


**Erratum: Semisimple extensions of the Standard Model gauge algebra  
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B. C. Allanach, Ben Gripaios, and Joseph Tooby-Smith 

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Algebra 32 contains the Pati-Salam algebra and so should not appear in Table I; similarly, algebras 23 and 26 are subalgebras of 22 and 7, respectively, and so should not appear in Table I. The counts of maximal and minimal algebras appearing on the second and the last pages should therefore be changed to 24 and 5, respectively. Updated Tables I and II and numberings are given below. Furthermore, updated Supplemental Material is provided.

TABLE I. All maximal and minimal anomaly-free algebras for exactly three generations of SM fermions plus three right-handed neutrinos.

| Maximal |   |
|---------|---|
| Algebra | Fermion representations corresponding to $\beta$  |
| 1       | $\mathfrak{so}(10) \oplus \mathfrak{su}(2)$ (16, 3)   |
| 2       | $\mathfrak{so}(10)^{\oplus 3}$ (16, 1, 1) $\oplus$ (1, 16, 1) $\oplus$ (1, 1, 16)   |
| 3       | $\mathfrak{so}(10)^{\oplus 2} \oplus \mathfrak{su}(2)$ (16, 1, 1) $\oplus$ (1, 16, 2)   |
| 4       | $\mathfrak{su}(4) \oplus \mathfrak{sp}(6)^{\oplus 2}$ ( $\bar{4}$ , 6, 1) $\oplus$ (4, 1, 6)  |
| 5       | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{sp}(6)$ ( $\bar{4}$ , 6, 1) $\oplus$ (4, 1, 6)  |
| 6       | $\mathfrak{su}(12) \oplus \mathfrak{su}(2)^{\oplus 2}$ ( $\bar{12}$ , 2, 1) $\oplus$ (12, 1, 2)   |
| 7       | $\mathfrak{su}(4) \oplus \mathfrak{sp}(4)^{\oplus 2} \oplus \mathfrak{so}(10)$ ( $\bar{4}$ , 4, 1, 1) $\oplus$ (4, 1, 4, 1) $\oplus$ (1, 1, 1, 16)  |
| 8       | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$ ( $\bar{5}$ , 3, 1, 1) $\oplus$ (10, 1, 3, 1) $\oplus$ (1, 1, 1, 2) $\oplus$ (1, 1, 1, 1)   |
| 9       | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$ ( $\bar{5}$ , 3, 1, 1) $\oplus$ (10, 1, 3, 1) $\oplus$ (1, 1, 1, 3)   |
| 10      | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$ ( $\bar{5}$ , 1, 1, 1) $\oplus$ ( $\bar{5}$ , 2, 1, 1) $\oplus$ (10, 1, 3, 1) $\oplus$ (1, 1, 1, 2) $\oplus$ (1, 1, 1, 1)   |
| 11      | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$ ( $\bar{5}$ , 1, 1, 1) $\oplus$ ( $\bar{5}$ , 2, 1, 1) $\oplus$ (10, 1, 3, 1) $\oplus$ (1, 1, 1, 3)   |
| 12      | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$ (10, 1, 1, 1) $\oplus$ ( $\bar{5}$ , 3, 1, 1) $\oplus$ (10, 1, 2, 1) $\oplus$ (1, 1, 1, 2) $\oplus$ (1, 1, 1, 1)  |
| 13      | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$ (10, 1, 1, 1) $\oplus$ ( $\bar{5}$ , 3, 1, 1) $\oplus$ (10, 1, 2, 1) $\oplus$ (1, 1, 1, 3)  |
| 14      | $\mathfrak{su}(5)^{\oplus 2} \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ ( $\bar{5}$ , 1, 1, 1) $\oplus$ (10, 1, 1, 1) $\oplus$ (1, $\bar{5}$ , 1, 1) $\oplus$ (1, 10, 1, 1) $\oplus$ (1, 1, 16, 1) $\oplus$ (1, 1, 1, 2)               |
