Cotton double copy for gravitational waves

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We construct a double copy relation between the Cotton spinor and the dual field strength spinor of topologically massive theories, as the three-dimensional analog of the Weyl double copy. The relationship holds in curved backgrounds for wave solutions. We give an explicit proof for type N spacetimes and show examples satisfying the Cotton double copy.

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I. INTRODUCTION

The nonlinearities of gravitational theories lead to involved perturbative calculations and intriguing features of exact classical solutions. In recent years, it has become apparent that the double copy [1–3], which allows us to write gravitational amplitudes as the square of Yang-Mills (YM) amplitudes, can be used to understand fundamental properties of gravity (such as cancellations of UV divergences) while allowing us to perform computations (such as loop amplitudes and gravitational radiation) in a simpler framework [4,5]. This has been observed: for example, it is known that this relationship holds beyond the scattering amplitudes case and it can relate classical solutions [6–26]. The different versions of the classical double copy share the philosophy of the amplitudes versions. For example, the Kerr-Schild double copy [7] uses the BCJ (Bern-Carrasco-Johansson) philosophy of exchanging color for kinematics [2,3]. Meanwhile, the Weyl double copy [11,12], follows the KLT (Kawai-Lewellen-Tye) approach [1], schematically, $\text{Gravity} = (\text{Yang} - \text{Mills})^2/\text{biadjoint scalar}$. The Weyl double copy holds for exact solutions of Petrov types D and N spacetimes when written in terms of spinors [11,12], it can be explained from a twistors perspective [27–29], and it has been formulated in tensorial form [30,31]. Altogether, the Weyl double copy provides a nonperturbative relation that can be extend asymptotically to generic spacetimes [32]. A simple version of the Weyl double copy, which gives the Weyl tensor $W_{\mu\nu\rho\lambda}$ as the square of the YM field strength $F^{\mu\nu}$, can be constructed for plane waves at linearized order

$$W^{\text{lin}}_{\mu\nu\rho\lambda} = \frac{1}{2}\frac{F^{\text{lin}}_{\mu\nu}F^{\text{lin}}_{\rho\lambda}}{e^{ip\cdot x}}. \tag{1}$$

An interesting question is whether the double copy holds beyond the massless case. As studied in [33–43], there are only a couple of known cases where the scattering amplitudes arising as the double copy of massive theories lead to a well-defined local theory. One example requires an infinite tower of massive states that satisfies a special relationship between the masses in the theory. The second example corresponds to topologically massive theories in three dimensions (3D). Note that a related double copy relation involving Chern-Simons terms can be found in [44]. We will now construct the analog of the Weyl double copy in 3D for type N solutions in topologically massive theories. This is a step forward towards understanding how the double copy operates in the massive case, which can help constructing exact solutions for theories driving the acceleration of the Universe and it can also provide hints on how to construct a double copy for cosmological correlators [5]. Just as in the massive case, the double copy of cosmological correlators contains nonphysical poles that in both cases can be traced to the lack of BCJ relations. While in the massive case we now have a clearer understanding of how to avoid such poles, that is not the case for cosmological correlators. Special cases with nonstandard BCJ relations, such as the topologically massive double copy, show how to generalize the double copy in more involved situations.

II. TOPOLOGICALLY MASSIVE THEORIES

We start by introducing the topologically massive theories that are related through the double copy. The action of topologically massive Yang-Mills (TMYM) in a curved spacetime is

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$$S_{\rm TMYM} = \int d^3x\sqrt{-g}\Big(-\frac{1}{4}F^{a\mu\nu}F_{a\mu\nu} + \frac{g}{\sqrt{2}}A^{\mu a}J_{\mu a} + \varepsilon_{\mu\nu\rho}\frac{m}{12}(6A^{a\mu}\partial^\nu A^\rho_a + g\sqrt{2}f_{abc}A^{a\mu}A^{b\nu}A^{c\rho})\Big), \quad (2)$$

