Fundamental scalar field with zero dimension from anomaly cancellations

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In this article a novel mechanism for dynamical electroweak symmetry breaking and the ensuing appearance of fermion mass terms in the action is proposed. The action contains massless fermions of the standard model coupled to gravity through a new type of nonminimal coupling to the vielbein field. The corresponding coupling constants in our approach become zero-dimension scalar fields. Such scalar fields provide the cancellation of the Weyl anomaly [Latham Boyle and Neil Turok, arXiv:2110.06258].

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I. INTRODUCTION

In this article a novel mechanism of dynamical symmetry breaking, thus leading to the appearance of fermion mass terms in the action, is put forward. The assumed action contains the full set of standard model (SM) fermions coupled to gravity through a new type of nonminimal coupling to the vielbein field. In contrast to the fermion-vielbein coupling via a constant term [1], we promote this coupling from a constant to being an actual field in its own right. Correspondingly, additional distinct scalar fields are introduced in the theory that furnish a set of coupling coefficients between the various different flavors of SM fermions to the vielbein. Adding scalar fields to the SM action is known to imply the cancellation of the Weyl anomaly [2]. We propose an action made up from the SM fermions coupled to the vielbein and fundamental scalar fields that (i) are the very coupling coefficients between the SM fermions and the vielbein, (ii) lead directly to the appearance of mass terms of the fermions, and (iii) generate a theory with a vanishing Weyl anomaly. This stands as a new model of the composite Higgs boson.

The Higgs boson itself has a rich history, beginning with the discovery that the *Higgs mechanism* [3–8] is responsible for the presence of mass terms in the SM, where the introduction of the Higgs boson in the action spontaneously breaks local symmetries thus ascribing mass terms to the massive bosons and fermions. Explicitly, the SM comprises the $SU(3) \times SU(2) \times U(1)$ gauge model of strong and electroweak interactions [9–14]. Through the Higgs mechanism the electroweak $SU(2) \times U(1)$ group is spontaneously broken down to the $U(1)$ symmetry of electromagnetism [3,6,7,15–17]. Couplings of the elementary Higgs scalar bosons also break quark and lepton chiral-flavor symmetries, giving rise to hard mass terms for the quarks and leptons. The Higgs boson itself has been detected as the scalar excitation with a mass found at 125 GeV [18–23], consistent with theoretical predictions.

Despite the success of the Higgs mechanism, the model admits a number of ambiguities [24]; perhaps the most striking one is the triviality of the model with fundamental scalar fields, as well an emerging hierarchy problem with vastly disparate energy scales in the theory, and more. This led to the question, if indeed the Higgs is a new fundamental particle of the SM in its own right, or rather is the Higgs itself composed of known fundamental particles of the SM?

Composite Higgs models began with the suggestion by Terazawa *et al.* that due to its large mass, the top quark is a natural candidate for being a constituent of the composite Higgs boson [25,26]. In 1989, the original top-quark condensate model of Terazawa *et al.* was revived by Miransky, Tanabashi, Yamawaki, and later by Bardeen *et al.* [27–30], who proposed the top-quark condensate as a means of electroweak symmetry breaking. In this approach the Higgs is composite at short distance scales. Nevertheless this model conflicts with experimental observations on a number of counts. In these models the energy scale of the new dynamics was assumed to be at about 10^{15} GeV. Consistent with this value, these models give naive predictions for the Higgs mass at approximately $2m_t \sim 350$ GeV [26–31], and present experimental data rule out such a value. A smaller value of the Higgs boson mass might be obtained if the calculations rely on large renormalization group corrections [27–30], due to the running of coupling constants between the working scale (10^{15} GeV) and the electroweak scale (100 GeV). At any rate this running is not able to explain the observed appearance of the Higgs mass at around 125 GeV. Even more, these models require fine-tuning of numerous parameters in order to match experimental results. But that aside, the models with four-fermion interactions should be taken as the phenomenological ones, in which only the leading order in the $1/N_c$ expansion is taken into account.

The *top seesaw* model was advocated initially by Chivukula *et al.* [32] in response to the difficulties of the earlier top-quark condensate models. The model admits, in addition to the top quark, an additional heavy fermion, χ [32,33]. In particular, in the top seesaw model there is no need for fine-tuning of parameters, and a later version of the model by Dobrescu and Cheng [34] includes a light composite Higgs boson.

In the 1970s the technicolor (TC) model was formulated [35,36] as an attempt to resolve the shortcomings of the SM already discussed, while providing sound theoretical predictions for the Higgs mass. TC is a gauge theory of fermions with no elementary scalars, yet at the same time inclusive of dynamical electroweak symmetry breaking and flavor symmetry breaking. In the TC model the Higgs, instead of being a fundamental scalar, is composed of a new class of fermions called technifermions interacting via technicolor gauge bosons. This interaction is attractive and as a result, by analogy with Bardeen-Cooper-Schrieffer (BCS) superconductor theory, leads to the formation of fermion condensates. However, TC theory fails to produce mass terms for the quarks and leptons. For this reason the model was extended to the extended technicolor (ETC) model [37,38], but this model is inconsistent with experimental constraints on flavor changing neutral currents, and precision electroweak measurements. Later on an improved TC model called the walking technicolor model [39,40] was constructed that resolves these issues, although it still fails to predict correctly the measured value of the top-quark mass. For a detailed pedagogical review of the TC and ETC models see Refs. [24,41].

After the technicolor models came the idea that the Higgs boson appears as a pseudo-Goldstone boson, as noted originally in Ref. [42]. This idea forms the basis of *little Higgs models* [43–48]. The model stems a well-known result of Goldstone's theorem, namely that spontaneous breaking of a global symmetry yields massless scalar particles, or *Goldstone bosons*. By selecting the right global symmetry, it is possible to have Goldstone bosons that correspond to the Higgs doublet in the SM.

Subsequently the partial compositeness model was conjectured, by Kaplan [49] initially. Therein each SM particle has a heavy partner that can mix with it. Like this, the SM particles are linear combinations of elementary and composite states via a mixing angle. The elegance of the partial compositeness model is its simplicity, without the need for deviations beyond the standard model, thus relying on the known fundamental particles only [50].

