Factorization of the forward-backward asymmetry and measurements of the weak mixing angle and proton structure at hadron colliders

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The forward-backward charge asymmetry (A_{FB}) at hadron colliders is sensitive to both the electroweak (EW) symmetry breaking represented by the effective weak mixing angle $\sin^2\theta^{\ell}_{\mathrm{eff}}$, and the proton structure information in the initial state modeled by the parton distribution functions (PDFs). Due to their strong correlation, the precisions of determination on $\sin^2\theta^{\ell}_{\mathrm{eff}}$ and PDFs using the measured A_{FB} spectrum are limited. In this paper, we define a set of structure parameters which factorize the unique proton information of the relative difference between quarks and antiquarks in the A_{FB} observation. Other than the conventional way of extracting $\sin^2\theta^{\ell}_{\mathrm{eff}}$ from the convolution of PDF and EW calculations, we propose a new method to simultaneously determine the value of $\sin^2\theta^{\ell}_{\mathrm{eff}}$ and the proton structure terms by fitting to the observed A_{FB} distribution, and point out the necessity of specifying additional observations to further reduce the uncertainties on the proton structure terms, respectively, so that model-independent high precision $\sin^2\theta^{\ell}_{\mathrm{eff}}$ and proton structure measurements can be achieved at the future LHC experiments.

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I. INTRODUCTION

The forward-backward asymmetry (A_{FB}) of the neutral current process $f_i\bar{f}_i \to Z/\gamma^* \to f_j\bar{f}_j$ describes the relative difference between cross sections of the forward and backward scattering. It arises from the vector and axial vector couplings of Z to fermions, and is governed by the effective weak mixing angle $\sin^2\theta^{\ell}_{\mathrm{eff}}$, which has been measured first in the electron-positron collisions at the LEP and the SLC [1], then in the proton-antiproton collisions at the Tevatron [2]. At the Large Hadron Collider (LHC), the initial fermions and antifermions of the corresponding Drell-Yan (DY) process $pp(q\bar{q}) \to Z/\gamma^* \to \ell^+\ell^-$ are quarks and antiquarks, acting as partons in the proton, with their momentum distributions modeled by the parton distribution functions (PDFs). Since quarks and antiquarks would come from either side of the proton beams, the observed asymmetry at hadron colliders, denoted as A^h_{FB}, is diluted from its original A_{FB} value. The reduction in magnitude from A_{FB} to A^h_{FB} directly reflects the difference between quark and antiquark momentum fractions in the proton, and can be used to constrain PDFs. Some of the earlier studies of using A^h_{FB} to constrain PDFs can be found in Refs. [3–6].

Though A^h_{FB} can be precisely measured at the LHC, it is difficult to use this observable to constrain the PDFs and extract a more precise value of $\sin^2\theta^{\ell}_{\mathrm{eff}}$, due to the strong correlation between them. On the one hand, as reported by the ATLAS, CMS and LHCb collaborations [7–9], the PDF-induced uncertainty is becoming the most dominant issue in the $\sin^2\theta^{\ell}_{\mathrm{eff}}$ measurements, and is same large as the total uncertainty on $\sin^2\theta^{\ell}_{\mathrm{eff}}$ measured at the LEP, SLC, and Tevatron. As the PDF-induced uncertainty is preventing the measurements of $\sin^2\theta^{\ell}_{\mathrm{eff}}$ at the LHC to achieve better precision in the future, the precision of the W boson mass measurement has been improved to a level of $\mathcal{O}(10^{-4})$ by the CDF collaboration [10], and the central value is significantly deviated from the Standard Model (SM) prediction. If such deviation hints new physics beyond the SM, it should also appear as a shift on $\sin^2\theta^{\ell}_{\mathrm{eff}}$. On the other hand, as noted in Ref. [6], directly using the A^h_{FB} observable in a typical global PDF analysis may yield a large bias induced by the limited precision of $\sin^2\theta^{\ell}_{\mathrm{eff}}$. In the latest global analysis of NNPDF4.0 [11], the A^h_{FB} distribution measured by ATLAS [12] is removed from its dataset to avoid the bias caused by the correlation. Therefore, it is essential to have a strategy to decouple the parton information from the EW prediction in the A^h_{FB}

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observation, so that the correlation can be properly considered, and $\sin^2\theta^{\ell}_{\rm eff}$ and parton information can be simultaneously determined using the LHC data instead of fitting for one and fixing the other in the conventional methods.