| 15      | $\mathfrak{su}(5)^{\oplus 3} \oplus \mathfrak{su}(2)$ ( $\bar{5}$ , 1, 1, 1) $\oplus$ (10, 1, 1, 1) $\oplus$ (1, $\bar{5}$ , 1, 1) $\oplus$ (1, 10, 1, 1) $\oplus$ (1, 1, $\bar{5}$ , 1) $\oplus$ (1, 1, 10, 1) $\oplus$ (1, 1, 1, 3)         |
| 16      | $\mathfrak{su}(8) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 2}$ (1, 16, 1, 1) $\oplus$ ( $\bar{8}$ , 1, 2, 1) $\oplus$ (8, 1, 1, 2)  |
| 17      | $\mathfrak{su}(4) \oplus \mathfrak{sp}(4) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 2}$ ( $\bar{4}$ , 4, 1, 1, 1) $\oplus$ (1, 1, 16, 1, 1) $\oplus$ (4, 1, 1, 2, 2)   |
| 18      | $\mathfrak{su}(4) \oplus \mathfrak{sp}(4) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 2}$ ( $\bar{4}$ , 4, 1, 1, 1) $\oplus$ (1, 1, 16, 1, 1) $\oplus$ (4, 1, 1, 2, 2)   |
| 19      | $\mathfrak{su}(4) \oplus \mathfrak{sp}(6) \oplus \mathfrak{su}(2)^{\oplus 3}$ ( $\bar{4}$ , 6, 1, 1, 1) $\oplus$ (4, 1, 2, 2, 1) $\oplus$ (4, 1, 1, 1, 2)   |
| 20      | $\mathfrak{su}(4) \oplus \mathfrak{sp}(6) \oplus \mathfrak{su}(2)^{\oplus 3}$ ( $\bar{4}$ , 6, 1, 1, 1) $\oplus$ (4, 1, 2, 2, 1) $\oplus$ (4, 1, 1, 1, 2)   |
| 21      | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 3}$ ( $\bar{4}$ , 6, 1, 1, 1) $\oplus$ (4, 1, 2, 2, 1) $\oplus$ (4, 1, 1, 1, 2)  |
| 22      | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 3}$ (1, 16, 1, 1, 1) $\oplus$ ( $\bar{5}$ , 1, 2, 1, 1) $\oplus$ (10, 1, 1, 2, 1) $\oplus$ (1, 1, 1, 1, 2)   |
| 23      | $\mathfrak{su}(5)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 3}$ (1, $\bar{5}$ , 1, 1, 1) $\oplus$ (1, 10, 1, 1, 1) $\oplus$ ( $\bar{5}$ , 1, 2, 1, 1) $\oplus$ (10, 1, 1, 2, 1) $\oplus$ (1, 1, 1, 1, 3)                                     |
| 24      | $\mathfrak{su}(4) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 4}$ (1, 16, 1, 1, 1, 1) $\oplus$ (4, 1, 2, 2, 1, 1) $\oplus$ ( $\bar{4}$ , 1, 1, 1, 2, 2)  |
| Minimal |   |
| 25      | $\mathfrak{su}(5)$ ( $\bar{5}$ ) <sup>3</sup> $\oplus$ (10) <sup>3</sup> $\oplus$ (1) <sup>3</sup>  |
| 26      | $\mathfrak{su}(4) \oplus \mathfrak{su}(2)^{\oplus 2}$ ( $\bar{4}$ , 2, 1) <sup>3</sup> $\oplus$ (4, 1, 2) <sup>3</sup>  |
| 27      | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)$ ( $\bar{4}$ , 6, 1) $\oplus$ (4, 1, 2) <sup>3</sup>   |
| 28      | $\mathfrak{su}(4) \oplus \mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ (1, $\bar{5}$ , 1, 1) <sup>2</sup> $\oplus$ (1, 10, 1, 1) <sup>2</sup> $\oplus$ ( $\bar{4}$ , 1, 2, 1) $\oplus$ (4, 1, 1, 2) $\oplus$ (1, 1, 1, 1) <sup>2</sup> |
| 29      | $\mathfrak{su}(4) \oplus \mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ (1, $\bar{5}$ , 1, 1) $\oplus$ (1, 10, 1, 1) $\oplus$ ( $\bar{4}$ , 1, 2, 1) <sup>2</sup> $\oplus$ (4, 1, 1, 2) <sup>2</sup> $\oplus$ (1, 1, 1, 1)              |

