W boson mass and muon $g-2$ in a lepton portal dark matter model

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We study a lepton portal dark matter model, motivated by the deviation of the W boson mass reported by the CDF collaboration. We introduce vectorlike leptons and a scalar dark matter (DM) that exclusively couples to the extra leptons and muon. The one-loop corrections induced by the new particles can shift the W boson mass. Besides, the discrepancy in the muon anomalous magnetic moment and the DM density can simultaneously be explained by this setup, if the vectorlike lepton is lighter than 200 GeV and nearly degenerate with the DM particle. We also see that the constraints on such a light extra lepton from the collider experiments can be evaded due to the existence of the DM particle.

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I. INTRODUCTION

The precision measurement of the electroweak (EW) interaction plays an important role in tests of new physics. In the Standard Model (SM), a Higgs doublet field develops a nonvanishing vacuum expectation value (VEV), and then the Z and W gauge bosons acquire their masses from the VEV. The mechanism can be tested by the prediction of the EW precision observables (EWPOs), such as the ρ parameter, defined as $\rho \coloneqq m_W^2/(m_Z \cos\theta_W)^2$, where θ_W is the weak mixing angle. It is well known that new fields that contribute to the EW symmetry breaking can, in general, affect to the ρ parameter at the tree level, and hence it can constrain new physics models. Besides, there are loop corrections to the EWPOs where new particles charged under the EW symmetry run in the loops.

The CDF collaboration recently announced a new result of the W boson mass measurement, $m_W^{\mathrm{CDF}} = 80.4335(94)$ GeV [1]. This value disagrees with the SM prediction $m_W^{\mathrm{SM}} = 80.361(6)$ GeV by about 7σ, and combinations of the other previous measurements, $m_W^{\mathrm{PDG}} = 80.379(12)$ GeV [2]. This new experimental result may suggest that the W boson mass deviates from the SM prediction due to the existence of new physics beyond the SM. It has already been pointed out that the deviation can be interpreted by the corrections to the oblique parameters [3–5]. In particular, the T parameter is relevant to this deviation. In order to explain this discrepancy in new physics models, there should be a new particle at the EW scale that is not neutral under the EW symmetry, and/or mixes with SM particles [3–40]. Such a light particle, however, tends to be excluded by the constraints from the LHC experiments if it decays to detectable SM particles. A way to avoid the limits is that an EW particle responsible for the m_W anomaly decays to an undetectable particle so that the signals are effectively invisible.

In this work, we propose a solution of the m_W anomaly by introducing vectorlike leptons that has not been studied yet. The real scalar dark matter (DM) is also introduced to relax the LHC limits on the vectorlike leptons, as discussed later. In this model, the scalar DM couples to the SM leptons via Yukawa couplings involving vectorlike leptons [41,42]. In Ref. [43], the authors show that the relic density of DM and another recent anomaly in muon anomalous magnetic moment, $\Delta a_\mu = 2.51(50) \times 10^{-9}$ [44–65], can be explained simultaneously, when the DM specifically couples to the muon. The muon anomalous magnetic moment in the lepton portal model is studied in Refs. [66–72]. We shall show that the discrepancy in the W boson mass can be solved in this model as well. The EWPOs have not been studied in Ref. [43], and hence this paper is a complement of the previous work.

The rest of this paper is organized as follows. The W boson mass in the lepton portal DM model is discussed in Sec. II, and then the DM physics and its relation to Δa_μ are studied in Sec. III. Section IV is devoted to summarize this

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paper. Loop functions used in our analysis are shown in the Appendix.

II. *W* BOSON MASS IN LEPTON PORTAL DM MODEL

A. Model

We briefly review our model in this section. The more details are shown in Ref. [43]. The terms relevant to new particles in the Lagrangian are given by

$$
\begin{aligned}
-\mathcal{L}_{\rm NP} = {} & \frac{1}{2} m_X^2 X^2 + m_L \bar{L}_L L_R + m_E \bar{E}_L E_R + \lambda_L \bar{\ell}_L X L_R \\
& + \lambda_R \bar{E}_L X \mu_R + \kappa \bar{L}_L H E_R + \bar{\kappa} \bar{E}_L i\sigma_2 H^\dagger L_R + \text{H.c.}, \qquad (1)
\end{aligned}
$$

where $L_{L,R}$ and $E_{L,R}$ are $SU(2)_L$ doublet and singlet vectorlike leptons, respectively. A real scalar DM field is denoted by X, which is a singlet under the SM gauge group. $\ell_L = (\nu_\mu, \mu_L)$, μ_R and H are the SM leptons in the second generation and the Higgs doublet, respectively. We neglect couplings of the scalar DM to the SM Higgs boson in the scalar potential. In our setup, we assume that the vectorlike leptons couple exclusively to the second generation leptons, so that the muon anomalous magnetic moment is explained without lepton flavor violations. We note that a Z_2 symmetry is assigned and only the vectorlike leptons and the DM particle X are odd to ensure the stability of the DM.