Such simultaneous determination is not available in the framework of PDF global analysis, with $\sin^2\theta^{\ell}_{\rm eff}$ as a free parameter. Many of the old experimental datasets included in the PDF analyses were performed at the leading order in EW interaction, and those data are still important for providing PDF information [13]. To achieve a high precision of $\sin^2\theta^{\ell}_{\rm eff}$ determination, all the Wilson coefficients of the scattering processes included in the PDF analysis have to be updated to higher-order in EW interactions. This feature is not yet available in the current PDF global analysis programs. Therefore, it would be helpful that decoupling the EW and parton information in a limited set of LHC observations can be developed, so that the simultaneous determination can practically be realized, instead of using comprehensive experimental results as input in the PDF global analysis.

In this paper, we propose a new method which factorizes the proton information relevant to the $A^h_{\rm FB}$ measurement into a set of structure parameters with limited number of freedom. They are defined to be generally independent of any specific PDF modeling, and thus can be viewed as some new experimental observables. These parameters, together with $\sin^2\theta^{\ell}_{\rm eff}$, can be determined from one single measurement of the $A^h_{\rm FB}$ at the LHC, through a carefully-designed fitting procedure, as to be described below. The correlation between the determination of $\sin^2\theta^{\ell}_{\rm eff}$ and proton structure is automatically taken into account in the proposed analysis.

II. FACTORIZATION OF $A^h_{\rm FB}$

The $A_{\rm FB}$ at hadron colliders is observed in the Collins-Soper (CS) frame [14], which is a center-of-mass frame with the $\hat{z}$-axis defined as the bisector of the angle formed by the direction of the momentum of one incoming hadron (H_A) and the negative direction of the other incoming hadron (H_B). To decouple the original asymmetry from the proton structure information, we define two scattering angles, i.e., $\cos\theta_q$ and $\cos\theta_h$, with different choices of H_A and H_B. For the $\cos\theta_q$, H_A, and H_B are assigned according to the directions of q and $\bar{q}$, respectively. The differential cross section of the DY process can be expressed in term of $\cos\theta_q$ as:

$$\frac{d\sigma}{d\cos\theta_q dY dM dQ_T} = \frac{3}{8}\sum_f \alpha_f(Y,M,Q_T)\times\{(1+\cos^2\theta_q) + \frac{1}{2}A_0^f(Y,M,Q_T)(1-3\cos^2\theta_q) + A_4^f(Y,M,Q_T)\cos\theta_q\}, \quad (1)$$

where Y, M, and Q_T are the rapidity, invariant mass and transverse momentum of the Z boson. The index f denote the flavor, and the term α_f is proportional to the cross section of the quark-antiquark subprocesses (with flavor f). The terms A_0^f and A_4^f are the normalized polar angular coefficient functions, while other angular functions are canceled when integrated over the azimuthal angle. Both A_0^f and A_4^f, as a function of Y, M, and Q_T, can be calculated in the perturbative expansion of the electroweak and QCD couplings. In this work, they are computed using the RESBOS [15] package at approximate next-to-next-to-leading order (NNLO) plus next-to-next-to-leading logarithm (NNLL) in QCD interaction.

The canonical scales [16,17] are set to the invariant mass of the lepton pair in the DY events. The EW calculation of RESBOS is similar to the effective born approximation used in ZFITTER [18], which corresponds to a calculation on the form factors with complete EW two-loop corrections. The DY events are categorized as forward (F) for $\cos\theta_q > 0$, and backward (B) for $\cos\theta_q < 0$. It is custom to define $A_{\rm FB}$ as

$$A_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \quad (2)$$

where σ_F and σ_B are the cross sections of F and B events. In this case, the parton densities of the initial state quarks and antiquarks contribute only to α_f terms, and A_0^f, A_4^f, and $A_{\rm FB}$ are independent of proton structure information. However, such $A_{\rm FB}$ cannot be experimentally observed because the directions of q and $\bar{q}$ are practically unknown.

At the LHC, the DY process is observed in terms of $\cos\theta_h$, of which H_A is defined as the hadron which points to the same hemisphere aligned to the reconstructed Z boson of the dilepton final state. Since the Z boson is boosted along with the direction of the parton which carries higher energy, $\cos\theta_h = \cos\theta_q$ when q has larger energy than $\bar{q}$, and $\cos\theta_h = -\cos\theta_q$ when $\bar{q}$ has the larger energy. We introduce the dilution factor $D_f(Y,M,Q_T)$ to represent the probability of having $\cos\theta_h = -\cos\theta_q$, i.e., the antiquark has higher energy than the quark in the hard scattering subprocess. In this manner, the differential cross section in term of $\cos\theta_h$ can be expressed as:

$$\frac{d\sigma}{d\cos\theta_h\, dY\, dM\, dQ_T} = \frac{3}{8}\sum_f \alpha_f(Y,M,Q_T) \times\{(1+\cos^2\theta_h) + \frac{1}{2}A_0^f(Y,M,Q_T)(1-3\cos^2\theta_h) + [1-2D_f(Y,M,Q_T)]A_4^f(Y,M,Q_T)\cos\theta_h\} \quad (3)$$