More recently, among a variety of theoretical papers that appeared between December 2015 and August 2016, there are several that consider both the 125-GeV Higgs boson, and the hypothetical new heavier Higgs boson as composite due to the new strong interaction, as noticed for example in Refs. [51,52]. In a different approach there are other papers devoted to the description of the composite nature of the heavier (750-GeV) Higgs boson only [53–66].

Other more modern composite Higgs models consist of the walking model [67,68], the ideal walking model [69–72], the technicolor scalar model [73], Sannino's model of the generalized orientafold gauge theory approach to electroweak symmetry breaking [74], and the walking model in higher-dimensional $SU(N)$ gauge theories of Dietrich and Sannino [75,76]. A more comprehensive review of modern composite Higgs models can be found in [77].

Quantum gravity in the first-order formalism with either the Palatini action [78,79] or the Holst action [80] shares a property with the above composite Higgs models: it leads to a four-fermion interaction. This four-fermion interaction is now between spinor fields coupled in a minimal way to the torsion field [81,82], but importantly this four-fermion interaction leads to fermion condensates of the type that could constitute the composite Higgs. A similar idea has already been suggested in Ref. [83], namely that the torsion field coupled in a nonminimal way to fermion fields [84–86] could be a source of dynamical electroweak symmetry breaking.

This type of action with fermion fields coupled to the vielbein is one of the main ingredients of our action. The other main ingredient flows from a theory that contains fundamental scalar fields with zero-mass dimension recently proposed in [2], as a way of simultaneously canceling both the vacuum energy and the Weyl anomaly. Starting with a theory of the SM coupled to gravity, the fermion and gauge fields are coupled to a background classical gravitational field, with a right-handed neutrino for each generation also coupled to the background field. This representation is a quantum field theory (QFT) on a classical background spacetime. The Weyl anomaly is a measure of the failure of the classically Weyl-invariant theory to define a Weyl-invariant quantum theory, and can be expressed in terms of the number of different fields present in the theory. In the SM there are $n_0 = 4$ ordinary real scalars in the usual complex Higgs doublet, $n_{1/2} = 3 \times 16 = 48$ Weyl spinors (16 per generation), $n_1 = 8 + 3 + 1 = 12$ gauge fields of $SU(3) \times SU(2) \times U(1)$, and a single gravitational field ($n_2 = 1$). With these combinations the Weyl anomaly is nonzero. However, suppose that the Higgs and graviton fields are not fundamental fields but rather composite, such that their contributions to the vacuum energy and Weyl anomaly can be dropped, and $n_0 = n_2 = 0$. The implication is that by introducing $n'_0 = 36$ scalars with zero-mass dimension, then not only does the Weyl anomaly vanish, but also the vacuum energy vanishes as well.

This embodies our motivation for the starting point of this work: a model comprising Dirac fermions of the SM with a nonminimal coupling to gravity, that includes 36 fundamental scalar fields. The details of the construction of the theory is described in full in Sec. II A. We find that in this model, the theory admits mass terms for the fermions. By using the Schwinger-Dyson equation for the fermion self-energy function, a relationship is derived between the top-quark mass, the Planck mass (the ultraviolet cutoff point of the loop integral), and the strength of the inverse coupling between the fermion fields and the vielbein. From this relation an estimation for the mass of the top-quark can be extracted.

This paper is organized as follows. In Sec. II the first main ingredient of the action is constructed, the piece that includes Dirac fermions coupled nonminimally to the vielbein field that describes the background gravitational field. of the action is formulated, namely the dynamical piece of the fundamental scalar fields described above. In Sec. IV the Schwinger-Dyson approach is implemented to calculate the leading-order piece of the self-energy of the dominant scalar field in the action, namely the scalar field that couples to the top quark, being the heaviest flavor. In this section the main result of the paper is derived, namely the emergence of a mass term for the top quark. A relationship between the top-quark mass, the coupling of the scalars to the vielbein (gravitational) field, and the cutoff energy of the loop integral (the Planck mass) is used to estimate the leading-order contribution to the top-quark mass. In Sec. V the main conclusions of the paper are summarized along with prospects for future research. There are two appendixes: Appendix A contains the main formulas for a Wick rotation used in the computation of the one-loop integral, and Appendix B contains full details of the angular integrals in the one-loop integral.

Throughout this article lowercase Latin letters $a, b, c\ldots = 0, 1, 2, 3$ label internal Lorentz indices, Greek letters $\mu, \nu, \ldots = 0, 1, 2, 3$ label spacetime indices, and ϵ_{abcd} is the completely antisymmetric Levi-Civita symbol. Lorentz indices are raised and lowered by the Minkowski metric η_{ab} and spacetime indices are raised and lowered by a spacetime metric $g_{\mu\nu}$. Metrics are assumed to have a mostly negative signature, and specifically the Minkowski metric is $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$.

II. FUNDAMENTAL SCALARS IN MODELS WITH THREE GENERATIONS OF SM FERMIONS

A. Nonminimal coupling of fermions with gravity

The action for Dirac fermions coupled to the vielbein field can be written as [81,87]

$$S = \frac{1}{6}\epsilon_{abcd}\int d^4x\theta^a \wedge e^b \wedge e^c \wedge e^d, \tag{1}$$

where

$$\theta^a = \frac{i}{2}[\bar{\psi}\gamma^a D_\mu\psi - \overline{D_\mu\psi}\gamma^a\psi]dx^\mu, \tag{2}$$

with γ^a the usual Dirac matrices, $\bar{\psi} = \psi^\dagger\gamma^0$, and $D_\mu\psi$ is the covariant derivative acting on the Dirac fermion field defined by [88]

$$D_\mu\psi = \partial_\mu\psi - \frac{i}{2}\gamma_{ab}\omega^{ab}{}_\mu\psi, \tag{3}$$

where $\gamma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ and $\omega^{ab}{}_\mu$ is the spin connection. The fields e^a are the vielbein fields that describe the background gravitational field, related to the spacetime metric g through the completion relation $\eta_{ab}e^a{}_\mu e^b{}_\nu = g_{\mu\nu}$. The action in (1) is a functional of e alone, as opposed to being a functional of both e and ω. Explicitly, ω becomes dependent on e by imposing Cartan's first structure equation.

