

Bhabha scattering in the gauge-Higgs unification

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We examine effects of Z' bosons in gauge-Higgs unification (GHU) models at $e^+e^- \rightarrow e^+e^-$ Bhabha scatterings. We evaluate differential cross sections in Bhabha scatterings including Z' bosons in two types of $SO(5) \times U(1) \times SU(3)$ GHU models. We find that deviations of differential cross sections in the GHU models from those in the SM can be seen at $\sqrt{s} = 250$ GeV. With 80%-longitudinally polarized electron and 30%-longitudinally polarized positron beams, the left-right asymmetries in the GHU A and B models are resolved at more than 3σ at $L_{\text{int}} = 250 \text{ fb}^{-1}$. We also show that Bhabha scattering with scattering angle less than 100 mrad can be safely used as luminosity measurements at e^+e^- colliders since the effects of Z' bosons are well suppressed for small scattering angle. We propose a new observable which can be measured at future TeV-scale e^+e^- linear colliders.

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I. INTRODUCTION

With the establishment of the standard model (SM) by the discovery of the Higgs boson, searching for physics beyond the SM (BSM) and understanding the electroweak phase transition have become main topics in particle physics. As one scenario of BSM, the gauge-Higgs unification (GHU) scenario has been studied [1–15]. In GHU, the Higgs boson is a part of the extra-dimensional component of gauge potentials, appearing as a fluctuation mode of an Aharonov-Bohm (AB) phase θ_H in the fifth dimension. Many GHU models are proposed for electroweak unification [16–35], and GHU models for grand unification are also proposed [36–45]. Among them, two types of $SU(3)_C \times SO(5) \times U(1)_X$ GHU models, the A model [16–22] and B model [23–28], in warped spacetime have been extensively studied. At low energies below the electroweak scale, the mass spectrum and gauge and Higgs couplings of SM particles are nearly the same as in the SM.

Couplings of the first Kaluza-Klein (KK) neutral gauge bosons to right-handed SM fermions are large in the A model, whereas those to left-handed SM fermions become large in the B model. In proton-proton collisions KK bosons

of photons and Z bosons appear as huge broad resonances of Z' bosons in the Drell-Yan process and can be seen in current and future hadron collider experiments [19,20,28]. The KK-excited states of the W boson are also seen as resonances of W' bosons. In the A model, the couplings of the first KK W boson to the SM fermions are small. In the B model, the couplings of the first KK W boson to right-handed fermions are negligible, while the couplings to left-handed fermions are much larger than the W boson couplings. Therefore, at the LHC, the first KK W boson appears as a narrow resonance of W' boson in the A model but appears as a broad resonance in the B model [20,28].

In e^+e^- collider experiments, effects of GHU can be examined by exploring interference effects among photon, Z boson, and Z' bosons. In the previous papers, we have studied effects of new physics (NP) on such observable quantities as cross section, forward-backward asymmetry, and left-right asymmetry in $e^+e^- \rightarrow f\bar{f}$ ($f \neq e$) with polarized and unpolarized e^+e^- beams [21,25,30,31,46–48]. In Ref. [21], we compared such observable quantities of GHU with those of the SM in LEP experiments at $\sqrt{s} = M_Z$, and LEP2 experiments for $130 \text{ GeV} \leq \sqrt{s} \leq 207 \text{ GeV}$ [49,50]. In Refs. [21,25], we also gave predictions of several signals of Z' bosons in GHU in e^+e^- collider experiments designed for future with collision energies $\sqrt{s} \geq 250 \text{ GeV}$ with polarized electron and positron beams. In the $e^+e^- \rightarrow f\bar{f}$ ($f \neq e$) modes, the deviations of total cross sections become large for right-polarized electrons in the A model, whereas

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in the B model, the deviations are large for left-polarized electrons.

Deviations from the SM can be seen in the Higgs couplings as well. HWW , HZZ , and Yukawa couplings deviate from those in the SM in a universal manner [16,17,32]. They are suppressed by a common factor

$$\frac{g_{HWW}^{\text{GHU}}}{g_{HWW}^{\text{SM}}}, \frac{g_{HZZ}^{\text{GHU}}}{g_{HZZ}^{\text{SM}}} \simeq \cos(\theta_H) \quad (1)$$

for W and Z bosons, and

$$\frac{y_{ff}^{\text{GHU}}}{y_{ff}^{\text{SM}}} \simeq \begin{cases} \cos(\theta_H) & \text{A model [17]} \\ \cos^2(\theta_H/2) & \text{B model [24]} \end{cases} \quad (2)$$

for SM fermions f . In the analysis of both Z' and W' bosons in hadron colliders [19,20,28], it is found that the AB phase is constrained as $\theta_H \lesssim 0.1$. For $\theta_H = \mathcal{O}(0.1)$, probable values in the model, the deviation of the couplings amounts to $(1 - \cos \theta_H) = \mathcal{O}(0.005)$ and is small. At the International Linear Collider (ILC) at $\sqrt{s} = 250$ GeV, the ZZH coupling can be measured in the 0.6% accuracy with 2ab^{-1} data [51]. Since the masses and Higgs couplings of the SM fields in the GHU models are very close to those in the SM, the electroweak phase transition (EWPT) occurs at $T_C \sim 160$ GeV and appears very weakly first order [22,27] in both the A and B models, which is very similar to EWPT in the SM [52].

In this paper, we study effects of Z' bosons in GHU models on the $e^+e^- \rightarrow e^+e^-$ Bhabha scattering. Measurements of Bhabha scattering at linear colliders have contributed to the establishment of the SM [53–55]. Bhabha scattering is also useful to explore NP [56,57]. Unlike $e^+e^- \rightarrow f\bar{f}$ ($f \neq e$) scattering, in Bhabha scattering, not only s -channel but also t -channel contributions enter the process. Since the t -channel contribution of photon exchange dominates in forward scattering amplitudes, the cross section becomes very large for small scattering angles, which improves the statistics of experiments. It will be seen below that effects of Z' bosons on cross sections can be measured with large significances.

Bhabha scattering at very small scattering angles is used for the determination of the luminosity of the e^+e^- beams. Since cross sections of all other scattering processes depend on the luminosity, one needs to know whether or not effects of Z' bosons on the $e^+e^- \rightarrow e^+e^-$ cross section are sufficiently small. Forward-backward asymmetry A_{FB} of the cross section in Bhabha scattering is no longer a good quantity for searching NP, since the backward scattering cross section is much smaller than the forward scattering cross section. We will propose a new quantity A_X to measure with polarized e^+e^- beams, which can be used for seeing NP effects instead of A_{FB} .

In Sec. II, we briefly review the GHU A and B models and discuss the $e^+e^- \rightarrow e^+e^-$ scattering in both the SM and GHU models. In Sec. III, we show the formulas of $e^+e^- \rightarrow e^+e^-$ scattering cross sections for longitudinally polarized $e^\pm$ beams and numerically evaluate the effects of Z' bosons in GHU models on differential cross sections and left-right asymmetries. We also show that effects of Z' bosons on the cross section are very small at the very small scattering angle.

