Neutron-mirror neutron conversion in the vacuum, a trap, material, and a neutron star

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The possibility of neutron swapping between the ordinary and mirror sectors is today a subject of a great many theoretical and experimental studies. In this paper, we investigate the neutron-mirror neutron transitions in different environments from a vacuum to a neutron star. Our approach is based on the density matrix formalism, Lindblad and Bloch equations and the implication of the seesaw mechanism to the Hamiltonian diagonalization.

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I. INTRODUCTION

The observation of neutron transformation to mirror neutron would be a discovery of fundamental importance. This would demonstrate the existence of a hidden copy of the ordinary particle sector. The idea goes back to a Lee and Yang suggestion [1] to compensate the parity violation in weak interactions by the introduction of the right-handed protons. The concept of mirror world as an independent parallel sector took a distinct form in a seminal paper by Kobzarev, Okun, and Pomeranchuk [2]. Ordinary and mirror particles can communicate via gravitation and via oscillations between neutral particles from both sectors [3–7]. To our knowledge this neutron-mirror neutron $n - n'$ mixing was first considered in [5], where it was also shown that such transitions do not destabilize nuclei. A comprehensive review of the status of the mirror matter concept for 50 years 1956–2006 was given in [8]. The review of the early searches for neutron to mirror neutron conversion was presented in [9]. At the present, mirror matter, and in particular the $n - n'$ transformation, is the focus of intense theoretical and experimental studies. It is not in the scope of the present work to give a review of a great number of relevant publications. An up-to-date situation with regard to performed and planning experiments and a comprehensive list of references may be found in [10]. The most important among the planned experiments is the HIBEAM/NNBAR [10] to be performed at the European Spallation Source [10]. Two significant experiments, namely STEREO [11] and MURMUR [12], deserve a special mention. These are passing-through-wall experiments with nuclear reactor acting as a source of neutrons/hidden neutrons. This experimental strategy is similar to a short-baseline search for sterile neutrinos [13–17].

The present limit for a free $n - n'$ oscillation time obtained by [18] is $\tau_{n-n'} \geq 448$ s (90% C.L.). This result implies the assumption of compensation the Earth's magnetic field and the absence of the mirror magnetic field. Results with these conditions relaxed may be found in [10,19–22]. Results of STEREO [11] and MURMUR [12] experiments are presented in terms of the swapping probability p (see below), which is correspondingly equal to $p < 3.1 \times 10^{-11}$ (95% C.L.) [11], and $p < 4.0 \times 10^{-10}$ (95% C.L.) [12].

An important motivation for the current interest to mirror neutron physics is the $n - n'$ transition inside the neutron star and putative neutron star-MNS transition, where MNS means a mixed neutron-mirror neutron star [23–27]. Possible cosmological and astrophysical manifestations of mirror particles have been discussed since the early days of the mirror matter concept [28–32]. Another reason why the $n - n'$ transition is now a point of attraction is the possibility that dark matter is a mirror twin of the Standard Model [4,29,33,34]. Finalizing this brief introductory review necessary to mention a conjecture that $n - n'$ transition can be responsible for the neutron lifetime anomaly [21,27,35–37]. The neutron lifetime is measured in two types of experiments, the bottle and the beam experiments, see the references above. There is a 4σ discrepancy between the results of the two methods with the beam experiment giving the value of the lifetime higher than the bottle one. Whether this discrepancy may be attributed to $n - n'$ oscillations is an open question. According to [35,37] the most probable reason of discrepancy is the

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drawback of the beam method. In [27] the upper bound for $n-n'$ mixing parameter is deduced from the binary pulsars data with the conclusion that the obtained limit excludes the possibility to explain the neutron lifetime anomaly.

The aim of the present work is to describe the neutron-mirror neutron conversion $n-n'$ using the density matrix formalism. The point is that when the two-state quantum system is embedded into the environment, this approach is the most adequate one. Interaction with the environment leads to the destruction of the density matrix off-diagonal elements and thus to the lost of the coherence. The present work relies on the Lindblad [38,39] and Bloch [40] equations for the density matrix and in one case on the seesaw diagonalization of the Hamiltonian. In our view the cornerstone of this approach is a seminal paper by G. Feinberg and S. Weinberg [41] on the conversion of muonium into antimuonium. A while later this work served as a basis for numerous investigations of the transitions into the mirror world and a related subject of hidden particles production. In recent years the Lindblad equation became a standard tool to describe the multilevel system embedded into the environment. Just two of a great many examples is the investigation of the neutrino oscillations in plasma [42] and heavy quark systems evolution in quark-gluon plasma [43]. Therefore some equations presented below will be either taken for granted or supplemented with minimal explanations. Our work has some overlap with [44–47] as will be indicated below. The detailed numerical calculations will be given elsewhere.