In turn, the action for the right-handed Weyl fermions may be written as Eq. (1) with $\theta^a \to \theta^a_R$, where

$$\theta^a_R = \frac{i}{2}[\bar{\psi}_R\sigma^a D_\mu\psi_R - \overline{D_\mu\psi}_R\sigma^a\psi_R]dx^\mu, \tag{4}$$

while the action for the left-handed Weyl fermions may be written as Eq. (1) with $\theta^a \to \theta^a_L$ and

$$\theta^a_L = \frac{i}{2}[\bar{\psi}_L\bar{\sigma}^a D_\mu\psi_L - \overline{D_\mu\psi}_L\bar{\sigma}^a\psi_L]dx^\mu. \tag{5}$$

A nonminimal coupling of fermions to gravity is achieved via the substitution [1]

$$\theta^a_R \to \theta^a_R + \frac{i}{2}[\bar{\psi}_R\xi_R\sigma^a D_\mu\psi_R - \xi^*_R\overline{D_\mu\psi}_R\sigma^a\psi_R]dx^\mu \tag{6}$$

and

$$\theta^a_L \to \theta^a_L + \frac{i}{2}[\bar{\psi}_L\xi_L\bar{\sigma}^a D_\mu\psi_L - \xi^*_L\overline{D_\mu\psi}_L\bar{\sigma}^a\psi_L]dx^\mu, \tag{7}$$

with complex-valued constants ξ_R and ξ_L. Notice that in the absence of the spin connection, $D_\mu = \partial_\mu$ and the imaginary parts of ξ_R and ξ_L decouple and do not interact with fermions. At the same time the real parts of these constants may be absorbed by a rescaling of the fermion fields.

That said, suppose that this construction is extended to the case where ξ_R and ξ_L become coordinate-dependent fields. In this case both the real and imaginary parts of ξ_R and ξ_L interact with the fermions of the theory, and cannot be removed by any rescaling of the fields. Furthermore, let ξ_R and ξ_L be assigned flavor indices. In the presence of $SU(3)\otimes SU(2)\otimes U(1)$ gauge fields, there are $n_1 = 8 + 3 + 1 = 12$ vector fields. In accordance the number of Weyl fermions must equal $n_{1/2} = 4n_1 = 48$. This number coincides with the number of Weyl fermions in the SM with three generations: $n_{1/2} = (1_{\text{leptons}} + 3_{\text{quarks}}) \times 2_{\text{up \& down}} \times 2_{\text{left \& right}} \times 3_{\text{generations}} = 48$.

B. Parity-breaking interactions with zero-dimension scalar fields

The theory can be extended one stage further to the case where ξ_L and ξ_R carry not only flavor indices but also generation indices. Even more they can be further distinguished by assigning different forms for the matrices ξ for the left-handed and the right-handed particles, while ξ the matrices for the quarks and leptons remain identical. In this approach the fermion term θ^a in (1) now reads

$$\theta^a = \theta^a_L + \theta^a_R, \tag{8}$$

where

$$\theta^a_R = \frac{i}{2}[\bar\psi_R(1+\xi_R)\sigma^a D_\mu\psi_R - (D_\mu\bar\psi_R)(1+\xi_R^\dagger)\sigma^a\psi_R]dx^\mu, \tag{9}$$

and

$$\theta^a_L = \frac{i}{2}[\bar\psi_L(1+\xi_L)\bar\sigma^a D_\mu\psi_L - (D_\mu\bar\psi_L)(1+\xi_L^\dagger)\bar\sigma^a\psi_L]dx^\mu. \tag{10}$$

This gives rise to $n_0' = 2_{\text{chiralities}} \times 2_{\text{real \& imaginary parts}} \times 3_{\text{generations}} \times 3_{\text{generations}} = 36$ components of the scalar fields. This is precisely the number of scalars $n_0' = 3n_1$ needed for the cancellation of the Weyl anomaly [2].

C. Interactions that conserve parity

There would be the same number of scalar fields as above if ξ_L were identical to ξ_R, but with the matrices ξ being different for quarks and leptons. In this case the interaction with ξ does not break parity. The fermion action is still defined as in Eq. (1), but instead of Eq. (8),

$$\theta^a = \theta^a_l + \theta^a_q, \tag{11}$$

where

$$\theta^a_q = \frac{i}{2}[\bar\psi_q(1+\xi_q)\gamma^a D_\mu\psi_q - (D_\mu\bar\psi_q)(1+\xi_q^\dagger)\gamma^a\psi_q]dx^\mu, \tag{12}$$

and

$$\theta^a_l = \frac{i}{2}[\bar\psi_q(1+\xi_l)\gamma^a D_\mu\psi_l - (D_\mu\bar\psi_l)(1+\xi_l^\dagger)\gamma^a\psi_l]dx^\mu. \tag{13}$$

Here both ξ_l and ξ_q are the 3×3 complex-valued matrices in flavor space. As before, $n_0' = 2_{\text{leps\&qrks}} \times 2_{\text{real\&imagparts}} \times 3_{\text{generations}} \times 3_{\text{generations}} = 36$ zero-dimension scalar fields.

D. Φ fields

It is worth mentioning that the θ field of Eq. (12) may be replaced in a different way in order to produce the zero-dimension scalar fields:

$$\theta^a \to \frac{i}{2}[\bar\psi\gamma^a D_\mu\psi - (D_\mu\bar\psi)\gamma^a\psi]dx^\mu + \frac{i}{2}[\bar\psi^A_L\Phi^a_{AB}D_\mu\psi^B_R - (D_\mu\bar\psi_R)^B\Phi^{+,a}_{AB}\psi^A_L]dx^\mu. \tag{14}$$

Here Φ is the $SU(2)$ doublet that carries in addition the generation index A and the flavor B of the right-handed fermions. (We may take e, μ, τ for leptons, and d, s, b, and/or u, c, t for quarks.) This expression is the further extension of the nonminimal coupling of fermions to gravity, in which the above-mentioned constants ξ not only become fields, but are, in addition, transformed under the $SU(2)_L$ group. For example, we can take $A = 1, 2, 3$ corresponding to the three generations of quarks, and $B = 1, 2, 3$ corresponding to the u, c, t quarks. As a result we obtain $4\times 4\times 3 = 144$ real-valued components of these scalar fields. This number is larger than the one needed for the cancellation of Weyl anomaly [2]. The minimal choice here is if we consider the singlets in flavor space, which gives only $4 = 16$ components of the scalars—the number that is smaller than the required 32 components. This scheme therefore cannot be used directly to build the theory with the cancellation of Weyl anomaly. The other ingredients should be added in order to match this condition. Therefore, in the following we will concentrate on the scheme proposed above in Sec. II C.