It is clear that only the A_4^f term would be affected by the dilution effect. Accordingly, the cross sections of the forward and backward event in terms of $\cos\theta_q$ in each $f\bar{f}$ subprocess are written as:

$$\begin{aligned}
\sigma_F^f(Y,M,Q_T) &= \frac{3}{8}\alpha_f(Y,M,Q_T)\int_0^1 d\cos\theta_q\Big[(1+\cos^2\theta_q) \\
&\quad + \frac{1}{2}A_0^f(Y,M,Q_T)(1-3\cos^2\theta_q) + A_4^f\cos\theta_q\Big]. \\
\sigma_B^f(Y,M,Q_T) &= \frac{3}{8}\alpha_f(Y,M,Q_T)\int_{-1}^0 d\cos\theta_q\Big[(1+\cos^2\theta_q) \\
&\quad + \frac{1}{2}A_0^f(Y,M,Q_T)(1-3\cos^2\theta_q) + A_4^f\cos\theta_q\Big] \\
&= \frac{3}{8}\alpha_f(Y,M,Q_T)\int_0^1 d\cos\theta_q\Big[(1+\cos^2\theta_q) \\
&\quad + \frac{1}{2}A_0^f(Y,M,Q_T)(1-3\cos^2\theta_q) - A_4^f\cos\theta_q\Big].
\end{aligned} \tag{4}$$

Thus the cross section of the observed forward events in terms of $\cos\theta_h$ can be expressed using $\sigma_F^f(Y,M,Q_T)$ and $\sigma_B^f(Y,M,Q_T)$ as:

$$\begin{aligned}
\sigma_F^h(Y,M,Q_T) &= \sum_f[1-D_f(Y,M,Q_T)]\cdot\sigma_F^f(Y,M,Q_T) \\
&\quad + \sum_f D_f(Y,M,Q_T)\cdot\sigma_B^f(Y,M,Q_T). \\
\sigma_B^h(Y,M,Q_T) &= \sum_f[1-D_f(Y,M,Q_T)]\cdot\sigma_B^f(Y,M,Q_T) \\
&\quad + \sum_f D_f(Y,M,Q_T)\cdot\sigma_F^f(Y,M,Q_T).
\end{aligned} \tag{5}$$

Using Eq. (5), the experimental observed asymmetry $A_{\rm FB}^h$ can be directly formulated as follow:

$$\begin{aligned}
&A_{\rm FB}^h(Y,M,Q_T) \\
&\quad = \frac{\sigma_F^h(Y,M,Q_T)-\sigma_B^h(Y,M,Q_T)}{\sigma_F^h(Y,M,Q_T)-\sigma_B^h(Y,M,Q_T)} \\
&\quad = \frac{\sum_f[1-2D_f(Y,M,Q_T)]\alpha_f(Y,M,Q_T)A_{\rm FB}^f(Y,M,Q_T)}{\sum_f\alpha_f(Y,M,Q_T)}
\end{aligned} \tag{6}$$

where $A_{\rm FB}^f(Y,M,Q_T)$ denotes the asymmetry calculated using $\cos\theta_q$ for each $f\bar{f}$ subprocess, which is independent with proton structure.

The derivation of Eq. (6) is straightforward, and reveals the nature of the observed $A_{\rm FB}^h$, which is an average of the asymmetry contributed from each $f\bar{f}$ subprocess, weighted by its relative cross section, and the associated dilution factor reflects our limited knowledge of the q and $\bar{q}$ PDFs of the proton.

Here, we assume that the parton distributions of strange quark and antiquark of the proton are the same, $s(x)=\bar{s}(x)$, though a small violation can be generated at the NNLO [19]. Similarly, $c(x)=\bar{c}(x)$ and $b(x)=\bar{b}(x)$, with c and b denoting charm and bottom flavor, respectively. Consequently, the dilution factors D_s, D_c and D_b equal to 0.5 in all Z boson kinematic configurations, and their contributions in the numerator of Eq. (6) vanish.

Based on the argument, we propose to factorize the observed $A_{\rm FB}^h$ as products of the proton structure parameters and independent EW-dominant parts:

$$\begin{aligned}
&A_{\rm FB}^h(Y,M,Q_T) \\
&= [\Delta_u(Y,M,Q_T)+P_0^u(Y,Q_T)]\cdot A_{\rm FB}^u(Y,M,Q_T;\sin^2\theta_{\rm eff}^\ell) \\
&\quad + [\Delta_d(Y,M,Q_T)+P_0^d(Y,Q_T)]\cdot A_{\rm FB}^d(Y,M,Q_T;\sin^2\theta_{\rm eff}^\ell),
\end{aligned} \tag{7}$$

