Confined Penrose process and black-hole bomb

O. B. Zaslavskii*

Department of Physics and Technology, Kharkov V.N. Karazin National University, 4 Svoboda Square, Kharkov 61022, Ukraine

(Received 27 April 2022; accepted 11 July 2022; published 20 July 2022)

We consider the decay of a particle with some energy $E_0 > 0$ inside the ergosphere of a black hole. After the first decay, one of particles with the energy $E_1 < 0$ falls towards a black hole while the second one with $E_2 > E_0$ moves in the outward direction. It bounces back from a reflecting shell and, afterwards, the process repeats. For radial motion of charged particles in the Reissner-Nordstöm metric, the result depends strongly on a concrete scenario. In particular, an indefinitely large growth of energy inside a shell is possible that gives rise to a black-hole bomb. We also consider a similar multiple process with neutral particles in the background of a rotating axially symmetric stationary black hole. We demonstrate that, if particle decay occurs in the turning point, a black-hole bomb in this case is impossible at all. For a generic point inside the ergoregion, there is a condition for a black-hole bomb to exist. It relates the to ratio of masses before and after decay and the velocity of a fragment in the center of mass frame.

DOI: 10.1103/PhysRevD.106.024037

I. INTRODUCTION

There exist two universal mechanisms of energy extraction from black holes. The first one is the Penrose process. In the original form, it was found [1] for rotating black-hole backgrounds. If, in a space-time, there exists a region where negative Killing energies $E < 0$ are possible, then a parent particle 0 can decay to two fragments 1 and 2 in such a way that $E_1 < 0$ while $E_2 > E_0$, so amplification of the original energy occurs. The ergosphere is realized in the region when the component of the metric tensor g_{tt} changes the sign as compared to infinity (t is time). Later on, it turned out that a similar process occurs in the background of the Reissner-Nordström (RN) metric as well [2]. In contrast to the aforementioned rotating case, now the ergosphere is not a pure geometric entity but depends on the parameters of a particle. For further development of the Penrose process with charged particles see, e.g., Refs. [3–9].

On the other hand, there exists a wave analogue of the Penrose process. This is a so-called super-radiance [10,11] (for a recent review and list of references see [12]). The Penrose process is realized with particles, super-radiance—with waves. In turn, super-radiance leads to the possibility of one more interesting physical effect—a black-hole bomb. It occurs if a black hole is surrounded by a reflecting shell and a wave bounces back in such a way that the process repeats endlessly; the energy is accumulated without bound giving rise to an explosion. This was shown for reflection by a concave mirror in [13] and for a convex one in [14]. (The relation between the existence of such a bomb and instabilities due to the properties of quasinormal modes was discussed in [15].)

Strange as it may seem, the question about the possibility of a black-hole bomb on the basis of processes with particles was posed only quite recently [16]. The authors considered the motion of charged particles in the background of the Reissner-Nordström metric with decay of a parent particle to two fragments with consequent reflection from a shell backward. The event of decay was chosen to occur in the turning point. The authors demonstrated convincingly that a black-hole bomb for such a scenario is impossible. In this sense, the difference between the standard Penrose process and its multiple version in a confined system is blurred.

However, such a situation is not universal. In the present paper we show that for other scenarios within the same system a black-hole bomb is indeed possible. Apart from this, we consider a similar problem for rotating axially symmetric metrics. It turns out that if the process occurs (similarly to the RN case) in the turning point, the energy remains bounded, and no black-hole bomb is possible. However, for more general scenarios, this may happen and we indicate a simple condition on the parameters of a system, necessary for an indefinitely large growth of energy.

We use the system of units in which fundamental constants $G = c = 1$.

II. PARTICLE MOTION IN THE REISSNER-NORDSTRÖM METRIC: BASIC EQUATORS

Let us consider the RN metric. It has the form

$$ds^2 = -dt^2 f + \frac{dr^2}{f} r^2 d\omega^2, \tag{1}$$

*zaslav@ukr.net

where

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \tag{2}$$

M is the black-hole mass, Q being its electric charge. We take $Q > 0$.