III. ELECTROWEAK SYMMETRY BREAKING AND MASS GENERATION

Consider the SM with three generations and the fundamental scalar ξ fields constructed in Sec. II C. With variations of the vielbein and spin connection restrained, define an action for the zero-dimension scalar fields as

$$S_B = \alpha^{uv}_{ABCD}\int d^4x(\xi^{AB}_u)^*\Box^2\xi^{CD}_v, \tag{15}$$

where $A, B, C, D = 1, 2, 3$ are generation indices, $u, v = q, l$ are flavor indices, and α^{uv}_{ABCD} are a set of coupling constants. The effective four-fermion interactions arise from the exchange by quanta of the fields ξ_q and ξ_l. The effective four-fermion interaction is nonlocal.

Consider now a certain sector of the theory that describes the dominant contributions of the field ξ^{33}_q. Add this piece of the action to the action in (1), with θ^a given by (11), to obtain

$$S = \int d^4x \left[\alpha\xi^*\Box^2\xi + \frac{i}{2}\bar{Q}(1+\xi)\not{D}Q - \frac{i}{2}\overline{\not{D}Q}(1+\xi^*)Q\right], \tag{16}$$

where $\not{D} = \gamma^\mu D_\mu$, $\xi = \xi_q^{33}$, $\alpha = \alpha_{3333,}^q$ and

$$Q_3(x) = \begin{pmatrix} t \\ b \end{pmatrix} \tag{17}$$

is the third generation of quarks. Note that the covariant derivative D contains gauge fields. Define an effective action, S_4 as $e^{iS_4} = \frac{1}{Z_0}\int D\xi D\xi^* e^{iS}$. After integrating out the scalar fields the corresponding effective action is found to be

$$S_4 = -\frac{1}{4\alpha}\int d^4x d^4y \bar{Q}_3(x)\gamma^\mu D_\mu Q_3(x) \times \Box^{-2}(x,y) D_\nu \bar{Q}_3(y)\gamma^\nu Q_3(y), \tag{18}$$

where $\Box^{-2}(x,y)$ is the square of the inverse propagator of the boson field ξ_q^{33}. The one-loop contribution to the two-point Green function is straightforwardly read off Eq. (18). The form of the self-energy of the thirdgeneration quarks then follows, as written in Eq. (19) below. The corresponding Feynman diagram is shown in Fig. 1.

IV. SCHWINGER-DYSON APPROACH FOR CALCULATING FERMION MASS

At this point the discussion follows the method of [89], itself based on [90]. The idea of this method is to write down the Schwinger-Dyson equation in order express the inverse of the full propagator, $D^{-1}(p)$ in terms of the self-energy function $\Sigma(p)$ in the form of a simplified ansatz, through which $\Sigma(p)$ can be determined. The self-energy function, $\Sigma(p)$ of the third-generation quarks through leading order, has the form

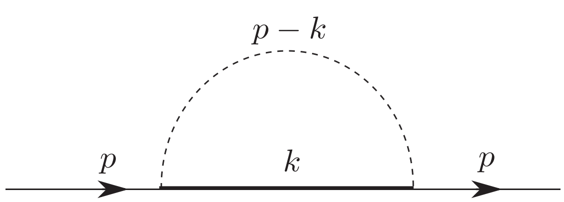

FIG. 1. Feynman diagram of the fermion self-energy corresponding to Eq. (19). The incoming and outgoing lines are initial and final fermion states with momentum p. The dashed line in the loop is a scalar boson propagator carrying virtual momentum $p-k$, and the thick horizontal line corresponds to the full fermion propagator with renormalized mass $\Sigma(k)$ inclusive of all self-energy corrections, as related in Eq. (19).

$$\Sigma(p) = \frac{1}{\alpha}\int đ^4k \gamma k \frac{i}{\gamma k - \Sigma(k)}\gamma k \frac{1}{(p-k)^4}, \tag{19}$$

where the standard notation $\int đ^4k(\ldots) = (2\pi)^{-4}\int d^4k(\ldots)$ has been used. Now apply a Wick rotation (see Appendix A for details) to obtain

$$i\Sigma(p) = \frac{1}{\alpha}\int đ^4k_E \gamma_E k_E \frac{1}{(\gamma_E k_E - i\Sigma(k))}\gamma_E k_E \frac{1}{(p_E-k_E)^4}. \tag{20}$$

Since all expressions from now on are in terms of Euclidean-space variables the subscript "E" can be dropped.

The ansatz for the Schwinger-Dyson equation has the form

$$D^{-1}(p) = A(p^2)\gamma p - iB(p^2) = \gamma p - i\Sigma(p). \tag{21}$$

The right-hand side of (21) may be substituted inside (20). To keep notation brief it is useful to use the shorthand $A = A(k^2)$ and $B = B(k^2)$. The resulting expression is

$$i\Sigma(p) = \frac{1}{\alpha}\int đ^4k \gamma k \frac{(A\gamma k + iB)}{(A^2k^2 + B^2)}\gamma k \frac{1}{(p-k)^4}. \tag{22}$$

By (A3), $(\gamma k)^2 = \gamma^\mu\gamma^\nu k_\mu k_\nu = \frac{1}{2}\delta^{\mu\nu}k_\mu k_\nu = k^2$, and hence (22) becomes

$$i\Sigma(p) = \Sigma_1(p) + \Sigma_2(p), \tag{23}$$

where

$$\Sigma_1(p) \coloneqq \frac{1}{\alpha}\int đ^4k \gamma k \frac{k^2A}{(A^2k^2+B^2)}\frac{1}{(p-k)^4}, \tag{24}$$

$$\Sigma_2(p) \coloneqq \frac{i}{\alpha}\int đ^4k \frac{k^2B}{(A^2k^2+B^2)}\frac{1}{(p-k)^4}. \tag{25}$$

