Is the resonance $X_0(2900)$ a ground-state or radially excited scalar tetraquark $[ud][\bar{c}\,\bar{s}]$?

S. S. Agaev,[1] K. Azizi,[2,3,4,*] and H. Sundu[5]

[1]Institute for Physical Problems, Baku State University, Az–1148 Baku, Azerbaijan
[2]Department of Physics, University of Tehran, North Karegar Avenue, Tehran 14395-547, Iran
[3]Department of Physics, Doğuş University, Acibadem-Kadiköy, 34722 Istanbul, Turkey
[4]School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531 Tehran, Iran
[5]Department of Physics, Kocaeli University, 41380 Izmit, Turkey

(Received 18 May 2022; accepted 5 July 2022; published 25 July 2022)

We investigate properties of the ground-state and first radially excited four-quark mesons X_0 and X_0' with a diquark-antidiquark structure $[ud][\bar{c}\,\bar{s}]$ and spin-parities $J^{\mathrm{P}}=0^+$. Our aim is to reveal whether or not one of these states can be identified with the resonance $X_0(2900)$, recently discovered by the LHCb Collaboration. We model X_0 and X_0' as tetraquarks composed of either axial-vector or scalar diquark and antidiquark pairs. Their spectroscopic parameters are computed by employing the QCD two-point sum rule method and including vacuum condensates up to dimension 15 in the analysis. For an axial-axial structure of $X_0^{(\prime)}$, we find partial widths of the decays $X_0^{(\prime)} \to D^- K^+$ and $X_0^{(\prime)} \to D^0 K^0$, and estimate full widths of the states $X_0^{(\prime)}$. To this end, we calculate the strong couplings at the vertices $X_0^{(\prime)} DK$ in the framework of the light-cone sum rule method. We also use technical approaches of the soft-meson approximation necessary to analyze tetraquark-meson-meson vertices. We obtain $m=(2545 \pm 160)$ MeV and $m'=(3320 \pm 120)$ MeV [$m_{\mathrm{S}}=(2663 \pm 110)$ MeV and $m_{\mathrm{S}}'=(3325 \pm 85)$ MeV for a scalar-scalar current] for the masses of the particles X_0 and X_0', as well as estimates for their full widths $\Gamma_0=(140 \pm 29)$ MeV and $\Gamma_0'=(110 \pm 25)$ MeV, which allow us to interpret that neither is the resonance $X_0(2900)$. At the same time, these predictions provide important information about the ground-state and radially excited diquark-antidiquark structures X_0 and X_0', which should be objects of future experimental and theoretical studies.

DOI: 10.1103/PhysRevD.106.014019

I. INTRODUCTION

One of the most important recent achievements in the physics of multiquark hadrons is the observation of the structures $X_0(2900)$ and $X_1(2900)$ by the LHCb Collaboration. These resonance-like peaks were discovered in the invariant mass distribution D^-K^+ of the decay channel $B^+ \to D^+D^-K^+$ [1,2]. The LHCb measured the masses and widths of these structures and fixed their spin-parities. It turned out that $X_0(2900)$ and $X_1(2900)$ are scalar and vector resonances with quantum numbers $J^{\mathrm{P}}=0^+$ and $J^{\mathrm{P}}=1^-$, respectively.

*Corresponding author.
kazem.azizi@ut.ac.ir

The appearance of the mesons D^- and K^+ in the final state of their decays implies that $X_0(2900)$ and $X_1(2900)$ are composed of the quarks $\bar{c}\,\bar{s}\,ud$, and may be considered as particles containing four quarks of different flavors. In other words, $X_0(2900)$ and $X_1(2900)$ presumably constitute new evidence for exotic mesons with full open-flavor structures. This is an important fact, because the existence of the resonance $X(5568)$—presumably built from $sd\bar{b}\,\bar{u}$ quarks and considered as a first candidate for a fully open-flavor four-quark state [3]—was not confirmed by other collaborations. Of course, this analysis is correct in the context of the four-quark model of $X_0(2900)$ and $X_1(2900)$, because there are theoretical analyses that claim to explain the LHCb data using hadronic rescattering effects. The LHCb Collaboration also did not exclude such an interpretation of the observed structures.

