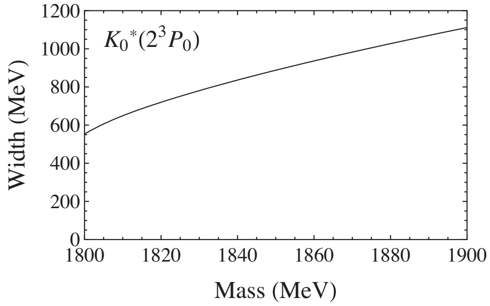

FIG. 4. The dependence of the total width of $K_0^*(2^3P_0)$ on the initial state mass.

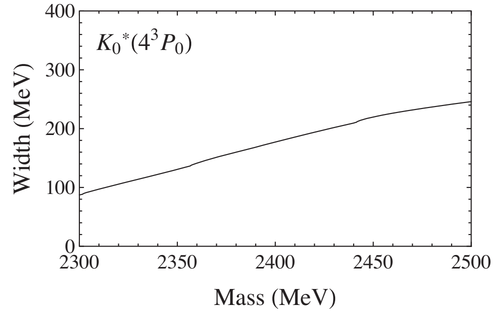

FIG. 5. The dependence of the total width of $K_0^*(4^3P_0)$ on the initial state mass.

state mass varies from 1800 to 1900 MeV, the total width of the $K_0^*(2^3P_0)$ varies from about 550 to 1100 MeV. The decay width varies greatly, since some decay modes are open gradually. The total width of $K_0^*(2^3P_0)$ with initial mass of 1811 MeV is expected to be about 656 MeV. The dominant decay modes of $K_0^*(2^3P_0)$ are $\pi(1300)K$, $K\eta(1295)$, and $\pi K(1460)$. The $K_0^*(2^3P_0)$ is predicted to be a broad state, in agreement with the calculations of Refs. [14,15], which could be the reason that the $K_0^*(2^3P_0)$ candidate is not yet observed experimentally.

TABLE VI. Decay widths of $K_0^*(4^3P_0)$ (in MeV). The initial state mass is 2404 MeV.

Channel	Mode	$\Gamma_i(4^3P_0)$	Mode	$\Gamma_i(4^3P_0)$
$0^+ \to 0^- + 0^-$	πK	0.60	$K\eta$	< 0.01
	$K\eta'$	0.28	$\pi(1300)K$	0.16
	$K\eta(1295)$	0.07	$\pi K(1460)$	0.12
	$K\eta(1475)$	< 0.01	$K(1460)\eta$	0.31
$0^+ \to 0^- + 1^+$	$Ka_1(1260)$	0.70	$Kb_1(1235)$	4.50
	$h_1(1415)K$	4.60	$h_1(1170)K$	0.06
	$Kf_1(1420)$	0.21	$Kf_1(1285)$	0.03
	πK_{1A}	0.02	πK_{1B}	1.69
	ηK_{1B}	2.74	ηK_{1A}	0.98
	$\eta' K_{1B}$	0.08	$\eta' K_{1A}$	0.01
	$Ka_1(1640)$	0.23		
$0^+ \to 0^+ + 1^-$	$K_0^*(1430)\rho$	0.09		
$0^+ \to 1^- + 1^-$	$K^*\rho$	8.76	$K^*\rho(1450)$	6.34
	$K^*\omega(1420)$	19.83	$K^*\phi(1020)$	0.02
	$K^*\omega$	8.15	$K^*(1410)\rho$	32.46
	$K^*(1410)\omega$	36.79		
$0^+ \to 1^+ + 1^-$	$K^*a_1(1260)$	2.50	$K^*b_1(1235)$	0.03
	$K^*h_1(1170)$	0.13	$K^*h_1(1415)$	< 0.01
	$K^*f_1(1420)$	0.69	$K^*f_1(1285)$	0.34
	ρK_{1A}	< 0.01	ωK_{1A}	< 0.01
	ϕK_{1B}	0.25	ρK_{1B}	1.29
	ωK_{1B}	0.22		
$0^+ \to 0^- + 2^-$	$\pi K_2(1820)$	0.17	$\pi K_2(1770)$	2.03
	$K\pi_2(1670)$	7.15	$\eta_2(1645)K$	4.02
	$\eta_2(1870)K$	0.07	$\eta K_2(1820)$	0.05
	$\eta K_2(1770)$	< 0.01		
$0^+ \to 1^- + 2^+$	$K^*f_2(1270)$	0.93	$K^*a_2(1320)$	12.56
	$K_2^*(1430)\rho$	17.92		
Total width		180.23		

We show the partial decay widths and the total decay width of the $K_0^*(4^3P_0)$ state with mass of 2404 MeV in Table VI. The total width of $K_0^*(4^3P_0)$ is expected to be about 180 MeV. The dominant decay modes of $K_0^*(4^3P_0)$ include $K^*(1410)\,\omega$, $K^*(1410)\,\rho$, and $K^*\omega(1420)$. The dependence of the total width of the $K_0^*(4^3P_0)$ state on the initial state mass is shown in Fig. 5. When the initial state mass varies from 2300 to 2500 MeV, the total width of the $K_0^*(4^3P_0)$ varies from about 80 to 240 MeV.

IV. SUMMARY

In this work, we have discussed the possible assignments of $K_0^*(1950)$ and $K_0^*(2130)$ by calculating the strong decay widths within the $^3P_0$ strong decay model.

We suggest that the $K_0^*(2130)$ could be assigned as $K_0^*(3^3P_0)$ based on its mass and width. However, the mass and the width of $K_0^*(1950)$ cannot be reasonably explained within the $q\bar{q}$ nature. The more precise measurement of $K_0^*(1950)$ mass and width should be helpful to shed light on its nature.

With the assignment of the $K_0^*(2130)$ as the $K_0^*(3^3P_0)$ state, we have roughly estimated the masses of $K_0^*(2^3P_0)$ and $K_0^*(4^3P_0)$ to be about 1811 and 2404 MeV, respectively, within the Regge trajectories. The total width of $K_0^*(2^3P_0)$ is predicted to be about 656 MeV, which implies that this state is not easy to be observed experimentally. The total width of $K_0^*(4^3P_0)$ is predicted to be about 180 MeV, which could be helpful to search for the $K_0^*(4^3P_0)$ state in the future.

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