where m is the mass of the gauge field, g the coupling strength, and $\varepsilon_{\mu\nu\rho}$ is the Levi-Civita tensor given by $\varepsilon_{\mu\nu\rho} = \sqrt{-g}\epsilon_{\mu\nu\rho}$, with $\epsilon_{\mu\nu\rho}$ as the Levi-Civita symbol. We consider gauge fields and sources of the form $A^{\mu a} = c^a A^\mu$ and $J^{\mu a} = c^a J^\mu$, with c^a a constant color charge and a an adjoint $U(N)$ index, so that the equations of motion become linear and read

$$\nabla_\mu F^{\mu\nu} + \frac{m}{2}\varepsilon^{\nu\rho\gamma}F_{\rho\gamma} = \frac{g}{\sqrt{2}}J^\nu, \quad (3)$$

where $F_{\mu\nu}$ is the linearized Yang-Mills field strength. The double copy of TMYM corresponds to topologically massive gravity (TMG) whose action is

$$S_{TMG} = \frac{1}{\kappa^2}\int d^3x\sqrt{-g}\Big(-R + 2\Lambda + \mathcal{L}_{\rm Matter} - \frac{1}{2m}\varepsilon^{\mu\nu\rho}\Big(\Gamma^\alpha_{\mu\sigma}\partial_\nu\Gamma^\sigma_{\alpha\rho} + \frac{2}{3}\Gamma^\alpha_{\mu\sigma}\Gamma^\sigma_{\nu\beta}\Gamma^\beta_{\rho\alpha}\Big)\Big), \quad (4)$$

where $\kappa^2 = 16\pi G$, and Λ is the cosmological constant. The equations of motion read

$$G_{\mu\nu} + C_{\mu\nu}/m = -\kappa^2\frac{T_{\mu\nu}}{2} - \Lambda g_{\mu\nu}, \quad (5)$$

where $G^{\mu\nu}$ is the Einstein tensor, $T^{\mu\nu}$ the stress-energy tensor, and $C^{\mu\nu} = \varepsilon^{\mu\alpha\beta}\nabla_\alpha(R^\nu_\beta - \frac{1}{4}g^\nu_\beta R)$ the Cotton tensor.

We proceed to understand whether one can construct an of the Weyl double copy for topologically massive theories. When trying to generalize this to 3D, one immediately hits a roadblock since the Weyl tensor is zero. Instead, we will look at the analog of the Weyl tensor in 3D, which is the Cotton tensor. In 3D, the Cotton tensor is invariant under conformal transformations and thus is zero for conformally flat spacetimes, just like the Weyl tensor for $d > 3$. Since the Cotton tensor appears in the TMG equations of motion, Eq. (5), this tells us that we could write it as a square of terms in the TMYM equations of motion. By a simple counting of derivatives, we see that an appropriate ansatz is

$$C^{\rm lin.}_{\mu\nu} = -\frac{1}{4}\frac{(\partial^\lambda F^{\rm lin.}_{\lambda(\nu})(\varepsilon_{\mu)\rho\gamma}(F^{\rho\gamma})^{\rm lin.})}{e^{ip\cdot x}}, \quad (6)$$

which is satisfied for plane waves. Note that we can use the TMYM equations of motion, (3), to rewrite this relation in a simpler form. Considering only localized sources, outside of the source we have

$$C^{\rm lin.}_{\mu\nu} = \frac{m}{2}\frac{{}^\star F^{\rm lin.}_{(\mu}{}^\star F_{\nu)}{}^{\rm lin.}}{e^{ip\cdot x}}, \quad (7)$$

where ${}^\star F_\rho = \varepsilon_{\mu\nu\rho}F^{\mu\nu}/2$ is the dual field strength. In the following section we will use this relation as motivation for the Cotton double copy.