II. DENSITY MATRIX FORMALISM. LINDBLAD EQUATION

We consider the time evolution of the $n-n'$ system embedded in the environment with the vacuum being a particular case. The density matrix of this system has the form

$$\hat{\rho}=\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\equiv\begin{pmatrix}\rho_{1} & x+iy\\ x-iy & \rho_{2}\end{pmatrix}, \qquad (1)$$

where indices 1 and 2 correspond to the neutron and mirror neutron. The second expression for $\hat{\rho}$ in (1) is a convenient form used previously in [45,46]. The relation between the two forms is

$$\rho_1=\rho_{11},\quad \rho_2=\rho_{22},\quad x=\frac{1}{2}(\rho_{12}+\rho_{21}),\quad y=\frac{i}{2}(\rho_{21}-\rho_{12}). \qquad (2)$$

The time evolution of the density matrix is described by the Von-Neumann–Liouville equation

$$\frac{d\hat{\rho}}{dt}=-i[\hat{H},\hat{\rho}], \qquad (3)$$

where $\hat{H}$ is the Hamiltonian of the system. But this is not the whole story if the system under consideration interacts with the environment. In this case the reduced density matrix evolution is given by the Lindblad equation. Prior to the introduction of this equation a historical remark is in order. To our knowledge, the first work in which such an equation was written down and used for the calculation of the muonium to antimuonium conversion is [41] by G. Feinberg and S. Weinberg mentioned in the Introduction. These authors derived the needed equation from the physical arguments long before it was formulated in the general form [38,39] on the mathematical grounds and started to be called Lindblad equation. However, in his later work [48] S. Weinberg used the term Lindblad equation, made reference to [38,39] and did not mention [41].

The Lindblad equation time evolution of the density matrix of an open system has the following form [38,39]

$$\frac{d\hat{\rho}}{dt}=-i[H\hat{\rho}]+L\rho L^{+}-\frac{1}{2}\{L^{+}L,\hat{\rho}\}, \qquad (4)$$

where L is the Lindblad operator, which should satisfy certain conditions [49] but is not known *a priori*, and $\{\ldots\}$ is an anticommutator. A pedagogical derivation of the Lindblad equation may be found in [49].

The Hamiltonian in (4) is Hermitian. Dissipation arising from the interaction with the environment is described by the two terms on the right-hand side of (4) containing the Lindblad operator L. This dissipation is called the Lindblad decoherence. In order to take into account the decay widths of the mass eigenstates (beta decay of n and n') one has to consider the non-Hermitian Hamiltonians. Such generalization of the Lindblad equation has been discussed in literature, see, e.g., [50], and reads

$$\frac{d\hat{\rho}}{dt}=-i(H\hat{\rho}-\hat{\rho}H^{+})+L\rho L^{+}-\frac{1}{2}\{L^{+}L,\hat{\rho}\}, \qquad (5)$$

For $n-n'$ conversion H and L in (4) are [44,51]

$$H=\begin{pmatrix}-\frac{2\pi}{k}nv\mathrm{Re}f(0)+E-i\frac{\gamma}{2} & \varepsilon\\ \varepsilon & E'-i\frac{\gamma'}{2}\end{pmatrix}, \qquad (6)$$

$$L=\sqrt{nv}F,\qquad F=\begin{pmatrix}f(\theta) & 0\\ 0 & 0\end{pmatrix}. \qquad (7)$$

Here E, E', γ, and γ' are the neutron and mirror neutron energies and widths. The energies and the widths may differ either due to the broken mirror symmetry or to the different external conditions like neutron interaction with matter and electromagnetic fields. Magnetic fields, both usual and mirror [19,20], will not be explicitly introduced into the equations. Magnetic field acting in our world is implicitly included into the energy difference $d=E-E'$, while the hypothetical mirror magnetic field, which may lead to intriguing effects [19,52,53], is beyond the scope of this work. The amplitude $f(\theta)$ is the neutron elastic scattering one, n stands for the number density of the surrounding matter, v is the mean relative velocity between the neutron and the matter particles. The term $\frac{2\pi}{k}nv\mathrm{Re}f(0)$ corresponds to the energy shift related to the forward scattering. The mirror neutron is considered as a sterile particle subjected only to the decay with the decay constant γ'. With Eqs. (6) and (7), Eq. (5) yields

$$\dot{\rho}_1 = -2\varepsilon y - (nv\sigma_r + \gamma)\rho_1, \tag{8}$$

$$\dot{\rho}_2 = +2\varepsilon y - \gamma'\rho_2, \tag{9}$$

$$\dot{x} = -Mx + (d+K)y - \frac{1}{2}(\gamma+\gamma')x, \tag{10}$$

$$\dot{y} = -My - (d+K)x + \varepsilon(\rho_1 - \rho_2) - \frac{1}{2}(\gamma+\gamma')y, \tag{11}$$