where the structure parameters are related to Eq. (6) via

$$\begin{aligned}
P_0^u(Y,Q_T) &= \int\frac{[1-2D_u(Y,M,Q_T)]\alpha_u(Y,M,Q_T)}{\sum_f\alpha_f(Y,M,Q_T)}dM \bigg/ \int dM \\
P_0^d(Y,Q_T) &= \int\frac{[1-2D_d(Y,M,Q_T)]\alpha_d(Y,M,Q_T)}{\sum_f\alpha_f(Y,M,Q_T)}dM \bigg/ \int dM \\
\Delta_u(Y,M,Q_T) &= \frac{[1-2D_u(Y,M,Q_T)]\alpha_u(Y,M,Q_T)}{\sum_f\alpha_f(Y,M,Q_T)} \\
&\quad - P_0^u(Y,Q_T) \\
\Delta_d(Y,M,Q_T) &= \frac{[1-2D_d(Y,M,Q_T)]\alpha_d(Y,M,Q_T)}{\sum_f\alpha_f(Y,M,Q_T)} \\
&\quad - P_0^d(Y,Q_T)
\end{aligned} \tag{8}$$

In Eq. (7), the $\sin^2\theta_{\rm eff}^\ell$-dependence of the light quark asymmetry $A_{\rm FB}^{u/d}(Y,M,Q_T;\sin^2\theta_{\rm eff}^\ell)$ is explicitly written out, which can be precisely predicted by the SM electroweak calculation. The structure parameters $P_0^{u/d}(Y,Q_T)$ represent the parton information averaged in a given mass range of M, while the parameters $\Delta_{u/d}(Y,M,Q_T)$ describe the variation around the averaged behavior provided by P_0^u and P_0^d, respectively, in that mass range. Both P_0^f and $\Delta_f(M)$ parameters are structure parameters, representing the parton information. When $A_{\rm FB}^h$ is observed in a narrow mass window around the Z boson mass pole, $\Delta_f(M)$ actually represents the evolution of the quark and antiquark

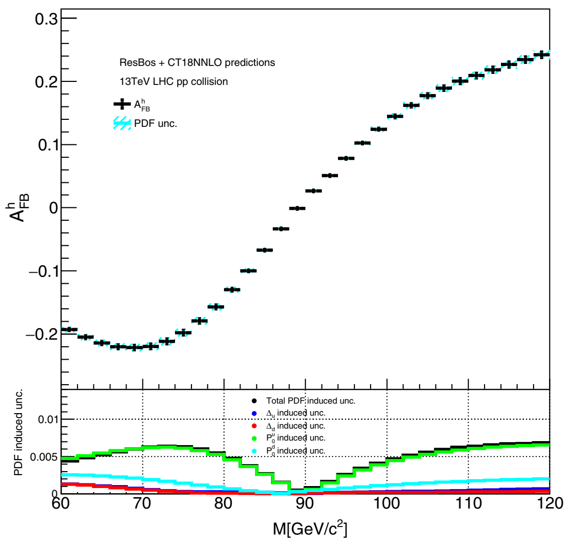

FIG. 1. The spectrum of $A^h_{\mathrm{FB}}(M)$, and the corresponding uncertainties from P^u_0, P^d_0, $\Delta_u(M)$, and $\Delta_d(M)$ terms, respectively. The central values and of A^h_{FB} and the uncertainties in each M bin are predicted by CT18 PDFs. The uncertainties correspond to 68% C.L. The observables are averaged over Y and Q_T in this figure.

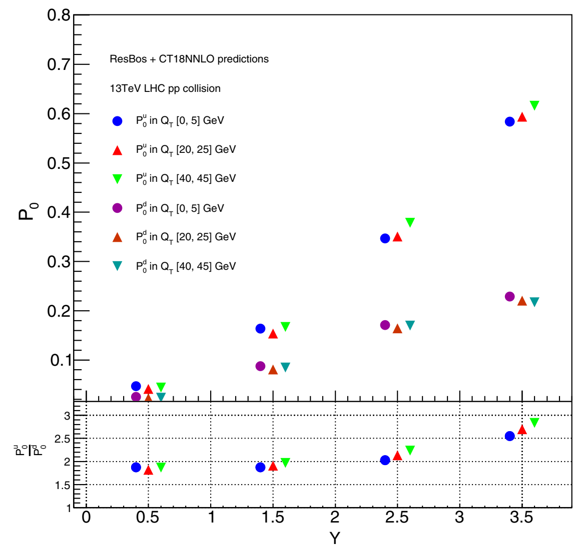

FIG. 2. The P^u_0 and P^d_0 values as a function of Q_T and Y predicted by ResBos+CT18. For the Y dependence, results in the plots correspond to the P_0 values averaged over $|Y|$ in [0, 1.0], [1.0, 2.0], [2.0, 3.0], and [3.0, 4.0] bins. For the Q_T dependence, the P_0 values are averaged in three regions of [0, 5], [20, 25], and [40, 45] GeV, as examples.

densities as a function of their momentum fraction x, which are related to M as $x_{1,2} = \frac{\sqrt{M^2+Q_T^2}}{\sqrt{s}} e^{\pm Y}$, where $\sqrt{s}$ is the collider energy. A mass window of $O(10)$ GeV corresponds to a very small x region, in which the variation and uncertainties of $\Delta_f(M)$ should be small, and therefore the proton structure information are dominated by P^f_0 parameters.