After the EW symmetry breaking, the vectorlike lepton masses are given by

$$
\mathcal{M}_E = \begin{pmatrix} m_L & \bar{\kappa} v_H \\ \kappa v_H & m_E \end{pmatrix}, \qquad \mathcal{M}_N = m_L, \qquad (2)
$$

where $v_H = 174$ GeV is the Higgs VEV. Note that there is no mixing between the vectorlike leptons and the SM leptons. We define the mass eigenstates and diagonalizing matrices as

$$
\begin{aligned}
& \begin{pmatrix} E'_L \\ E_L \end{pmatrix} = U_L \begin{pmatrix} E_{L_1} \\ E_{L_2} \end{pmatrix}, \quad \begin{pmatrix} E'_R \\ E_R \end{pmatrix} = U_R \begin{pmatrix} E_{R_1} \\ E_{R_2} \end{pmatrix}, \\
& U_L^\dagger \mathcal{M}_E U_R = \mathrm{diag}(m_{E_1}, m_{E_2}), \qquad (3)
\end{aligned}
$$

and we parametrize the diagonalizing matrices as

$$
U_L = \begin{pmatrix} c_L & s_L \\ -s_L & c_L \end{pmatrix}, \qquad U_R = \begin{pmatrix} c_R & s_R \\ -s_R & c_R \end{pmatrix}, \qquad (4)
$$

where $c_A^2 + s_A^2 = 1 (A = L, R)$. $E'_{L,R}$ is a charged component in the doublet $L_{L,R}$. The neutral component in the gauge basis, $N_{L,R}$, is already in the mass basis N_1.

B. Muon anomalous magnetic moment

In this model, the new physics contribution to the muon anomalous magnetic moment, Δa_μ, is originated from the 1-loop effects induced by the Yukawa couplings of muon with the vectorlike leptons and X. Δa_μ is evaluated as [73–75]

$$
\begin{aligned}
\Delta a_\mu = \frac{m_\mu}{16\pi^2 m_X^2} [& (c_R^2 |\lambda_L|^2 + s_L^2 |\lambda_R|^2) m_\mu F(x_1) \\
& + c_R s_L \mathrm{Re}(\lambda_L \lambda_R) m_{E_1} G(x_1) \\
& + (s_R^2 |\lambda_L|^2 + c_L^2 |\lambda_R|^2) m_\mu F(x_2) \\
& - c_L s_R \mathrm{Re}(\lambda_L \lambda_R) m_{E_2} G(x_2)], \qquad (5)
\end{aligned}
$$

where $x_i = m_{E_i}^2 / m_X^2$ $(i = 1, 2)$. The functions F and G are defined in the Appendix. For $m_{E_i} = \mathcal{O}(100)$GeV, $s_L \neq 0$ or $s_R \neq 0$ is required to explain the discrepancy due to the chirality enhanced effect. This means that both singlet and doublet vectorlike leptons are necessary to explain Δa_μ.

C. Oblique parameters

New physics effects to the EWPOs are well described by the oblique parameters [76,77]. The one-loop contribution of the vectorlike leptons, E_1, E_2, and N_1, to the oblique parameter T is given by [78]

$$
\begin{aligned}
16\pi s_W^2 c_W^2 T = {} & (c_L^2 + c_R^2) \theta_+(x, y_1) + 2 c_L c_R \theta_-(x, y_1) \\
& + (s_L^2 + s_R^2) \theta_+(x, y_2) + 2 s_L s_R \theta_-(x, y_2) \\
& - (c_L^2 s_L^2 + c_R^2 s_R^2) \theta_+(y_1, y_2) \\
& - 2 c_L s_L c_R s_R \theta_-(y_1, y_2), \qquad (6)
\end{aligned}
$$

