

W-exchange contribution to the decays $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ using light-cone sum rules

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We calculate the W -exchange contribution to the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ decay using light-cone sum rules. The two-particle light-cone distribution amplitudes of the pion are used as nonperturbative input for the sum rules calculation, and the perturbative kernel is calculated at the leading order. We obtain the corresponding decay branching fractions by combining our W -exchange amplitudes with the factorizable amplitudes given by various theoretical methods from the literature. It is shown that with the factorizable amplitudes from heavy quark effective theory, we obtain the branching fraction ratio $\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^{+'} \pi^+) / \mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) = 1.42 \pm 0.78$, which is consistent with the experimental value of $1.41 \pm 0.17 \pm 0.1$.

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I. INTRODUCTION

The conventional quark model predicted the existence of doubly heavy baryons consisting of two heavy quarks (bottom or charm quarks) [1–6]. After pursuing the doubly charmed baryons for decades, in 2017 the LHCb collaboration announced the observation of the lowest-lying state Ξ_{cc}^{++} with mass $3620.6 \pm 1.5(\text{stat}) \pm 0.4(\text{syst}) \pm 0.3(\Xi_c^+) \text{ MeV}/c^2$ [7]. This new baryon was observed via the decay channel $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$, which is consistent with the prediction given by Ref. [8]. One year later, in 2018, a two-body decay channel $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ was observed [9], which further confirms the existence of this doubly charmed baryon. Recently, the LHCb collaboration has observed a similar decay channel, $\Xi_{cc}^{++} \rightarrow \Xi_c^{+'} \pi^+$, and measured the branching fraction ratio [10]:

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^{+'} \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)} \equiv \frac{\mathcal{B}'}{\mathcal{B}} = 1.41 \pm 0.17 \pm 0.1, \quad (1)$$

which means that the branching fraction of the decay into $\Xi_c^{+'}$ is larger than that into Ξ_c^+ .

Nowadays, there are two main thrusts for the theoretical research on doubly-heavy baryons. One is to find the optimal decay channels where new doubly heavy states can be observed by future experiments [11,12]. The other is to study the recently observed decay processes, especially the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ decay. The weak decay $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ receives contributions from the two topological diagrams as shown in Fig. 1, the W -emission diagram (left) and the W -exchange diagram (right). Generally, according to the naive factorization [13–15], the W -emission diagram is approximately factorizable so that it can be calculated by considering the baryon transformation matrix element and the pion matrix element independently. For the decay of the Ξ_{cc}^{++} , the transformation matrix element is parametrized by form factors that have been evaluated in various theoretical works based on QCD sum rules (QCDSR) [16], light-cone sum rules (LCSR) [17–21], QCD factorization (QCDF) [22,23], diquark effective theory [24], light-front quark model (LFQM) [25–30], constituent quark model (CQM) [31–33], nonrelativistic quark model (NRQCD), and heavy quark effective theory (HQET) [22,34]. Further, the pion

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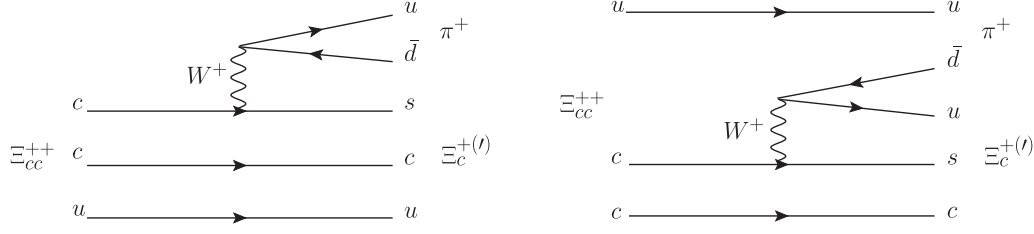


FIG. 1. W -emission diagram (left) and W -exchange diagram (right) for the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ decay.

matrix element can be simply expressed by the pion decay constant.

Unlike the weak decays of mesons, the W -exchange diagrams in the baryon decays are generally nonfactorizable, and this difficulty increases when we are facing the decays of doubly heavy baryons. Nowadays, except an $SU(3)$ symmetry analysis [35,36] and a phenomenological study [37], there is no QCD based or model-independent study on the W -exchange contribution in doubly charmed baryon decays. The mostly used approach to evaluate such a contribution is the pole model [22,23,27,31,34]. Recently, the combination of the factorizable contribution and the nonfactorizable contribution from the pole-model leads to the ratio \mathcal{B}'/\mathcal{B} between 0.81 and 0.83 [22,23], which means that the branching fraction of the decay into $\Xi_c^{+'}$ is smaller, which is obviously contrary to the experimental result. Furthermore, if one includes the interference between the W -emission and the W -exchange contributions, the \mathcal{B}'/\mathcal{B} will become much larger, namely 6.74 [27].

This deviation of the theoretical prediction from the experimental measurement implies that a more precise theoretical calculation for the W -exchange contribution in the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ decay is necessary. In this work, we will use the method of LCSR to solve this problem. LCSR were firstly proposed to study the transition form factors of the radiative or semileptonic hadron decays [38–40]. In the framework of LCSR, the required transition matrix element can be extracted from a suitable correlation function at the hadron level. The quark-hadron duality enables us to relate this correlations function with the one at the quark-gluon level, where it can be calculated by the operator-product expansion (OPE), and all the nonperturbative contributions come from the light-cone distribution amplitudes (LCDAs) of a certain hadron in the decay. After decades of development, a new technique of LCSR was proposed to study the nonleptonic decay of the B mesons into two light mesons [41–43]. This new technique of LCSR can be extended to the case of heavy or doubly heavy baryon decays, and in this work, we will use it to calculate the W -exchange contribution in the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ decay.

This paper is organized as follows. In Sec. II, we introduce a suitable correlation function to extract the decay amplitude of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$. In Sec. III, we perform the hadron level calculation for the correlation function and extract the required decay amplitude. In Sec. IV, we

perform the quark-gluon level calculation for the correlation function with the use of two-particle LCDAs of the pion. In Sec. V, we give the numerical results on the decay amplitudes and branching fractions of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ and compare our results with those from the literature. Section VI contains a brief summary of this work.

II. THE CORRELATION FUNCTION IN LCSR

In this section, we give a suitable correlation function for the study of the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ decay in the framework of the LCSR. The relevant effective Hamiltonian for this decay is

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2), \\ \mathcal{O}_1 &= \bar{s} \gamma_\mu (1 - \gamma_5) c \bar{u} \gamma^\mu (1 - \gamma_5) d, \\ \mathcal{O}_2 &= \bar{s}_a \gamma_\mu (1 - \gamma_5) c_b \bar{u}_b \gamma^\mu (1 - \gamma_5) d_a, \end{aligned} \quad (2)$$

where the C_1, C_2 are Wilson coefficients, and the subscripts a, b are color indices. Generally, the transition matrix element of the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ induced by $\mathcal{O}_{1,2}$ can be parametrized as

$$\begin{aligned} \langle \Xi_c^{+(l)}(p-q) \pi^+(q) | \mathcal{O}_i(0) | \Xi_{cc}^{++}(p) \rangle \\ = i \bar{u}(p-q) (A^{(l)i} + B^{(l)i} \gamma_5) u(p). \end{aligned} \quad (3)$$

Since the initial and final states are on shell, $A^{(l)i}$ and $B^{(l)i}$ are just constants. In this work, our main task is to obtain the W -exchange contribution to $A^{(l)i}$ and $B^{(l)i}$, denoted as $A_{\text{WE}}^{(l)i}$ and $B_{\text{WE}}^{(l)i}$ in what follows.