II. GAUGE-HIGGS UNIFICATION

In GHU A and B models, the electroweak $\text{SU}(2) \times \text{U}(1)$ symmetry is embedded in $\text{SO}(5) \times \text{U}(1)_X$ symmetry in the Randall-Sundrum warped space [58], whose metric is given by

$$ds^2 = \frac{1}{z^2} \left[\eta_{\mu\nu} dx^\mu dx^\nu + \frac{dz^2}{k^2} \right], \quad 1 \leq z \leq z_L = e^{kL}, \quad (3)$$

where $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ and k is the anti-de Sitter (AdS) curvature. We refer to four-dimensional (4D) hyperplanes at $z = 1$ and $z = z_L$ as the UV and IR branes, respectively. The $\text{SO}(5)$ symmetry is broken to $\text{SO}(4) \simeq \text{SU}(2)_L \times \text{SU}(2)_R$ by the boundary conditions at $z = 1, z_L$, and the $\text{SU}(2)_R \times \text{U}(1)_X$ symmetry is broken to $\text{U}(1)_Y$ by a scalar field localized on the UV brane. $\text{SU}(2)_L \times \text{U}(1)_Y$ symmetry is broken to the electromagnetic $\text{U}(1)_{\text{EM}}$ symmetry by the vacuum expectation value (VEV) of the z -component gauge fields of $\text{SO}(5)/\text{SO}(4)$. The VEV is related to the gauge-invariant AB phase θ_H .

The difference between the A and B models lies in the content of fermions as tabulated in Table I. In the A model, quarks and leptons in the SM are embedded in the $\text{SO}(5)$ -vector representation. In the B model, quarks and leptons are embedded in the $\text{SO}(5)$ -spinor, vector, and singlet representations, which are naturally derived from spinor and vector multiplets in the $\text{SO}(11)$ gauge-Higgs grand unification [37,38]. We also note that the bulk mass parameter for the bulk electron field, c_e , is positive in the A model [18], whereas c_e in the B model has to be negative [23]. In the B model, positive c_e leads to an exotic light neutrino state, and therefore negative c_e must be chosen. Zero modes of fermion fields with positive bulk mass parameters are localized near the UV brane, whereas zero modes of fermion fields with negative bulk mass parameters are localized near the IR brane.

Interactions of the electron and gauge bosons are given by

$$\int d^4x \int_1^{z_L} \frac{dz}{k} \{ \bar{\Psi} \gamma^\mu (\partial_\mu - igA_\mu) \Psi \}, \quad (4)$$

where $A_\mu(x, z)$ is a four-dimensional component of the five-dimensional (5D) gauge field and $\Psi(x, z) = z^2 \check{\Psi}$ is the 5D electron field. The electron corresponds to the zero mode of

TABLE I. The fermion content in the first generation of the lepton sector is shown. In the A model, bulk fermions are introduced in the vector representation of $SO(5)$, which are decomposed of a $SO(4)$ vector and singlet. Zero modes of fermions appear in the left-handed components of $SO(4)$ vectors and in the right-handed components of $SO(4)$ singlets. The extra zero modes of L_j ($j = 1, 2, 3$) couple to brane fermions at the UV brane to become massive [18]. In the B model, bulk fermions are introduced in the spinor representation of $SO(5)$. Zero modes appear in the left-handed components of $SU(2)_L$ doublet (ν_e, e) and in the right-handed components of $SU(2)_R$ doublet (ν'_e, e') [23].

	A model	B model
Bulk fermion	$\left(\begin{pmatrix} \nu_e & L_{1X} \\ e & L_{1Y} \end{pmatrix}, e' \right),$ $\left(\begin{pmatrix} L_{2X} & L_{3X} \\ L_{2Y} & L_{3Y} \end{pmatrix}, \nu'_e \right)$	$\begin{pmatrix} \nu_e \\ e \\ \nu'_e \\ e' \end{pmatrix}$

$\tilde{\Psi}(x, z)$. In the A model, the left-handed electron is localized in the vicinity of the UV brane, and the right-handed component is localized near the IR brane. In the B model, the right-handed electron is localized in the vicinity of the UV brane, and the left-handed component is localized near the IR brane. $A_\mu(x, z)$ has a KK expansion, which contains the photon, Z boson, and their KK excited modes. The wave function of the photon is constant in the fifth-dimension coordinate z . The wave function of the Z boson is almost constant in z but has nontrivial dependence near the IR brane. Couplings of the electron to the Z boson are very close to those in the SM. Wave functions of the first KK-excited modes of gauge bosons are localized near the IR brane so that the first KK-excited gauge bosons couple strongly with fermions localized near the IR brane. In the A model, right-handed electrons have large couplings to the first KK-excited gauge bosons, whereas in the B model, left-handed electrons couple strongly to the first KK-excited gauge bosons.

In Tables II and III, parameters and couplings in the A and B models are tabulated. Here, model parameters

$(\theta_H, m_{KK}$ and $z_L)$, masses, widths, and couplings of Z' bosons are referred from Refs. [25,26]. The big difference in the magnitude of z_L in the A and B models originates from the formulas of top-quark mass. In the A model, $m_{\text{top}}^A \simeq (m_{KK}/(\sqrt{2}\pi))\sqrt{1 - 4c_{\text{top}}^2} \sin \theta_H$ [18], whereas in the B model, $m_{\text{top}}^B \simeq (m_{KK}/\pi)\sqrt{1 - 4c_{\text{top}}^2} \sin \frac{1}{2}\theta_H$ [23]. In both models, the W boson mass is given by $m_W \simeq m_{KK}/(\pi\sqrt{kL}) \sin \theta_H$ so that the lower bound of z_L becomes $z_L \gtrsim 8 \times 10^3$ in the A model and $z_L \gtrsim 7 \times 10^7$ in the B model. In Table II, the bulk mass parameter for the electron field c_e is given. As explained before, c_e is positive in the A model, whereas c_e is negative in the B model. In Table III, the left- and right-handed electron couplings to Z' bosons, r_V, ℓ_V ($V = Z, Z^{(1)}, Z_R^{(1)}, \gamma^{(1)}$), are tabulated. In the table, $g_w \equiv g_A/\sqrt{L}$ is the 4D gauge coupling of the $SO(5)$, where g_A is the 5D $SO(5)$ coupling. In terms of g_A and the 5D $U(1)_X$ coupling g_B , a mixing parameter is defined as [19,25]

$$e/g_w = \sin \theta_W^0 \equiv \frac{s_\phi}{\sqrt{1 + s_\phi^2}}, \quad s_\phi \equiv g_B/\sqrt{g_A^2 + g_B^2}. \quad (5)$$

The value of $\sin \theta_W^0$ is determined so as to reproduce the experimental value of the forward-backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ scattering at the Z pole. In the A model, the electron's right-handed couplings to Z' bosons are larger than left-handed couplings. In the B model, the electron's left-handed couplings to Z' bosons are larger than right-handed couplings.

III. BHABHA SCATTERING IN e^+e^- COLLIDERS

We consider the $e^+e^- \rightarrow e^+e^-$ scattering in the center-of-mass frame. In this frame, the Mandelstam variables (s, t, u) are given by

TABLE II. Parameters in GHU models. c_e in the rightmost column is the bulk parameter for the electron field.