As an example, we show in the upper panel of Fig. 1 the distribution of $A^h_{\mathrm{FB}}(M)$, in the mass region of $60 < M < 120$ GeV$/c^2$, for a pseudodata sample of the DY process produced at the 13 TeV LHC, generated using the ResBos program with CT18 NNLO PDFs [20]. In the lower panel of Fig. 1, the PDF-induced uncertainty of A^h_{FB} for each invariant mass bin is decomposed according to Eq. (7), for illustration. As expected, the uncertainty is dominated by those of P^u_0 and P^d_0, while the contributions from $\Delta_u(M)$ and $\Delta_d(M)$ are quite small.

By its definition in Eq. (8), the P^f_0 parameters contain information of the dilution effects of the light quarks and their relative cross sections. Accordingly, the values of P^f_0 are expected to be increasing as a function of Y, for the dilution effects should be reduced in the kinematic region where the difference in the quark and antiquark energy is large. At the LHC energies, both $(1-2D_u)$ and $(1-2D_d)$ approach to 0 when Y is zero, and toward 1 as the magnitude of Y increases. This trend is demonstrated in Fig. 2, where the P^f_0 values predicted by CT18 PDFs are shown as a function of Y and Q_T. Additionally, in the high Y region where the dilution effects of the u and d quarks are similarly small, the separation of P^u_0 and P^d_0 could be attributed to the difference in the light quark parton abundance and their corresponding cross sections. Note that if the ratio of P^u_0 and P^d_0 is employed, the information on s, c and b cross sections cancels out. Therefore, the experimental observation on such ratio can directly probe the ratio of u and d (anti)quark parton densities of the proton as a function of x.

These phenomenal conclusions are also checked using predictions of MSHT20 [21] and NNPDF3.1 [22], and found to be independent with the choice of PDFs. In Fig. 3, predictions on P_0 parameters from MSHT20 and NNPDF3.1 are demonstrated in the same format as Fig. 2. In Fig. 4, we compare the predictions and corresponding PDF uncertainties on the Δ_f parameters predicted by CT18, MSHT20 and NNPDF3.1.

In the above discussions, we assume $s = \bar{s}$, $c = \bar{c}$ and $b = \bar{b}$. In fact, the form of Eq. (7) does not rely on that assumption. The EW predictions of A^d_{FB}, A^s_{FB}, and A^b_{FB} coincide, as they all correspond to the down-type quark. Similarly, A^u_{FB} is consistent with A^c_{FB} as they correspond to the up-type quark. Therefore, the two-terms-form of Eq. (7) can be derived anyway. The only difference comes from the definition of P^f_0 and Δ_f parameters. When the assumption of $s = \bar{s}$, $c = \bar{c}$ and $b = \bar{b}$ is taken, the structure parameters depend only on the parton information of u and d quarks; while if the difference between quark and antiquark densities is taken into account, the structure parameters are dominated by u and d quark information but also contain contributions from s, c, and b. As a conclusion,