$\Sigma_1(p)$ needs to be in a form in which γp appears explicitly such that $A(p^2)$ can be easily read off (21) by comparing coefficients. Multiply Σ_1 by γp to obtain

$$\frac{1}{4}\mathrm{Tr}\gamma p\Sigma_1(p) = \frac{-1}{2\alpha}\int đ^4k \frac{k^2A}{(A^2k^2+B^2)} \times \left(\frac{1}{(p-k)^2} - \frac{p^2+k^2}{(p-k)^4}\right). \tag{26}$$

Now multiply (26) by γp, bearing in mind that by (A3) $(\gamma p)^2 = p^2$, to obtain

$$p^2\Sigma_1(p) = \frac{-1}{2\alpha}\gamma p \int đ^4k \frac{k^2 A}{(A^2k^2+B^2)} \times \left(\frac{1}{(p-k)^2} - \frac{p^2+k^2}{(p-k)^4}\right). \quad (27)$$

In this form the γp term is explicit and (23) reads

$$i\Sigma(p) = \frac{-1}{2\alpha p^2}\gamma p \int đ^4k \frac{k^2 A}{(A^2k^2+B^2)}\left(\frac{1}{(p-k)^2} - \frac{p^2+k^2}{(p-k)^4}\right) + \frac{i}{\alpha}\int đ^4k \frac{k^2 B}{(A^2k^2+B^2)}\frac{1}{(p-k)^4}. \quad (28)$$

Substitute (28) back into (21) and compare coefficients on both sides to obtain the integral equations

$$A(p^2) = 1 + \frac{1}{2\alpha p^2}\int đ^4k \frac{k^2 A}{(A^2k^2+B^2)} \times \left(\frac{1}{(p-k)^2} - \frac{p^2+k^2}{(p-k)^4}\right), \quad (29)$$

$$B(p^2) = \frac{1}{\alpha}\int đ^4k \frac{k^2 B}{(A^2k^2+B^2)}\frac{1}{(p-k)^4}. \quad (30)$$

Write (29) and (30) in terms of four-dimensional polar coordinates:

$$\begin{aligned} A(p^2) &= 1 + \frac{1}{2\alpha p^2}\frac{1}{(2\pi)^4}\int_0^\infty dk \int_0^\pi d\theta_1 \int_0^\pi d\theta_2 \\ &\quad \times \int_0^{2\pi} d\theta_3 k^3 \sin^2\theta_1 \sin\theta_2 \frac{k^2 A}{(A^2k^2+B^2)}\left(\frac{1}{p^2+k^2-2pk\cos\theta_1} - \frac{p^2+k^2}{(p^2+k^2-2pk\cos\theta_1)^2}\right) \\ &= 1 + \frac{\pi}{\alpha p^2}\frac{1}{(2\pi)^4}\int_0^\infty dk^2 \int_0^\pi d\theta_1 \sin^2\theta_1 \frac{k^4 A}{(A^2k^2+B^2)}\left(\frac{1}{p^2+k^2-2pk\cos\theta_1} - \frac{p^2+k^2}{(p^2+k^2-2pk\cos\theta_1)^2}\right). \end{aligned} \quad (31)$$

$$\begin{aligned} B(p^2) &= \frac{1}{\alpha}\frac{1}{(2\pi)^4}\int_0^\infty dk \int_0^\pi d\theta_1 \int_0^\pi d\theta_2 \int_0^{2\pi} d\theta_3 k^3 \sin^2\theta_1 \sin\theta_2 \frac{k^2 B}{(A^2k^2+B^2)}\frac{1}{(p^2+k^2-2pk\cos\theta_1)^2} \\ &= \frac{2\pi}{\alpha}\frac{1}{(2\pi)^4}\int_0^\infty dk^2 \int_0^\pi d\theta_1 \sin^2\theta_1 \frac{k^4 B}{(A^2k^2+B^2)}\frac{1}{(p^2+k^2-2pk\cos\theta_1)^2}. \end{aligned} \quad (32)$$

The θ_1 integrals have the forms

$$I_1 = \int_0^\pi d\theta_1 \frac{\sin^2\theta_1}{a - b\cos\theta_1}, \quad (33)$$

$$I_2 = \int_0^\pi d\theta_1 \frac{\sin^2\theta_1}{(a - b\cos\theta_1)^2}, \quad (34)$$

where $a = p^2 + k^2 > 0$ and $b = 2pk$, such that (31) and (32) read

$$A(p^2) = 1 + \frac{\pi}{\alpha p^2}\frac{1}{(2\pi)^4}\int_0^\infty dk^2 \times \frac{k^4 A}{(A^2k^2+B^2)}(I_1 - (p^2+k^2)I_2), \quad (35)$$

$$B(p^2) = \frac{2\pi}{\alpha}\frac{1}{(2\pi)^4}\int_0^\infty dk^2 \frac{k^4 B}{(A^2k^2+B^2)} I_2. \quad (36)$$

The values of the integrals I_1 and I_2 are calculated in Appendix B. Their final forms are given in Eqs. (B16) and (B17). Substitute them in (35) and (36) to yield

$$A(p^2) = 1 + \frac{1}{16\pi^2\alpha p^2}\int_0^\infty dk^2 \frac{Ak^4}{(A^2k^2+B^2)}\left[\frac{p^2}{k^2(p^2-k^2)}\theta(k-p) + \frac{k^2}{p^2(k^2-p^2)}\theta(p-k)\right] \quad (37)$$

and

$$B(p^2) = \frac{1}{16\pi^2\alpha}\int_0^\infty dk^2 \frac{Bk^4}{(A^2k^2+B^2)}\left[\frac{1}{k^2(k^2-p^2)}\theta(k-p) + \frac{1}{p^2(p^2-k^2)}\theta(p-k)\right]. \quad (38)$$