Model	θ_H (rad)	m_{KK} (TeV)	z_L	$m_{\gamma^{(1)}}$ (TeV)	$\Gamma_{\gamma^{(1)}}$ (TeV)	$m_{Z^{(1)}}$ (TeV)	$\Gamma_{Z^{(1)}}$ (TeV)	$m_{Z_R^{(1)}}$ (TeV)	$\Gamma_{Z_R^{(1)}}$ (TeV)	c_e
A	0.08	9.54	1.01×10^4	7.86	0.99	7.86	0.53	7.31	1.01	2.0342
B	0.10	13.0	3.87×10^{11}	10.2	3.25	10.2	7.84	9.95	0.816	-1.0067

TABLE III. Left-handed and right-handed couplings of the electron, ℓ_V, r_V ($V = Z, Z^{(1)}, Z_R^{(1)},$ and $\gamma^{(1)}$), in units of $g_w \equiv g_A/\sqrt{L}$ (see the text). Ratios of e and g_w are shown in the second column.

Model	$(e/g_w)^2$	ℓ_Z	r_Z	$\ell_{Z^{(1)}}$	$r_{Z^{(1)}}$	$\ell_{Z_R^{(1)}}$	$r_{Z_R^{(1)}}$	$\ell_{\gamma^{(1)}}$	$r_{\gamma^{(1)}}$
A	0.2312	-0.3066	0.2638	0.1195	0.9986	0.0000	-1.3762	0.1879	-1.8171
B	0.2306	-0.3058	0.2629	-1.7621	-0.0584	-1.0444	0.0000	-2.7587	0.1071

$$\begin{aligned}
s &= 4E^2, \\
t &= -\frac{s}{2}(1 - \cos \theta) = -s \sin^2 \frac{\theta}{2}, \\
u &= -\frac{s}{2}(1 + \cos \theta) = -s \cos^2 \frac{\theta}{2},
\end{aligned} \quad (6)$$

where E is the energy of initial electron and positron and θ is the scattering angle of the electron. Since the $e^+e^- \rightarrow e^+e^-$ scattering process consists both s - and t -channel processes, the scattering amplitude is written in terms of the six building blocks

$$\begin{aligned}
S_{LL} &= S_{LL}(s) \equiv \sum_i \frac{\ell_{V_i}^2}{s - M_{V_i}^2 + iM_{V_i}\Gamma_{V_i}}, \\
S_{RR} &= S_{RR}(s) \equiv \sum_i \frac{r_{V_i}^2}{s - M_{V_i}^2 + iM_{V_i}\Gamma_{V_i}}, \\
S_{LR} &= S_{LR}(s) \equiv \sum_i \frac{\ell_{V_i} r_{V_i}}{s - M_{V_i}^2 + iM_{V_i}\Gamma_{V_i}}, \\
T_{LL} &= T_{LL}(s, \theta) \equiv \sum_i \frac{\ell_{V_i}^2}{t - M_{V_i}^2 + iM_{V_i}\Gamma_{V_i}}, \\
T_{RR} &= T_{RR}(s, \theta) \equiv \sum_i \frac{r_{V_i}^2}{t - M_{V_i}^2 + iM_{V_i}\Gamma_{V_i}}, \\
T_{LR} &= T_{LR}(s, \theta) \equiv \sum_i \frac{\ell_{V_i} r_{V_i}}{t - M_{V_i}^2 + iM_{V_i}\Gamma_{V_i}},
\end{aligned} \quad (7)$$

where M_{V_i} and Γ_{V_i} are the mass and width of the vector boson V_i . ℓ_{V_i} and r_{V_i} are left- and right-handed couplings of electrons to the vector boson V_i ($V_0 = \gamma$, $V_1 = Z$), respectively. In particular, we have $\ell_\gamma = r_\gamma = Q_e e$, $Q_e = -1$, $\ell_Z = \frac{e}{\sin \theta_W^0 \cos \theta_W^0} [I_e^3 - Q_e \sin^2 \theta_W^0]$, $r_Z = \frac{e}{\sin \theta_W^0 \cos \theta_W^0} [-Q_e \sin^2 \theta_W^0]$, $I_e^3 = -\frac{1}{2}$ in the SM. Here, e , I_e^3 , and θ_W^0 are the electromagnetic coupling, weak isospin of the electron, and the bare Weinberg angle defined in (5), respectively.

When initial-state electrons and positrons are longitudinally polarized, the differential cross section is given by

$$\begin{aligned}
\frac{d\sigma}{d\cos \theta}(P_{e^-}, P_{e^+}) &= \frac{1}{4} \left\{ (1 + P_{e^-})(1 + P_{e^+}) \frac{d\sigma_{e_R^- e_R^+}}{d\cos \theta} \right. \\
&\quad + (1 - P_{e^-})(1 - P_{e^+}) \frac{d\sigma_{e_L^- e_L^+}}{d\cos \theta} \\
&\quad + (1 + P_{e^-})(1 - P_{e^+}) \frac{d\sigma_{e_R^- e_L^+}}{d\cos \theta} \\
&\quad \left. + (1 - P_{e^-})(1 + P_{e^+}) \frac{d\sigma_{e_L^- e_R^+}}{d\cos \theta} \right\}, \quad (8)
\end{aligned}$$

where P_{e^-} and P_{e^+} are the polarization of the electron and positron beam, respectively. $P_{e^-} = +1$ ($P_{e^+} = +1$) denotes purely right-handed electrons (positrons) [59,60]. $\sigma_{e_X^- e_Y^+}$ ($X, Y = L, R$) denotes the cross section for left-handed or right-handed electron and positron. When the electron mass is neglected, these cross sections are given by

$$\begin{aligned}
\frac{d\sigma_{e_L^- e_R^+}}{d\cos \theta} &= \frac{1}{8\pi s} (u^2 |S_{LL} + T_{LL}|^2 + t^2 |S_{LR}|^2), \\
\frac{d\sigma_{e_R^- e_L^+}}{d\cos \theta} &= \frac{1}{8\pi s} (u^2 |S_{RR} + T_{RR}|^2 + t^2 |S_{LR}|^2), \\
\frac{d\sigma_{e_L^- e_L^+}}{d\cos \theta} &= \frac{d\sigma_{e_R^- e_R^+}}{d\cos \theta} = \frac{1}{8\pi s} \cdot (s^2 |T_{LR}|^2).
\end{aligned} \quad (9)$$

When $s, t \ll M_Z^2$, the cross section is approximated by the one at the QED level, where we obtain $S_{LL} = S_{RR} = S_{LR} = e^2/s$ and $T_{LL} = T_{RR} = T_{LR} = e^2/t$, and

$$\begin{aligned}
\frac{d\sigma_{\text{QED}}}{d\cos \theta}(P_{e^-}, P_{e^+}) &= \frac{e^4}{16\pi s} \left\{ (1 - P_{e^-} P_{e^+}) \frac{t^4 + u^4}{s^2 t^2} \right. \\
&\quad \left. + (1 + P_{e^-} P_{e^+}) \frac{s^2}{t^2} \right\}. \quad (10)
\end{aligned}$$