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TABLE II. All algebras for exactly three generations of SM fermions plus three right-handed neutrinos which are neither maximal nor minimal.

| Nonmaximal and nonminimal algebras |   |
|------------------------------------|---|
| Algebra                            | Fermion representations corresponding to $\beta$  |
| 30                                 | $\mathfrak{so}(10)$ <span style="float: right;"><math>(16)^{\oplus 3}</math></span>   |
| 31                                 | $\mathfrak{so}(10)^{\oplus 2}$ <span style="float: right;"><math>(16, 1) \oplus (1, 16)^{\oplus 2}</math></span>  |
| 32                                 | $\mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(16, 1) \oplus (16, 2)</math></span>  |
| 33                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (10, 1) \oplus (1, 16)^{\oplus 2} \oplus (1, 1)</math></span>   |
| 34                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 2} \oplus (10, 1)^{\oplus 2} \oplus (1, 16) \oplus (1, 1)^{\oplus 2}</math></span>                           |
| 35                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 3) \oplus (10, 3) \oplus (1, 1)^{\oplus 3}</math></span>   |
| 36                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (\bar{5}, 2) \oplus (10, 3) \oplus (1, 1)^{\oplus 3}</math></span>   |
| 37                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 3) \oplus (10, 3) \oplus (1, 2) \oplus (1, 1)</math></span>  |
| 38                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 3) \oplus (10, 3) \oplus (1, 3)</math></span>  |
| 39                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1) \oplus (\bar{5}, 3) \oplus (10, 2) \oplus (10, 1)^{\oplus 3}</math></span>   |
| 40                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 3) \oplus (1, 1)^{\oplus 3}</math></span>  |
| 41                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (\bar{5}, 2) \oplus (10, 3) \oplus (1, 2) \oplus (1, 1)</math></span>  |
| 42                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (\bar{5}, 2) \oplus (10, 3) \oplus (1, 3)</math></span>  |
| 43                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (10, 1) \oplus (\bar{5}, 2) \oplus (10, 2) \oplus (1, 1)^{\oplus 3}</math></span>                              |
| 44                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1) \oplus (\bar{5}, 3) \oplus (10, 2) \oplus (1, 2) \oplus (10, 1)</math></span>  |
| 45                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1) \oplus (\bar{5}, 3) \oplus (10, 2) \oplus (1, 3)</math></span>   |
| 46                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1)^{\oplus 3} \oplus (\bar{5}, 3) \oplus (10, 1)^{\oplus 3}</math></span>   |
| 47                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 1) \oplus (10, 2) \oplus (1, 1)^{\oplus 3}</math></span>                                       |
| 48                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 3) \oplus (1, 2) \oplus (1, 1)</math></span>   |
| 49                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 3) \oplus (1, 3)</math></span>   |
| 50                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (10, 1) \oplus (\bar{5}, 2) \oplus (10, 2) \oplus (1, 2) \oplus (1, 1)</math></span>                           |
| 51                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (10, 1) \oplus (\bar{5}, 2) \oplus (10, 2) \oplus (1, 3)</math></span>   |
| 52                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (10, 1)^{\oplus 3} \oplus (\bar{5}, 2) \oplus (1, 1)^{\oplus 3}</math></span>                                  |
| 53                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1)^{\oplus 3} \oplus (\bar{5}, 3) \oplus (1, 2) \oplus (10, 1)</math></span>  |
| 54                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1)^{\oplus 3} \oplus (\bar{5}, 3) \oplus (1, 3)</math></span>   |
| 55                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 1) \oplus (10, 2) \oplus (1, 2) \oplus (1, 1)</math></span>                                    |
| 56                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 1) \oplus (10, 2) \oplus (1, 3)</math></span>  |
| 57                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (10, 1)^{\oplus 3} \oplus (\bar{5}, 2) \oplus (1, 2) \oplus (1, 1)</math></span>                               |