Using the LCSR to calculate a transition matrix element, one begins with an appropriate correlation function which will be calculated both at the hadron and the quark-gluon level. In our case, the correlation function corresponding to the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(l)} \pi^+$ decay is chosen as

$$\begin{aligned} \Pi^{\mathcal{O}_i}(p, q, k) &= i^2 \int d^4 x e^{-i(p-q) \cdot x} \int d^4 y e^{i(p-k) \cdot y} \\ &\times \langle 0 | T \{ J_{\Xi_c^{(l)}}(y) \mathcal{O}_i(0) \bar{J}_{\Xi_{cc}^{++}}(x) \} | \pi^-(q) \rangle, \end{aligned} \quad (4)$$

where the hadron currents are defined as [16]

$$\begin{aligned}
J_{\Xi_c} &= \frac{1}{\sqrt{2}} \varepsilon_{abc} (u_a^T C \gamma_5 s_b - s_a^T C \gamma_5 u_b) Q_c, \\
J_{\Xi_c} &= \frac{1}{\sqrt{2}} \varepsilon_{abc} (u_a^T C \gamma^\mu s_b + s_a^T C \gamma^\mu u_b) \gamma_\mu \gamma_5 Q_c, \\
J_{\Xi_{cc}} &= \varepsilon_{abc} (Q_a^T C \gamma^\mu Q_b) \gamma_\mu \gamma_5 u_c.
\end{aligned} \tag{5}$$

Following Ref. [41], we have temporarily included the pion in the initial state instead of the final state as in the real decay. The advantage of this is to enable us to factorize out the matrix element $\langle 0 | J_{\Xi_c} | \Xi_c^{+(\prime)} \rangle$ without any ambiguity after inserting the $\Xi_c^{+(\prime)}$ state into the right-hand side of J_{Ξ_c} . Actually, during the calculation at the hadron level, by suitable analytic continuation this initial pion state will be moved to the final state. We have also introduced an

$$\begin{aligned}
\Pi_H^{\mathcal{O}_i}(p, q, k)_{\text{WE}} &= i^2 \int d^4x d^4y e^{-i(p-q)\cdot x} e^{i(p-k)\cdot y} \sum_{\pm', \sigma'} \int \frac{d^3\vec{l}}{(2\pi)^3} \frac{1}{2E_l} \langle 0 | J_{\Xi_c}(y) | l, \sigma', \pm' \rangle \langle l, \sigma', \pm' | \mathcal{O}_i(0) \bar{J}_{\Xi_{cc}}(x) | \pi^-(q) \rangle \\
&\quad + \int_{s_{\Xi_c}}^\infty ds' \frac{\rho_{\Xi_c}(s', (p-q)^2, P^2)}{s' - (p-k)^2},
\end{aligned} \tag{6}$$

where the integration over ρ_{Ξ_c} represents the contribution from the continuous spectrum, \pm' corresponds to the positive or negative parity of the Ξ_c states, namely $\Xi_c(\frac{1}{2}^\pm)$, and σ' denotes the spin of the Ξ_c . The momentum of the one-particle state in the complete set l should be on shell, $l^2 = m_{\Xi_c}^{\pm'2}$. This means that the first matrix element in Eq. (6) can be simply parametrized by the Ξ_c decay constants $\lambda_{\Xi_c}^\pm$:

$$\begin{aligned}
\langle 0 | J_{\Xi_c}(y) | l, \sigma', + \rangle &= \lambda_{\Xi_c}^+ u(l, \sigma') e^{-il\cdot y}, \\
\langle 0 | J_{\Xi_c}(y) | l, \sigma', - \rangle &= \lambda_{\Xi_c}^- i\gamma_5 u(l, \sigma') e^{-il\cdot y},
\end{aligned} \tag{7}$$

while the second matrix element in Eq. (6) is a function of $p^2, q^2, k^2, (p-q)^2$, and $P^2 = (p-k-q)^2$. Generally speaking, a matrix element like this should also depend on l^2 , however, the on-shell condition has reduced such a dependence.

Now it can be understood better why, we have to set the pion as an initial state at the beginning. If the pion is set as a final state, then the complete set of states inserted between J_{Ξ_c} and \mathcal{O}_i must be a composite state of Ξ_c and π , instead of Ξ_c as Eq. (6) shows. As a result, in order to factorize out a Ξ_c matrix element as shown in Eq. (7), we must separate the Ξ_c and π in this composite state so that Ξ_c can be annihilated by J_{Ξ_c} independently. However, this requires the further assumption on the decoupling of the inserted Ξ_c and π , which is difficult to prove at the hadron level. Therefore, to avoid this problem, we set the pion as an

auxiliary momentum k in the correlation function. This momentum is unphysical and should be set to zero at the end of the calculation. The reason why it must be introduced is closely related to the analytic continuation mentioned above, which will be explained in the next section.

III. HADRON LEVEL CALCULATION IN THE LCSR

In this section, we derive the calculation of the correlation function in Eq. (4) at the hadron level. For simplicity, we take the case of the Ξ_c^+ as an example; the derivation for the decays into $\Xi_c^{+\prime}$ is similar. We insert a complete set of states with the same quantum numbers as $J_{\Xi_c}(y)$ into the correlation function,

initial state, but later it must be transformed to the final state by the analytical continuation.

Furthermore, since one can replace the $d^3\vec{l}$ integration by a four-momentum integration with the use of residue theorem,

$$\int \frac{d^3\vec{l}}{(2\pi)^3} \frac{1}{2E_l} |l\rangle \langle l| e^{-il\cdot y} = \int \frac{d^4l}{(2\pi)^4} e^{-il\cdot y} \frac{1}{l^2 - m_{\Xi_c}^{\pm'2}} |l\rangle \langle l|, \tag{8}$$

after integrating over d^4y , one arrives at

$$\begin{aligned}
\Pi_H^{\mathcal{O}_i}(p, q, k)_{\text{WE}} &= i^3 \int d^4x e^{-i(p-q)\cdot x} \sum_{\pm', \sigma'} \frac{1}{(p-k)^2 - m_{\Xi_c}^{\pm'2}} \\
&\quad \times \lambda_{\Xi_c}^{\pm'} u^{\pm'}(p-k, \sigma') \langle p-k, \sigma', \pm' | \mathcal{O}_i(0) \bar{J}_{\Xi_{cc}}(x) | \pi^-(q) \rangle \\
&\quad + \int_{s_{\Xi_c}}^\infty ds' \frac{\rho_{\Xi_c}(s', (p-q)^2, P^2)}{s' - (p-k)^2},
\end{aligned} \tag{9}$$

where, we have defined $u^+ = u, u^- = i\gamma_5 u$. To simplify the calculation, we have chosen $p^2 = k^2 = 0$ and $q^2 = m_\pi^2 \approx 0$. Now the correlation function depends on three Lorentz invariants $(p-k)^2, (p-q)^2$, and P^2 , while the matrix element $\langle p-k, \sigma', \pm' | \mathcal{O}_i(0) \bar{J}_{\Xi_{cc}}(x) | \pi^-(q) \rangle$ only depends on $(p-q)^2$ and P^2 .

Further, the same correlation function can be calculated at the quark-hadron level by the OPE in the deep Euclidean region, $(p-k)^2 \sim (p-q)^2 \sim P^2 \ll 0$, which will be explicitly done in the next section. In principle, the expression of the same correlation function at these two levels should be equivalent:

$$\begin{aligned}\Pi_H^{\mathcal{O}_i}(p, q, k)_{\text{WE}} &= \Pi_{\text{QCD}}^{\mathcal{O}_i}(p, q, k)_{\text{WE}} \\ &= \frac{1}{\pi} \int_{(m_c+m_s)^2}^{\infty} ds' \frac{\text{Im}\Pi_{\text{QCD}}^{\mathcal{O}_i}(s', (p-q)^2, P^2)_{\text{WE}}}{s' - (p-k)^2},\end{aligned}\quad (10)$$

where the $(p-k)^2$ dependence of $\Pi_{\text{QCD}}^{\mathcal{O}_i}$ has been described as in the form of a dispersion integral. We have omitted the u quark mass and $(m_c + m_s)^2$ is the quark level threshold to produce a Ξ_c baryon. According to the quark-hadron duality, the integration over the continuous spectrum at the hadron level is canceled by the corresponding integration above a certain threshold s_{Ξ_c} at the quark-gluon level, which leads to the following equation:

$$\begin{aligned}i^3 \int d^4x e^{-i(p-q)\cdot x} \sum_{\pm', \sigma'} \frac{1}{(p-k)^2 - m_{\Xi_c}^{\pm'2}} \lambda_{\Xi_c}^{\pm'} u^{\pm'}(p-k, \sigma') \langle p-k, \sigma', \pm' | \mathcal{O}_i(0) \bar{J}_{\Xi_{cc}}(x) | \pi^-(q) \rangle_{\text{WE}} \\ = \frac{1}{\pi} \int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' \frac{\text{Im}\Pi_{\text{QCD}}^{\mathcal{O}_i}(s', (p-q)^2, P^2)_{\text{WE}}}{s' - (p-k)^2}.\end{aligned}\quad (11)$$

Generally, this threshold parameter should be slightly larger than the mass squared of the corresponding hadron state. Here, it is chosen to be the same as in Ref. [16], where a QCDSR was used to study the semileptonic decay of the $\Xi_{cc} \rightarrow \Xi_c$. It should be mentioned that in principle s_{Ξ_c} is a universal parameter that is process independent so that this procedure should be reasonable.