Clearly the integrands in both expressions diverge at $k = p$. However, the integration limits will be adjusted to take values that agree with current experimental data, such that this singular point will lie outside of the integration range. This is elucidated in the next paragraph. The next step is to drop the contributions to the loop integrals in (37) and (38) from the region $k < p$ while retaining just the pieces from the region $k > p$. This too is justified in the following paragraph. To aid the discussion below it is useful to expand the remaining terms in A and B in powers of p^2 to yield

$$A(p^2) = 1 + \frac{1}{16\pi^2\alpha}\int_0^\infty dk^2 \frac{A}{(A^2k^2 + B^2)}\left[-1 - \frac{p^2}{k^2} - \frac{p^4}{k^4}\right] + O(p^6) \tag{39}$$

and

$$B(p^2) = \frac{1}{16\pi^2\alpha}\int_0^\infty dk^2 \frac{B}{(A^2k^2+B^2)}\left[1+\frac{p^2}{k^2}+\frac{p^4}{k^4}\right] + O(p^6). \tag{40}$$

The integrals in (37) and (38) are ultraviolet divergent. In contrast, notice that the terms in the expansions in the integrals, (39) and (39), except for the first term (independent of p) are infrared divergent. However, in our scheme we implement an infrared cutoff in addition to an ultraviolet cutoff. Physically the presence of this infrared cutoff is needed in order to satisfy present experimental bounds on the existence of extra scalar excitations. From this bound we derive that the infrared cutoff in the above integral should be of the order $\lambda \sim 1$ TeV. Evidently, the dominant contribution to $B(p^2)$ originates from the large-k region, provided that $\lambda \gg m_t$. The implication is that B is log divergent. The integral over k^2 is assumed to have an upper limit at $k = \Lambda$ with Λ finite. This essentially means that the four-momentum in the loop is bound from above (see Fig. 1). The upshot is that Λ is large to the extent that the value of the integral is saturated by the region where k is close to Λ. In physical terms the loop is dominated by large values of k, or equivalently, the loop is dominated by small distances of the order $1/\Lambda$, with Λ of the order of the Planck mass. As a result it is reasonable to take p to be of the order of 100 GeV, close to the fermion (top-quark) mass. Since 1 GeV is much smaller than the Planck scale, p is effectively zero in the integral.

The leading order (LO) in p^2 contributions to A and B are straightforwardly read off Eqs. (39) and (40). By inserting a cutoff at the ultraviolet and at the infrared end of the spectrum, these LO contributions have the forms

$$A_0 = 1 - \frac{1}{16\pi^2\alpha}\int_{\lambda^2}^{\Lambda^2} dk^2 \frac{A_0}{(A_0^2k^2 + B_0^2)}, \tag{41}$$

$$B_0 = \frac{1}{16\pi^2\alpha}\int_{\lambda^2}^{\Lambda^2} dk^2 \frac{B_0}{(A_0^2k^2 + B_0^2)}. \tag{42}$$

By canceling a constant factor of B_0 on both sides of (42) and substituting a reparametrization of the form $B_0 = m_tA_0$, Eqs. (41) and (42) reduce to

$$A_0 = 1 - \frac{1}{16\pi^2\alpha}\int_{\lambda^2}^{\Lambda^2} dk^2 \frac{A_0}{A_0^2(k^2 + m_t^2)}, \tag{43}$$

and

$$1 = \frac{1}{16\pi^2\alpha}\int_{\lambda^2}^{\Lambda^2} dk^2 \frac{1}{A_0^2(k^2 + m_t^2)}, \tag{44}$$

respectively. Substitute (44) in (43) to obtain

$$A_0 = 1 - A_0 \Rightarrow A_0 = \frac{1}{2}. \tag{45}$$

Now substitute (45) in (44) to find

$$1 = \frac{1}{4\pi^2\alpha}\ln\left(\frac{\Lambda^2 + m_t^2}{\lambda^2 + m_t^2}\right). \tag{46}$$

This gives rise to the existence of a critical value of the coupling constant

$$\alpha_c = \frac{1}{4\pi^2}\ln\left(\frac{\Lambda^2}{\lambda^2}\right) \sim 1.86641. \tag{47}$$

It can thus be inferred that for $\alpha < \alpha_c$ the fermion mass is generated dynamically. As mentioned above, we assume that $\Lambda \gg \lambda \gg m_t$, where λ is the infrared cutoff of the momentum k in the loop. In this regime

$$m_t = \Lambda e^{-2\pi^2\alpha}\sqrt{1 - e^{4\pi^2(\alpha-\alpha_c)}}. \tag{48}$$

It follows from (48) that the generated top mass varies from 0 at $\alpha = \alpha_c$ to a value that approaches Λ at strong coupling $1/\alpha \gg 1$. All other intermediate values of α correspond to physical values of the fermion masses. For example, the top quark mass, which is approximately 175 GeV, is generated for $\alpha = 1.86564$. By comparing this value with the critical value of α, we see that a certain type of the fine-tuning is needed in the given model.

As a consistency check, take the integrals in (37) and (38) but with the cutoffs $k^2 = \lambda^2$ at the lower end and $k^2 = \Lambda^2$ at the upper end of the spectrum, where recall that $\lambda \gg p$ is assumed such that only the $k > p$ contribution survives. Then substitute for A and B inside the integrands, the leading-order values found in (45) and (48). Select the values $\Lambda = 10^{19}$ Gev (the Planck mass), $\lambda = 1$ TeV, $m_t = 175$ GeV, and $\alpha = 1.86564$; then evaluate the integrals over k^2. In this regime, the integrals as functions of p labeled as $\tilde{A}(p)$ and $\tilde{B}(p)$, have the forms

$$\tilde{A}(p) = 1 + \frac{1}{16\pi^2\alpha}\int_{\lambda^2}^{\Lambda^2} dk^2 \frac{A_0k^4}{A_0^2k^2 + B_0^2}\frac{1}{(p^2 - k^2)}, \tag{49}$$

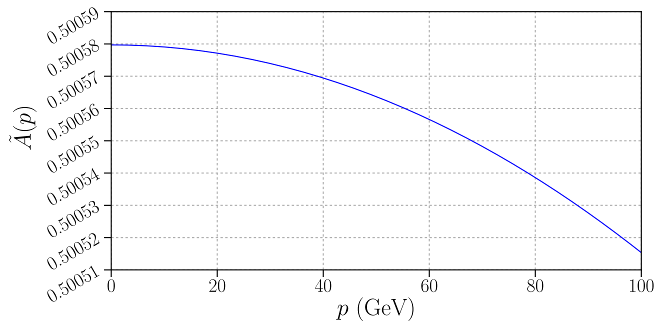

FIG. 2. The function $\tilde{A}(p)$ found by evaluating the integral in Eq. (49) for the values $\Lambda = 10^{19}$ Gev (the Planck mass), $\lambda = 1$ TeV, $m_t = 175$ GeV, and $\alpha = 1.86564$. The values of $\tilde{A}(p)$ are very close to $A_0 = \frac{1}{2}$, as expected.