where $x = m_{N_1}^2 / m_Z^2$ and $y_i = m_{E_i}^2 / m_Z^2$. Here, $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$. The formula for $-2\pi U$ is given by replacing the loop functions $\theta_\pm \to \chi_\pm$. The one-loop contribution to the S parameter is given by

$$
\begin{aligned}
2\pi S = {} & (c_L^2 + c_R^2) \psi_+(x, y_1) + 2 c_L c_R \psi_-(x, y_1) \\
& + (s_L^2 + s_R^2) \psi_+(x, y_2) + 2 s_L s_R \psi_-(x, y_2) \\
& - (c_L^2 s_L^2 + c_R^2 s_R^2) \chi_+(y_1, y_2) - 2 c_L s_L c_R s_R \chi_-(y_1, y_2). \qquad (7)
\end{aligned}
$$

The loop functions $\theta_\pm$, $\chi_\pm$ and $\psi_\pm$ are defined in the Appendix.

Figure 1 shows the S and T parameters on the left and right panels, respectively. The horizontal and vertical axes correspond to m_L and m_E, respectively. We take $\kappa = \bar{\kappa} = 1.0$ in this figure. The W boson mass reported by the CDF collaboration is explained on the blue line. Note that the deviation of the W boson mass from the SM prediction is related to the oblique parameters as [79,80]

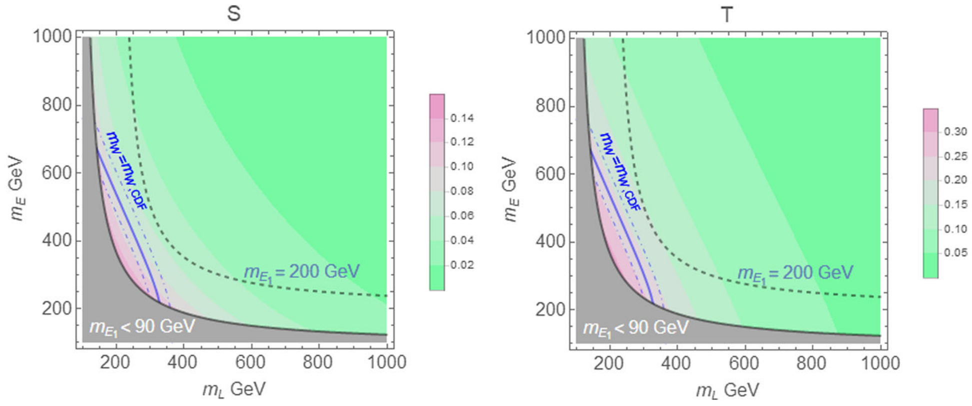

FIG. 1. Values of S and T parameters for $\kappa = \bar{\kappa} = 1.0$ with varying the vectorlike masses.

$$\frac{\delta m_W^2}{m_W^2|_{\mathrm{SM}}} = \frac{\alpha}{c_W^2 - s_W^2}\left(-\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U\right)$$
$$\sim -0.007S + 0.011T + 0.0087U, \tag{8}$$

where $\alpha = 1/128$ and $s_W^2 = 0.22337$ are used in the second equality. The dot-dashed lines correspond to 1σ range of the CDF result. The lightest vectorlike lepton mass is less than 90 GeV in the gray region, and is 200 GeV on the dashed line. As discussed later, the vectorlike lepton lighter than 90 GeV may be excluded by the LEP experiment. $T \gtrsim 0.15$ can be realized by the light vectorlike lepton, $m_{E_1} \lesssim 200$ GeV. S is positive in our parameter region. We also find $U < 0.03$, which is much smaller than the other oblique parameters.

We also compare our predictions of the oblique parameters with the results based on the CDF measurement and the previous works of the PDG. In Ref. [3], the EW fit with the new CDF measurement and the PDG value are studied. The favored values of the oblique parameters with $U = 0$ are given by $(S, T) = (0.15 \pm 0.08, 0.27 \pm 0.06)$ with the correlation coefficient 0.93 for the analysis with the CDF result and $(S, T) = (0.05 \pm 0.08, 0.09 \pm 0.07)$ with the correlation coefficient 0.92 for the analysis based on the PDG data. Similar values are obtained in Refs. [4,5]. In the SM, $S = T = U = 0$, the values of χ^2 are 60 and 3.7 with the CDF and PDG data, respectively. Thus, the new data strongly favors the existence of new physics.