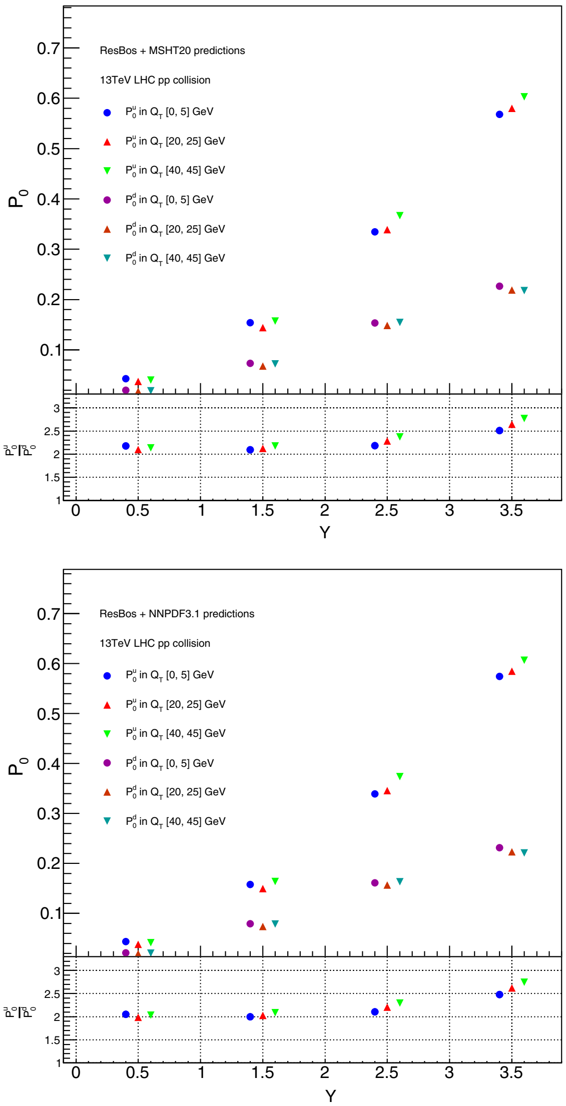

FIG. 3. Predictions of P_0^u and P_0^d in MSHT20 (up) and NNPDF3.1 (bottom) as a function of Q_T and Y. For the Y dependence, results in this plots correspond to the P_0 values averaged over $|Y|$ in [0, 1.0], [1.0, 2.0], [2.0, 3.0], and [3.0, 4.0] bins. For the Q_T dependence, the P_0 values are averaged in three regions of [0, 5], [20, 25], and [40, 45] GeV, as examples.

such assumption does not change the fact that P_0^u and P_0^d can be used as model independent experimental observable.

III. SIMULTANEOUS FIT OF $\sin^2\theta_{\mathrm{eff}}^{\ell}$ AND P_0^f

Some progresses on reducing the PDF and EW correlation and potential bias have been made in the past. In Ref. [6], it was found that excluding the A_{FB}^h data around the Z pole region in a PDF global analysis could reduce the bias. To further reduce the bias, at the expense of sensitivity, a new observable was proposed later in Ref. [23], which uses the gradient information of $A_{\mathrm{FB}}^h(M)$ spectrum to perform the PDF global fitting, while using its average value around the Z-pole to determine $\sin^2\theta_{\mathrm{eff}}^{\ell}$. However, the residual bias given by the above two methods is still sizable. Alternatively, Ref. [24] explored the impact of the imperfect knowledge of the proton structure on the determination of EW parameters (e.g., M_W) via nuisance parameter formalism, by including the bin-by-bin correlation of the kinematic distributions with respect to PDF variations. Though the analysis would effectively reduce the PDF uncertainty, it relies on the information of specific PDF sets, which are not automatically updated in the process of measuring the EW parameter. In short summary, none of these analyses provided a strategy to simultaneously fit $\sin^2\theta_{\mathrm{eff}}^{\ell}$ and parton information.

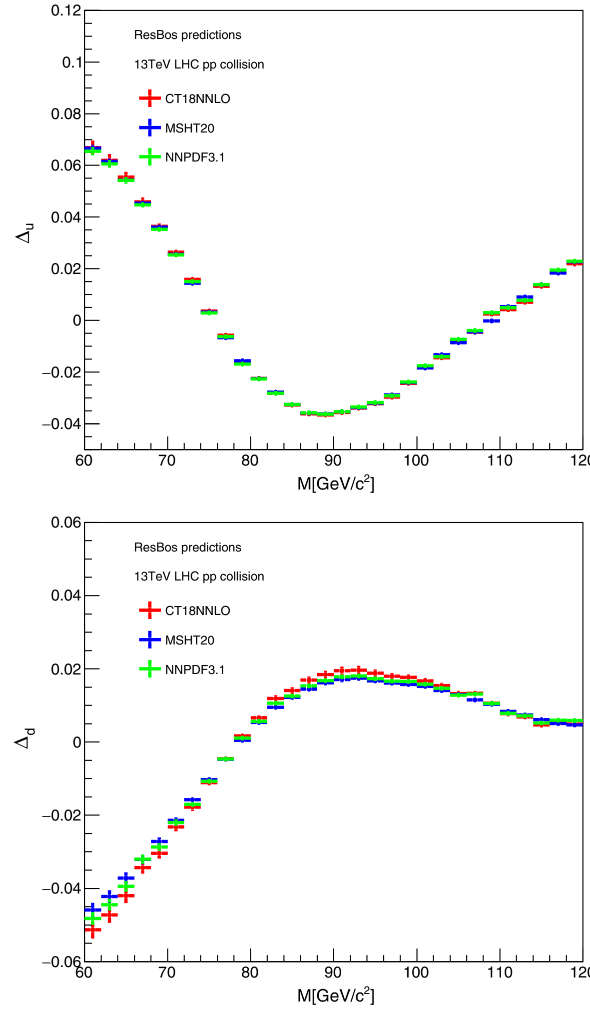

FIG. 4. Predictions of $\Delta_u(Y, M, Q_T)$ (up) and $\Delta_d(Y, M, Q_T)$ (bottom) in CT18, MSHT20 and NNPDF3.1 PDFs. Error bars correspond to the PDF uncertainties at the 68% confidence level. The observables in the two figures are averaged over Y and Q_T.