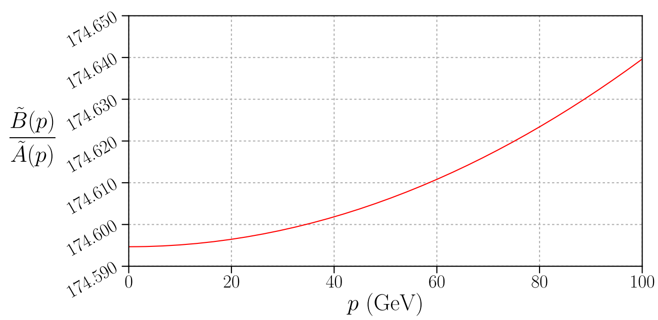

FIG. 3. The function $\tilde{B}(p)/\tilde{A}(p)$ found by evaluating the integral in (50) with the same numerical parameters as Fig. 2. The values of $\tilde{B}(p)/\tilde{A}(p)$ are very close to $m_t = 175$ GeV, as expected.

$$\tilde{B}(p) = \frac{1}{16\pi^2\alpha}\int_{\lambda^2}^{\Lambda^2} dk^2 \frac{B_0 k^2}{A_0^2 k^2 + B_0^2}\frac{1}{(k^2 - p^2)}, \qquad (50)$$

where in Eqs. (49) and (50), $A_0 = \frac{1}{2}$ and $B_0 = \frac{m_t}{2} = 87.5$ GeV should be inserted. The values for $\tilde{A}(p)$ and $\tilde{B}(p)/\tilde{A}(p)$ as functions of p are shown in the plots in Figs. 2 and 3. The values of $\tilde{A}(p)$ are very close to $A_0 = \frac{1}{2}$ as expected, while the values of $\tilde{B}(p)/\tilde{A}(p)$ are very close to $m_t = 175$ GeV, also as expected.

V. CONCLUSION

In this paper we have proposed a novel scheme that could pave the way for the construction of the ultraviolet completion of the SM. The said mechanism, instead of the conventional Higgs boson field, features a set of 36 zero-dimension scalar fields that give rise to dynamical symmetry breaking. In consequence we have presented a mechanism for obtaining fermion mass terms in the SM action. We have implemented this mechanism to the top-quark sector of the action, and shown that it is capable of predicting a value for the top-quark mass in agreement with the measured value, within specific limits imposed on the energy scale of the interactions between the scalars and the top-quark.

These scalar fields are precisely the couplings between the fermions and the vielbein field of the background gravitational field. The form of the coupling we propose is a direct extension of the nonminimal coupling of fermions to the vielbein [1], in which the constant coupling terms are promoted to fields in their own right. We further generalize to a scenario where the coupling terms between the vielbein and the various different fermion flavors are distinctly different fields for each different fermion in the SM action. The outcome is that we obtain a total of 36 scalar fields that constitute the coupling terms. At the same time this is the precise number of zero-dimension scalar fields required to have a vanishing Weyl anomaly [2], this being the case for the action we have formulated.

But more fundamentally, in the model that we propose, the zero-dimension scalar fields have manifestly geometrical origins. We assume that the sector of the theory containing these zero-dimension scalar fields is valid within the range of energy scales between $\lambda \sim 1$ TeV and $\Lambda \sim m_P \sim 10^{19}$ GeV. The presence of the infrared cutoff, λ is necessary in order to satisfy the currently known lower bound on extra scalar fields in the SM. The ultraviolet cutoff, Λ, is naturally of the order of the Plank mass, the scale at which quantum gravity effects are expected to play a significant role.

On grounds that the Weyl anomaly vanishes in the theory presented in this work, then arguably this theory offers a solution for the cosmological constant problem. We argue that within this theory the fermion masses are generated dynamically due to the exchange of such scalars between the SM fermions. We have tested our model by implementing a simplified toy-model version, containing just a doublet of the top quark and bottom quark interacting via the exchange of the zero-dimension scalars. Specifically, we used an approach based on the truncated Schwinger-Dyson equations (the so-called rainbow approximation), which enables a description of dynamical symmetry breaking.Our observations show that there exists a certain critical value of the coupling constant α. For smaller values (strong coupling) the dynamical fermion mass is generated. In order for the fermion mass to be close to the top-quark mass, the value of α must be close to the critical value.

Thinking beyond the results in this article, the toy model discussed here yields equal masses for both the top and bottom quarks. Without doubt this model should be generalized to incorporate different masses for the top and bottom quarks. In particular, a prospective generalized model should have a suppression of the bottom-quark mass in comparison with the top-quark mass. Above all, the interactions between other components of the scalars should be taken into account, which give rise to the smaller masses of these fermions. Further afield the question arises of how to incorporate the generation of the difference in masses between the u and d quarks, the c and s quarks, as well as the difference in masses between the charged leptons and their neutrinos. This generalized theory is beyond the scope of this paper but will be addressed in a follow-up paper.