For unpolarized electron or positron beams, the above expression reduces to a familiar formula:

$$\frac{d\sigma_{\text{QED}}^{\text{unpolarized}}}{d\cos \theta} = \frac{e^4}{16\pi s} \frac{s^4 + t^4 + u^4}{s^2 t^2}. \quad (11)$$

We also note that in terms of building blocks (7) we can write down components of s and t channels and interference terms as

$$\frac{d\sigma}{d\cos \theta} = \frac{d\sigma^{\text{s-channel}}}{d\cos \theta} + \frac{d\sigma^{\text{t-channel}}}{d\cos \theta} + \frac{d\sigma^{\text{interference}}}{d\cos \theta}, \quad (12)$$

where each component is given by

$$\begin{aligned}
\frac{d\sigma^{\text{s-channel}}}{d\cos\theta}(P_{e^-}, P_{e^+}) &= \frac{1}{32\pi s} \{ (1+P_{e^-})(1-P_{e^+})[u^2|S_{RR}|^2 + t^2|S_{LR}|^2] \\
&\quad + (1-P_{e^-})(1+P_{e^+})[u^2|S_{LL}|^2 + t^2|S_{LR}|^2] \}, \\
\frac{d\sigma^{\text{t-channel}}}{d\cos\theta}(P_{e^-}, P_{e^+}) &= \frac{1}{32\pi s} \{ (1+P_{e^-})(1+P_{e^+})s^2|T_{LR}|^2 + (1-P_{e^-})(1-P_{e^+})s^2|T_{LR}|^2 \\
&\quad + (1+P_{e^-})(1-P_{e^+})u^2|T_{RR}|^2 + (1-P_{e^-})(1+P_{e^+})u^2|T_{LL}|^2 \}, \\
\frac{d\sigma^{\text{interference}}}{d\cos\theta}(P_{e^-}, P_{e^+}) &= \frac{1}{16\pi s} u^2 \{ (1+P_{e^-})(1-P_{e^+})\text{Re}(S_{RR}T_{RR}^*) \\
&\quad + (1-P_{e^-})(1+P_{e^+})\text{Re}(S_{LL}T_{LL}^*) \}.
\end{aligned} \tag{13}$$

When initial electrons and/or positrons are longitudinally polarized, one can measure left-right asymmetries. The left-right asymmetry of polarized cross sections is given by

$$\begin{aligned}
A_{\text{LR}}(P_-, P_+) &\equiv \frac{\sigma(P_{e^-} = -P_-, P_{e^+} = -P_+) - \sigma(P_{e^-} = +P_-, P_{e^+} = +P_+)}{\sigma(P_{e^-} = -P_-, P_{e^+} = -P_+) + \sigma(P_{e^-} = +P_-, P_{e^+} = +P_+)} \\
&= (P_- - P_+) \cdot \frac{\sigma_{e_L^- e_R^+} - \sigma_{e_R^- e_L^+}}{(1+P_-P_+)(\sigma_{e_L^- e_L^+} + \sigma_{e_R^- e_R^+}) + (1-P_-P_+)(\sigma_{e_L^- e_R^+} + \sigma_{e_R^- e_L^+})}, \\
1 \geq P_- \geq 0, \quad 1 \geq P_+ \geq -1,
\end{aligned} \tag{14}$$

where the cross section in a given bin $[\theta_1, \theta_2]$ is given by $\sigma \equiv \int_{\cos\theta_1}^{\cos\theta_2} \frac{d\sigma}{d\cos\theta} d\cos\theta$. We have used $\sigma_{e_L^- e_L^+} = \sigma_{e_R^- e_R^+}$ because $\frac{d\sigma_{e_L^- e_L^+}}{d\cos\theta} - \frac{d\sigma_{e_R^- e_R^+}}{d\cos\theta} = 0$. We can also define the left-right asymmetry of the differential cross section as

$$\begin{aligned}
A_{\text{LR}}(P_-, P_+, \cos\theta) &\equiv \frac{\frac{d\sigma}{d\cos\theta}(P_{e^-} = -P_-, P_{e^+} = -P_+) - \frac{d\sigma}{d\cos\theta}(P_{e^-} = +P_-, P_{e^+} = +P_+)}{\frac{d\sigma}{d\cos\theta}(P_{e^-} = -P_-, P_{e^+} = -P_+) + \frac{d\sigma}{d\cos\theta}(P_{e^-} = +P_-, P_{e^+} = +P_+)} \\
&= \frac{(P_- + P_+) \left(\frac{d\sigma_{e_L^- e_L^+}}{d\cos\theta} - \frac{d\sigma_{e_R^- e_R^+}}{d\cos\theta} \right) + (P_- - P_+) \left(\frac{d\sigma_{e_L^- e_R^+}}{d\cos\theta} - \frac{d\sigma_{e_R^- e_L^+}}{d\cos\theta} \right)}{(1+P_-P_+) \left(\frac{d\sigma_{e_L^- e_L^+}}{d\cos\theta} + \frac{d\sigma_{e_R^- e_R^+}}{d\cos\theta} \right) + (1-P_-P_+) \left(\frac{d\sigma_{e_L^- e_R^+}}{d\cos\theta} + \frac{d\sigma_{e_R^- e_L^+}}{d\cos\theta} \right)} \\
&= (P_- - P_+) \cdot \frac{\Sigma_{\text{LR-RL}}}{(1+P_-P_+)\Sigma_{\text{LL+RR}} + (1-P_-P_+)\Sigma_{\text{LR+RL}}},
\end{aligned} \tag{15}$$

where we have used $d\sigma_{e_L^- e_L^+}/d\cos\theta = d\sigma_{e_R^- e_R^+}/d\cos\theta$ and defined

$$\begin{aligned}
\Sigma_{\text{LL+RR}} &\equiv 2s^2|T_{LR}|^2, \\
\Sigma_{\text{LR+RL}} &\equiv u^2(|S_{LL} + T_{LL}|^2 + |S_{RR} + T_{RR}|^2) + 2t^2|S_{LR}|^2, \\
\Sigma_{\text{LR-RL}} &\equiv u^2(|S_{LL} + T_{LL}|^2 - |S_{RR} + T_{RR}|^2).
\end{aligned} \tag{16}$$

In linear colliders, we can measure the cross sections for $(P_{e^-}, P_{e^+}) = (P_-, P_+)$, $(P_-, -P_+)$, $(-P_-, P_+)$, and $(-P_-, -P_+)$. Combining these quantities, one can make a new observable quantity which does not depend on the value of $P_{\pm}$. In $e^+e^- \rightarrow e^+e^-$ scatterings, we have $A_{\text{LR}}(P_-, +P_+, \cos\theta)$ and $A_{\text{LR}}(P_-, -P_+, \cos\theta)$ as independent observables, and one may define the nontrivial quantity

$$\begin{aligned}
A_X(\cos\theta) &\equiv \frac{\Sigma_{\text{LL+RR}} - \Sigma_{\text{LR+RL}}}{\Sigma_{\text{LL+RR}} + \Sigma_{\text{LR+RL}}} \\
&= \frac{1}{P_-P_+} \cdot \frac{(P_- - P_+)A_{\text{LR}}(P_-, -P_+, \cos\theta) - (P_- + P_+)A_{\text{LR}}(P_-, +P_+, \cos\theta)}{(P_- - P_+)A_{\text{LR}}(P_-, -P_+, \cos\theta) + (P_- + P_+)A_{\text{LR}}(P_-, +P_+, \cos\theta)},
\end{aligned} \tag{17}$$

where the second equality holds only when $P_{\pm} \neq 0$ and $|P_{+}| \neq |P_{-}|$. As is evident in the first line of (17), $A_X(\cos\theta)$ is independent of the magnitudes of polarization $P_{\pm}$. This quantity may be used to explore NP beyond the SM as discussed below.