For simplicity, we consider pure radial motion, so equations of motion read

$$m\dot{t} = \frac{X}{f}, \tag{3}$$

$$p^r \equiv m\dot{r} = \sigma P, \qquad P = \sqrt{X^2 - m^2 f}, \tag{4}$$

$$X = E - q\varphi, \tag{5}$$

where the dot denotes differentiation with respect to the proper time. Here, E is the particle energy, m being its mass. The electric Coulomb potential

$$\varphi = \frac{Q}{r}. \tag{6}$$

The forward-in-time condition implies $\dot{t} > 0$, whence

$$X > 0 \tag{7}$$

outside the horizon. Hereafter, we will use the notations

$$\varepsilon = \frac{E}{m}, \qquad \tilde{q} = \frac{q}{m}. \tag{8}$$

The system can have a turning point r_t, where $P = 0$. Then

$$r_t = \frac{1}{\varepsilon^2 - 1}(\varepsilon\tilde{q}Q - M \pm \sqrt{C}), \tag{9}$$

$$C = (M - \varepsilon\tilde{q}Q)^2 + (1 - \tilde{q}^2)Q^2(\varepsilon^2 - 1). \tag{10}$$

In what follows we will be interested in the case when a particle has the energy $E > m$, so $\varepsilon > 1$.

III. GENERAL SCENARIO OF DECAY

Let a parent particle 0 decay in the point with $r = r_0$ to particles 1 and 2. We assume the conservation laws

$$E_0 = E_1 + E_2, \tag{11}$$

$$q_0 = q_1 + q_2, \tag{12}$$

$$p_0^r = p_1^r + p_2^r, \tag{13}$$

whence

$$X_0 = X_1 + X_2. \tag{14}$$

Then, using these equations and (4) and (5), one can obtain

$$X_1 = \frac{1}{2m_0^2}(X_0 b_1 + P_0\delta\sqrt{d}), \tag{15}$$

$$X_2 = \frac{1}{2m_0^2}(X_0 b_2 - P_0\delta\sqrt{d}), \tag{16}$$

$$b_{1,2} = m_0^2 \pm (m_1^2 - m_2^2), \tag{17}$$

where $i = 0$, 1, 2,

$$d = b_1^2 - 4m_0^2 m_1^2 = b_2^2 - 4m_0^2 m_2^2, \tag{18}$$

$\delta = \pm 1$. For radial momenta one obtains

$$P_1 = \left|\frac{P_0 b_1 + \delta X_0\sqrt{d}}{2m_0^2}\right|, \tag{19}$$

$$P_2 = \left|\frac{P_0 b_2 - \delta X_0\sqrt{d}}{2m_0^2}\right|. \tag{20}$$

Alternatively, one can take advantage of the results already obtained in Eqs. (19)–(30) of [17]. We use particle labels 1 and 2 instead of 3 and 4, respectively, in [17].

The direction of motion is characterized by a quantity σ, where $\sigma = +1$ for motion in the outward direction and $\sigma = -1$ for the inward case. We are mainly interested in the situation when particle 0 moves towards a black hole from large radii to smaller ones. Then, at least one of the particles falls into a black hole.

IV. PARTICULAR SCENARIO OF DECAY

We will mainly concentrate on the case when decay occurs in the turning point for all three particles, so $P_i = 0$ for $i = 0$, 1, 2 and $r_0 = r_t$, so that

$$X_i = m_i\sqrt{f(r_0)}, \tag{21}$$

$i = 0$, 1, 2. It is clear from (14) that this requires

$$m_0 = m_1 + m_2. \tag{22}$$

Then, it follows from (4), (17), and (18) that $b_1 = 2m_1 m_0$, $b_2 = 2m_0 m_2$, $d = 0$.

Let, in addition, a reflecting shell be placed at some point at $r_B > r_0$. Particle 0 decays to 1 and 2; particle 1 moves towards a black hole, while particle 2 moves in the outward direction, bounces back from the shell and decays again to particles 3 and 4, etc. Similarly to [16], we consider a scenario in which decay happens in the same point r_0.

We can use the formulas from the previous sections in which substitutions $0 \to 2n$, $1 \to 2n+1$, and $2 \to 2n+2$ are made, $n = 0, 1, 2\ldots$ Then, using (5) and (21), we have

$$E_{2n+1} = m_{2n+1}\sqrt{f(r_0)} + \frac{q_{2n+1}Q}{r_0}, \tag{23}$$

$$E_{2n+2} = m_{2n+2}\sqrt{f(r_0)} + \frac{q_{2n+2}Q}{r_0}. \tag{24}$$

The energy $E_{2n+1} < 0$, provided $q_{2n+1} = -|q_{2n+1}|$, where $|q_{2n+1}| > q^*_{2n+1}$,

$$q^*_{2n+1} = \gamma m_{2n+1}, \qquad \gamma = \frac{r_0}{Q}\sqrt{f(r_0)}. \tag{25}$$

As $m_{2n} < m_0$ is finite, the final result depends crucially on q_{2n}. If $\lim_{n\to\infty} q_n = q_\infty < \infty$, E_{2n} is finite as well. This is just what happens in the scenario considered in [16] (we call it Scenario 1). Below we compare two qualitatively different types of scenarios.