Here $\sigma_r = \sigma_t - \sigma_e$ is the neutron reaction cross section. It will be assumed that the neutron cross section is saturated by an s wave. The quantities K and M in (10), (11) stand for

$$K = -\frac{2\pi}{k}nv\mathrm{Re}f(0), \qquad M = \frac{2\pi}{k}nv\mathrm{Im}f(0), \tag{12}$$

and $d = E - E'$. Finally, we note that one can write the same set of Eqs. (8)–(11) taking for granted Eq. (23) of [41], namely

$$\frac{d\hat{\rho}}{dt} = -i\tilde{H}\hat{\rho} + i\hat{\rho}\tilde{H}^{+} + 2\pi nv\int d(\cos\theta)\hat{F}(\theta)\hat{\rho}\hat{F}^{*}(\theta), \tag{13}$$

with $\hat{F}(\theta)$ given by (7) and where $\tilde{H}$ differs from (6) by omitting the symbol Re in the amplitude $f(0)$. This way of reasoning has been adopted, e.g., in [45,54]. Still, we wanted to present the Lindblad equation in its original form since it is presently commonly used in particle physics.

The set of differential Eqs. (8)–(11) do not allow simple closed form solutions [55]. Therefore in the next sections we shall resort to the approximations adequate for the given environment.

III. CONVERSION IN VACUUM AND IN THE TRAP

The well-known expression for $n - n'$ oscillations in vacuum can be immediately received from Eqs. (8)–(11). To this end we use the truncated set of equations with $K = M = \gamma = \gamma' = 0$. Taking the derivative of the Eq. (11) for $\dot{y}$ and making obvious substitutions, one gets

$$\ddot{y} + (d^2 + 4\varepsilon^2)y = 0. \tag{14}$$

We consider the transition n to n'. Therefore the initial conditions at $t = 0$ are $\rho_1 = 1, \rho_2 = \rho_{12} = \rho_{21} = 0$. Correspondingly according to (2) the initial condition for y is $y(0) = 0$, and from (11) $\dot{y}(0) = \varepsilon$. With this initial condition, one obtains

$$y = \frac{\varepsilon}{\Omega}\sin\Omega t, \qquad \Omega^2 = d^2 + 4\varepsilon^2. \tag{15}$$

Then Eq. (9) for $\dot{\rho}_2$ yields the result

$$\rho_2 = \frac{4\varepsilon^2}{\Omega^2}\sin^2\frac{\Omega}{2}t. \tag{16}$$

Next we consider transitions in a trap. This process for ultracold neutrons has been experimentally studied in a number of works [56,57]. On the theoretical side we resort to [44,45,58]. A general remark is in order. Decoherence drastically suppresses oscillations if the collision rate with environment is much higher than the oscillation frequency. There are two sources of decoherence in the trap experiments, namely, collisions with the trap walls and with the residual gas inside the trap. Here we consider only collisions with the walls, decoherence due to the presence of low density gas inside the trap was studied in [44].

Let τ_i be the time interval between $(i-1)$th and ith collisions with the walls. It is convenient to introduce a variable $R_z = \rho_1 - \rho_2$, which is the zth component of the Bloch three-vector $\mathbf{R}$ [40]. We shall return to the discussion of the decoherence due to the collisions with the trap walls within the R-matrix formalism at the end of this section. In [45] this variable is called s. Following collisions step by step one arrives at the obvious result [44,45]:

$$R_z(\tau_{n^+}) = \prod_{k=1}^{n}\cos(2\varepsilon\tau_{k^+}). \tag{17}$$

Note that $\varepsilon\tau \ll 1$, where τ is the average collision time, $\tau = t/n$, n is the number of collisions and we assume $n \gg 1$, $\tau \simeq 0.1s$, $\varepsilon^{-1} \geq 448s$ according to [18]. On account of $2\varepsilon\tau_k \ll 1$ and in the approximation of equal time intervals between the collisions, one can represent (17) as

$$R_z \simeq \prod_{k=1}^{n}\left(1 - \frac{(2\varepsilon\tau)^2}{2}\right) \simeq \left(1 - \frac{2\varepsilon^2\tau t}{n}\right)^n \simeq \exp(-2\varepsilon^2\tau t), \tag{18}$$

which coincides with (32) of [45]. We see that collisions with the walls exponentially suppress oscillations. Transition probability after n collisions is equal to