Note that since $\text{Im}\Pi_{\text{QCD}}^{\mathcal{O}_i}(s', (p-q)^2, P^2)_{\text{WE}}$ is an analytic function of P^2 , the left-hand side of Eq. (11) shares this property. Therefore one can extend P^2 to the physical region, namely $P^2 > 0$, which allows us to replace the initial state $|\pi^-(q)\rangle$ of the matrix element on the left-hand side by a final state $\langle \pi^+(-q)|$. After that, a Borel transformation for $(p-k)^2$ on both sides of Eq. (11) is performed, and we obtain

$$\begin{aligned}-i^3 \int d^4x e^{-i(p-q)\cdot x} \sum_{\pm', \sigma'} e^{-m_{\Xi_c}^{\pm'2}/T^2} \lambda_{\Xi_c}^{\pm'} u^{\pm'}(p-k, \sigma') \langle p-k, \sigma', \pm'; \pi^+(-q) | \mathcal{O}_i(0) \bar{J}_{\Xi_{cc}}(x) | 0 \rangle_{\text{WE}} \\ = \frac{1}{\pi} \int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' e^{-s'/T^2} \text{Im}\Pi_{\text{QCD}}^{\mathcal{O}_i}(s', (p-q)^2, P^2)_{\text{WE}}.\end{aligned}\quad (12)$$

Now it can be understood why, we have to introduce the auxiliary momentum k . The matrix element on the left-hand side of Eq. (11) depends on two Lorentz invariants, $(p-q)^2$ and P^2 . Since $(p-q)^2$ is used for the Borel transformation, it must stay in the deep Euclidean region. Thus the only variable that can be used for analytical continuation is P^2 . Note that $P^2 = (p-q-k)^2$ depends on k . If there is no k , then there will be no P^2 dependence and the analytical continuation cannot be realized.

Next, we insert another complete set of states with the same quantum numbers as $\bar{J}_{\Xi_{cc}}$ into the left-hand side of Eq. (12), which becomes

$$\begin{aligned}- \sum_{\pm', \pm, \sigma', \sigma} e^{-m_{\Xi_c}^{\pm'2}/T^2} \frac{1}{(p-q)^2 - m_{\Xi_{cc}}^{\pm2}} \lambda_{\Xi_c}^{\pm'} \lambda_{\Xi_{cc}}^{\pm} u^{\pm'}(p-k, \sigma') \langle p-k, \sigma', \pm'; \pi^+(-q) | \mathcal{O}_i(0) | p-q, \sigma, \pm \rangle_{\text{WE}} \bar{u}^{\pm}(p-q, \sigma) \\ + \int_{s_{\Xi_{cc}}}^{\infty} ds \frac{\rho_{\Xi_{cc}}(s, P^2)}{s - (p-q)^2}.\end{aligned}\quad (13)$$

For the same reason as discussed before, the matrix element $\langle p-k, s', \pm'; \pi^+(-q) | \mathcal{O}_i(0) | p-q, s, \pm \rangle_{\text{WE}}$ only depends on P^2 . The $(p-q)^2$ dependence on the right-hand side of Eq. (12) can be further expressed as a dispersion integral,

$$\frac{1}{\pi^2} \int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' e^{-s'/T^2} \int_{4m_c^2}^{\infty} ds \frac{1}{s - (p-q)^2} \text{Im}^2 \Pi_{\text{QCD}}^{\mathcal{O}_i}(s', s, P^2)_{\text{WE}}.\quad (14)$$

Now using the quark-hadron duality again to cancel out the ds' integration above a certain threshold $s_{\Xi_{cc}}$ corresponding to the lowest Ξ_{cc} state, and performing the Borel transformation for $(p-q)^2$, we arrive at

$$\begin{aligned}\sum_{\pm', \pm, \sigma', \sigma} e^{-m_{\Xi_c}^{\pm'2}/T^2 - m_{\Xi_{cc}}^{\pm2}/T^2} \lambda_{\Xi_c}^{\pm'} \lambda_{\Xi_{cc}}^{\pm} u^{\pm'}(p-k, \sigma') \langle p-k, \sigma', \pm'; \pi^+(-q) | \mathcal{O}_i(0) | p-q, \sigma, \pm \rangle_{\text{WE}} \bar{u}^{\pm}(p-q, \sigma) \\ = \frac{1}{\pi^2} \int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' \int_{4m_c^2}^{s_{\Xi_{cc}}} ds e^{-s'/T^2} e^{-s/T^2} \text{Im}^2 \Pi_{\text{QCD}}^{\mathcal{O}_i}(s', s, P^2).\end{aligned}\quad (15)$$

Due to the existence of the auxiliary momentum k , unlike Eq. (3), the matrix element appearing in Eq. (15) must be parametrized by four terms:

$$\begin{aligned} & \langle p - k, \sigma', \pm'; \pi^+(-q) | \mathcal{O}_i(0) | p - q, \sigma, \pm \rangle_{\text{WE}} \\ &= i\bar{u}^{\pm'}(p - k, \sigma') \left[A_{1,i}^{\pm'\pm}(P^2) + B_{1,i}^{\pm'\pm}(P^2)\gamma_5 + A_{2,i}^{\pm'\pm}(P^2) \frac{\not{q}}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{\pm'\pm}(P^2) \frac{\not{q}\gamma_5}{m_{\Xi_{cc}}^{\pm}} \right] u^{\pm}(p - q, \sigma). \end{aligned} \quad (16)$$

The form factors $A_{1,2,i}^{\pm'\pm}$ and $B_{1,2,i}^{\pm'\pm}$ are functions of P^2 . Using the sum rules equation given in Eq. (15), we can extract these four functions. Summing up the spin indices, we have

$$\begin{aligned} & ie^{-m_{\Xi_c}^{\pm 2}/T^2 - m_{\Xi_{cc}}^{\pm 2}/T^2} \lambda_{\Xi_c}^{\pm} \lambda_{\Xi_{cc}}^{\pm} (\not{p}_2 + m_{\Xi_c}^{\pm}) \left[A_{1,i}^{++} + B_{1,i}^{++}\gamma_5 + A_{2,i}^{++} \frac{\not{q}}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{++} \frac{\not{q}\gamma_5}{m_{\Xi_{cc}}^{\pm}} \right] (\not{p}_1 + m_{\Xi_{cc}}^{\pm}) \\ &+ ie^{-m_{\Xi_c}^{\pm 2}/T^2 - m_{\Xi_{cc}}^{\pm 2}/T^2} \lambda_{\Xi_c}^{\pm} \lambda_{\Xi_{cc}}^{\pm} (\not{p}_2 - m_{\Xi_c}^{\pm}) \left[A_{1,i}^{+-} + B_{1,i}^{+-}\gamma_5 + A_{2,i}^{+-} \frac{\not{q}}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{+-} \frac{\not{q}\gamma_5}{m_{\Xi_{cc}}^{\pm}} \right] (\not{p}_1 + m_{\Xi_{cc}}^{\pm}) \\ &+ ie^{-m_{\Xi_c}^{\pm 2}/T^2 - m_{\Xi_{cc}}^{\pm 2}/T^2} \lambda_{\Xi_c}^{\pm} \lambda_{\Xi_{cc}}^{\pm} (\not{p}_2 + m_{\Xi_c}^{\pm}) \left[A_{1,i}^{-+} + B_{1,i}^{-+}\gamma_5 + A_{2,i}^{-+} \frac{\not{q}}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{-+} \frac{\not{q}\gamma_5}{m_{\Xi_{cc}}^{\pm}} \right] (\not{p}_1 - m_{\Xi_{cc}}^{\pm}) \\ &+ ie^{-m_{\Xi_c}^{\pm 2}/T^2 - m_{\Xi_{cc}}^{\pm 2}/T^2} \lambda_{\Xi_c}^{\pm} \lambda_{\Xi_{cc}}^{\pm} (\not{p}_2 - m_{\Xi_c}^{\pm}) \left[A_{1,i}^{--} + B_{1,i}^{--}\gamma_5 + A_{2,i}^{--} \frac{\not{q}}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{--} \frac{\not{q}\gamma_5}{m_{\Xi_{cc}}^{\pm}} \right] (\not{p}_1 - m_{\Xi_{cc}}^{\pm}) \\ &= \frac{1}{\pi^2} \int_{(m_c+m_s)^2}^{S_{\Xi_c}} ds' \int_{4m_c^2}^{S_{\Xi_{cc}}} ds e^{-s'/T^2} e^{-s/T^2} \text{Im}^2 \Pi_{\text{QCD}}^{\mathcal{O}_i}(s', s, P^2)_{\text{WE}}, \end{aligned} \quad (17)$$