A. Scenario 1

To make presentation self-contained, in this Section we outline briefly the scenario studied in [16]. One can assume that splitting of the charge to two fragments occurs in such a way that

$$q_{2n+1} = -(1+\Delta)q^*_{2n+1}, \qquad q_{2n+2} = q_{2n} - q_{2n+1}, \tag{26}$$

where Δ is some constant. We assume that in each act of decay

$$m_{2n+1} = \alpha_1 m_{2n}, \qquad m_{2n+2} = \alpha_2 m_{2n}, \tag{27}$$

where

$$\alpha_1 + \alpha_2 = 1. \tag{28}$$

As a result,

$$m_{2n} = \alpha_2^n m_0, \tag{29}$$

$$m_{2n+1} = \alpha_1 \alpha_2^n m_0. \tag{30}$$

To avoid confusion, we use here another notation as compared to [16], quantities without a tilde and with it are interchanged. Then, repeating transformations carried out in [16], we arrive at the expressions

$$q_{2n+1} = -(1+\Delta)\frac{m_{2n+1}\sqrt{f}}{Q}r_0 = -m_0(1+\Delta)\gamma\alpha_1\alpha_2^n, \tag{31}$$

$$E_{2n+1} = -m_{2n+1}\sqrt{f}\Delta, \tag{32}$$

$$\varepsilon_{2n+1} = -\sqrt{f(r_0)}\Delta. \tag{33}$$

This means that $\frac{q_{2n+1}}{m_{2n+1}} = \mathrm{const}$ does not depend on n as well as ε_{2n+1}. Then, it follows from the conservation laws that

$$q_{2n} = q_0 + (1+\Delta)\gamma(1-\alpha_2^n), \tag{34}$$

$$E_{2n} = E_0 + m_0\gamma\Delta(1-\alpha_2^n)\frac{Q}{r_0}. \tag{35}$$

Equations (34) corresponds to Eq. (3.32) of [16] and Eq. (35) corresponds (in our notations) to Eq. (3.31) of [16]. Then,

$$\lim_{n\to\infty} E_{2n} = E_0 + m_0\gamma\Delta\frac{Q}{r_0} = \lim_{\alpha_2\to 0} E_{2n}, \tag{36}$$

$$\lim_{n\to\infty} q_{2n} = q_0 + \gamma(1+\Delta) = \lim_{\alpha_2\to 0} q_{2n}. \tag{37}$$

Here, the limit $\alpha_2 \to 0$ means that all even particles are photons,

$$\lim_{n\to\infty} m_{2n} = \lim_{\alpha_2\to 0} m_{2n} = 0. \tag{38}$$

One can define the efficiency

$$\eta = \frac{E_{2n}}{E_0}. \tag{39}$$

Then,

$$\lim_{n\to\infty} \eta_n = 1 + \gamma\Delta\frac{m_0 Q}{E_0 r_0} = 1 + \frac{m_0\Delta}{E_0}\sqrt{f(r_0)} \tag{40}$$

which corresponds to Eq. (3.34) of [16].

B. Scenario 2

However, there are also other scenarios. We will consider one such example. Let us assume the same law for masses (27)–(30). However, for electric charges we take another dependence:

$$q_{2n+1} = q_{2n}\beta_1, \tag{41}$$

$$q_{2n+2} = q_{2n}\beta_2, \tag{42}$$

$$\beta_1 + \beta_2 = 1, \tag{43}$$

due to the conservation of charge. Here, $n = 0, 1, 2\ldots$ As a result,

$$q_{2n+2} = \beta_2^{n+1} q_0 \tag{44}$$

$$q_{2n+1} = \beta_1\beta_2^n q_0. \tag{45}$$

We choose

$$\beta_1 < 0, \qquad \beta_2 > 1. \tag{46}$$

Then,

$$E_{2n+1} = m_{2n+1}\sqrt{f(r_0)} + \frac{\beta_1\beta_2^n q_0 Q}{r_0}, \tag{47}$$