Figure 2 shows values of χ from the vectorlike leptons, L and E, for $\kappa = \bar{\kappa} = 1.0$. In the (light) red region, $\chi < 1(2)$ for the CDF data, while the blue region are those for the PDG data. We see that the new CDF result favors the light vectorlike leptons, and $\chi < 2$ is achieved where $m_{E_1} < 200$ GeV, whereas the PDG result favors heavier vectorlike leptons. The red region almost coincides with the region where the new m_W is explained in Fig. 1. The values at the benchmark points are shown in Table I. Point (A) fits to the new CDF result and $\chi = 0.80$, while point (B) fits to the combined PDG result and $\chi = 0.65$. These points are plotted as the dots in Fig. 2. We also note that the PDG data also prefers a relatively light vectorlike lepton, and the point (B) has a 400 GeV vectorlike lepton.

D. Limits from the collider experiments

The vectorlike lepton should be lighter than 200 GeV to explain the new CDF result. Such a light vectorlike lepton may be excluded by direct searches in the collider experiments. If there is no DM particle and a vectorlike lepton decays to a muon and a SM boson, then the run-1 data at the LHC [81] rule out most cases [82]. Furthermore, the recent studies using the run-2 data exclude doubletlike vectorlike leptons, favored to explain the m_W anomaly; the lower bound on the mass is about 800 GeV [83,84], under the assumptions that the lepton flavor is different in these analyses. Therefore the light vectorlike lepton is possibly excluded by the LHC searches.

In our setup, the vectorlike lepton decays to a DM particle and muon. In this case, the signal is $2\mu + E_T^{\mathrm{miss}}$, where E_T^{miss} comes from the DM. This signal is studied in the slepton searches at the LHC [85–89]. As we have already shown in Ref. [43], the limit for the doublet vectorlike lepton is about 900 GeV if the DM mass is sufficiently light to produce

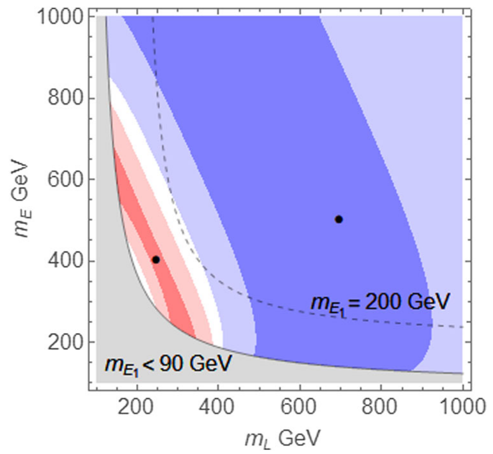

FIG. 2. Values of χ from the vectorlike leptons.

energetic muons. The limit is, however, much weaker if $\Delta m \coloneqq m_{E_1} - m_X \lesssim 100$ GeV since the muons become soft in this region. There are dedicated searches for signals with soft leptons and $E_T^{\rm miss}$ associated with an additional jet [86]. If we use the limit on degenerate sleptons [86] as a conservative one, then the limit is about 250 GeV for $\Delta m \sim 10$ GeV, while the limit is less than 100 GeV for $\Delta m \lesssim 1$ or $\gtrsim 30$ GeV.

The muons are too soft to be detected in the detector for even smaller Δm, and hence there is no detectable signal from the vectorlike lepton decays. In this case, we refer to the limits from the Higgsino searches since the Higgsino has the same quantum number as the doublet vectorlike lepton. At the LHC, the monojet searches [90–92] cannot constrain the Higgsinos due to its large backgrounds.[1] Therefore, the current lower limit of the vectorlike lepton may be about 90 GeV at the LEP experiment [94,95].

To sum up, the vectorlike lepton with $90 < m_{E_1} < 200$ GeV can still be viable if the DM is nearly degenerate with the vectorlike lepton. Limits in the degenerate region are very sensitive to the mass difference. For instance, there will be no limits stronger than 90 GeV from the degenerate slepton searches [86] for $\Delta m \lesssim 1$ GeV or $\gtrsim 30$ GeV. A dedicated study for the degenerate vectorlike lepton is an interesting subject but beyond the scope of this paper, and thus we only take the LEP bound into account while bearing in mind that there may be an exclusion limit on certain mass differences.

III. DARK MATTER PHYSICS AND Δa_μ

The vectorlike leptons can participate in the DM thermal production via the Yukawa couplings to the SM leptons and DM. With the couplings to muon, it also contributes to Δa_μ. We have figured out in Ref. [43] that Δa_μ is explained consistently with the DM relic abundance if the double and singlet vectorlike leptons have a mass mixing. In this section, we examine the compatibility of the DM production with the muon $g-2$ and W boson mass anomalies on the two benchmark points in Table I.