with $p_1 = p - q$ and $p_2 = p - k$. Note that there are 16 independent spinor structures, and this number is equal to that of the form factors $A_{1,2}^{\pm'\pm}$ and $B_{1,2}^{\pm'\pm}$. This matching enables us to solve all of these form factors from the equation above. Finally, what, we really

care about is the transition matrix element of positive parity baryons with $k = 0$ and $P^2 = m_{\Xi_{cc}}^{\pm 2}$. Therefore, considering the $k \rightarrow 0$ limit, and using the equation of motion, the matrix element in Eq. (16) can be simplified as

$$\begin{aligned} & \langle p - k, \sigma', +; \pi^+(-q) | \mathcal{O}_i(0) | p - q, \sigma, + \rangle_{k \rightarrow 0, P^2 = m_{\Xi_{cc}}^{\pm 2}} \\ &= i\bar{u}^+(p - k, \sigma') \left\{ \left[A_{1,i}^{++}(m_{\Xi_{cc}}^{\pm 2}) + \left(1 - \frac{m_{\Xi_c}^{\pm}}{m_{\Xi_{cc}}^{\pm}} \right) A_{2,i}^{++}(m_{\Xi_{cc}}^{\pm 2}) \right] + \left[B_{1,i}^{++}(m_{\Xi_{cc}}^{\pm 2}) - \left(1 + \frac{m_{\Xi_c}^{\pm}}{m_{\Xi_{cc}}^{\pm}} \right) B_{2,i}^{++}(m_{\Xi_{cc}}^{\pm 2}) \right] \gamma_5 \right\} u^+(p - q, \sigma), \\ &\equiv i\bar{u}^+(p, \sigma') (A_{\text{WE}}^i + B_{\text{WE}}^i \gamma_5) u^+(p - q, \sigma). \end{aligned} \quad (18)$$

Now the four unknown form factors are reduced to two constants A_{WE}^i and B_{WE}^i (A_{WE}^i and B_{WE}^i for the case of Ξ_c'), which has the same form as Eq. (27).

IV. QUARK-GLUON LEVEL

In this section, we use the OPE to calculate the correlation function in Eq. (4) at the quark-gluon level. In the deep Euclidean region, $(p - k)^2 \sim (p - q)^2 \sim P^2 \ll 0$, the W -exchange contribution to the correlation function can be expressed as a convolution of the perturbative kernel and a nonperturbative matrix element of the pion:

$$\begin{aligned} \Pi_{\text{QCD}}^{\mathcal{O}_1}(p, q, k)_{\alpha\sigma} &= -\Pi_{\text{QCD}}^{\mathcal{O}_2}(p, q, k)_{\alpha\sigma}, \\ &= -2\sqrt{2} \varepsilon_{abc} \varepsilon_{ebc} \int d^4x d^4y e^{-i(p-q) \cdot x} e^{i(p-k) \cdot y} \\ &\quad \times [S_Q(y-x) \gamma^\nu C S_Q^T(-x) C (1 - \gamma_5) \gamma_\mu C S_S^T(y) C \gamma_5 S_u(y) \gamma^\mu (1 - \gamma_5)]_{\alpha\beta} (\gamma_\nu \gamma_5)_{\rho\sigma} \langle 0 | \bar{u}_e^\rho(x) d_a^\beta(0) | \pi^-(q) \rangle, \end{aligned} \quad (19)$$

where $\alpha, \beta, \rho, \sigma$ are spinor indices and $S_{Q,u,s}$ are the free propagators of the c, u, s quarks. The superscript “ T ” denotes the transposed in spinor space. The last matrix element can be expressed by the LCDAs of the pion. Figure 2 shows the Feynman diagram of the W -exchange effect in the correlation function, where the gray bubble denotes the pion LCDAs. The contribution from two-particle LCDAs of pion up to the twist-3 order are defined as [44,45]

$$\begin{aligned} & \langle 0 | \bar{u}_e^\rho(x) d_a^\rho(0) | \pi^-(q) \rangle \\ &= -\frac{i}{12} \delta_{ae} f_\pi \int_0^1 du e^{-i\bar{u}q \cdot x} \left[(\not{p}\gamma_5)_{\beta\rho} \varphi_\pi(u) + (\gamma_5)_{\beta\rho} \mu_\pi \phi_{3\pi}^p(u) \right. \\ & \quad \left. + \frac{1}{6} (\gamma_5 \sigma_{\mu\nu})_{\beta\rho} q^\mu x^\nu \mu_\pi \phi_{3\pi}^\sigma(u) \right], \end{aligned} \quad (20)$$

where

$$\begin{aligned} \varphi_\pi(u) &= 6u\bar{u}(1 + a_2 C_2^{3/2}(u - \bar{u}) + a_4 C_4^{3/2}(u - \bar{u})), \\ \phi_{3\pi}^p(u) &= 1 + 30 \frac{f_{3\pi}}{\mu_\pi f_\pi} C_2^{1/2}(u - \bar{u}) - 3 \frac{f_{3\pi} \omega_{3\pi}}{\mu_\pi f_\pi} C_4^{1/2}(u - \bar{u}), \\ \phi_{3\pi}^\sigma(u) &= 6u(1 - u) \left(1 + 5 \frac{f_{3\pi}}{\mu_\pi f_\pi} \left(1 - \frac{\omega_{3\pi}}{10} \right) C_2^{3/2}(u - \bar{u}) \right), \end{aligned} \quad (21)$$

are the twist-2, twist-3p, and twist-3 σ LCDAs, respectively. $a_2 = 0.27$, $a_4 = 0.179$, $\mu_\pi = 2.87$ GeV, $f_{3\pi} = 0.0045$ GeV²,

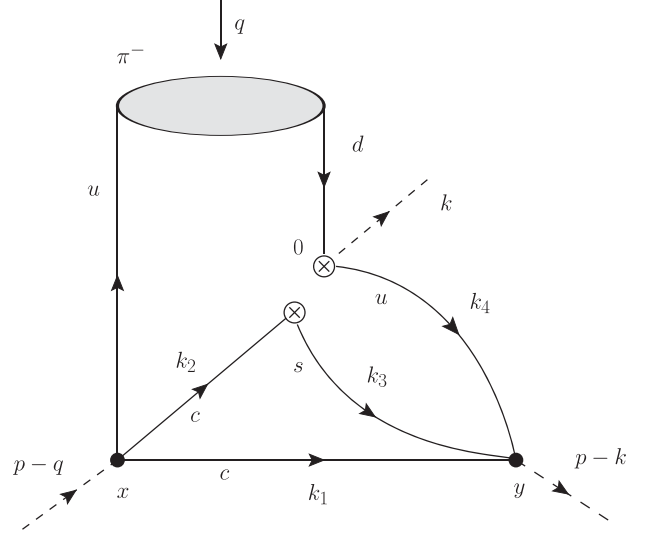


FIG. 2. Feynman diagram of the W -exchange contribution to the correlation function in Eq. (4). The gray bubble denotes the pion LCDAs, the black dots denote the baryon currents and the white double crossed dot represents the four-fermion interaction vertex from $\mathcal{O}_{1,2}$. The dashed lines with arrows denote the external momentum flows.