$$E_{2n+2} = m_{2n+2}\sqrt{f(r_0)} + \frac{\beta_2^{n+1} q_0 Q}{r_0}. \tag{48}$$

These expressions can be rewritten in the form

$$E_{2n+1} = m_{2n+1}\left[\sqrt{f(r_0)} - \frac{|\beta_1| q_0 Q}{\alpha_1 r_0 m_0}\left(\frac{\beta_2}{\alpha_2}\right)^n\right], \tag{49}$$

$$E_{2n+2} = m_{2n+2}\left[\sqrt{f(r_0)} + \left(\frac{\beta_2}{\alpha_2}\right)^{n+1}\frac{q_0 Q}{m_0 r_0}\right]. \tag{50}$$

There is a difference between Scenarios 1 [16] and 2 in the following sense. In Scenario 1, the parameters of the process are chosen in such a way that $E_1 < 0$, so amplification happens from the very beginning. Meanwhile, for Scenario 2 this is not mandatory. If one assumes that

$$\frac{|\beta_1| q_0 Q}{\alpha_1 r_0 m_0} > \sqrt{f(r_0)}, \tag{51}$$

amplification occurs at every stage starting from the first decay ($n = 0$). It can happen that there is no amplification for all $n \leq n_0$, where n_0 is some number, if

$$\sqrt{f(r_0)} - \frac{|\beta_1| q_0 Q}{\alpha_1 r_0 m_0}\left(\frac{\beta_2}{\alpha_2}\right)^n > 0. \tag{52}$$

As the negative term in (49) grows with n due to the fact that $\beta_2 > 1$ and $\alpha_2 < 1$, for sufficiently large n and for any such β_1, β_2, the energy E_{2n} becomes arbitrarily large anyway, so $\lim_{n\to\infty} E_{2n}$ and $\lim_{n\to\infty} \eta_n$ diverge. In doing so, E_{2n} grows exponentially with n, so we have a black-hole bomb.

V. ROTATING BLACK HOLES: METRIC AND EQUATIONS OF MOTION

Now, we will consider the confined Penrose process for rotating black holes. We will see that there are some qualitative differences as compared to the RN case. The metric has the form

$$ds^2 = -N^2 dt^2 + g_\phi(d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2, \tag{53}$$

where for shortness we use notations $g_\phi = g_{\phi\phi}$ and $g_\theta = g_{\theta\theta}$. We assume that the metric coefficients may depend on r and θ only and there is a symmetry with respect to the plane $\theta = \frac{\pi}{2}$. We consider motion of particles within this plane. Then, the equations of motion read

$$m\dot{t} = \frac{X}{N^2}, \tag{54}$$

$$X = E - \omega L, \tag{55}$$

$$p^r = m\dot{r} = \sigma P, \tag{56}$$

$$P = \sqrt{X^2 - \tilde{m}^2 N^2}, \tag{57}$$

$$m\dot{\phi} = \frac{L}{g_\phi} + \frac{\omega X}{N^2}. \tag{58}$$

Here,

$$\tilde{m}^2 = m^2 + \frac{L^2}{g_\phi}, \tag{59}$$

and Eq. (7) should be satisfied. Inside the ergoregion $g_{tt} > 0$, the particle energy can be negative.

VI. THE TURNING POINT INSIDE THE ERGOSPHERE

In this Section, we will consider the general scenario similar to that analyzed above for the RN metric. Namely, we choose the point of decay to be a turning point in the radial direction for all three particles 0, 1, 2. (However, inside the ergoregion the motion along the ϕ direction is inevitable.) It is seen from (57) that this implies

$$X_i = \tilde{m}_i N \tag{60}$$

for $i = 0, 1, 2$. It follows from the conservation laws for the energy (11) and angular momentum,

$$L_0 = L_1 + L_2, \tag{61}$$

that (14) is valid, whence

$$\tilde{m}_0 = \tilde{m}_1 + \tilde{m}_2. \tag{62}$$

For what follows, we will need some general properties of particle dynamics inside the ergoregion. It is characterized by the condition $g_{tt} > 0$. According to (53), this entails

$$\omega\sqrt{g_\phi} > N. \tag{63}$$

Taking into account (63), one can check that inside the ergoregion the expression $\omega L + \tilde{m}N$ is a monotonically increasing function of L, so Eq. (60) has only one root. This root obeys the condition (7). Then,

$$\frac{L_i}{\sqrt{g_\phi}} = \frac{\omega\sqrt{g_\phi}E_i - N\sqrt{E_i^2 + m_0^2 g_{tt}}}{g_{tt}}, \tag{64}$$

$i = 0, 1, 2$.