| 58                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1) \oplus (10, 1)^{\oplus 3} \oplus (\bar{5}, 2) \oplus (1, 3)</math></span>   |
| 59                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 1)^{\oplus 3} \oplus (1, 2) \oplus (1, 1)</math></span>  |
| 60                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 3} \oplus (10, 1)^{\oplus 3} \oplus (1, 3)</math></span>  |
| 61                                 | $\mathfrak{su}(5)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 1)^{\oplus 2} \oplus (10, 1)^{\oplus 2} \oplus (1, \bar{5}) \oplus (1, 10) \oplus (1, 1)^{\oplus 3}</math></span>                     |
| 62                                 | $\mathfrak{su}(4) \oplus \mathfrak{sp}(6) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{4}, 6, 1) \oplus (4, 1, 2)^{\oplus 3}</math></span>  |
| 63                                 | $\mathfrak{su}(4) \oplus \mathfrak{sp}(6) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{4}, 6, 1) \oplus (4, 1, 2)^{\oplus 3}</math></span>  |
| 64                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 1, 1) \oplus (10, 1, 1) \oplus (1, 16, 1) \oplus (1, 1, 16) \oplus (1, 1, 1)</math></span>                   |
| 65                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(1, 16, 1) \oplus (\bar{5}, 1, 2) \oplus (10, 1, 2) \oplus (1, 11, 1)^{\oplus 2}</math></span>            |
| 66                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1, 1) \oplus (10, 1, 1) \oplus (1, 16, 2) \oplus (1, 1, 1)</math></span>                        |
| 67                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(1, 16, 1) \oplus (\bar{5}, 1, 2) \oplus (10, 1, 2) \oplus (1, 1, 2)</math></span>                        |
| 68                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1, 1)^{\oplus 2} \oplus (1, 16, 1) \oplus (10, 1, 2) \oplus (1, 1, 1)^{\oplus 2}</math></span>  |
| 69                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1, 1)^{\oplus 2} \oplus (1, 16, 1) \oplus (\bar{5}, 1, 2) \oplus (10, 1, 1)^{\oplus 2}</math></span> |
| 70                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1, 1)^{\oplus 2} \oplus (1, 16, 1) \oplus (10, 1, 2) \oplus (1, 1, 2)</math></span>             |
| 71                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(10, 1, 1)^{\oplus 2} \oplus (1, 16, 1) \oplus (\bar{5}, 1, 2) \oplus (1, 1, 2)</math></span>             |
| 72                                 | $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$ <span style="float: right;"><math>(\bar{5}, 1, 1)^{\oplus 2} \oplus (10, 1, 1)^{\oplus 2} \oplus (1, 16, 1) \oplus (1, 1, 2)</math></span>  |
| 73                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 3, 1) \oplus (10, 1, 3) \oplus (1, 1, 1)^{\oplus 3}</math></span>   |
| 74                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 1, 1) \oplus (\bar{5}, 2, 1) \oplus (10, 1, 3) \oplus (1, 1, 1)^{\oplus 3}</math></span>                      |
| 75                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 3, 1) \oplus (10, 1, 3) \oplus (1, 1, 2) \oplus (1, 1, 1)</math></span>                                       |
| 76                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 3, 1) \oplus (10, 1, 3) \oplus (1, 1, 3)</math></span>  |
| 77                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 3, 1) \oplus (10, 1, 3) \oplus (1, 2, 1) \oplus (1, 1, 1)</math></span>                                       |
| 78                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 3, 1) \oplus (10, 1, 3) \oplus (1, 3, 1)</math></span>  |
| 79                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 3, 1) \oplus (10, 3, 1) \oplus (1, 1, 2) \oplus (1, 1, 1)</math></span>                                       |
| 80                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 3, 1) \oplus (10, 3, 1) \oplus (1, 1, 3)</math></span>  |
| 81                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(10, 1, 1) \oplus (\bar{5}, 3, 1) \oplus (10, 1, 2) \oplus (10, 1, 1)^{\oplus 3}</math></span>                          |
| 82                                 | $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 2}$ <span style="float: right;"><math>(\bar{5}, 1, 1) \oplus (\bar{5}, 2, 1) \oplus (10, 1, 3) \oplus (1, 1, 2) \oplus (1, 1, 1)</math></span>                |