TABLE I. Benchmark points (A) and (B). $\kappa = \bar{\kappa} = 1.0$. $\chi_{\rm CDF}$ and $\chi_{\rm PDG}$ are the values of χ with the new CDF and PDG results, respectively.

	A	B
(m_L, m_E) GeV	(250,400)	(700,500)
(m_{E_1}, m_{E_2}) GeV	(135.5, 514.5)	(399.3, 800.7)
$s_L = s_R$	0.549623	0.86553
(S, T, U)	(0.087, 0.228, 0.012)	(0.019, 0.050, 0.0004)
m_WGeV	80.4371	80.3740
$(\chi_{\rm CDF}, \chi_{\rm PDG})$	(0.80, 4.00)	(6.03, 0.65)

In the thermal freeze-out scenario, DM relic abundance is controlled by DM pair annihilation cross section at the freeze-out temperature $T_f \simeq m_X/20$, where DM particles can be considered as nonrelativistic. In the presence of the doublet-singlet mixing, the DM couples to both left- and right-handed muons and the pair annihilation $XX \to \mu\bar{\mu}$ has the s-wave part,

$$(\sigma v)_{XX\to\mu\bar{\mu}} \simeq \frac{(\lambda_L\lambda_R)^2}{\pi}\left(\frac{c_R s_L m_{E_1}}{m_X^2 + m_{E_1}^2} - \frac{c_L s_R m_{E_2}}{m_X^2 + m_{E_2}^2}\right)^2 + \mathcal{O}(v^2), \tag{9}$$

where we neglect the muon mass and assume that λ_L and λ_R are real. Here, v is relative velocity of DM particles. This process is expressed in terms of a coupling combination $\lambda_L\lambda_R$ like the chirality enhanced contribution in Δa_μ. This suggests that the DM abundance can be highly correlated to Δa_μ in this model. In fact, as we discuss in Ref. [43] in detail, a large new physics contribution $\Delta a_\mu \sim 10^{-8}$ is predicted in this model when the s-wave part is mainly responsible for the DM production. Thus, one needs to make the s-wave contribution subdominant in the DM annihilation by, e.g., employing sizable coannihilation or introducing a large hierarchy between two coupling constants $\lambda_{L,R}$ to invoke the velocity suppressed d-wave annihilation instead of the s wave.

Figure 3 shows the predicted value of Δa_μ on the benchmark A (left) and B (right). The black solid, dashed, and dotted lines correspond to $\lambda_R = \lambda_L, 0.01\lambda_L$, and $5\times10^{-4}\lambda_L$, respectively, and the value of λ_L is fixed to explain the observed DM abundance in each case. The purple band represents the 2σ range of Δa_μ. We employ micrOMEGAs_5_2_4 [96] to calculate the DM relic abundance including all coannihilation processes.

We see in Fig. 3 that Δa_μ and DM can be consistently explained on both benchmarks only if the DM is degenerate to the lighter vectorlike lepton E_1. In this mass region, the DM pair annihilation is subdominant and the dominant process is the coannihilation of E_1; $E_1\bar{E}_1 \to \gamma h, WW, ZZ, f\bar{f}$, where f is the SM fermions. Although all of the coannihilation processes are comparable, $E_1\bar{E}_1 \to \gamma h$ gives the largest contribution for our benchmark points due to the large κ and $\bar{\kappa}$, which are favored to deviate the T parameter. Apart from the coannihilation region, a large Δa_μ is predicted due in part to a large Yukawa coupling constant λ_L being required for the DM production, and as a result such a mass region is strongly disfavored. It also follows from Fig. 3(left) that for $\lambda_R/\lambda_L \gtrsim 0.01$, $\Delta m \simeq 10$ GeV is favored by Δa_μ and DM, and the LHC result is likely to exclude the 135 GeV vectorlike lepton. While, we find $\Delta m \gtrsim 30$ GeV to explain Δa_μ if $\lambda_R/\lambda_L \sim 5\times10^{-4}$. In this case, the current LHC limit can be evaded. It should be noted, however, that λ_L is sizable with a small λ_R/λ_L and $\lambda_L \simeq 1.8$ is required

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[1] Recently, it has been proposed that mono-W/Z signal may cover light Higgsinos ~ 110 GeV with the full run-2 data [93].