III. 3D SPINOR FORMALISM AND THE COTTON DOUBLE COPY

The spinor formalism in 3D has been considered in [45–47]. It uses the fact that the 3D Lorenz group $SO(1,2)$ is isomorphic to $SL(2,\mathrm{R})/\mathrm{Z}_2$ to rewrite the tangent space Lorentz transformations. This allows us to write a vector in tangent space as $v_{\rm a} = -(\sigma_{\rm a})_{AB}v^{AB}$, where the sigma matrices, $\sigma_{\rm a}$, form a basis of $SL(2,\mathrm{R})$ that satisfy the Clifford algebra. To move between coordinate space and tangent space we use the frame $e^\mu_{\rm a}$ that satisfies $\eta_{\rm ab} = e^\mu{}_{\rm a}e^\nu{}_{\rm b}g_{\mu\nu}$. Thus, we can write a vector in coordinate space as $v^\mu = -e^\mu{}_{\rm a}(\sigma^{\rm a})_{AB}v^{AB}$. The $SL(2,\mathrm{R})$ indices, $A, B = 1, 2$, are lowered and raised with the 2D Levi-Civita symbol ϵ_{AB} according to the following conventions $\psi^A = \psi_B\epsilon^{BA}, \psi_A = \epsilon_{AB}\psi^B$ [46]. In the following, it will be useful to work with a spinor basis given by a dyad (ι, o) that satisfies $\iota_A\iota^A = o_Ao^A = 0, \iota_Ao^A = -1$. Thus we can write $\epsilon_{AB} = 2\iota_{[A}o_{B]}$.

In a vacuum 3D spacetime, the TMG equations of motion together with the definition of the Cotton tensor and the Bianchi identities tell us that

$$\nabla^{EA}C_{BECD} = \frac{m}{\sqrt{2}}C^A{}_{CDE}, \quad (8)$$

where $C^{\mu\nu} = \sigma^\mu_{AB}\sigma^\nu_{CD}C_{ABCD}$. Meanwhile, the equation of motion for linearized TMYM and the Bianchi identity for the field strength give

$$\nabla^{CA}f_{BC} = \frac{m}{\sqrt{2}}f^A{}_B, \quad (9)$$

where the dual field strength is given by $f^\mu = -\sigma^\mu_{AB}f^{AB}$. Motivated by the linear relationship found in Eq. (7), we propose that the analog of the Weyl double copy between the Cotton and field strength spinors is[1]

$$\boxed{C_{ABCD} = \frac{m}{2}\frac{f_{(AB}f_{CD)}}{S}}. \quad (10)$$

Below we will prove that this relationship is satisfied for type N spacetimes with a scalar field S satisfying the

[1]The mass factor in Eq. (10) is a choice of conventions. It could be absorbed in S or we could write the relation for the traceless Ricci spinor since the TMG equations tell us that $C_{ABCD} = -m\Phi_{ABCD}$, where the traceless Ricci tensor is given as $S^{\mu\nu} \equiv R^{\mu\nu} - Rg^{\mu\nu}/3 = \sigma^\mu_{AB}\sigma^\mu_{CD}\Phi_{ABCD}$.

massive Klein Gordon equation with a nonminimal coupling in curved spacetimes. Note that (10) follows the KLT double copy philosophy. In our case S plays the role of the KLT kernel and can be thought of as a linearized solution of the massive biadjoint scalar when considering the ansatz $S^{ab} = c^a c^b S$; this is commonly referred to as the zeroth copy.

IV. TYPE N SOLUTIONS

For type N solutions, which encode transverse radiation, the Cotton spinor and field strength spinor can be written as

$$C_{ABCD} = \psi_4 o_A o_B o_C o_D, \qquad f_{AB} = \Phi_2 o_A o_B, \tag{11}$$

where ψ_4 and Φ_2 are Newman-Penrose (NP) scalars. In this case, the double copy can simply be expressed as

$$\psi_4 = \frac{m\Phi_2^2}{2S}. \tag{12}$$

We will now prove that the Cotton double copy holds for type N spacetimes in curved backgrounds by deriving the equation of motion of the zeroth copy S.