After simple but somewhat lengthy algebraic transformations, one finds from (62) and (61) that

$$L_1 = \frac{L_0 b_1}{2m_0^2} - \frac{\sqrt{d}\sqrt{g_\phi}}{2m_0^2}\tilde{m}_0, \tag{65}$$

$$L_2 = \frac{L_0 b_2}{2m_0^2} + \frac{\sqrt{d}\sqrt{g_\phi}}{2m_0^2}\tilde{m}_0, \tag{66}$$

$$E_1 = E_0\frac{b_1}{2m_0^2} - \frac{\sqrt{d}}{2m_0^2}\sqrt{E_0^2 + m_0^2 g_{tt}}, \tag{67}$$

$$E_2 = E_0\frac{b_2}{2m_0^2} + \frac{\sqrt{d}}{2m_0^2}\sqrt{E_0^2 + m_0^2 g_{tt}}, \tag{68}$$

where b_1, b_2, and d are defined according to (17) and (18), and g_{tt} is the corresponding component of the metric tensor in (53). In Eqs. (65)–(68) we chose the signs before square roots in such a way that it is particle 1 for which $E_1 < 0$ is possible. Correspondingly, for $E_1 \le 0$, we have also $L_1 \le 0$ in agreement with (7) and (64) (see below for more details).

The condition $d \ge 0$ entails

$$m_0 \ge m_1 + m_2. \tag{69}$$

Although in (62) the equality sign stands for effective masses $\tilde{m}_i$, a strict inequality is quite possible in (69).

Also, in the turning point one can calculate

$$\tilde{m} = \frac{X}{N} = -\frac{NE}{g_{tt}} + \frac{\omega\sqrt{g_\phi}\sqrt{E^2 + m^2 g_{tt}}}{g_{tt}} > 0 \tag{70}$$

independently of the sign of E. In the particular case $E_0 = m_0$, $m_1 = m_2 = m$ we obtain

$$E_{1,2} = \frac{m_0}{2} \mp \frac{m_0}{2}\sqrt{1 - 4\frac{m^2}{m_0^2}}\sqrt{1 + g_{tt}}, \tag{71}$$

which coincides with Eq. (3.47) of [12]. The analogues of the corresponding formulas for the splitting of particles into two fragments in the turning point in the Schwarzschild case are listed in [18].

Equations (67) and (68) generalize previously known formulas for the case of arbitrary E_0, m_0, m_1, m_2, and can be of some use for applications in somewhat different contexts.

The requirement $E_1 < 0$ leads to the condition

$$E_0 < E_0^* \equiv \frac{\sqrt{g_{tt}d}}{2m_1}. \tag{72}$$

When $E_0 \to E_0^*$, the energy $E_1 \to 0$,

$$L_0 \to L_0^* = \frac{\omega g_\phi\sqrt{d} - N b_1\sqrt{g_\phi}}{2m_1\sqrt{g_{tt}}}, \tag{73}$$

$$L_1 \to L_1^* = -\frac{N m_1\sqrt{g_\phi}}{\sqrt{g_{tt}}} < 0. \tag{74}$$

It is seen from (64) that for $E_1 = -|E_1| < 0$ the angular momentum L_1 is a decreasing function of $|E_1|$. In this sense, the existence of the Penrose process requires the inequality $L_1 < L_1^*$.

One may ask how to obtain, for given m_0 and E_0, the maximum E_2, i.e., the maximum efficiency (39). Let us introduce notations $y_i = m_i^2$. One can check that

$$m_0^2\frac{\partial E_2}{\partial y_2} = \frac{Y}{\sqrt{d}},$$
$$Y = E_0\sqrt{d} - \sqrt{E_0^2 + m_0^2 g_{tt}}(y_0 - y_2 + y_1) < 0. \tag{75}$$

Therefore, the maximum of η occurs when particle 2 has a minimum possible mass $m_2 = 0$ (photon). This is completely similar to the situation with the scenario considered in [16] for the RN case.

The dependence of E_2 on coordinates is encoded in (68) in the term with g_{tt}. If g_{tt} is a monotonically decreasing function of r (like in the Kerr metric), the maximum is achieved if decay occurs on the horizon.