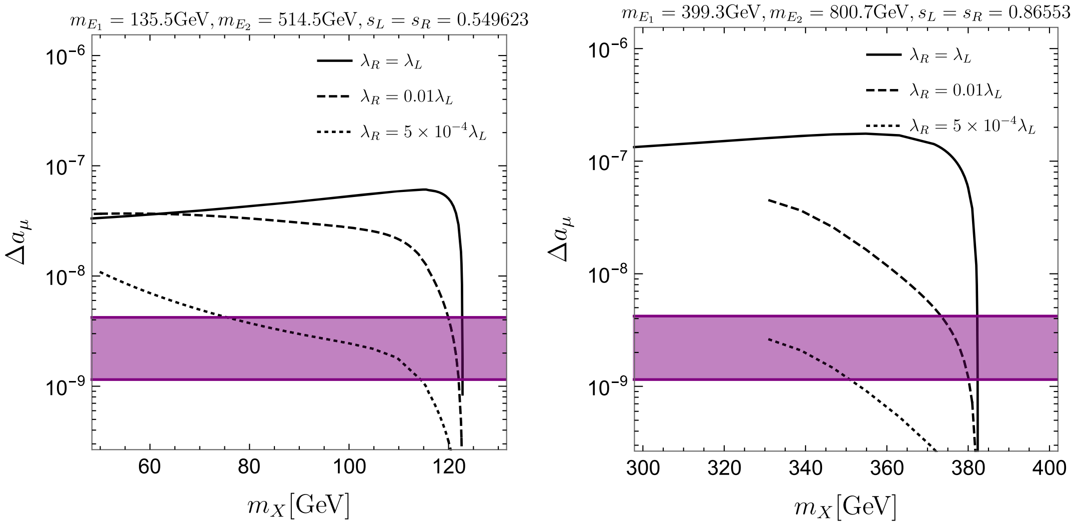

FIG. 3. The predicted value of Δa_μ on the benchmark point A (left) and B (right). The purple band represents the 2σ favored value of the muon $g-2$. For further details, see the text.

for the DM production when $m_X = 100$ GeV and $\lambda_R/\lambda_L \lesssim 0.01$. Otherwise, $\lambda_L = \mathcal{O}(0.01-0.1)$ is enough to fit the DM abundance. In the benchmark (B), the value of λ_L tends to be larger than that of the benchmark (A) mainly because m_{E_1} is heavier. This tendency is pronounced for a small λ_R/λ_L and, in fact, we find that λ_L exceeds the perturbative value $\sqrt{4\pi}$ below $m_X \simeq 330$ GeV when $\lambda_R/\lambda_L \lesssim 0.01$. That is why the black dashed and dotted lines are interrupted in $m_X \lesssim 330$ GeV.

We briefly comment on direct and indirect detection constraints. The DM candidate is a SM gauge singlet and couples only to the muon, so DM-nucleon scattering arises at the loop level. Furthermore, it is known that, in the lepton portal models, a real scalar DM starts the scattering at two-loop level via diphoton exchanging [97]. The resulting cross section is highly suppressed and thus well below the reach of the direct detection experiments. As regards the indirect detection, gamma-ray searches at Fermi-LAT [98,99] and H.E.S.S [100], and positron flux measurements at AMS [101–103] are important. In particular, a real scalar DM with the lepton portal couplings predicts a sharp photon flux via virtual internal bremsstrahlung and one-loop processes, and can be effectively searched by the Fermi/H.E.S.S line gamma searches [104–106]. If there is no doublet-singlet mixing, the bound reads $m_{E_1} \lesssim 1.1(1.2)m_X$ at best for a purely singlet (doublet) vectorlike lepton [43]. Thus, the mass region of our interest may be within the reach of the gamma-ray searches. One should note, however, that there is no detailed study on the gamma ray bound in the case with both left- and right-handed couplings, so that this Fermi/H.E.S.S limit cannot be applied directly to this model. For the AMS positron measurements, the current conservative lower limit on DM mass is 30 GeV [107], assuming the cross section of $XX \to \mu\bar{\mu}$ has the canonical size for thermal production, i.e., $(\sigma v)_{XX\to\mu\bar{\mu}} \simeq 3 \times 10^{-26}$ cm^3/s. Hence, the AMS measurements do not provide the relevant limit in this case.