We start by substituting (11) into (8) and (9), and contracting the equations with ι and o to get

$$o_A \nabla^A{}_C \log \Psi_4 + 4 o_A \iota^B \nabla^A{}_C o_B - \iota_A o^B \nabla^A{}_C o_B = \frac{m}{\sqrt{2}} o_c, \tag{13}$$

$$o_A \nabla^A{}_C \log \Phi_2 + 2 o_A \iota^B \nabla^A{}_C o_B - \iota_A o^B \nabla^A{}_C o_B = \frac{m}{\sqrt{2}} o_c, \tag{14}$$

$$o^B o_C \nabla^{CA} o_B = 0. \tag{15}$$

From the Cotton double copy in (12), together with Eqs. (13) and (14), we find

$$o_A \nabla^A{}_C \log S - \iota_A o^B \nabla^A{}_C o_B = -\frac{m}{\sqrt{2}} o_c. \tag{16}$$

To show that S satisfies the Klein-Gordon equation with a nonminimal coupling term first we write $\nabla_\mu \nabla^\mu S$ as

$$-\nabla_{AB}\nabla^{AB} S = -\epsilon_{AC} \nabla^C{}_B \nabla^{AB} S = 2\iota_C o_A \nabla^C{}_B \nabla^{AB} S. \tag{17}$$

Then, one can use the Leibniz rule and (16) to eliminate the derivatives of S:

$$2\iota_C o_A \nabla^C{}_B \nabla^{AB} S = -2\iota_C \nabla^C{}_B o_A \nabla^{AB} S + 2\iota_C \nabla^C{}_B \left(S \iota_A o^D \nabla^{AB} o_D - \frac{m}{\sqrt{2}} S o^B \right). \tag{18}$$

Expanding (18) and using (16) to eliminate ∇S terms we get

$$\begin{aligned} -\nabla_{AB}\nabla^{AB} S = S(&2\iota_C \nabla^C{}_B \iota_A o^D \nabla^{AB} o_D + 2\iota_C \iota_A \nabla^C{}_B o^D \nabla^{AB} o_D + 2\iota_D \iota_A \nabla^A{}_C o^D \iota_E o^F \nabla^{EC} o_F \\ &- m\sqrt{2}(\iota_C \nabla^C{}_B o^B + \iota_D \iota_A \nabla^A{}_C o^D o^C - \iota_C \iota_E o^F \nabla^{EC} o_F) + m^2 + 2\iota_C \iota_A o^D \nabla^C{}_B \nabla^{AB} o_D). \end{aligned} \tag{19}$$

The first three terms as well as the terms linear in m add up to zero by (15). The term with the second derivative of o can be related to curvature spinors by the following relation [46]:

$$\nabla_{D(A} \nabla_{B)}{}^D o_C = \frac{1}{2} \Phi_{ABCD} o^D + \frac{1}{24} R (\epsilon_{AC} o_B + \epsilon_{BC} o_A), \tag{20}$$

where Φ_{ABCD} is the spinor equivalent of the traceless Ricci tensor, which is proportional C_{ABCD}, see footnote 1. By substituting (11), we see that the term proportional to Φ_{ABCD} does not contribute. Therefore we find that

$$2\iota_C \iota_A o^D \nabla^C{}_B \nabla^{AB} o_D = \frac{1}{6} R.$$

Finally, substituting everything into (19) we get

$$-\nabla_{AB} \nabla^{AB} S = \Box S = \left(m^2 + \frac{1}{6} R \right) S. \tag{21}$$

This proves that the Cotton double copy is satisfied for type N solutions with the zeroth copy given by a linearized massive biadjoint scalar with a nonminimal coupling. Note that we obtained the same nonminimal coupling as in the 4D zeroth copy [14,15,27], but in 3D it does not give a conformally invariant equation. We will now show explicit examples of type N spacetimes where the Cotton double copy holds.

A. Parallel propagation waves

We analyze the double copy relation for plane-fronted waves with parallel propagation (pp waves). For TMG, any solution that admits a null Killing vector, well defined through all space, is a pp-wave solution[2] [49]. In flat space the metric of pp waves can always be written as [50]

[2]The nomenclature of pp waves for the nonzero cosmological constant case can be misleading since the null Killing vector is not covariantly conserved [48].

TABLE I. In this table we show the NP scalars for the Cotton spinor and the dual field strength spinor. We also show the scalar S constructed from the Cotton double copy in Eq. (12).