$$\rho_2 = \frac{1}{2}(1 - R_z) \simeq \varepsilon^2\tau t, \tag{19}$$

in line with (35) of [45] and with (39) of [58]. It is interesting to note that one can came to the same result by using the evolution equation for the Bloch three-vector $\mathbf{R}$ defined as

$$\hat{\rho} = \frac{1}{2}(1 + \mathbf{R}\boldsymbol{\sigma}), \tag{20}$$

$$\mathbf{R} = \begin{pmatrix} \rho_{12} + \rho_{21} \\ -i(\rho_{21} - \rho_{12}) \\ \rho_1 - \rho_2 \end{pmatrix} = \begin{pmatrix} 2x \\ -2y \\ \rho_1 - \rho_2 \end{pmatrix}. \tag{21}$$

We shall use the **R**-matrix formalism in the form proposed by L. Stodolsky [59]. Direct examination shows that the Lindblad equation in the form (5), or (8)–(11) is equivalent to

$$\dot{\mathbf{R}} = \mathbf{V} \times \mathbf{R} - D_T \mathbf{R}_T - \gamma \mathbf{R}, \tag{22}$$

where

$$\mathbf{V} = \begin{pmatrix} 2\varepsilon \\ O \\ d + K \end{pmatrix}, \quad D_T = \begin{pmatrix} M & O \\ O & M \end{pmatrix}, \quad R_T \begin{pmatrix} R_x \\ R_y \end{pmatrix}. \tag{23}$$

The physical meaning of the three terms in (22) is quite different. The contribution K in V_Z corresponds to the energy shift due to the refraction index. Alternatively, it may be considered as a supplementary "magnetic field" along the Z axis [59]. The third term in (22) is rather trivial. It corresponds to the shrinking of the Bloch vector **R** in length. It may be set equal to zero without distortion of the physical picture. The most important is the second term with D_T giving the quantum friction. It leads to destroying the off-diagonal elements of the density matrix and the lost of coherence. Using the optical theorem the matrix elements M of D_T are related to the total neutron cross section

$$M = \frac{2\pi}{k} n v \mathrm{Im} f(0) = \frac{1}{2} n v \sigma_t. \tag{24}$$

In order to recover the result (19) for the transition in the trap we take Eq. (11) for $\dot{y}$ in the form

$$\dot{y} = -My + \varepsilon(\rho_1 - \rho_2). \tag{25}$$

Taking the derivative once more, one obtains the following equation for R_z

$$\ddot{R}_z + M\dot{R}_z + 4\varepsilon^2 R_z = 0. \tag{26}$$

This is the equation for oscillator with friction. When $M \gg 4\varepsilon$ it leads to the overdamping solution that at "long" times $t \gg 1/M$ is proportional to

$$R_z \sim \exp\left(-\frac{4\varepsilon^2}{M} t\right), \tag{27}$$

and we return to (19) provided $M = 2/\tau$. Therefore the transition probability is

$$\rho_2(t) = \frac{1}{2}[1 - \exp(-2\varepsilon^2 \tau t)] \simeq \varepsilon^2 \tau t \tag{28}$$

in line with [58]. The general problem of the equivalence between the Lindblad equation and the Bloch vector evolution equation is discussed in [60].

IV. SEESAW MECHANISM IN STRONG ABSORPTION REGIME

Consider now the $n - n'$ conversion in the media with strong neutron absorption. Within the density matrix formalism similar problem has been solved in [44]. Here we turn to the Hamiltonian diagonalization method closely following [61–63]. At the end of this section we shall show how the Lindblad equation works in this case. Consider the limiting case $M \gg |K|$ [see (12)]. This means that we neglect the neutron rescattering. It makes the use of Lindblad equation not obligatory. The quantity M can be presented as

$$M = \frac{2\pi}{k} n v \mathrm{Im} f(0) = \frac{1}{2} n v \sigma \simeq \Gamma/2, \tag{29}$$

where σ is the total neutron cross section. The last equality in (29) may be explained in the following way. The mean free path of the neutron is $L = (n\sigma)^{-1}$. The corresponding propagation time is $t = L/v$, so that $\Gamma \simeq t^{-1} = n v \sigma$. The Hamiltonian reads

$$H = \begin{pmatrix} -iM & \varepsilon \\ \varepsilon & \omega \end{pmatrix}, \tag{30}$$

where we subtracted the part proportional to the unity matrix, $\omega = E' - E = -d$. Diagonalization results in the two eigenvalues

$$\mu_1 \simeq -iM + \varepsilon^2 \frac{iM}{M^2 + \omega^2} - \varepsilon^2 \frac{\omega}{M^2 + \omega^2}, \tag{31}$$

$$\mu_2 \simeq \omega - \varepsilon^2 \frac{iM}{M^2 + \omega^2} + \varepsilon^2 \frac{\omega}{M^2 + \omega^2}. \tag{32}$$