For a concrete metric, one can find the additional conditions for the existence of a turning point. However, now the goal is different. We simply assume, not specifying the form of a metric, that a turning point does exist and analyze whether or not a black-hole bomb due to the multiple Penrose process is possible in this scenario. We will see that the main general conclusions can be derived without appeal to the explicit form of a metric.

VII. MULTIPLE PENROSE PROCESS

Now, let a particle 1 with $E_1 < 0$ fall in the black hole, particle 2 with $E_2 > E_0$ moves in the outward direction. We can place a reflecting shell, so particle 2 bounces back and decays again. For simplicity, we assume that new decay happens in the point with the same r_0. Repeating the process again and again, we obtain the rotating version of the confined Penrose process. Now, our basic equations follow from (65)–(68) and read

$$L_{2n+1} = \frac{L_0 b_{2n+1}}{2m_{2n}^2} - \frac{\sqrt{d_{2n+2}}\sqrt{g_\phi}}{2m_{2n}^2}\tilde{m}_{2n}, \quad (76)$$

$$L_{2n+2} = \frac{L_{2n} b_{2n+2}}{2m_{2n}^2} + \frac{\sqrt{d_{2n+2}}\sqrt{g_\phi}}{2m_n^2}\tilde{m}_{2n}, \quad (77)$$

$$E_{2n+1} = \frac{b_{2n+1}}{2m_{2n}^2}E_{2n} - \frac{\sqrt{d_{2n+2}}}{2m_{2n}^2}\sqrt{E_{2n}^2 + m_{2n}^2 g_{tt}}, \quad (78)$$

$$E_{2n+2} = \frac{b_{2n+2}}{2m_{2n}^2}E_{2n} + \frac{\sqrt{d_{2n+2}}}{2m_{2n}^2}\sqrt{E_{2n}^2 + m_{2n}^2 g_{tt}}, \quad (79)$$

$$b_{2n+1} = m_{2n}^2 + m_{2n+1}^2 - m_{2n+2}^2, \quad (80)$$

$$b_{2n+2} = m_{2n}^2 + m_{2n+2}^2 - m_{2n+1}^2, \quad (81)$$

$$d_{2n+2} = b_{2n+2}^2 - 4m_{2n}m_{2n+2} = b_{2n+1}^2 - 4m_{2n}^2 m_{2n+1}^2. \quad (82)$$

Equation (72) turns into

$$E_{2n} < E_{2n}^* = \frac{\sqrt{g_{tt} d_{2n+2}}}{2m_{2n+1}}. \quad (83)$$

Is it possible to obtain divergent E_{2n} when $n \to \infty$? As $m_{2n} \le m_0$ is finite, we would have

$$E_{2n+2} \approx E_{2n} s_n, \quad (84)$$

where

$$s_n = \frac{b_{2n} + \sqrt{d_{2n+2}}}{2m_{2n}^2}. \quad (85)$$

However, $\sqrt{d_{2n+2}} \le b_{2n+1}$. As $b_{2n+1} + b_{2n+2} = 2m_{2n}^2$, we obtain that $s_n < 1$, so the growth of E_{2n} stops for $m_{2n+1} \neq 0$. According to (83), this happens when E_{2n} reaches E_{2n}^*.

If $m_{2n+1} = 0$, $m_{2n} = m_0$, $\sqrt{d_{2n}} = b_{1n}$, $s_n = 1$. Again, there is no growth of E_{2n}.

VIII. WALD BOUND AND DECAY IN AN ARBITRARY POINT

Now we relax the condition that decay inside the ergoregion occurs just in the turning point. Can this improve the efficiency to the extent that the confined Penrose process would lead to a black-hole bomb? In the present context, it makes sense to remind one of a universal inequality that is valid when a parent particle 0 decays to two fragments 1 and 2. According to [19], the upper bound on E_2 satisfies the condition

$$E_{\max} = \frac{m_2}{m_0}\gamma\left(E_0 + v\sqrt{E_0^2 + m_0^2 g_{tt}}\right). \quad (86)$$

Here, v is the velocity of fragment 2 in the frame comoving with particle 0 before decay (after decay this is the center of mass frame of debris), $\gamma = \frac{1}{\sqrt{1-v^2}}$. This corresponds to the ejection strictly in the direction of particle 0. If we compare this to (68), it is clear that

$$v = \frac{\sqrt{d}}{b_2} = \sqrt{1 - \frac{4m_2^2 m_0^2}{b_2^2}}. \quad (87)$$

One can verify this statement independently. If we introduce the tetrad attached, say, to the zero angular momentum observer [20], then

$$v^{(3)} = \frac{L}{\sqrt{L^2 + m^2 g_\phi}}. \quad (88)$$

In particular, this formula can be taken from Eq. (12) of [21] in combination with (60).