	Ψ_4	Φ_2	S
Minkowski	$\frac{m^3}{4}\mathrm{e}^{-my}f(u)$	$-\frac{m}{\sqrt{2}}\mathrm{e}^{-my}g(u)$	$\frac{g(u)^2}{f(u)}\mathrm{e}^{-my}$
AdS	$\frac{m^3}{4}\mathrm{e}^{-(\frac{3}{L}+m)y}f(u)$	$-\frac{m}{\sqrt{2}}\mathrm{e}^{-(\frac{2}{L}+m)y}g(u)$	$\frac{g(u)^2}{f(u)}\mathrm{e}^{-(\frac{1}{L}-m)y}$

$$\mathrm{d}s^2 = \mathrm{d}y^2 - 2\mathrm{d}u\mathrm{d}v + \mathrm{e}^{-my}f(u)\mathrm{d}u^2, \tag{22}$$

where u, v are light cone coordinates, while in anti–de Sitter (AdS) terms it reads

$$\mathrm{d}s^2 = \mathrm{d}y^2 - 2\mathrm{e}^{2\frac{y}{L}}\mathrm{d}u\mathrm{d}v + \mathrm{e}^{\frac{(1-mL)}{L}y}f(u)\mathrm{d}u^2, \tag{23}$$

where L is the AdS radius. Note that we can obtain the dS solution by taking $L \to iL$. On the TMYM side, we can write the pp-wave solution as

$$A^a = c^a \mathrm{e}^{-my}g(u)du \tag{24}$$

for the Minkowski, AdS, and dS cases. In Table I we show the NP scalars for the corresponding pp waves in Minkowski and AdS. One can easily see that the scalar S, which is computed using the Cotton double copy in Eq. (12) satisfies

$$\left(\nabla^2 - m^2 - \frac{R}{6}\right)S(u,y) = \left(\partial_y^2 - m^2 + \frac{1}{L^2}\right)S = 0. \tag{25}$$

B. Shock waves and gyratons

1. Minkowski

We now consider solutions with a source corresponding to a fast moving particle whose stress tensor is traceless and is given by

$$T_{\mu\nu} = (Ek_\mu k_\nu + \sigma k_{(\mu}\epsilon_{\nu)}^{\alpha\beta}k_\alpha\partial_\beta)\delta(u)\delta(y), \tag{26}$$

where the null vector k^μ is defined as $k_\mu dx^\mu = du$, E is the energy of the source particle, and σ is its classical spin. Note that this source can be thought of as a boosted gravitational anyon. If the particle has no classical spin ($\sigma = 0$), then it generates shockwaves; otherwise, the solutions are dubbed gyratons. In flat space, both of these solutions have a metric of the form[3]

$$\mathrm{d}s^2 = \mathrm{d}y^2 - 2\mathrm{d}u\mathrm{d}v + \kappa F(u,y)\mathrm{d}u^2. \tag{27}$$

For these solutions, we have that the only nonzero NP Cotton scalar is

$$\psi_4 = -\frac{1}{4}\partial_y^3 F(u,y), \tag{28}$$

where F satisfies the following equation of motion

$$\partial_y^3 F(u,y) + m\partial_y^2 F(u,y) = \kappa m\delta(u)(E\delta(y) - \sigma\delta'(y)). \tag{29}$$

On the gauge theory side, we will also consider a boosted spinning source whose current is given by

$$J_\mu = (Qk_\mu + Q'\epsilon_\mu^{\alpha\beta}k_\alpha\partial_\beta)\delta(u)\delta(y), \tag{30}$$

where Q is the electric charge and Q' contributes, together with Q, to the magnetic flux. We consider the following gauge field

$$A^a = c^a G(u,y)du, \tag{31}$$

which linearizes the TMYM equations of motion and gives only one nonvanishing component of field strength $F_{uy} = -\partial_y G(u,y)$. Hence the only nonzero NP field strength scalar is

$$\Phi_2 = \frac{1}{\sqrt{2}}\partial_y G(u,y), \tag{32}$$

where G satisfies

$$\partial_y^2 G(u,y) + m\partial_y G(u,y) = g\delta(u)(Q\delta(y) - Q'\delta'(y)). \tag{33}$$