$\omega_\pi = -1.5$, and $f_\pi = 0.13$ GeV [44]. C_n^α are the Gegenbauer polynomials. Now the derivation becomes straightforward, and we take the twist-2 LCDA as an example. Its contribution to the correlation function is

$$\begin{aligned} \Pi_{\text{QCD}}^{\mathcal{O}_1}(p, q, k)_{(2)} &= -4\sqrt{2} N_c \left(-\frac{i}{12} \right) f_\pi \int_0^1 du \varphi_\pi(u) \int d^4 x d^4 y \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \frac{d^4 k_4}{(2\pi)^4} \\ & \quad \times e^{-i\bar{u}q \cdot x} e^{-ik_1 \cdot (y-x)} e^{ik_2 \cdot x} e^{-ik_3 \cdot y} e^{-ik_4 \cdot y} e^{-i(p-q) \cdot x} e^{i(p-k) \cdot y} \frac{1}{(k_1^2 - m_c^2)(k_2^2 - m_c^2)(k_3^2 - m_s^2)k_4^2} \\ & \quad \times [(\not{k}_1 + m_c)\gamma^\nu (\not{k}_2 - m_c)(1 - \gamma_5)\gamma_\mu (\not{k}_3 - m_s)\gamma_5 \not{k}_4 \gamma^\mu (1 - \gamma_5)\not{q}\gamma_5 \gamma_\nu \gamma_5], \end{aligned} \quad (22)$$

with N_c the number of colors.

The double imaginary part of the correlation function is related to its double discontinuity, which can be extracted by the cutting rules. Setting the momentum of each propagator on shell, we have

$$\begin{aligned} \text{Im}^2 \Pi_{\text{QCD}}^{\mathcal{O}_1}(s', s, P^2)_{(2)} &= \frac{1}{(2i)^2} \text{Disc}^2 \Pi_{\text{QCD}}^{\mathcal{O}_1}(s', s, P^2)_{(2)} \\ &= -4\sqrt{2} N_c \left(-\frac{i}{12} \right) f_\pi (-2\pi i)^4 \frac{1}{(2i)^2} \frac{1}{(2\pi)^4} \int_0^1 du \varphi_\pi(u) \int dm_{34}^2 \int d\Phi_\Delta(P_1^2, p_2^2) \int d\Phi_2(m_{34}^2) \\ & \quad \times [(\not{k}_1 + m_c)\gamma^\nu (\not{k}_2 - m_c)(1 - \gamma_5)\gamma_\mu (\not{k}_3 - m_s)\gamma_5 \not{k}_4 \gamma^\mu (1 - \gamma_5)\not{q}\gamma_5 \gamma_\nu \gamma_5]. \end{aligned} \quad (23)$$

Here, we have introduced an extra integration on $m_{34}^2 = (k_3 + k_4)^2$ to express the double discontinuity of the correlation function as a convolution of a two-body phase space integration $\int d\Phi_2(m_{34}^2)$ and a triangle diagram integration $\int d\Phi_\Delta(P_1^2, p_2^2)$,

$$\begin{aligned} \int d\Phi_2(m_{34}^2) &= \int \frac{d^3 k_3}{(2\pi)^3} \frac{1}{2E_{k_3}} \frac{d^3 k_4}{(2\pi)^3} \frac{1}{2E_{k_4}} \delta^4(k_{34} - k_3 - k_4), \quad k_{34}^2 = m_{34}^2, \\ \int d\Phi_\Delta(P_1^2, p_2^2) &= \int d^4 k_{34} d^4 k_1 d^4 k_2 \delta(k_1^2 - m_c^2) \delta(k_2^2 - m_c^2) \delta(k_{34}^2 - m_{34}^2) \delta^4(P_1 - k_1 - k_2) \delta^4(p_2 - k_1 - k_{34}), \end{aligned} \quad (24)$$

TABLE I. The masses and decay constants of the charmed baryons with positive or negative parity. The decay constants are defined as in Eq. (7).

Baryon	$\Xi_{cc}(\frac{1}{2}^+)$	$\Xi_{cc}(\frac{1}{2}^-)$	$\Xi_c(\frac{1}{2}^+)$	$\Xi_c(\frac{1}{2}^-)$	$\Xi'_c(\frac{1}{2}^+)$	$\Xi'_c(\frac{1}{2}^-)$
Mass [GeV]	3.62 [47]	3.77 [48]	2.47 [47]	2.79 [49]	2.58 [47]	2.87 [48]
λ [GeV ³]	0.109 [50,51]	0.159 [48]	0.038 [50,51]	0.042 [52]	0.076 [50,51]	0.084 [48]

where $P_1 = p - uq = p_1 + \bar{u}q$ and $p_1^2 = s$, $p_2^2 = s'^2$. This factorization for the momentum flows can also be understood intuitively by Fig. 2. It is seen that the momentum flowing into the left-lower corner of the triangle diagram is not p_1 itself but $P_1 = p - uq = p_1 + \bar{u}q$ instead, and these momenta are related as $P_1^2 = up_1^2 = us$.

For the contribution from the twist-3 p and twist-3 σ LCDAs the calculation is similar. The only difference is that the twist-3 σ LCDA contains a term proportional to the coordinate x^ν . Note that from Eq. (22) there is an exponential term $\exp(iuq \cdot x)$ in the correlation function. One can use it to express x^ν as $(-i/u)(\partial/\partial q_\nu) \exp(iuq \cdot x)$ so that the calculation can still be done in momentum space.

V. NUMERICAL RESULTS

In this section, we first give the numerical results for the amplitudes of the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(\prime)} \pi^+$ decays, namely the two constants $A_{\text{WE}}^{(i)}$ and $B_{\text{WE}}^{(i)}$ in Eq. (18). In this work, we use the $\overline{\text{MS}}$ masses for the quarks, $m_c(\mu) = 1.27$ GeV and $m_s(\mu) = 0.103$ GeV with $\mu = 1.27$ GeV [46]. The masses of the u quark and the pion are neglected. The masses and decay constants of the charm baryons with positive or negative parity are listed in Table I, where the decay constants are defined as in Eq. (7).

On the other hand, the LCSR contains two kinds of extra parameters, the thresholds $s_{\Xi_{cc}}, s_{\Xi_c}, s_{\Xi'_c}$ and the Borel parameters T^2, T'^2 . For the threshold parameters, we have argued that they are process independent and will be taken from Ref. [41], $s_{\Xi_{cc}} = (4.1 \pm 0.1)^2$ GeV², $s_{\Xi_c} = (3.2 \pm 0.1)^2$ GeV², and $s_{\Xi'_c} = (3.3 \pm 0.1)^2$ GeV². These are about 0.5^2 GeV² larger than the squared masses of the corresponding baryons. Since this difference to the squared mass is only an empirical value proposed in Ref. [53], we will consider the uncertainty of the thresholds when evaluating the error of numerical results.