Then, the relativistic law of addition of velocities gives us that in our stationary frame

$$v = \frac{v_2^{(3)} - v_0^{(3)}}{1 - v_2^{(3)} v_0^{(3)}}. \quad (89)$$

Straightforward calculations with (66) taken into account show that (87) indeed holds true.

Now we can derive the necessary condition for the black-hole bomb phenomenon to exist. Let decay occur in some intermediate point, not in the turning one. Then, v (or, say, L_2) is a free parameter. We want to have the behavior with $E_{2n} \to \infty$ in the multiple decay. Then, it follows from (86) (where necessary replacements are made for the n-th event) that

$$E_{2n+2} \approx s_n E_{2n}, \quad (90)$$

where now

$$s_n = \frac{m_{2n+2}}{m_{2n}}\gamma_n(1 + v_n) = \frac{m_{2n+2}}{m_{2n}}\sqrt{\frac{1+v_n}{1-v_n}}. \quad (91)$$

The process of amplification continues provided $s_n > 1$, so

$$v_n > \frac{1-\alpha_n^2}{1+\alpha_n^2}, \qquad \alpha_n = \frac{m_{2n+2}}{m_{2n}} < 1. \quad (92)$$

If the velocity $v \to 1$ is close to that of light, this can be satisfied easily. However, this is a quite trivial case in which there is no crucial difference between a single decay or a multiple one. Additionally, such a condition is not realistic astrophysically [19]. To have a nontrivial possibility for a bomb (not too big v), we must choose $\alpha \sim 1$ or, better, α close to 1.

We can check the inequality under discussion for the process of decay in the turning point. When particle 0 decays, it follows from extension of (87) to an arbitrary n that

$$s_n = \frac{b_{2n+2} + \sqrt{b_{2n+1}^2 - 4m_{2n}^2 m_{2n+1}^2}}{2m_{2n}^2} \leq 1; \qquad (93)$$

a similar formula holds if we replace 0 with 2n and 2 with $2n + 2$. Thus, a black-hole bomb is impossible in this case in agreement with the material of the previous Section. One can say that the scenario with decay in the turning point is too constrained and dictates quite definite values of particle characteristics (too low velocity of a fragment) not compatible with the existence of a black-hole bomb.

One can try to apply the counterpart of (86) to the RN case. Then, a standard replacement $E \to X = E - q\varphi$ should be made. For pure radial motion $v = 0$ in the turning point, so the corresponding formula gives us $X_{2n+2} = \frac{m_{2n+2}}{m_{2n}} X_{2n}$ which can be obtained from (60) directly. In this sense, it does not give us new information; the analysis from Sec. IV applies.

IX. SUMMARY AND CONCLUSIONS

We showed that a black-hole bomb is indeed possible due to the confined Penrose process but this requires some additional constraints. For the RN metric, the analysis of multiple decays in the turning point showed that existence of a black-hole bomb depends crucially on the type of scenario. There exist scenarios (like Scenario 2 in our paper) that do give rise to a black-hole bomb.

We also considered a similar process with neutral particles in the background of rotating black holes. When decay occurs in the turning point, the most efficient process develops when an escaping particle is massless (photon) and decay occurs just near the horizon. These features are similar to those in the RN metric [16].

However, as far as the multiple process is concerned, the situation is different. In the case of particle decay in the turning point, a black-hole bomb is absent at all. We would like to stress that this result is model-independent and did not require the knowledge of the details of metric.

How does one explain this crucial difference between the RN metric and rotating black holes? If one compares the static charged black holes and rotating neutral ones, the role similar to the electric particle charge q is played by the angular momentum. Meanwhile, the electric charge does not enter the effective mass $\tilde{m} = m$ in the first case. By contrast, for the rotating background, it enters the mass $\tilde{m}$ (59) and, through a set of coupled equations, affects dynamics crucially. As a result, q_i remains a free parameter for the scenario under discussion in the RN metric, whereas L_i are unambiguously defined by Eqs. (65) and (66) for the case of rotating black holes.