Then the scalar S in the Cotton double copy, Eq. (10), is given as

$$S = -m\frac{(\partial_y G(u,y))^2}{\partial_y^3 F(u,y)}. \tag{34}$$

Equations (29) and (33) imply that outside the sources the following is true:

$$(\nabla^2 - m^2)S(u,y) = 0. \tag{35}$$

To see the double copy for an explicit gyraton or shock wave solution, we need to pick boundary conditions for the metric. As realized in [52], we cannot have the same coordinate chart on both sides of the shockwave. In [37], we have shown that a useful prescription to observe the double copy relation is to a consider boundary conditions where the metric is flat for $y < 0$ and Cartesian for $y > 0$ [53]. Then we can solve (29) by imposing these boundary conditions:

[3] Note that the gyraton metric is generically written as $ds^2 = -2dudv + dy^2 + \kappa F(u,y)du^2 + 2\kappa\alpha(u,y)dudy$, where the cross term proportional to $\alpha(u,y)$ allows us to see the rotation explicitly [51]. Here we have chosen a gauge where $\alpha = 0$.

$$F(u,y)=\frac{\kappa}{m}(E+m\sigma)e^{-my}\theta(y)+\frac{\kappa}{m}(E+m\sigma-Emy)\theta(-y), \tag{36}$$

Choosing the analog boundary condition in TMYM, namely $F_{\mu\nu}=0$ for $y<0$ and $\lim_{y\to\infty}A^\mu=0$, leads to

$$G(u,y)=g\frac{Q+mQ'}{m}(\theta(y)e^{-my})+g\frac{Q}{m}\theta(-y). \tag{37}$$

On the $y<0$ side of the gyraton, the double copy is trivial since $\Psi_4=\Phi_2=0$. On the other hand, in the $y>0$ side the NP scalars are proportional to those of the flat space pp waves in Table I times $\delta(u)$. Making the replacement $(Q+mQ')^2\to(E+m\sigma)$ leads to the double copy relation in Eq. (12). Note that the shockwave solutions can be obtained by setting $\sigma=0$ and $Q'=0$. We then see that the gyraton NP scalar is obtained from the shockwave one by the shift $E\to E(1+m\frac{\sigma}{E})$ in TMG and $Q\to Q(1+m\frac{Q'}{Q})$ in TMYM, which arise from spin deformations of on shell 3-point amplitudes [40] and was originally found for gravitational anyons [54].

2. AdS

We proceed to consider gyraton solutions of TMG, TMYM in an AdS background. Just like gravitational shockwaves in AdS, the gyraton solution can be written in Poincare coordinates as

$$ds^2=\frac{L^2}{y^2}(-2dudv+dy^2+\delta(u)F(y)du^2), \tag{38}$$

where F satisfies

$$\frac{y}{L}F'''+mF''-m\frac{F'}{y}=\kappa m\left(E\frac{L}{y}\delta(y-y_0)-\sigma\delta'(y-y_0)\right), \tag{39}$$

where $y_0\neq0$ is the location of the source in the bulk and we will assume $mL>1$. As before, we need to fix the boundary conditions to find the explicit solution. We choose to have the same boundary conditions as in the Minkowski case in the flat space limit. This is equivalent to imposing Brown-Henneaux boundary conditions and requiring a regular solution in the bulk. The explicit solution with these boundary conditions is

$$\begin{aligned}F(y)=-\frac{\kappa L^2mE}{2(1-(Lm)^2)}\Bigg[&2\left(1+\frac{\sigma}{E}((1+mL)/L)\right)\left(\frac{y}{y_0}\right)^{1-Lm}\theta(y-y_0)\\&+\left((1-Lm)\left(\frac{y}{y_0}\right)^2+\left(1+\frac{2\sigma}{EL}\right)(1+Lm)\right)\theta(-y+y_0)\Bigg].\end{aligned} \tag{40}$$

On the nontrivial side of the gyraton solution, $y>y_0$, we have that the only nonzero Cotton NP scalar is

$$\begin{aligned}\Psi_4=-\frac{1}{2}\frac{y}{L}\delta(u)F'''(y)&=\frac{\kappa}{2}E\left(1+\frac{\sigma}{E}((1+mL)/L)\right)\\&\times L^2m^2\left(\frac{y}{y_0}\right)^{-1-Lm}\delta(u).\end{aligned} \tag{41}$$