Since $e^+e^- \rightarrow e^+e^-$ scattering contains t -channel processes, forward scatterings dominate. Therefore, unlike the $e^+e^- \rightarrow f\bar{f}$ ($f \neq e^-$) scattering, the forward-backward asymmetry of $e^+e^- \rightarrow e^+e^-$ scattering is a less-meaningful quantity.

We note that all of the above formulas can be applied to $\ell^+\ell^- \rightarrow \ell^+\ell^-$ ($\ell = \mu, \tau$) scatterings.

IV. NUMERICAL STUDY

In the following, we calculate the $e^+e^- \rightarrow e^+e^-$ cross sections in both the SM and GHU models, and we evaluate effects of Z' bosons in GHU models on observables given in the previous section. As benchmark points, we have chosen typical parameters of the A and B models in Tables II and III. For experimental parameters, we choose $\sqrt{s} = 250$ GeV and $L_{\text{int}} = 250 \text{ fb}^{-1}$ as typical value of linear e^+e^- colliders like the ILC [61]. We also choose $L_{\text{int}} = 2 \text{ ab}^{-1}$, which will be achieved at circular e^+e^- colliders like a high-energy lepton collider at the future circular collider (FCC-ee) [62] and circular electron positron collider (CEPC) [63]. For the new asymmetry $A_X(\cos\theta)$ in (17), we consider $\sqrt{s} = 3$ TeV for future linear colliders like the compact linear collider (CLIC) [64]. As for the longitudinal polarization, we set the parameter ranges $-0.8 \leq P_{e^-} \leq +0.8$ and $-0.3 \leq P_{e^+} \leq 0.3$, which can be achieved at the ILC [61].

In Fig. 1, $e^+e^- \rightarrow e^+e^-$ differential cross sections in the SM are plotted. In the forward-scattering region ($\cos\theta > 0$), the magnitudes of cross sections of the t channel and the interference parts are much larger than those of the s -channel part.

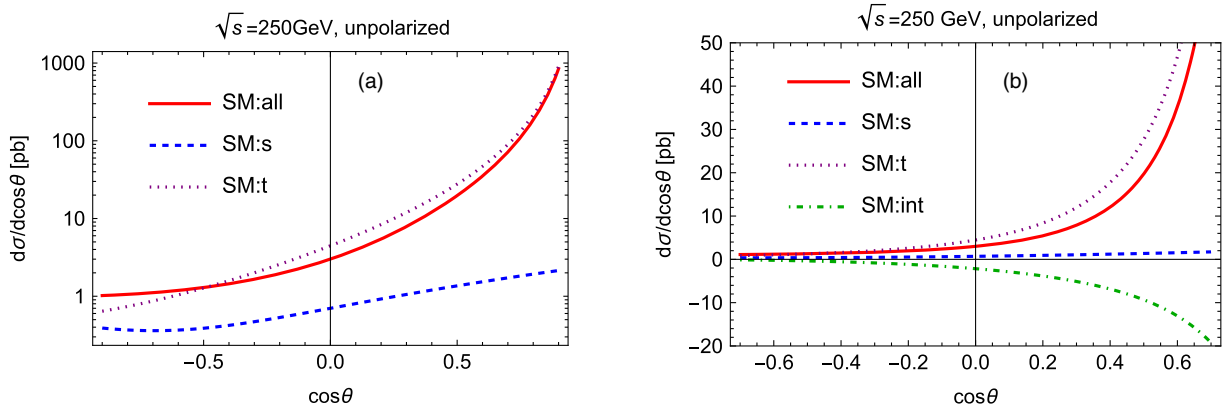


FIG. 1. Differential cross sections for unpolarized e^+e^- initial states in the SM. (a) Log-scale plot with $-0.9 \leq \cos\theta \leq 0.9$. (b) Linear plot with $-0.7 \leq \cos\theta \leq 0.7$. In both plots, red-solid lines indicate the total of s and t channels and interferences. The s -channel and t -channel contributions are drawn as blue-dashed and purple-dotted lines, respectively. In (b), the negative contribution from interferences is plotted with the green-dashed line.

In Figs. 2 and 3, the differences of differential cross sections of the GHU from the SM for unpolarized and polarized beams are plotted, respectively. In the figures, differences of the s -channel and t -channel and interference contributions are also plotted. In the s channel, the NP effects contribute destructively in the forward scattering. On the other hand, in the t channel, NP effects contribute constructively. Since the cross section is dominated by the t channel, the total of s , t , and interference channels increases due to the NP effects.

In the A model, Z' bosons have larger couplings to right-handed electrons than to left-handed electrons. Therefore, the cross section of the $e_R^-e_L^+$ initial states becomes larger than that of $e_L^-e_R^+$. On the other hand, in the B model, Z' bosons have larger couplings to left-handed electrons than to right-handed electrons, and the cross section of $e_L^-e_R^+$ initial states becomes larger.

NP effects become smaller when θ becomes smaller. The statistical uncertainty, however, also becomes small since the cross section becomes very large. Therefore, deviations of the cross section relative to statistical uncertainties may become large.

For unpolarized e^+e^- beams (Fig. 3), the new physics effect in both models tends to enhance the cross section at forward scattering with almost the same magnitude. In the B model, the suppression of NP effects due to larger Z' masses is compensated by larger couplings of Z' bosons than the couplings in the A model. The enhancement of the differential cross section due to the NP effects at $\cos\theta \sim 0.3$ is around 1%.

For polarized beams, deviations can be much larger. In the A model, electrons have large right-handed couplings to Z' bosons, and for right-handed polarized beam, relative deviations of the cross section become as much as 2% [Fig. 3(b)], whereas for right-handed beams, relative deviations are around 0.1% [Fig. 3(a)]. Contrarily, in the B model, a left-handed electron has large couplings to Z'