What concerns the Borel parameters, to simplify the problem, we apply the following equation to relate the two Borel parameters corresponding to the s and s' channels [54]:

$$\frac{T^2}{T'^2} \approx \frac{M_1^2 - m_1^2}{M_2^2 - m_2^2} \equiv \frac{1}{\kappa}, \quad (25)$$

where $M_{1(2)}$ is the mass of the initial (final) baryon and $m_{1(2)}$ is the mass of the quark before (after) the weak decay. In Figs. 3 and 4, we present the Borel parameter dependence of

the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$ decay amplitudes $A^{(\prime)(2)}(B^{(\prime)(2)})$, $A^{(\prime)(3p)}(B^{(\prime)(3p)})$, and $A^{(\prime)(3\sigma)}(B^{(\prime)(3\sigma)})$ contributed from the twist-2, twist-3 p , and twist-3 σ LCDAs, respectively. In each diagram, the blue band denotes the uncertainty from the error of the threshold. The upper and lower red bands denote the uncertainty from the error of the Monte Carlo integrations. Generally, the window of Borel parameters is chosen to satisfy three requirements. The first one is that they must be small enough so that the contribution from the continuous spectrum can be suppressed, which determines their upper bound. The second one is that they must be large enough to ensure the OPE convergence, which determines their lower bound. The last one is that the result must be stable in this window. To determine the upper bound of the Borel parameter, we require that the pole contribution must be larger than the continuous spectrum contribution, namely:

$$\frac{\int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' \int_{4m_c^2}^{s_{\Xi_{cc}}} ds e^{-s'/\kappa T^2} e^{-s/T^2} \text{Im}^2 \Pi_{\text{QCD}}^{\mathcal{O}_i}(s', s, P^2)}{\int_{(m_c+m_s)^2}^{\infty} ds' \int_{4m_c^2}^{\infty} ds e^{-s'/\kappa T^2} e^{-s/T^2} \text{Im}^2 \Pi_{\text{QCD}}^{\mathcal{O}_i}(s', s, P^2)} > 0.5. \quad (26)$$

The numerator represents the pole contribution, which is just the integral on the right-hand side of Eq. (15). The denominator is the same integral but the upper limits of s and s' are extended to infinity, so that it contains both pole and continuous spectrum contributions. This requirement shows that the upper bound of the Borel parameter is around $T^2 = 7$ GeV² for $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ and $T^2 = 6$ GeV² for $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$.

The lower bounds of the Borel parameters are determined in principle by the ratio between the contribution from the leading order and next-to-leading order QCD corrections to the perturbative kernel of OPE. However, in this work, we have only considered the leading order contribution so that this method cannot be used. From Figs. 3 and 4, it can be seen that the upper bounds given above are in a relatively stable region. Therefore, although, we cannot determine the lower bound quantitatively, we can take a range below the upper bound of the Borel parameter. Here, we set the window as $6 < T^2 < 8$ GeV² for $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ and $5 < T^2 < 7$ GeV² for $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$. The amplitudes and the corresponding errors from the uncertainties of $s_{\Xi_{cc}}, s_{\Xi_c}, s_{\Xi'_c}$, and T^2 are listed in Table II. Note that most of the contributions from the

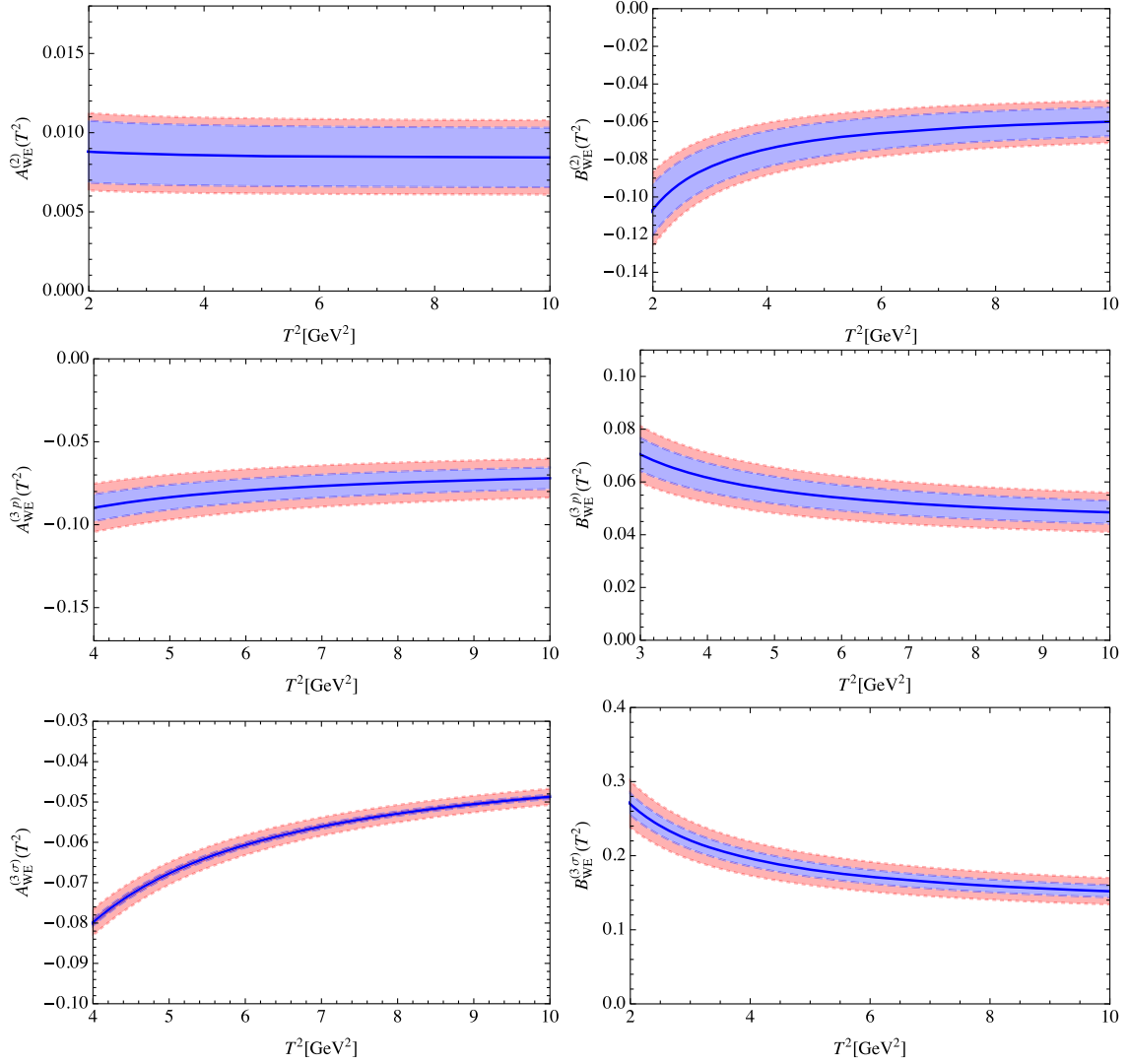


FIG. 3. The Borel parameter dependence of the $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ decay amplitudes $A^{(2)}(B^{(2)})$, $A^{(3p)}(B^{(3p)})$, and $A^{(3\sigma)}(B^{(3\sigma)})$ from the twist-2, twist-3p, and twist-3 σ LCDAs, respectively. In each diagram, the blue band denotes the uncertainty from the error of the threshold: $s_{\Xi_c} = (4.1 \pm 0.1)^2 \text{ GeV}^2$ and $s_{\Xi_c} = (3.2 \pm 0.1)^2 \text{ GeV}^2$. The upper and lower red bands denote the uncertainty from the error of Monte Carlo integrations.

twist-3 LCDAs are larger than that from twist-2. In principle, to make the twist-2 contribution dominate, we have to set a much larger Borel parameter, which corresponds with the convergence of the light-cone expansion. However, such a large Borel parameter will exceed the upper bound that we have determined using Eq. (26), where we demanded that the continuous spectrum contribution must be suppressed. Taking this into account, we have to keep the Borel parameter below or near this upper bound. Otherwise the pole contribution will be polluted from the continuous spectrum contribution, which may lead to a considerable uncertainty. Therefore, choosing the Borel parameter around this upper bound is the best choice though it may sacrifice the twist-2 dominance to some extent. On the other hand, this feature is also common in the LCSR studies on the heavy-to-light decays.

For example, in the LCSR calculation of the $B \rightarrow \pi$ form factors [44,55], the contribution of twist-3 is generally of the same order or even larger than that of twist-2. The highly suppressed contributions should come from the LCDAs of twist-4 or higher.