(Table continued)











TABLE II. (Continued)

| Nonmaximal and nonminimal algebras |  |   |
|------------------------------------|--|---|
| Algebra                            | Fermion representations corresponding to $\beta$   |   |
| 323                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 5}$                         | $(4, 1, 2, 2, 1, 1, 1) \oplus (\bar{4}, 1, 1, 1, 2, 1, 1)^{\oplus 2} \oplus (1, \bar{4}, 1, 1, 1, 2, 1) \oplus (1, 4, 1, 1, 1, 1, 2)$   |
| 324                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 5}$                         | $(\bar{4}, 1, 2, 1, 1, 1, 1) \oplus (1, \bar{4}, 2, 1, 1, 1, 1) \oplus (\bar{4}, 1, 1, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1)$<br>$\oplus (4, 1, 1, 1, 1, 2, 1) \oplus (1, 4, 1, 1, 1, 1, 2)$                                     |
| 325                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 5}$                         | $(\bar{4}, 1, 2, 1, 1, 1, 1) \oplus (1, \bar{4}, 2, 1, 1, 1, 1) \oplus (\bar{4}, 1, 1, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1)$<br>$\oplus (4, 1, 1, 1, 1, 2, 1) \oplus (1, 4, 1, 1, 1, 1, 2)$                                     |
| 326                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 5}$                         | $(\bar{4}, 1, 2, 1, 1, 1, 1)^{\oplus 2} \oplus (4, 1, 1, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1) \oplus (1, \bar{4}, 1, 1, 1, 2, 1)$<br>$\oplus (1, 4, 1, 1, 1, 1, 2)$   |
| 327                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 5}$                         | $(\bar{4}, 1, 2, 1, 1, 1, 1)^{\oplus 2} \oplus (4, 1, 1, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1) \oplus (1, \bar{4}, 1, 1, 1, 2, 1)$<br>$\oplus (1, 4, 1, 1, 1, 1, 2)$   |
| 328                                | $\mathfrak{su}(4)^{\oplus 3} \oplus \mathfrak{su}(2)^{\oplus 4}$                         | $(\bar{4}, 1, 1, 2, 1, 1, 1) \oplus (1, \bar{4}, 1, 2, 1, 1, 1) \oplus (1, 1, \bar{4}, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1)$<br>$\oplus (1, 4, 1, 1, 1, 2, 1) \oplus (1, 1, 4, 1, 1, 1, 2)$                                     |
| 329                                | $\mathfrak{su}(4)^{\oplus 3} \oplus \mathfrak{su}(2)^{\oplus 4}$                         | $(\bar{4}, 1, 1, 2, 1, 1, 1) \oplus (1, \bar{4}, 1, 2, 1, 1, 1) \oplus (1, 1, \bar{4}, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1)$<br>$\oplus (1, 4, 1, 1, 1, 2, 1) \oplus (1, 1, 4, 1, 1, 1, 2)$                                     |
| 330                                | $\mathfrak{su}(4)^{\oplus 3} \oplus \mathfrak{su}(2)^{\oplus 4}$                         | $(\bar{4}, 1, 1, 2, 1, 1, 1) \oplus (1, \bar{4}, 1, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1) \oplus (1, 1, \bar{4}, 1, 2, 1, 1)$<br>$\oplus (1, 4, 1, 1, 1, 2, 1) \oplus (1, 1, 4, 1, 1, 1, 2)$                                     |
| 331                                | $\mathfrak{su}(4)^{\oplus 3} \oplus \mathfrak{su}(2)^{\oplus 4}$                         | $(\bar{4}, 1, 1, 2, 1, 1, 1) \oplus (1, \bar{4}, 1, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1) \oplus (1, 4, 1, 1, 2, 1, 1)$<br>$\oplus (1, 1, \bar{4}, 1, 1, 2, 1) \oplus (1, 1, 4, 1, 1, 1, 2)$                                     |