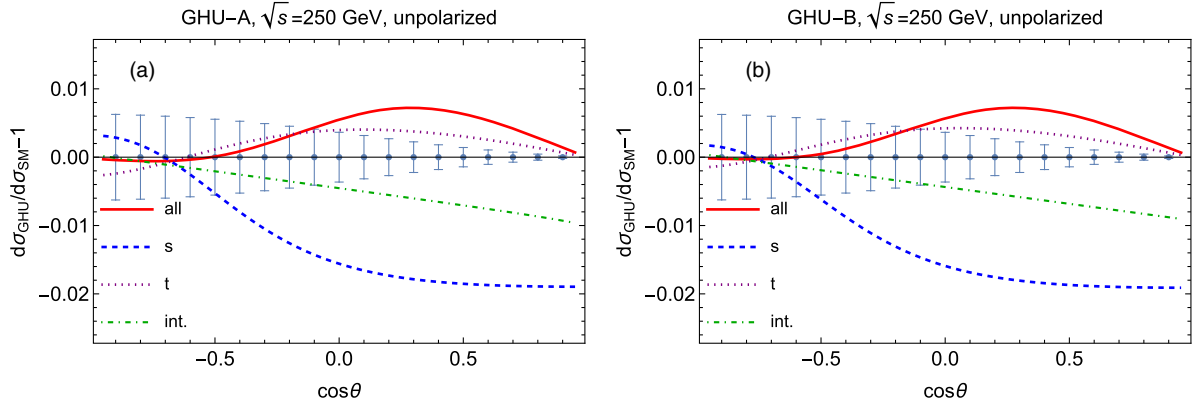


FIG. 2. Deviations of differential cross sections of GHU from those in the SM, $\frac{d\sigma_{\text{GHU}}}{d\cos\theta} / \frac{d\sigma_{\text{SM}}}{d\cos\theta} - 1$, for unpolarized e^+e^- beams are plotted. The left plot is for the A model, and the right plot is for the B model. In each plot, the red-solid curve represents the deviation of the sum of all the components of the differential cross section. Blue-dashed, purple-dotted, and green dot-dashed curves correspond to the deviations of s and t channels and interference components of differential cross sections, respectively. Error bars are estimated for $L_{\text{int}} = 250 \text{ fb}^{-1}$.

bosons. Therefore, in the B model, deviations become large for left-polarized beam [Figs. 3(c) and 3(d)]. In Fig. 2, we have also shown statistical 1σ relative errors at $L_{\text{int}} = 250 \text{ fb}^{-1}$ for bins $[\cos\theta_0 - 0.05, \cos\theta_0 + 0.05]$ ($\cos\theta_0 = -0.90, -0.80, \dots, +0.90$) as vertical bars. In each bin, the observed number of events and statistical uncertainty are

given by N and $\sqrt{N}$, respectively. Therefore, the relative error of the cross section is estimated as the inverse of the square of the number of events N .

In Fig. 4, the statistical significances in the GHU models are plotted. An estimated significance of the deviation of the cross section in a bin is given by

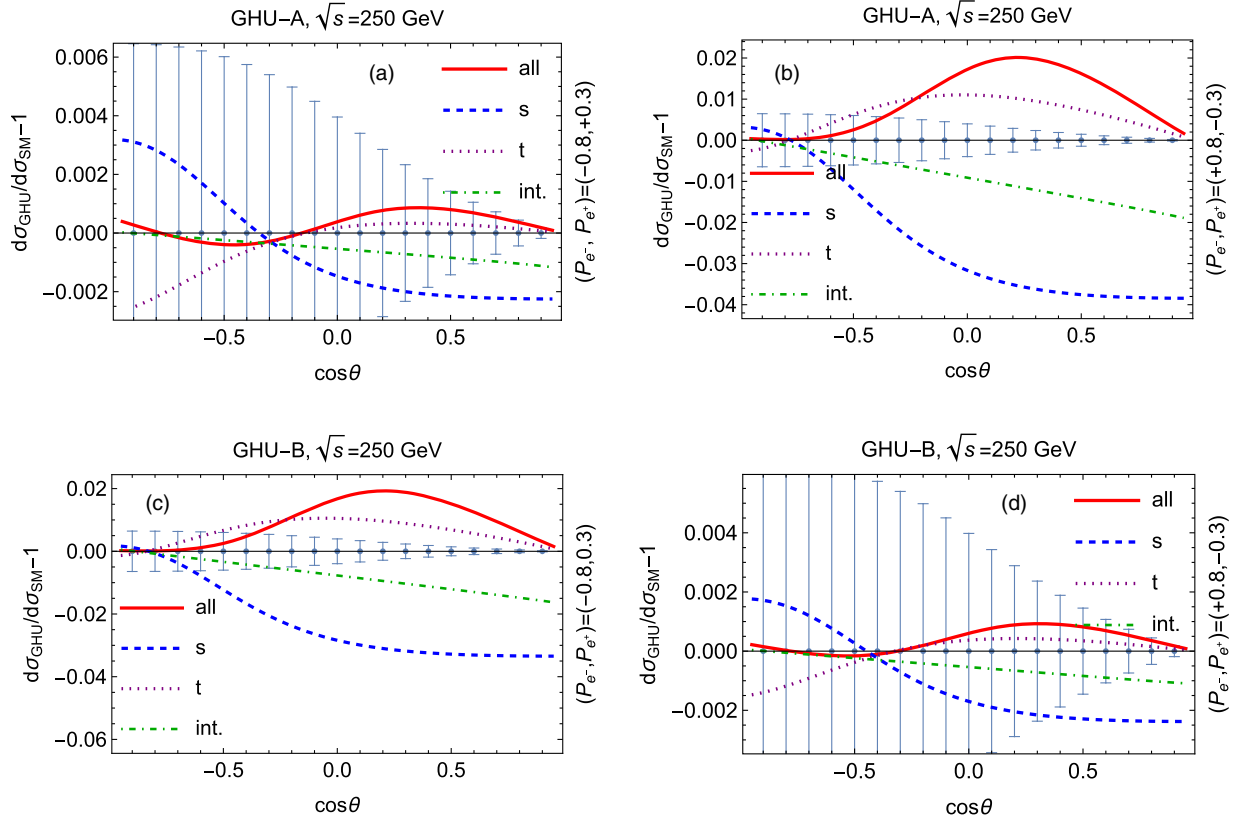


FIG. 3. Deviations of differential cross sections of GHU models from the SM for polarized e^+e^- beams. GHU-A [(a) and (b)] and GHU-B [(c) and (d)]. (a) and (c) are for $(P_{e^-}, P_{e^+}) = (-0.8, +0.3)$. (b) and (d) are for $(P_{e^-}, P_{e^+}) = (+0.8, -0.3)$. The meanings of the curves and error bars are the same as in Fig. 2.

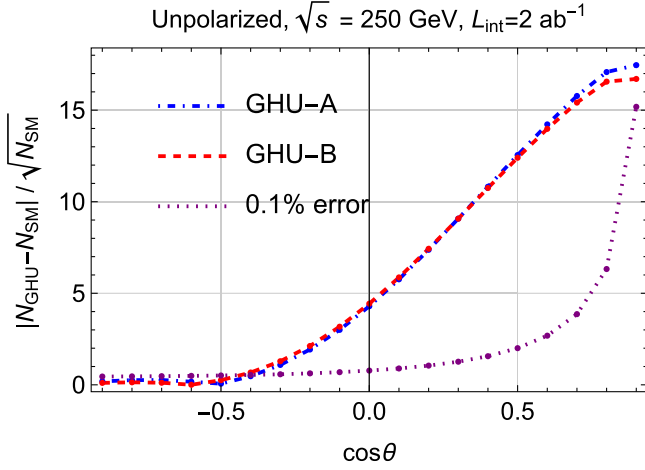


FIG. 4. Estimated significances on differential cross sections of GHU models with unpolarized e^+e^- beam with the integrated luminosity $L_{\text{int}} = 2 \text{ ab}^{-1}$. A statistical significance of 0.1% nonstatistical error is plotted with a purple dot-dashed line.

$$\frac{|N_{\text{GHU}} - N_{\text{SM}}|}{N_{\text{SM}}} / \frac{\sqrt{N_{\text{SM}}}}{N_{\text{SM}}} = \frac{|N_{\text{GHU}} - N_{\text{SM}}|}{\sqrt{N_{\text{SM}}}}, \quad (18)$$

where N_{GHU} and N_{SM} are observed numbers of events in a bin. In Fig. 4, significances are larger than 5σ for $\cos\theta \gtrsim 0.1$. Significances are very large for forward scatterings but are very small for backward scatterings. In Fig. 4, relative 0.1% errors are also shown. Errors due to the NP effects become very small and are around 0.1% for $\cos\theta \simeq 0.9$. A similar analysis has been given in Ref. [56].