For the degenerate $n - n'$ levels ($\omega = 0$) there is a huge disparity between the two eigenvalues. This is a typical seesaw picture [61]. The wave function evolution is described by [63]

$$\psi(t) = \left(\frac{H - \mu_2}{\mu_1 - \mu_2} e^{-i\mu_1 t} + \frac{H - \mu_1}{\mu_2 - \mu_1} e^{-i\mu_2 t}\right) \begin{pmatrix} a \\ b \end{pmatrix}, \tag{33}$$

where a stands for $\psi_n(0)$ and b for $\psi_{n'}(0)$. In the leading order in ε and assuming $|\varepsilon(\omega + iM)^{-1}| \ll 1$ and $\varepsilon^2(\omega^2 + M^2)^{-1} ML \ll 1$, one obtains

$$\psi(t) = \begin{pmatrix} a e^{-\frac{\Gamma}{2}t} - b \frac{\varepsilon}{\omega + i\frac{\Gamma}{2}} (e^{-\frac{\Gamma}{2}t} - e^{-i\omega t}) \\ b e^{-i\omega t} - a \frac{\varepsilon}{\omega + i\frac{\Gamma}{2}} (e^{-\frac{\Gamma}{2}t} - e^{-i\omega t}) \end{pmatrix}. \tag{34}$$

We are interested in mirror neutron production, so that $a = 1$, $b = 0$, and

$$\psi_{n'}(t) = -\frac{\varepsilon}{\omega + i\frac{\Gamma}{2}}(e^{-\frac{\Gamma}{2}t} - e^{-i\omega t}). \tag{35}$$

For $|\psi_{n'}(t)|^2 = \rho_2(t)$ this yields

$$|\psi_{n'}(t)|^2 = \frac{\varepsilon^2}{\omega^2 + \frac{\Gamma^2}{4}}(1 + e^{-\Gamma t} - 2e^{-\frac{\Gamma}{2}t}\cos\omega t). \tag{36}$$

Our results (35) and (36) coincide with those of [62–64]. The solution (36) corresponds to the dominant role of the eigenvalue μ_1 [see (31)]. If we remove assumptions allowing to obtain (34) from (33), and consider the solution of (33) for the times

$$\frac{1}{M} \ll t \ll \frac{1}{M}\left(\frac{M}{\varepsilon}\right)^2, \tag{37}$$

we arrive to another solution with the dominant role the eigenvalue μ_2 (32) provided $\omega = 0$, or very small. Then $\rho_2(t) = |\psi_{n'}(t)|^2$ reads [44,61]

$$|\psi_{n'}(t)|^2 = \frac{\varepsilon^2}{\omega^2 + \frac{\Gamma^2}{4}}e^{-\delta t}, \tag{38}$$

where $\delta = \varepsilon^2\Gamma(\omega^2 + \frac{\Gamma^2}{4})^{-1}$. This is the limit of "long" times or distances close to or exceeding the absorption length $L = vt$, where v is neutron velocity. Formally, the distance (or time) independent factor in (38) is obtained from (36) at $\Gamma t \gg 1$, or $(n\sigma L) \gg 1$. The regime (38) means that at some thickness L of the absorber such that $(nv\sigma)t \gg 1$ or $(n\sigma L) \gg 1$ the attenuation of the neutron beam due to the $n - n'$ transition becomes almost constant.

One can solve Eq. (33) in the next order in ε/M and get the solution that incorporates both limiting regimes (36) and (38). It has the following form:

$$|\psi_{n'}(t)|^2 = \frac{\varepsilon^2}{\omega^2 + \frac{\Gamma^2}{4}}e^{-\delta t}(1 + e^{-\Gamma'' t} - 2e^{-\frac{\Gamma''}{2}t}\cos\omega'' t), \tag{39}$$

where

$$\delta = \varepsilon^2\frac{\Gamma}{\omega^2 + M^2}, \qquad \omega_\varepsilon = \varepsilon^2\frac{\omega}{\omega^2 + M^2}, \tag{40}$$

$$\Gamma'' = \Gamma - 2\delta, \qquad \omega'' = \omega + 2\omega_\varepsilon. \tag{41}$$