On the other hand, the linearized gyraton solution for TMYM in an AdS background is given by

$$A^a=c^a\delta(u)G(y)du, \tag{42}$$

where the function G satisfies

$$\begin{aligned}&\frac{y^4}{L^4}\left(G''+\frac{1+Lm}{y}G'\right)\\&=\frac{y^3}{L^3}g\left(Q\delta(y-y_0)-Q'\frac{y}{L}\delta'(y-y0)\right).\end{aligned} \tag{43}$$

The explicit gyraton solution with boundary condition analog to the gravitational case, namely $F_{\mu\nu}=0$ for $y<y_0$ and $\lim_{y\to\infty}A^\mu=0$, is given by

$$\begin{aligned}G(y)=-\frac{gQ}{m}\Bigg[&\left(1+\frac{Q'}{QL}(1+mL)\right)\left(\frac{y}{y_0}\right)^{-Lm}\theta(y-y_0)\\&+\left(1+\frac{Q'}{QL}\right)\left(\frac{y_0}{L}\right)^3\theta(-y+y_0)\Bigg].\end{aligned} \tag{44}$$

Thus we have that for $y>y_0$ the only nonzero dual field strength NP scalar is

$$\begin{aligned}\Phi_2&=2\delta(u)\frac{y}{L}f'(y),\\&=2gQ\left(1+\frac{Q'}{Q}((1+mL)/L)\right)\delta(u)\left(\frac{y}{y_0}\right)^{-Lm}.\end{aligned} \tag{45}$$

Lastly, we consider the linearized biadjoint scalar, $S^{a\tilde{a}}=c^ac^{\tilde{a}}S$, living in an AdS background with a nonminimal coupling as in Eq. (21) and sourced by

$(\lambda + \lambda' \frac{y}{L} \frac{\partial}{\partial_y})\delta(y-y_0)\delta(u)$ The gyraton solution is now given by

$$S = -\frac{\lambda L}{2my_0}\left[\left(1 - m\frac{\lambda'}{\lambda}\right)\left(\frac{y}{y_0}\right)^{1-Lm}\theta(y-y_0) + \left(1 + m\frac{\lambda'}{\lambda}\right)\left(\frac{y}{y_0}\right)^{1+Lm}\theta(-y+y_0)\right]\delta(u), \quad (46)$$

where we chose boundary conditions by requiring that the field vanishes deep in the bulk and as we approach the AdS boundary. We note that this solution corresponds to the scalar that arises from Eq. (12), which shows that the Cotton double copy is satisfied for AdS shock waves as expected. Again the shockwave solutions can be obtained by setting σ and Q' to zero. In a similar manner to the flat space case, one can consider shifts of the charge and energy to obtain the gyraton double copy from shockwaves. In this case the shifts are given $E \to E(1 + \frac{\sigma}{E}((1+mL)/L))$ for the TMG case and $Q \to Q(1 + \frac{Q'}{Q}((1+mL)/L))$ in TMYM. In future work, we will explore whether the origin of these shifts can be traced down to spin deformations of three-point correlators.

V. CONCLUSIONS AND FUTURE DIRECTIONS

We have constructed a double copy relation for topologically massive theories that gives the Cotton spinor as the square of the dual field strength spinor in curved spacetime backgrounds. This generalizes the 4D Weyl double copy to 3D spacetimes. In this paper, we have focused on type N spacetimes, which correspond to radiative solutions. We gave a proof of the Cotton double copy for gravitational waves and showed explicit examples. Other examples that we did not look at explicitly can be found in [48,55]. It would be interesting to understand whether the double copy holds for type D solutions, which describe fields around isolated objects such as black holes. Previous analysis studying the scattering of massive fields, which represent isolated objects, through topologically massive mediators [37,39,40] have shown that this is not straightforward, and further investigations should clarify this intriguing case.

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Note added.—Recently, we were made aware of the work by W. Emond and N. Moynihan that contains some overlapping results [56].

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