However, new options appear if one relaxes the condition that decay happens in the turning point. Then, under some additional conditions that relate the velocity of fragments in the center of mass frame and the ratio of masses before and after decay, a black-hole bomb is indeed possible. In doing so, the issue under discussion has an interesting overlap with the Wald bounds.

One reservation is in order. What is required for a bomb in the present context is the existence of the ergosphere, the horizon itself is not required. Therefore, in principle, the notion of the bomb under discussion is wider than a black-hole bomb in a narrow sense. However, one should bear in mind that the horizonless objects with the ergoregion are, as a rule, unstable themselves [22].

It would be of interest to generalize our analysis to combine both the electric charge and rotation.

[1] R. Penrose, Gravitational collapse: The role of general relativity, Riv. Nuovo Cimento **1**, 252 (1969).

[2] G. Denardo and R. Ruffini, On the energetics of Reissner Nordstrøm geometries, Phys. Lett. **45B**, 259 (1973).

[3] G. Denardo, L. Hively, and R. Ruffini, On the generalized ergosphere of the Kerr-Newman geometry, Phys. Lett. **50B**, 270 (1974).

[4] M. Bhat, S. Dhurandhar, and N. Dadhich, Energetics of the Kerr-Newman black hole by the Penrose process, J. Astrophys. Astron. **6**, 85 (1985).

[5] S. Parthasarathy, S. M. Wagh, S. V. Dhurandhar, and N. Dadhich, High efficiency of the Penrose process of energy extraction from rotating black holes immersed in electromagnetic fields, Astron. J. **307**, 38 (1986).

[6] S. M. Wagh and N. Dadhich, The energetics of black holes in electromagnetic fields by the Penrose process, Phys. Rep. **183**, 137 (1989).

[7] N. Dadhich, The Penrose process of energy extraction in electrodynamics, ICTP Report No. IC80/98, 1980.

[8] O. B. Zaslavskii, Pure electric Penrose and super-Penrose processes in the flat space-time, Int. J. Mod. Phys. D **28**, 1950062 (2019).

[9] L. Comisso and F. A. Asenjo, Magnetic reconnection as a mechanism for energy extraction from rotating black holes, Phys. Rev. D **103**, 023014 (2021).

[10] A. A. Starobinskii, Amplification of waves during reflection from a rotating “black hole,” Sov. Phys. JETP, **37**, 28 (1973).

[11] A. A. Starobinskil and S. M. Churilov, Amplification of electromagnetic and gravitational waves scattered by a rotating "black hole," Sov. Phys. JETP, **38**, 1 (1974).

[12] R. Brito, P. Pani, and V. Cardoso, *Superradiance: New Frontiers in Black Hole Physics*, 2nd ed., Lecture Notes in Physics (Springer, New York, 2015).

[13] W. H. Press and S. A. Teukolsky, Floating orbits, superradiant scattering and the black-hole bomb, Nature (London) **238**, 211 1972.

[14] A. V. Vilenkin, Exponential amplification of waves in the gravitational field of ultrarelativistic rotating body, Phys. Lett. **78B**, 301 (1978).

[15] V. Cardoso, O. J. C. Dias, J. P. S. Lemos, and S. Yoshida, The black-hole bomb and super-radiant instabilities, Phys. Rev. D **70**, 044039 (2004); **70**, 049903(E) (2004).

[16] T. Kokubu, S.-L. Li, P. Wu, and Hongwei Yu, Confined Penrose process with charged particles, Phys. Rev. D **104**, 104047 (2021).

[17] O. B. Zaslavskii, Center of mass energy of colliding electrically neutral particles and super-Penrose process, Phys. Rev. D **100**, 024050 (2019).

[18] Yu. V. Pavlov and O. B. Zaslavskii, The Oberth effect and relativistic rocket in the Schwarzschild background, arXiv: 2111.09240.

[19] R. M. Wald, Energy limits on the Penrose process, Astrophys. J. **191**, 231 (1974).

[20] J. M. Bardeen, W. H. Press, and S. A. Teukolsky, Rotating black holes: Locally nonrotating frames, energy extraction, and scalar synchrotron radiation, Astrophys. J. **178**, 347 (1972).

[21] O. B. Zaslavskii, Acceleration of particles by black holes: Kinematic explanation, Phys. Rev. D **84**, 024007 (2011).

[22] J. L. Friedman, Ergosphere instability, Commun. Math. Phys. **63**, 243 (1978).