To treat the strong absorbtion regime within the Lindblad equation approach one can take Eqs. (9) and (11) in an oversimplified form

$$\dot{\rho}_2 = 2\varepsilon y, \tag{42}$$

$$\dot{y} = -My + \varepsilon(\rho_1 - \rho_2). \tag{43}$$

Taking the time derivative in (43) once more we obtain

$$\frac{d^2y}{dt^2} + M\frac{dy}{dt} + 4\varepsilon^2 y = 0. \tag{44}$$

The initial conditions are $y(0) = 0, \dot{y}(0) = \varepsilon$. Solving (44) in the same approximation that lead to (34), putting the result into (42), we obtain

$$\rho_2(t) = \frac{\varepsilon^2}{M^2}(1 - e^{-Mt})^2 = \frac{4\varepsilon^2}{\Gamma^2}\left(1 - e^{-\frac{\Gamma}{2}t}\right)^2, \tag{45}$$

which is the same as (36) for $\omega = 0$.

Striving for an analytical solution in the strong absorption regime we were forced to make a drastic approximation $M \gg |K|$, or $\mathrm{Im}f(0) \gg |\mathrm{Re}f(0)|$. Looking into the NIST table of the neutron scattering length [65] one concludes that this is not the most adequate assumption. As noted already the system of Lindblad equations (8)–(11) does not allow a transparent solution without approximations. For illustrative purposes we present a solution in a regime "opposite" to the previous one, namely with the negligible absorption $|K| \gg M$. Equations (9) and (11) take the form

$$\dot{\rho}_2 = 2\varepsilon y, \tag{46}$$

$$\dot{y} = -Kx + \varepsilon(\rho_1 - \rho_2). \tag{47}$$

Invoking Eq. (10) for $\dot{x}$ we arrive at the following equation for $\ddot{y}$

$$\frac{d^2y}{dt^2} + (K^2 + 4\varepsilon^2)y = 0. \tag{48}$$

Solving (46)–(48) with the same initial conditions for $y(0)$ and $\dot{y}(0)$ we obtain

$$\rho_2(t) = \frac{4\varepsilon^2}{K^2 + 4\varepsilon^2}\sin^2\frac{1}{2}\sqrt{K^2 + 4\varepsilon^2}t. \tag{49}$$

As expected, the neutron undergoes oscillations with the time-averaging swapping probability [46]

$$P = \frac{2\varepsilon^2}{K^2 + 4\varepsilon^2}. \tag{50}$$

The actual experimental regime most probably corresponds to $|K| \gtrsim M$ [65] so that both these quantities should be kept in (8)–(11). With a given set of physical parameters a compete solution of (8)–(11) is a cumbersome but tractable task.

V. A TOY MODEL OF $n-n'$ CONVERSION IN NEUTRON STARS

As stated, the system of Eqs. (8)–(11) does not allow a simple analytical solution. There is a physical situation when it is of minor importance to control the $n-n'$ conversion at every moment. The process might be slow, long-lasting, and the goal is to predict the final outcome, namely the balance between n and n' components at an asymptotically long time. This is exactly what may happen in a neutron star, which could gradually transform into a mixed star consisting of normal neutron and mirror neutron components [23,24]. What follows is a preliminary outline of a future work on this problem.

The system (8)–(11) cannot be solved in a closed form but can be integrated in time from 0 to ∞ with given initial conditions. This procedure has been previously performed in [41,45].

We return to (8)–(11) and introduce the following notations:

$$X=\int_0^\infty dtx,\qquad Y=\int_0^\infty dty,\quad P_i=\int_0^\infty dt\rho_i,\quad i=1,2, \tag{51}$$

$$R=nv\sigma_r+\gamma,\qquad \Gamma=\frac{1}{2}(\gamma+\gamma'+2M),\qquad \Delta=d+K. \tag{52}$$

This mode of action and similar notations were first introduced by G. Feinberg and S. Weinberg [41]. The initial conditions read $P_1=1, P_2=X=Y=0$. The set of equations to be solved are

$$RP_1+2\varepsilon Y=1, \tag{53}$$

$$\gamma' P_2-2\varepsilon Y=0, \tag{54}$$

$$\Gamma X-\Delta Y=0, \tag{55}$$

$$\Gamma Y+\Delta X-\varepsilon(P_1-P_2)=0. \tag{56}$$

The branching ratio we seek for is

$$Br=\frac{P_2}{P_1+P_2}. \tag{57}$$

Br is easily found from (53)–(56) in the approximation $\varepsilon^2\ll\Gamma^2$ with the result

$$Br\simeq\frac{\Gamma}{\gamma'}\frac{2\varepsilon^2}{\Gamma^2+\Delta^2}. \tag{58}$$

As might be expected, we obtained the same result as [41,45]. The minor difference from (53)–(56) of [41] is because the problem considered here is not completely the same as in [41]. The next task would have been to implement the neutron star parameters from, e.g., [23,24] and to get the mirror matter admixture under different conditions. This will be the subject of the work in preparation.