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APPENDIX A: WICK ROTATIONS

In Minkowski space the γ matrices satisfy the Clifford algebra given by

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \quad (\mu, \nu = 0, 1, 2, 3), \tag{A1}$$

where $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. A wick rotation comprises a transformation of the γ matrices to new matrices, γ_E defined by

$$\gamma_E^4 = \gamma^0, \qquad \gamma_E^i = -i\gamma^i, \quad (i = 1, 2, 3) \tag{A2}$$

such that

$$\{\gamma_E^\mu, \gamma_E^\nu\} = 2\delta^{\mu\nu}, \quad (\mu, \nu = 1, 2, 3, 4). \tag{A3}$$

Four-vector components get transformed under a Wick rotation to new components. For example, given components k^μ of the four-vector k in Minkowski space,

$$k_E^4 = ik^0, \qquad k_E^i = k^i, \quad (i = 1, 2, 3). \tag{A4}$$

The implication is that

$$k^2 = (k^0)^2 - (k^i)^2 = -(k_E^4)^2 - (k_E^1)^2 - (k_E^2)^2 - (k_E^3)^2 \equiv -k_E^2. \tag{A5}$$

$$\gamma k = \gamma^0 k^0 - \gamma^i k_i = -i\gamma_E^4 k_E^4 - i\gamma_E^1 k_E^1 - i\gamma_E^2 k_E^2 - i\gamma_E^3 k_E^3 \equiv -i\gamma_E k_E. \tag{A6}$$

APPENDIX B: ANGULAR INTEGRAL

The integrals in (B1) and (B4) are solved in this appendix, whose forms are

$$I_1 = \int_0^\pi d\theta \frac{\sin^2\theta}{(a - b\cos\theta)}, \qquad I_2 = \int_0^\pi d\theta \frac{\sin^2\theta}{(a - b\cos\theta)^2}, \tag{B1}$$

where

$$a = p^2 + k^2, \qquad b = 2pk. \tag{B2}$$

They both have the same structure, namely

$$I_n = \int_0^\pi d\theta \frac{\sin^2\theta}{(a - b\cos\theta)^n}, \tag{B3}$$

and are evaluated in the same way.

First write it as an integral from 0 to 2π so that it can be converted into a contour integral along a closed circular path in the complex plane. Since $\cos\theta = \cos(2\pi - \theta)$ and $\sin^2\theta = \sin^2(2\pi - \theta)$, then I_n can equally be written as

$$I_n = \int_0^\pi d\theta \frac{\sin^2(2\pi - \theta)}{(a - b\cos(2\pi - \theta))^n} = \int_\pi^{2\pi} d\xi \frac{\sin^2\xi}{(a - b\cos\xi)^n}, \tag{B4}$$

such that

$$I_n = \frac{1}{2}\int_0^{2\pi} d\theta \frac{\sin^2\theta}{(a - b\cos\theta)^n}. \tag{B5}$$

Now write this as a closed integral over the variable $z = e^{i\theta}$ in the complex plane, with $\sin\theta = (-i/2)(z - z^{-1})$ and $\cos\theta = (1/2)(z + z^{-1})$:

$$I_n = \frac{i}{8}\oint dz \frac{2^n z^{n-3}(z^2 - 1)^2}{(2az - bz^2 - b)^n}. \tag{B6}$$

Write the denominator of the integrand as

$$(2az - bz^2 - b)^n = (-b)^n (z - z_1)^n (z - z_2)^n, \tag{B7}$$

where

$$z_1 = \frac{a + \sqrt{a^2 - b^2}}{b}, \qquad z_2 = \frac{a - \sqrt{a^2 - b^2}}{b}, \tag{B8}$$

to bring the integral into the form

$$I_n = 2^{n-3} i \oint dz \frac{z^{n-3}(z^2 - 1)^2}{(-b)^n (z - z_1)^n (z - z_2)^n}. \tag{B9}$$

Now the integral has the familiar form $\oint dz \frac{f(z)}{(z - z_1)^n (z - z_2)^m}$ where the contour is the unit circle with $f(z)$ analytic and continuous at z_1 and z_2, and it can be solved by the standard method of summing over the residues of $f(z)$.

Note carefully the location of the poles. From (B2) it follows that

$$a - b = p^2 + k^2 - 2pk = (p - k)^2 > 0, \tag{B10}$$

and there it is always true that

$$a > b. \tag{B11}$$

Accordingly

$$z_1 = \frac{k}{p}\theta(k-p) + \frac{p}{k}\theta(p-k), \tag{B12}$$

$$z_2 = \frac{p}{k}\theta(k-p) + \frac{k}{p}\theta(p-k). \tag{B13}$$

In conclusion $|z_2| < 1$ while $|z_1| > 1$, so the pole at z_1 is discounted because it is not inside the unit circle. This leaves two poles in the expression (B9): one at $z = 0$ and one at $z = z_2$.

For $n = 2$ the integral in (B9), by the Cauchy formula, has the form

$$\begin{aligned} I_2 &= \frac{i}{2}(2\pi i)\frac{1}{b^2}\left[\left(\frac{(z^2-1)^2}{(z-z_1)^2(z-z_2)^2}\right)\bigg|_{z=0} + \frac{d}{dz}\left(\frac{(z^2-1)^2}{z(z-z_1)^2}\right)\bigg|_{z=z_2}\right] \\ &= -\frac{\pi}{(2pk)^2}\left[\frac{1}{(z_1z_2)^2} - \frac{(z_2^2-1)^2}{z_2^2(z_2-z_1)^2} - \frac{2(z_2^2-1)^2}{z_2(z_2-z_1)^3} + \frac{4(z_2^2-1)}{(z_2-z_1)^2}\right], \end{aligned} \tag{B14}$$

while for $n = 1$ it is

$$\begin{aligned} I_1 &= \frac{i}{4}(2\pi i)\frac{1}{(-b)}\left[\frac{d}{dz}\left(\frac{(z^2-1)^2}{(z-z_1)(z-z_2)}\right)\bigg|_{z=0} + \left(\frac{(z^2-1)^2}{z^2(z-z_1)}\right)\bigg|_{z=z_2}\right] \\ &= \frac{\pi}{4pk}\left[\frac{z_1+z_2}{{z_1}^2{z_2}^2} + \frac{({z_2}^2-1)^2}{{z_2}^2(z_2-z_1)}\right]. \end{aligned} \tag{B15}$$

Finally. substitute the explicit forms of z_1 and z_2 to obtain

$$I_1 = \frac{\pi}{2k^2}\theta(k-p) + \frac{\pi}{2p^2}\theta(p-k), \tag{B16}$$

and

$$I_2 = -\frac{\pi}{2k^2(p^2-k^2)}\theta(k-p) - \frac{\pi}{2p^2(k^2-p^2)}\theta(p-k). \tag{B17}$$

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