For small scattering angles θ , the scattering amplitude is dominated by t -channel contributions which are constructed with the blocks T_{LL} , T_{RR} , and T_{LR} . When $|t| \simeq s\theta^2/4 \ll M_Z^2, M_{Z'}^2$, we can approximate the SM and NP contributions to the block T_{LL} in the scattering amplitude as

$$T_{LL}^{\text{NP}} \equiv \sum_{Z'} \frac{\ell_{Z'}^2}{t - M_{Z'}^2 + iM_{Z'}\Gamma_{Z'}} \simeq \sum_{Z'} \frac{\ell_{Z'}^2}{-M_{Z'}^2 + iM_{Z'}\Gamma_{Z'}}. \quad (19)$$

When $s\theta^2/4 \ll M_Z^2$, T_{LL} is dominated by the QED part, $T_{LL}^{\text{QED}} \simeq -4e^2/(\theta^2 s)$, and the NP effects are estimated as

$$\frac{T_{LL}^{\text{NP}}}{T_{LL}^{\text{QED}}} \simeq \theta^2 \sum_{Z'} \frac{(\ell_{Z'}^2/4e^2)s}{M_{Z'}^2 - iM_{Z'}\Gamma_{Z'}} = \theta^2 \cdot \mathcal{O}(s/M_{Z'}^2), \quad (20)$$

and similar analysis is applied to T_{LR} and T_{RR} . Consequently, this correction arises not only in amplitudes but also in differential cross sections. For $\sqrt{s} = 250 \text{ GeV}$ and $\theta \lesssim 300 \text{ mrad}$, the QED t -channel contribution dominates, and corrections due to Z' bosons are suppressed by a factor $\theta^2 s/M_{Z'}^2$. In Fig. 5, deviations of differential cross sections of GHU from the SM for $\theta < 300 \text{ mrad}$ are

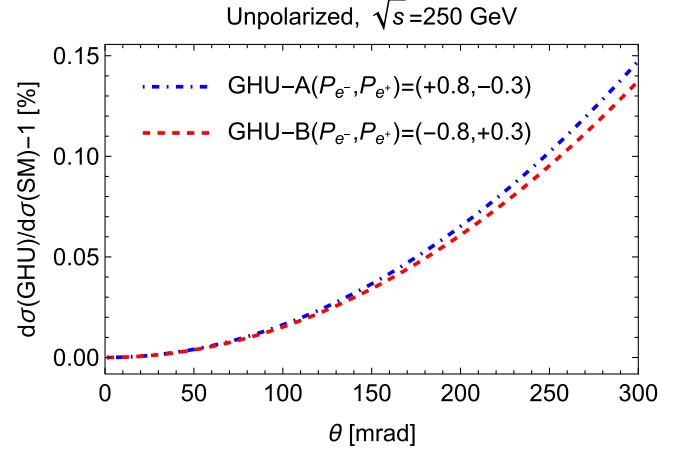


FIG. 5. Deviations of differential cross sections of GHU models from the SM in the forward-scattering region ($\theta \leq 300 \text{ mrad}$).

plotted. Deviations of cross sections from the SM are proportional to the square of θ and become smaller than 0.1% when $\theta < 250 \text{ mrad}$. The measurement of Bhabha scatterings at small scattering angle is used for the determination of the luminosity of e^+e^- collision, and uncertainties of the luminosity should be smaller than 0.1%. In GHU models, the NP effects on such uncertainty are well suppressed when $\theta \lesssim 100 \text{ mrad}$. At the ILC, the luminosity calorimeter in International Large Detector [ILD] (Silicon Detector [SiD]) operates between 43 and 68 (40 and 90) mrad [61], where the influence of the Z' bosons is below 0.1%.

When the initial electron and positron beams are longitudinally polarized, the left-right asymmetry A_{LR} (14) can be measured. In Fig. 6, the left-right asymmetries of the SM and GHU models are plotted. The measured asymmetries become larger when $|P_{e^-} - P_{e^+}|$ are larger. As seen in Fig. 2, in the A-model cross section of $e_R^- e_L^+$ initial states becomes large, whereas in the B-model cross section of $e_L^- e_R^+$ initial states is enhanced due to the large left-handed Z' couplings. Therefore, A_{LR} of B models are larger than the SM, whereas A_{LR} of the A models are smaller. Since the A_{LR} is proportional to $|P_{e^-} - P_{e^+}|$, the asymmetries in Fig. 6(a) are almost twice as large as in Fig. 6(b).

In Fig. 6, an asymmetry A_{LR} in a bin and the statistic error ΔA_{LR} are also shown. Here,

$$A_{\text{LR}} = \frac{N_L - N_R}{N_L + N_R}, \quad \Delta A_{\text{LR}} = \sqrt{\frac{2(N_L^2 + N_R^2)}{(N_L + N_R)^3}} \quad (21)$$

with N_L and N_R being the observed number of events for the left-handed ($P_{e^-} < 0$) and right-handed ($P_{e^-} > 0$) electron beams, respectively. For small scattering angle $\cos\theta \gtrsim 0.8$, both $A_{\text{LR}}^{\text{GHU}}$ and $A_{\text{LR}}^{\text{SM}}$ become close to each other.

To see how NP effects against statistical uncertainty grow for small θ , we plotted in Fig. 7 the averages and

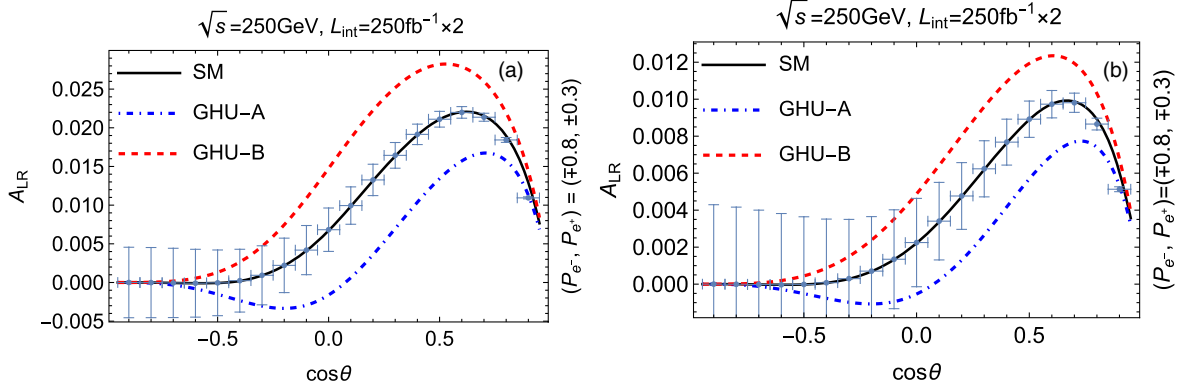


FIG. 6. Left-right asymmetries. In both plots, blue dot-dashed, red dashed, and black solid curves correspond to the GHU-A, GHU-B, and the SM, respectively. Error bars are indicated for $L_{\text{int}} = 250\text{ fb}^{-1}$ in each polarization. (a) Asymmetries for $(P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$. (b) Asymmetries for $(P_{e^-}, P_{e^+}) = (\mp 0.8, \mp 0.3)$.