VI. CONCLUSIONS AND OUTLOOK

We have considered the neutron transition into the mirror world in different conditions from vacuum to neutron stars. It is shown that the reduced density matrix formalism, the Lindblad and Bloch equations are the most efficient tools to solve this problem. The reason is that the contact with the surroundings leads to the destruction of the density matrix off-diagonal elements and consequently the loss of the coherence. It is important to note that deconherence and the resulting collapse of the wave function is a phenomenon beyond the standard quantum mechanics based on the Schrodinger equation with optical potential. The correct description was proposed by G. Feinberg and S. Weinberg [41]. Eventually this approach was coined the name Lindblad equation [38,39]. Relation between the optical potential and the Lindblad equation was discussed in [51,66]. A correct way to incorporate the optical potential into the Lindblad equation presented in [51] is

$$\frac{d\rho}{dt}=-i[H,\rho]-i(W_{\rm opt}\rho-\rho W^*_{\rm opt})+L\rho L^+, \tag{59}$$

$$\text{where } W_{\rm opt}=-2\pi\frac{n}{m_*}f(0). \tag{60}$$

Subject to minor differences, this equation is equivalent to (8)–(11) of the present work.

To summarize, we may say that the problem of the $n-n'$ conversion in a trap is completely solved. The transition in the absorbing material has been studied in [11,12,46,67,68] and the present work. Our approach is close to that used in [12,46]. In some limiting approximations the results basically coincide, like (9) of [46] and (50) of the present work. On the other hand, we can not find an immediate correspondence between (24) of [67] and our results. This poses a serious problem to work at in order to provide a clear guidance to the experiment. As for the intriguing problem of neutron-mirror-neutron star transition, it gets a close attention [23,24] but is far from a complete solution.