Table III shows the comparison of our results with those from the literature. Here, we have unified the definition of the amplitudes from all of these works. The amplitudes presented in the table are the parameters of the matrix element induced by the effective Hamiltonian from Eq. (2) instead of $\mathcal{O}_{1,2}$:

$$\begin{aligned} & \langle \Xi_c^{+(l)}(p-q)\pi^+(q) | \mathcal{H}_{\text{eff}}(0) | \Xi_{cc}^{++}(p) \rangle_{\text{fac.nf}} \\ & = i\bar{u}(p-q)[A^{(l)\text{fac.nf}} + B^{(l)\text{fac.nf}}\gamma_5]u(p), \end{aligned} \quad (27)$$

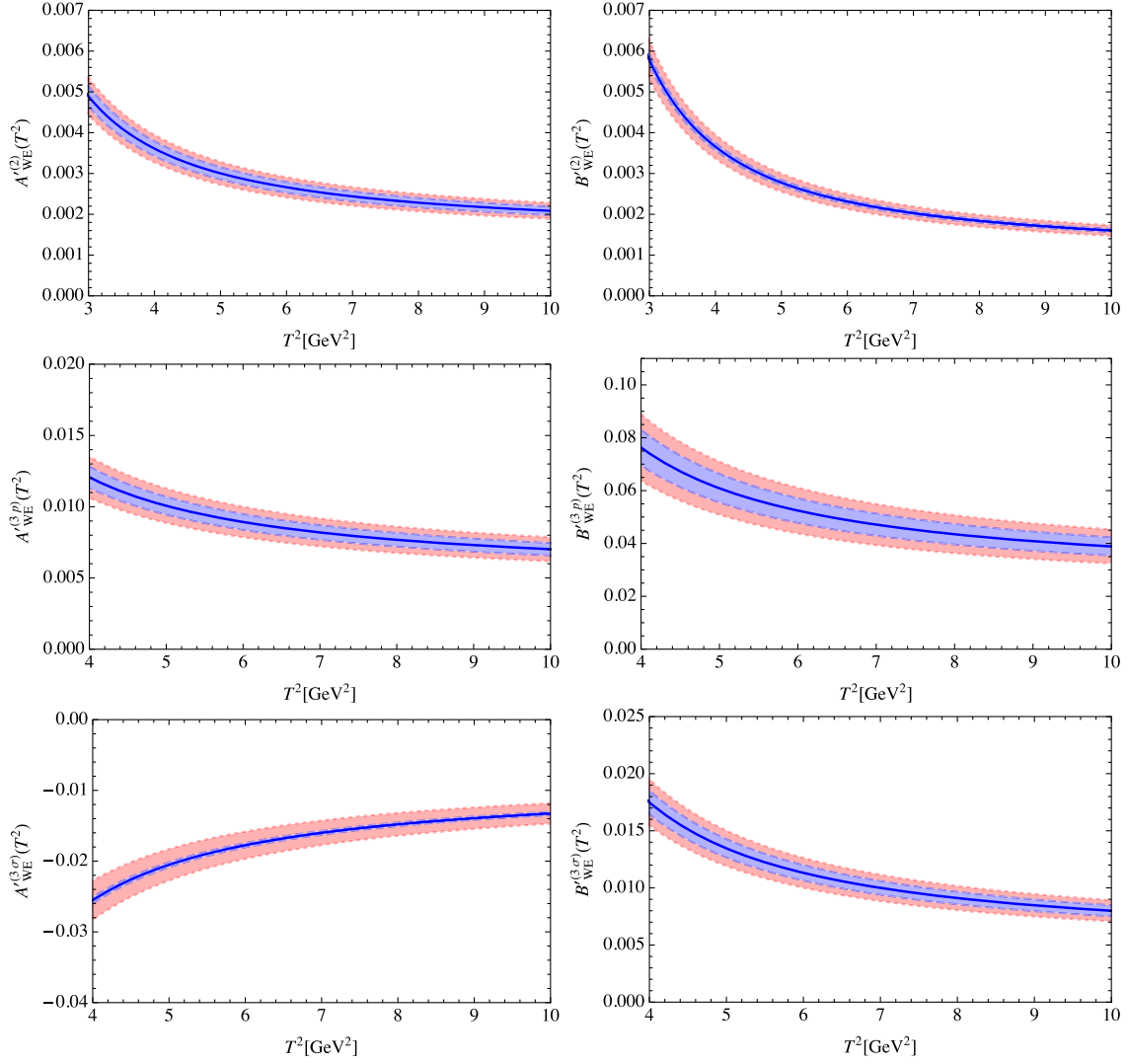


FIG. 4. The Borel parameter dependence of the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+/\prime} \pi^+$ decay amplitudes $A^{(2)}(B^{(2)})$, $A^{(3p)}(B^{(3p)})$, and $A^{(3\sigma)}(B^{(3\sigma)})$ from the twist-2, twist-3p, and twist-3 σ LCDAs, respectively. In each diagram, the blue band denotes the uncertainty from the error of the threshold: $s_{\Xi_c} = (4.1 \pm 0.1)^2 \text{ GeV}^2$ and $s_{\Xi_c'} = (3.3 \pm 0.1)^2 \text{ GeV}^2$. The upper and lower red bands denote the uncertainty from the error of Monte Carlo integrations.

where “fac” means the factorizable or W -emission contribution, while “nf” denotes the nonfactorizable or equivalently the W -exchange contribution, and $(A, B)^{(l)\text{tot}} = (A, B)^{(l)\text{fac}} + (A, B)^{(l)\text{nf}}$. All the amplitudes in Table III

are in units of $10^{-2} G_F \text{ GeV}^2$. Here, we have listed the W -exchange amplitudes from the pole model (PM) [22,27,34] and three-loop constituent quark model (3LCQM) [31]. We have also listed the corresponding factorizable amplitudes

TABLE II. Decay amplitudes of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+/\prime} \pi^+$ from the W -exchange contribution. The Borel parameters are set in the region $6 < T^2 < 8 \text{ GeV}^2$ for $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ and $5 < T^2 < 7 \text{ GeV}^2$ for $\Xi_{cc}^{++} \rightarrow \Xi_c^{\prime+} \pi^+$.

$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$	Twist-2	Twist-3p	Twist-3 σ	Total
A_{WE}	0.0084 ± 0.0024	-0.077 ± 0.01	-0.056 ± 0.002	-0.124 ± 0.011
B_{WE}	-0.064 ± 0.01	0.052 ± 0.01	0.165 ± 0.025	0.153 ± 0.029
$\Xi_{cc}^{++} \rightarrow \Xi_c^{\prime+} \pi^+$	Twist-2	Twist-3p	Twist-3 σ	Total
A'_{WE}	0.0027 ± 0.0005	0.0089 ± 0.002	-0.018 ± 0.0003	-0.0062 ± 0.002
B'_{WE}	0.0023 ± 0.0006	0.052 ± 0.016	0.011 ± 0.003	0.066 ± 0.016

TABLE III. Comparison of the decay amplitudes of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$ from this work with those from the literature. All the amplitudes below are in unit $10^{-2} G_F \text{ GeV}^2$. Here, we list the W -exchange amplitudes from the PM [22,27,34] and three-loop quark model (3LQM) [31]. We also list the factorizable amplitudes from QCDSR [16], LFQM [27], NRQM [22,34], and HQET [22,34]. The notation LFQM + PM for example in the first column means the factorizable amplitude is from LFQM while the W -exchange amplitude is from PM, otherwise both of them are from the same theoretical approach.