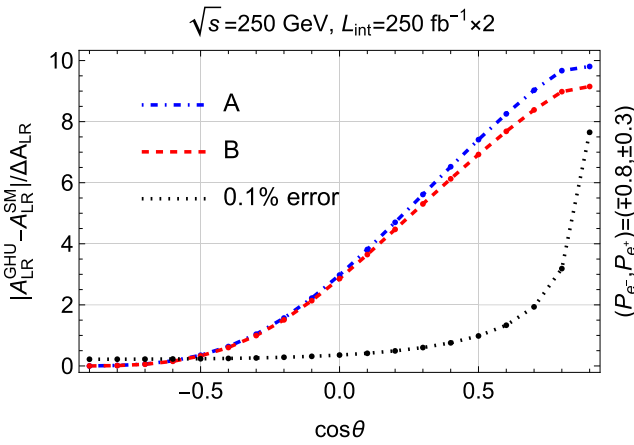


FIG. 7. Estimated statistical significance on left-right asymmetries in GHU models. The blue dot-dashed, red dashed, and black dotted lines indicate GHU-A, GHU-B, and 0.1% nonstatistical errors, respectively.

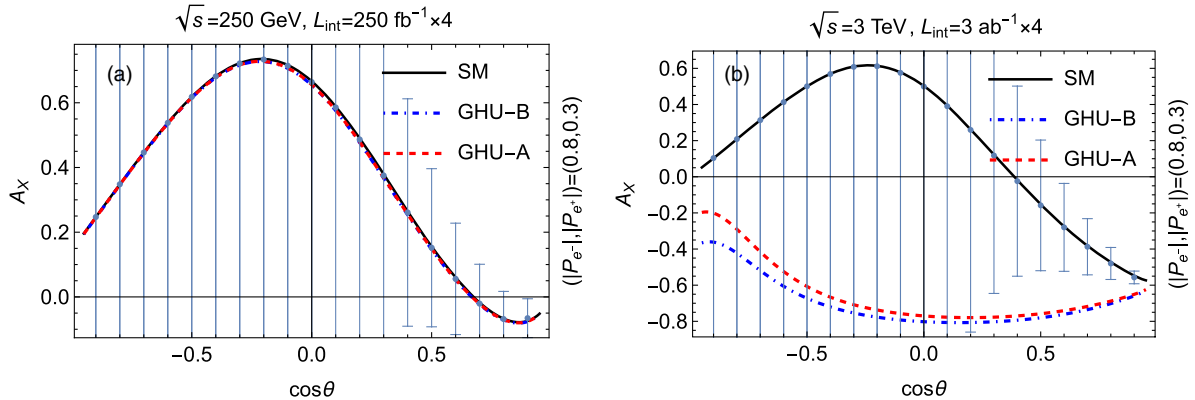


FIG. 8. Plot of an asymmetry A_X in Eq. (17). The left plot (a) is for $\sqrt{s} = 250\text{ GeV}$ with $L_{\text{int}} = 250\text{ fb}^{-1}$ for each set of (P_{e^-}, P_{e^+}) . The right plot (b) is for $\sqrt{s} = 3\text{ TeV}$ with $L_{\text{int}} = 3\text{ ab}^{-1}$ for each set of (P_{e^-}, P_{e^+}) . The red dashed, blue dot-dashed, and black solid curves correspond to the A model, B model, and the SM, respectively. Error bars are indicated for $L_{\text{int}} = 250\text{ fb}^{-1}$ in (a) and $L_{\text{int}} = 3\text{ ab}^{-1}$ in (b) in each set of (P_{e^-}, P_{e^+}) .

statistical significances of left-right asymmetries in GHU models in each bin, which are estimated as

$$\frac{A_{LR}^{\text{GHU}} - A_{LR}^{\text{SM}}}{\Delta A_{LR}}. \quad (22)$$

For the forward scattering with $\cos\theta \gtrsim 0.2$, the deviations are bigger than several times of standard deviations. In Fig. 7, the significance is larger than 5σ for $\cos\theta \gtrsim 0.2$. Both models are well distinguished from the SM. Using the magnitude and sign of deviations, the A model and B model can be distinguished.

In Fig. 8, the asymmetry A_X defined in Eq. (17) is plotted for $\sqrt{s} = 250\text{ GeV}$ and $\sqrt{s} = 3\text{ TeV}$. At $\sqrt{s} = 250\text{ GeV}$, the NP effect on A_X is very small. For $\sqrt{s} = 3\text{ TeV}$, the asymmetry A_X of the SM and GHU models is clearly different and may be discriminated experimentally. In the present analysis of NP effects, only first KK excited states of neutral bosons are taken into account. At $\sqrt{s} \sim 3\text{ TeV}$,

effects of second KK modes on A_X are estimated as a few percent. These effects are much smaller than the effects of the first KK modes.

V. SUMMARY

In this paper, we examined the effects of Z' bosons in the GHU models in the $e^+e^- \rightarrow e^+e^-$ (Bhabha) scatterings. We first formulated differential cross sections in Bhabha scattering including Z' bosons. We then numerically evaluated the deviations of differential cross sections in the two $SO(5) \times U(1) \times SU(3)$ GHU models (the A and B models) at $\sqrt{s} = 250$ GeV. We found that at $L_{\text{int}} = 2 \text{ ab}^{-1}$ with unpolarized e^+e^- beams the deviation due to Z' bosons in the GHU models from the SM can be clearly seen. We also found that for 80%-longitudinally polarized electron and 30%-polarized positron beams deviations of the differential cross sections from the SM become as large as a few percent for $\cos \theta \sim 0.2$ and that the A model and the B model are well distinguished at more than 3σ significance at $L_{\text{int}} = 250 \text{ fb}^{-1}$. We also checked the effects of Z' bosons are negligible for the scattering angle smaller than 100 mrad at $\sqrt{s} = 250$ GeV. Therefore, Bhabha

scatterings at very small θ can be safely used as the measurements of the luminosity in the e^+e^- collisions. Finally, we introduced the new observable A_X . We numerically evaluated it at $\sqrt{s} = 250$ GeV and 3 TeV. Effects of the GHU models on A_X can be seen at future TeV-scale e^+e^- colliders.

In this paper, the effects of Z' bosons are calculated at the Born level. Higher-order QED effects should be taken into account for more precise evaluation [65,66].

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