IV. SUMMARY

In this paper, we studied corrections to the oblique parameters, which are strongly correlated with the *W* boson mass, in the extended SM with the real scalar DM and the vectorlike leptons. The sizable mixing between the singlet and doublet vectorlike leptons is crucial to deviate the oblique parameters. Since the mixing is induced by the Higgs VEV, the vectorlike leptons should be close to the EW symmetry breaking scale. In fact, we found that the lightest vectorlike lepton has to be lighter than 200 GeV to explain the new CDF result, see Fig. 2. The light vectorlike lepton may be excluded by the direct searches at the LHC if it decays to the SM particles. The new lepton, however, can evade the limit if it decays to a DM particle whose mass is close to the lepton, so that the signals are effectively invisible.

As discussed in Ref. [43], the Yukawa couplings involving muon, a real scalar DM and vectorlike leptons can resolve the discrepancy in the muon anomalous magnetic moment, Δa_μ. The simultaneous explanation of the DM and Δa_μ requires the sizable mixing in the vectorlike leptons and the mass degeneracy of the lighter vectorlike lepton and DM. Interestingly, the former is required to achieve the shift in *W* boson mass and the latter is to avoid the LHC limits. The mass region favored by those anomalies would be covered by the future gamma-ray searches utilizing the sharp photon flux from the galactic DM annihilation.

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APPENDIX: LOOP FUNCTIONS

The loop functions for Δa_μ are defined as

$$F(x) = \frac{2+3x-6x^2+x^3+6x\log x}{6(1-x)^4},$$
$$G(x) = \frac{3-4x+x^2+2\log x}{(1-x)^3}. \tag{A1}$$

The loop functions for the oblique parameters are defined as

$$\theta_+(y_1,y_2) = y_1+y_2-\frac{2y_1y_2}{y_1-y_2}\log\frac{y_1}{y_2},$$
$$\theta_-(y_1,y_2) = 2\sqrt{y_1y_2}\left(\frac{y_1+y_2}{y_1-y_2}\log\frac{y_1}{y_2}-2\right), \tag{A2}$$

and

$$\begin{aligned}\chi_+(y_1,y_2) = {}& \frac{y_1+y_2}{2}-\frac{(y_1-y_2)^2}{3} \\ &+\left(\frac{(y_1-y_2)^3}{6}-\frac{1}{2}\frac{y_1^2+y_2^2}{y_1-y_2}\right)\log\frac{y_1}{y_2} \\ &+\frac{y_1-1}{6}f(y_1,y_1)+\frac{y_2-1}{6}f(y_2,y_2) \\ &+\left(\frac{1}{3}-\frac{y_1+y_2}{6}-\frac{(y_1-y_2)^2}{6}\right)f(y_1,y_2),\end{aligned} \tag{A3}$$

$$\begin{aligned}\chi_-(y_1,y_2) = -\sqrt{y_1y_2}\bigg[&2+\left(y_1-y_2-\frac{y_1+y_2}{y_1-y_2}\right)\log\frac{y_1}{y_2} \\ &+\frac{f(y_1,y_1)+f(y_2,y_2)}{2}-f(y_1,y_2)\bigg],\end{aligned} \tag{A4}$$

$$\begin{aligned}\psi_+(y_1,y_2) = {}&\frac{2y_1+10y_2}{3}+\frac{1}{3}\log\frac{y_1}{y_2}+\frac{y_1-1}{6}f(y_1,y_1) \\ &+\frac{5y_2+1}{6}f(y_2,y_2),\end{aligned} \tag{A5}$$

$$\psi_-(y_1,y_2) = -\sqrt{y_1y_2}\left(4+\frac{f(y_1,y_1)+f(y_2,y_2)}{2}\right). \tag{A6}$$

Here,

$$f(y_1,y_2) = \begin{cases} \sqrt{d}\log\left|\frac{y_1+y_2-1+\sqrt{d}}{y_1+y_2-1-\sqrt{d}}\right| & d>0 \\ 0 & d=0, \\ -2\sqrt{|d|}\left[\tan^{-1}\frac{y_1-y_2+1}{\sqrt{|d|}}-\tan^{-1}\frac{y_1-y_2-1}{\sqrt{|d|}}\right] & d<0 \end{cases} \tag{A7}$$

with $d := (1+y_1-y_2)^2-4y_1$. Note that $\psi_\pm$ are different from those for vectorlike quarks shown in Ref. [78].

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