| 332                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 4}$ | $(1, 1, \bar{5}, 1, 1, 1, 1) \oplus (1, 1, 10, 1, 1, 1, 1) \oplus (\bar{4}, 1, 1, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1)$<br>$\oplus (1, \bar{4}, 1, 1, 1, 2, 1) \oplus (1, 4, 1, 1, 1, 1, 2) \oplus (1, 1, 1, 1, 1, 1, 1)$       |
| 333                                | $\mathfrak{su}(4) \oplus \mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 5}$            | $(\bar{4}, 1, 2, 1, 1, 1, 1) \oplus (4, 1, 1, 2, 1, 1, 1) \oplus (1, \bar{5}, 1, 1, 2, 1, 1) \oplus (1, 10, 1, 1, 1, 2, 1)$<br>$\oplus (1, 1, 1, 1, 1, 1, 2)$   |
| 334                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 6}$                         | $(4, 1, 2, 2, 1, 1, 1, 1) \oplus (\bar{4}, 1, 1, 1, 2, 2, 1, 1) \oplus (1, \bar{4}, 1, 1, 1, 1, 2, 1) \oplus (1, 4, 1, 1, 1, 1, 1, 2)$  |
| 335                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 6}$                         | $(4, 1, 2, 2, 1, 1, 1, 1) \oplus (\bar{4}, 1, 1, 1, 2, 1, 1, 1) \oplus (\bar{4}, 1, 1, 1, 1, 2, 1, 1) \oplus (1, \bar{4}, 1, 1, 1, 1, 2, 1)$<br>$\oplus (1, 4, 1, 1, 1, 1, 1, 2)$   |
| 336                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 6}$                         | $(4, 1, 2, 2, 1, 1, 1, 1) \oplus (\bar{4}, 1, 1, 1, 2, 1, 1, 1) \oplus (\bar{4}, 1, 1, 1, 1, 2, 1, 1) \oplus (1, \bar{4}, 1, 1, 1, 1, 2, 1)$<br>$\oplus (1, 4, 1, 1, 1, 1, 1, 2)$   |
| 337                                | $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 6}$                         | $(\bar{4}, 1, 2, 1, 1, 1, 1, 1) \oplus (\bar{4}, 1, 1, 2, 1, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 2, 1, 1, 1) \oplus (4, 1, 1, 1, 1, 2, 1, 1)$<br>$\oplus (1, \bar{4}, 1, 1, 1, 1, 2, 1) \oplus (1, 4, 1, 1, 1, 1, 1, 2)$                |
| 338                                | $\mathfrak{su}(4)^{\oplus 3} \oplus \mathfrak{su}(2)^{\oplus 5}$                         | $(\bar{4}, 1, 1, 2, 1, 1, 1, 1) \oplus (1, \bar{4}, 1, 2, 1, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1, 1) \oplus (1, 4, 1, 1, 1, 2, 1, 1)$<br>$\oplus (1, 1, \bar{4}, 1, 1, 1, 2, 1) \oplus (1, 1, 4, 1, 1, 1, 1, 2)$                   |
| 339                                | $\mathfrak{su}(4)^{\oplus 3} \oplus \mathfrak{su}(2)^{\oplus 5}$                         | $(\bar{4}, 1, 1, 2, 1, 1, 1, 1) \oplus (1, \bar{4}, 1, 2, 1, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1, 1) \oplus (1, 4, 1, 1, 1, 2, 1, 1)$<br>$\oplus (1, 1, \bar{4}, 1, 1, 1, 2, 1) \oplus (1, 1, 4, 1, 1, 1, 1, 2)$                   |
| 340                                | $\mathfrak{su}(4)^{\oplus 3} \oplus \mathfrak{su}(2)^{\oplus 6}$                         | $(\bar{4}, 1, 1, 2, 1, 1, 1, 1, 1) \oplus (4, 1, 1, 1, 2, 1, 1, 1, 1) \oplus (1, \bar{4}, 1, 1, 1, 2, 1, 1, 1)$<br>$\oplus (1, 4, 1, 1, 1, 1, 2, 1, 1) \oplus (1, 1, \bar{4}, 1, 1, 1, 1, 2, 1) \oplus (1, 1, 4, 1, 1, 1, 1, 1, 2)$ |

Separately, the algebra  $\mathfrak{su}(4) \oplus \mathfrak{sp}(6) \oplus \mathfrak{sp}(6)$  had been discussed prior to the publication of this paper in Refs. [1,2].

[1] T. K. Kuo and N. Nakagawa, The generation problem and the symplectic group, *Nucl. Phys.* **B250**, 641 (1985).

[2] T. K. Kuo and N. Nakagawa, An  $Sp_L(6) \times U_Y(1)$  extension of the electroweak theory, *Phys. Rev. D* **30**, 2011 (1984).