Based on the factorization of Eq. (7), we propose a new method to simultaneously determine the EW and proton structure parameters by fitting to the observed $A_{\mathrm{FB}}^h(Y, M, Q_T)$ data. One can employ Eq. (7) to build theoretical templates, with $\sin^2\theta_{\mathrm{eff}}^{\ell}$, $P_0^u(Y, Q_T)$, and $P_0^d(Y, Q_T)$ as fitting parameters. Their values are therefore determined by requiring the best agreement between theory templates and the observed data. Due to lack of sufficient constraints by using the $A^h_{\rm FB}$ distribution only, the $\Delta_f(Y, M, Q_T)$ terms would have to be fixed to the prediction of current PDFs, and thus induce extra theoretical uncertainties to the fitted parameters.

For numerical test, a RESBOS+CT18 pseudo-data sample of 2.5 billion DY events is used, corresponding to 1 ab^{-1} integrated luminosity at the LHC 13 TeV pp collisions. The fitted results of P^u_0, P^d_0, and $\sin^2\theta^\ell_{\rm eff}$ parameters, as a function of Y, are depicted in Fig. 5. The statistical fluctuations of the fitted P^u_0, P^d_0, and $\sin^2\theta^\ell_{\rm eff}$ values (labeled as "Fitted unc."), are comparable to the difference between the fitted values and their input or predicted values in the pseudodata (labeled as "δ"), which manifests the closure of the factorization and the fitting method.

The theoretical uncertainties associated with the input Δ_f terms in the fit (labeled as "Δ_f-induced unc.") are estimated by repeating the fitting procedure with different $\Delta_f(Y, M, Q_T)$ predictions given by the CT18 PDF error sets, instead of the central set, with P^f_0 and $\sin^2\theta^\ell_{\rm eff}$ as free fitting parameters, and quoting the variations of their fitted values, respectively. For comparison, the variations of P^f_0 values predicted by different CT18 PDF error sets, and the PDF-induced uncertainty on the $\sin^2\theta^\ell_{\rm eff}$ extracted from the conventional method of setting $\sin^2\theta^\ell_{\rm eff}$ as the only fitting parameter, are also depicted (labeled as "Original PDF unc."), respectively. The factorization method provides a new perspective for the issue of the correlation between PDFs and $\sin^2\theta^\ell_{\rm eff}$. First, the Δ_f-induced uncertainties arise from the variation of proton structure information in a small region of the Bjorken variable x, while the original PDF ones are dominated by the average magnitude of the

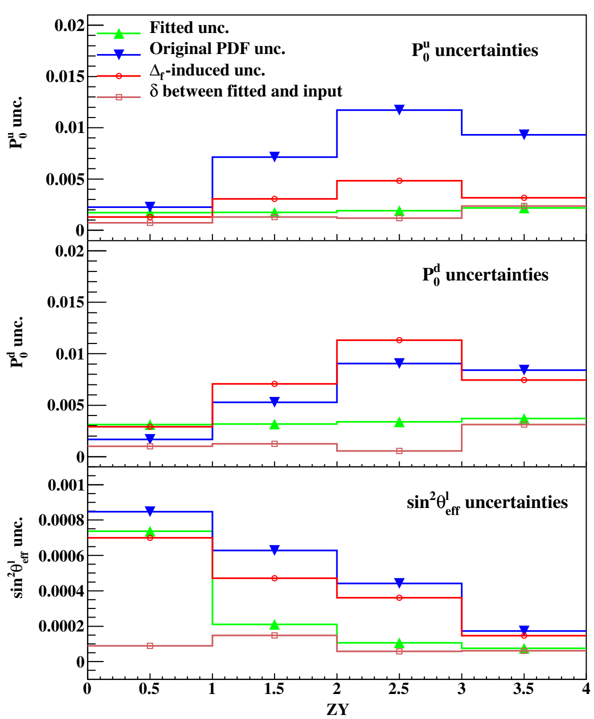

FIG. 5. Various uncertainties of P^u_0 (upper), P^d_0 (middle) and $\sin^2\theta^\ell_{\rm eff}$ (lower), as detailed in the main text. Results are given in the Q_T range of [5, 10] GeV, as an example.