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[2] I. Kobzarev, L. Okun, and I. Pomeranchuk, Sov. J. Nucl. Phys. **3**, 837 (1966).
[3] Z. G. Berezhiani and R. N. Mohapatra, Phys. Rev. D **52**, 6607 (1995).
[4] Z. G. Berezhiani, A. D. Dolgov, and R. N. Mohapatra, Phys. Lett. B **375**, 26 (1996).
[5] Z. Berezhiani and L. Bento, Phys. Rev. Lett. **96**, 081801 (2006).
[6] Z. Berezhiani and L. Bento, Phys. Rev. Lett. **B635**, 253 (2006).
[7] K. S. Babu and R. N. Mohapatra, Symmetry **14**, 731 (2022).
[8] L. B. Okun, Phys. Usp. **50**, 380 (2007).
[9] Yu. N. Pokotilovski, Phys. Lett. B **639**, 214 (2006).
[10] A. Addazi *et al.*, J. Phys. G **48**, 070501 (2021).
[11] H. Almazan *et al.*, Phys. Rev. Lett. **128**, 061801 (2022).
[12] C. Stasser, G. Terwagne, J. Lamblin, O. Méplan, G. Pignol, B. Coupé, S. Kalcheva, S. Van Dyck, and M. Sarrazin, Eur. Phys. J. C **81**, 17 (2021).
[13] N. Allemandou *et al.*, J. Instrum. **13**, P07009 (2018).
[14] I. Alekseev *et al.*, Phys. Lett. B **787**, 56 (2018).
[15] A. Serebrov *et al.*, Phys. Rev. D **104**, 032003 (2021).
[16] J. Ashenfelter *et al.*, Phys. Rev. Lett. **121**, 251802 (2018).
[17] Y. Abreu *et al.*, J. Instrum. **16**, P02025 (2021).
[18] A. P. Serebrov *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **611**, 137 (2009).
[19] Z. Berezhiani and F. Nesti, Eur. Phys. J. C **72**, 1974 (2012).
[20] P. Mohanmurthy, A. R. Yang, J. Winger, and G. Zsigmond, Symmetry **14**, 487 (2022).
[21] L. J. Broussard *et al.*, Phys. Rev. Lett. **128**, 212503 (2022).
[22] N. J. Ayres *et al.*, Symmetry **14**, 503 (2022).
[23] I. Goldman, R. N. Mohapatra, and S. Nussinov, Phys. Rev. D **100**, 123021 (2019).
[24] Z. Berezhiani, R. Biondi, M. Manarelli, and F. Tonelli, Eur. Phys. J. C **81**, 1036 (2021).
[25] I. Goldman, R. N. Mohapatra, S Nussinov, and Y. Zhang, arXiv:2203.08473.
[26] D. McKeen, M. Pospelov, and N. Raj, Phys. Rev. Lett. **127**, 061805 (2021).
[27] I. Goldman, arXiv:2011.12070.
[28] S. I. Blinnikov and M. Yu. Khlopov, Sov. Astron. **27**, 371 (1983).
[29] R. Foot, Int. J. Mod. Phys. A **29**, 1430013 (2014).
[30] S. Blinnikov and M. Khlopov, Sov. J. Nucl. Phys. **36**, 472 (1982).
[31] A. Yu.Ignatiev and R. R. Volkas, Phys. Rev. D **68**, 023518 (2003).
[32] L. Bento and Z. Berezhiani, Fortschr. Phys. **50**, 489 (2002).
[33] E. W. Kolb, D. Seckel, and M. Turner, Nature (London) **314**, 415 (1985).
[34] R. Foot, H. Lew, and R. Volkas, Phys. Lett. B **272**, 67 (1991).
[35] A. P. Serebrov, Phys. Usp. **62**, 596 (2019).
[36] Z. Berezhiani, Eur. Phys. J. C **79**, 484 (2019).
[37] A. P. Serebrov, M. E. Chaikovskii, G. N. Klyushnikov, O. M. Zherebtsov, and A. V. Chechkin, Phys. Rev. D **103**, 074010 (2021).
[38] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).
[39] A. Kossakowski, Rep. Math. Phys. **3**, 247 (1972); V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, J. Math. Phys. (N.Y.) **17**, 821 (1976).
[40] F. Bloch, Phys. Rev. **70**, 460 (1946).
[41] G. Feinberg and S. Weinberg, Phys. Rev. **123**, 1439 (1961).
[42] A. D. Dolgov, Phys. Rep. **370**, 333 (2002).
[43] Y. Akamatsu, Phys. Rev. D **91**, 056002 (2015).
[44] B. O. Kerbikov, Nucl. Phys. **A975**, 59 (2018).
[45] S. V. Demidov, D. S. Gorbunov, and A. A. Tokareva, Phys. Rev. D **85**, 015022 (2012).
[46] M. Sarrazin, G. Pignol, J. Lamblin, F. Petit, G. Terwagne, and V. V. Nesvizhevsky, Phys. Rev. D **91**, 075013 (2015).
[47] Y. Grossman and Wee Hao Ng, Phys. Rev. D **98**, 035020 (2018).
[48] S. Weinberg, Phys. Rev. A **94**, 042117 (2016).
[49] P. Pearle, Eur. J. Phys. **33**, 805 (2012).
[50] T. Ohlsson and Shun Zhou, Phys. Rev. A **103**, 022218 (2021).
[51] B. Vacchini and K. Hornberger, Phys. Rep. **478**, 71 (2009).
[52] Z. Berezhiani, Eur. Phys. J. C **64**, 421 (2009).
[53] Z. Berezhiani, R. Biondi, P. Geltenbort, I. A. Krasnoschekova, V. E. Varlamov, A. V. Vassiljev, and O. M. Zherebtsov, Eur. Phys. J. C **78**, 717 (2018).
[54] B. O. Kerbikov, M. S. Lukashov, Y. A. Kamyshkov, and L. J. Varriano, arXiv:1512.03398.
[55] T. Skinner, arXiv:1704.00826.
[56] G. Ban *et al.*, Phys. Rev. Lett. **99**, 161603 (2007).
[57] A. Serebrov, Phys. Lett. B **663**, 181 (2008).
[58] B. Kerbikov and O. Lychkovskiy, Phys. Rev. C **77**, 065504 (2008).
[59] L. Stodolsky, Quantum damping and its paradoxes, in *Quantum Coherence*, edited by I. S. Anandan (World Scientific, Singapore, 1990).
[60] D. Salgado and J. L. Sanchez-Gomez, Phys. Lett. A **323**, 365 (2004).
[61] A. Gal, Phys. Rev. C **61**, 028201 (2000).
[62] S. Demidov, S. Gninenko, and D. Gorbunov, J. High Energy Phys. 07 (2019) 162.
[63] B. Kerbikov, Phys. At. Nucl. **66**, 2178 (2003).
[64] J. Redondo, J. Cosmol. Astropart. Phys. 07 (2015) 024.
[65] V. F. Sears, Neutron News **3**, 29 (1992).
[66] L. Lanz and B. Vacchini, Phys. Rev. A **56**, 4826 (1997).
[67] Yu. Kamyshkov, J. Ternullo, L. Varriano, and Z. Berezhiani, Symmetry **14**, 230 (2022).
[68] M. Hostert, D. McKeen, M. Pospelov *et al.*, arXiv:2201.02603.