$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$	A^{fac}	A^{nf}	A^{tot}	B^{fac}	B^{nf}	B^{tot}
This work	...	-16.67 ± 1.41	20.47 ± 3.89	...
QCDSR [16]	-8.74 ± 2.91	16.76 ± 5.36
LFQM + PM [27]	7.40	-10.79	-3.38	15.06	-18.91	-3.85
3LQM [31]	-8.13	10.50	3.37	-12.97	18.53	5.56
NRQM + PM [22,34]	7.38	0	7.38	16.77	24.95	41.72
HQET + PM [22,34]	9.52	0	9.52	19.45	24.95	44.40

$\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$	A^{fac}	A^{nf}	A^{tot}	B^{fac}	B^{nf}	B^{tot}
This Work	...	-0.83 ± 0.28	8.86 ± 2.16	...
QCDSR [16]	-3.55 ± 0.68	34.13 ± 11.6
LFQM + PM [27]	4.49	-0.04	4.45	48.50	-0.06	48.44
3LCQM [31]	-4.34	-0.11	-4.45	-37.59	-1.37	-38.96
NRQM + PM [22,34]	4.29	0	4.29	53.65	0	53.65
HQET + PM [22,34]	5.10	0	5.10	62.37	0	62.37

from QCDSR [16], LFQM [27], nonrelativistic quark model (NRQM) [22,34] and HQET [22,34]. The notation LFQM + PM for example in the first column means that the factorizable amplitude is from LFQM while the W -exchange amplitude is from PM. On the other hand, 3LCQM alone means that the factorizable and nonfactorizable amplitudes are both from 3LCQM. From the comparison in Table III, one can find that our results for the W -exchange amplitudes of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ are consistent with most of the results from literatures. However, in terms of the W -exchange amplitudes of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$, our results are much larger than those from the literature, which provides a possibility to explain the anomaly of B'/B . Two issues deserve further discussion:

- (i) We have only used the two-particle LCDAs of the pion for the calculation up to the leading order. Although the contribution from the higher-twist LCDAs or QCD loop corrections is expected to be suppressed, in principle they are still necessary for improving the accuracy of the W -exchange contribution, which will be included in a future study.
- (ii) From Table III, we see that the various predictions on the factorizable contribution from the literature are not totally consistent with each other. These works assume the naive factorization which is based on color transparency [13]. It states that in the bottom hadron decays, the b quark is heavy enough so that the emitted light meson flies quickly and decouples from other hadrons before it is caught up by the soft gluons. However, since the charm quark

is lighter, in the charmed hadron decays the effect of soft gluon exchange may be important. Therefore, such effects need to be worked out in the future.

Now, we calculate the decay branching fractions by combining each factorizable amplitude from the literature and the nonfactorizable amplitudes from this work, and make a comparison among them. The decay width is expressed as

$$\Gamma^{(\prime)} = \frac{p_c}{8\pi} \left[\frac{(m_{\Xi_{cc}} + m_{\Xi_c^{(\prime)}})^2 - m_\pi^2}{m_{\Xi_{cc}}^2} |A^{\text{tot}(\prime)}|^2 + \frac{(m_{\Xi_{cc}} - m_{\Xi_c^{(\prime)}})^2 - m_\pi^2}{m_{\Xi_{cc}}^2} |B^{\text{tot}(\prime)}|^2 \right], \quad (28)$$

where p_c is the magnitude of the pion three-momentum in the rest frame of the Ξ_{cc} . The lifetime of the Ξ_{cc}^{++} is chosen as $\tau(\Xi_{cc}^{++}) = 2.56 \times 10^{-13} \text{ s}$, the Wilson coefficients are chosen as $C_1 = 1.35$ and $C_2 = -0.64$ [27], $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$, and $V_{cs} = 0.975$, $V_{ud} = 0.973$ [46]. The branching fractions are listed in Table IV. In the first five lines the branching fractions are evaluated by the nonfactorizable amplitudes from this work and the factorizable amplitudes from the literature. The next four lines present the branching fractions with both kinds of amplitudes being evaluated in the literature. In the last three lines, we also list the branching fraction calculated by considering the FSR [56], where the decay amplitudes are dressed by the FSR effect so that their explicit value are not presented. η is a parameter introduced to account for the

TABLE IV. Comparison of the decay branching fractions of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(\prime)} \pi^+$ from this work with those from the literature. The lifetime of the Ξ_{cc}^{++} is chosen as $\tau(\Xi_{cc}^{++}) = 2.56 \times 10^{-13} s$ [27]. In the first five lines the branching fractions are evaluated by the nonfactorizable amplitudes from this work and the factorizable amplitudes from the literature. The next four lines present the branching fractions with both these two kinds of amplitudes evaluated in the literature. In the last three lines, we also list the branching fraction calculated by considering the final-state rescattering (FSR) [56]. η is a parameter introduced for the off-shell effect of particle exchange. In the last column, we list the values of \mathcal{B}'/\mathcal{B} from all the theoretical methods mentioned in Table III.

Method	A^{tot}	B^{tot}	$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)$	A'^{tot}	B'^{tot}	$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^{+(\prime)} \pi^+)$	\mathcal{B}'/\mathcal{B}
QCDSR + LCSR	-25.4 ± 4.32	37.23 ± 9.25	$40 \pm 14\%$	-4.38 ± 0.96	42.99 ± 13.76	$3.91 \pm 2.5\%$	0.098 ± 0.14
LFQM + LCSR	-9.27 ± 1.41	35.53 ± 3.89	$7.54 \pm 2.22\%$	3.66 ± 0.28	57.36 ± 2.16	$5.83 \pm 0.5\%$	0.77 ± 0.42
3LCQM + LCSR	-24.8 ± 1.41	7.5 ± 3.89	$35.55 \pm 4.29\%$	-5.17 ± 0.28	-28.73 ± 2.16	$2.75 \pm 0.35\%$	0.08 ± 0.02
NRQM + LCSR	-9.29 ± 1.41	37.24 ± 3.89	$7.82 \pm 2.25\%$	3.46 ± 0.28	62.51 ± 2.16	$6.70 \pm 0.54\%$	0.85 ± 0.44
HQET + LCSR	-7.18 ± 1.41	39.92 ± 3.89	$6.22 \pm 1.94\%$	4.27 ± 0.28	71.23 ± 2.16	$8.85 \pm 0.62\%$	1.42 ± 0.78
LFQM + PM	-3.83	3.85	0.69%	4.45	48.44	4.65%	6.74
3LCQM	3.37	5.56	0.71%	-4.45	-38.96	3.39%	4.77
HQET + PM	7.38	41.72	6.64%	4.29	53.65	5.39%	0.81
NRQM + PM	9.52	44.40	9.19%	5.1	62.37	7.34%	0.8
FSR($\eta = 1.0$)	7.11%	4.72%	0.66
FSR($\eta = 1.5$)	8.48%	4.72%	0.56
FSR($\eta = 2.0$)	10.75%	4.74%	0.44

off-shell effect of the particle exchange [56,57]. In the last column, we list the values of \mathcal{B}'/\mathcal{B} from all the theoretical methods mentioned above. We note that most of the calculated \mathcal{B}'/\mathcal{B} are much larger or smaller than the experimental value $(\mathcal{B}'/\mathcal{B})_{\text{exp}} = 1.41 \pm 0.17 \pm 0.1$. However, the value obtained from HQET + LCSR leads to the fraction

$$(\mathcal{B}'/\mathcal{B})_{\text{HQET+LCSR}} = 1.42 \pm 0.78, \quad (29)$$

which agrees amazingly well with the experimental value. Although there exists non-negligible uncertainty for this theoretical result, it still implies that $(\mathcal{B}'/\mathcal{B})_{\text{exp}} = 1.41 > 1$ can be understood and realized theoretically without introducing any physics beyond the Standard Model. The absolute branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ from HQET + LCSR is

$$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)_{\text{HQET+LCSR}} = 6.22 \pm 1.94\%. \quad (30)$$

Up to now, there is no experimental announcement on this absolute branching fraction. Instead, in Ref. [27] the authors have given an evaluation of it by phenomenological methods:

$$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) = 1.83 \pm 1.01\%, \quad (31)$$

which is consistent with our result within the uncertainties. For the details on this phenomenological evaluation, we refer to the discussion around the Eq. (38) in Ref. [27]. However, due to the lack of experimental measurements all

the theoretical predictions given above are still waiting to be tested by future experiments.

VI. CONCLUSION

We have calculated the W -exchange contribution in the $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(\prime)} \pi^+$ decay with the use of the LCSR. The two-particle LCDAs of pion are used as the nonperturbative inputs for the sum rules calculation, and the perturbative kernel is calculated at the leading order. We obtain the relative decay branching fraction \mathcal{B}'/\mathcal{B} of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+(\prime)} \pi^+$ by combining our W -exchange amplitudes with the factorizable amplitudes from various theoretical methods in the literature. We find that using the factorizable amplitudes from heavy quark effective theory, we obtain the ratio $\mathcal{B}'/\mathcal{B} = 1.42 \pm 0.78$, which is consistent with the experimental value, $1.41 \pm 0.17 \pm 0.1$. The corresponding absolute decay branching fraction is consistent with that evaluated in the literature, which should be tested by future experiments.

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