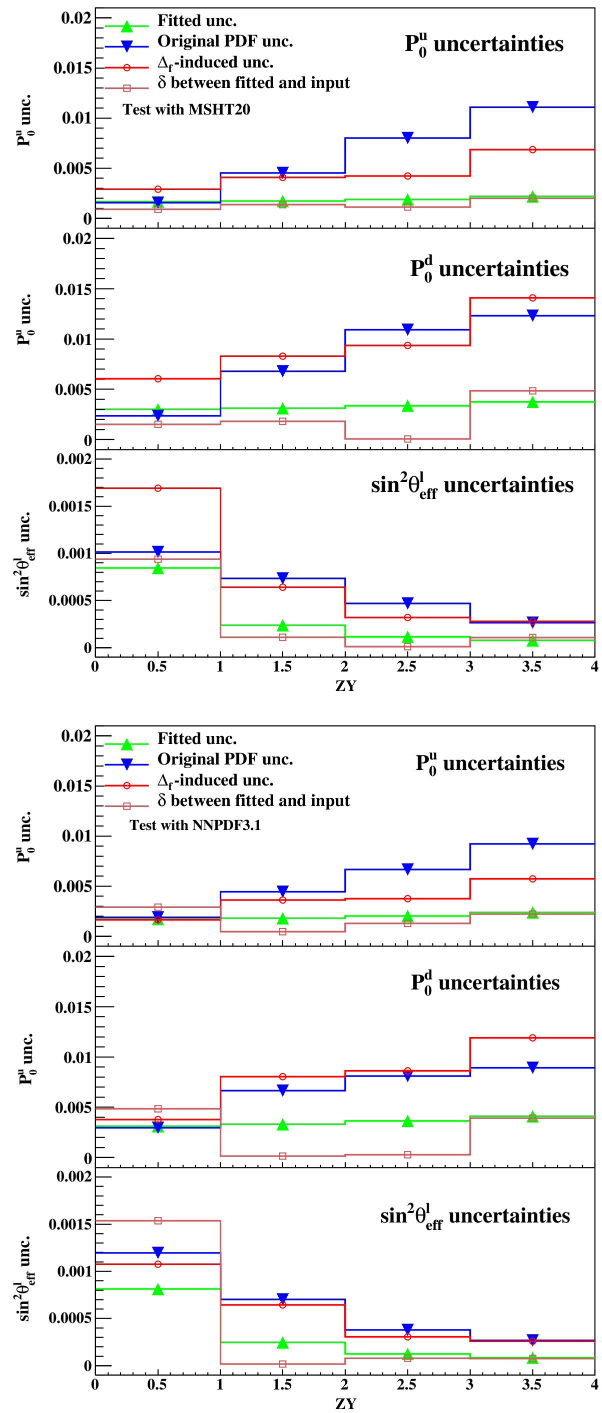

FIG. 6. Various uncertainties of P^u_0, P^d_0, and $\sin^2\theta^\ell_{\rm eff}$, as detailed in the main text, corresponding to the test using MSHT20 (up) and NNPDF3.1 (bottom). Results are given in the Q_T range of [5, 10] GeV, as an example.

structure parameters. Second, we note that the Δ_f terms do not contribute large uncertainty on A_{FB}^h distribution itself, cf. Fig. 1. However, when A_{FB}^h is the sole data included in the fit, the P_0^f terms and $\sin^2\theta_{\mathrm{eff}}^\ell$ are higly (negatively) correlated. To improve the resolution power of the proposed factorization form of A_{FB}^h to the determination of the P_0^f terms and $\sin^2\theta_{\mathrm{eff}}^\ell$, we could either introduce new Δ_f-sensitive observables in the PDF global fitting to reduce the uncertainty on Δ_f itself, or add additional P_0^f-sensitive data in the simultaneous fit, so that the correlation between $\sin^2\theta_{\mathrm{eff}}^\ell$ and P_0^f can be reduced. Of course, this kind of new Δ_f or P_0^f-sensitive observables, preferably at the LHC, ought to be $\sin^2\theta_{\mathrm{eff}}^\ell$-independent.

Similar tests are also made using MSHT20 and NNPDF3.1, of which the results are given in Fig. 6.

IV. CONCLUSION

The A_{FB}^h measurement at hadron colliders not only is an ideal observable for the determination of $\sin^2\theta_{\mathrm{eff}}^\ell$, but also provides unique proton structure information on the relative difference between quarks and antiquarks. It is essential to factorize the proton structure information with well defined observables, so that both $\sin^2\theta_{\mathrm{eff}}^\ell$ and the parton information can be simultaneously determined. In this article, we proposed a novel method to do just that, which is based on the factorization property of Eq. (7), and demonstrated how the method works. The A_{FB}^h factorization provides a novel method to handle the proton distribution uncertainties in the electroweak measurement at the LHC, in contrast to the conventional PDF error estimation. It should also be pointed out that by introducing other observables in the same LHC data, which are sensitive to either of the two types of factorized Δ_f or P_0^f terms, the precision of the simultaneous fit based on the A_{FB}^h factorization could be further improved, so that the proton structure information and the EW $\sin^2\theta_{\mathrm{eff}}^\ell$ parameter could be determined model-independently in the future LHC experiment.

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