New experimental information triggered intensive theoretical activities aimed at revealing the internal organization of these resonances, calculating their parameters, and studying the processes in which $X_0(2900)$ and $X_1(2900)$

can be produced [4–26]. In an overwhelming majority of investigations the resonances $X_0(2900)$ and $X_1(2900)$ were modeled as diquark-antidiquark states or hadronic molecules. In fact, the resonance $X_0(2900)$ was explored as a scalar tetraquark $[sc][\bar{u}\,\bar{d}]$ in Refs. [4,5] using a phenomenological model and the sum rule method, respectively. The predictions for the mass [(2863 ± 12) and (2910 ± 120) MeV] obtained in these papers allowed the authors to interpret $X_0(2900)$ as the ground-state scalar tetraquark $[sc][\bar{u}\,\bar{d}]$. An interesting assumption about the nature of $X_0(2900)$ was made in Ref. [6], where it was studied as a radially excited state $[ud][\bar{c}\,\bar{s}]$. In Refs. [7–10] the resonance $X_0(2900)$ was examined as the S-wave molecule $D^{*-}K^{*+}$. The tetraquark and molecular models were used for the resonance $X_1(2900)$, as well [6,7,11]. But two resonance-like peaks in the D^-K^+ mass distribution may have a different nature and emerge due to triangle singularities in the rescattering diagrams $\chi_{c1}D^{*-}K^{*+}$ and $D_{sJ}\bar{D}_1^0K^0$ [12].

In Ref. [27], we investigated $X_0(2900)$ as the molecule $\bar{D}^{*0}K^{*0}$ and evaluated its spectroscopic parameters and width. Comparing our results for the mass [(2868 ± 198) MeV] and width [(49.6 ± 9.3) MeV] of $\bar{D}^{*0}K^{*0}$ with the corresponding LHCb data [$m = (2866 \pm 7 \pm 2)$ MeV and $\Gamma = (57 \pm 12 \pm 4)$ MeV], we decided that a molecular model is acceptable for the resonance $X_0(2900)$.

The vector resonance $X_1(2900)$ was considered in the context of the diquark-antidiquark model in our article [28]. We studied it as a vector tetraquark composed of a diquark $u^TC\gamma_5 d$ and an antidiquark $\bar{c}\gamma_\mu\gamma_5 C\bar{s}^T$, and computed relevant parameters. Though the predictions for the mass [(2890 ± 122) MeV] and width [(93 ± 13) MeV] of this tetraquark are smaller than the relevant LHCb data, we interpreted it as the resonance $X_1(2900)$ by keeping in mind that theoretical and experimental investigations suffer from certain errors.

Over the last few years, diquark-antidiquark states containing four quarks/antiquarks (c, s, u, and d) in different configurations have been a subject of investigation. Thus, a scalar tetraquark $X_c = [su][\bar{c}\,\bar{d}]$ was considered in our article [29], where it was modeled as an exotic meson made of scalar-scalar and axial-axial diquarks with $C\gamma_5 \otimes \gamma_5 C$ and $C\gamma_\mu \otimes \gamma^\mu C$ type interpolating currents, respectively. The mass of X_c found using these two structures was (2634 ± 62) and (2590 ± 60) MeV, respectively. The result (2.55 ± 0.09) GeV for the mass of X_c was also obtained in Ref. [30].

Though X_c and $X_0 = [ud][\bar{c}\,\bar{s}]$ have similar content, there are two differences between them: X_c consists of a relatively heavy diquark $[su]$ and heavy antidiquark $[\bar{c}\,\bar{d}]$, whereas X_0 has a light diquark $[ud]$ and heavy antidiquark $[\bar{c}\,\bar{s}]$. The second difference is the decay channels of these particles. While the dominant decay mode of X_c is $X_c \to D_s^-\pi^+$, in the case of X_0 we have $X_0 \to D^-K^+$. Nevertheless, as we shall see below, the masses and widths of X_0 and X_c are similar mainly due to their quark contents.

In the current work, we explore the scalar tetraquark $X_0 = [ud][\bar{c}\,\bar{s}]$ in a detail. Thus, we compute the masses of the ground-state $1S$ and radially excited $2S$ tetraquarks X_0 and X_0', using the QCD two-point sum rule method, and two interpolating currents. The widths of X_0 and X_0' are calculated in the framework of the light-cone sum rules (LCSR) method. This is necessary to find the strong couplings at the vertices $X_0^{(\prime)}D^-K^+$ and $X_0^{(\prime)}\bar{D}^0K^0$, which determine the partial widths of the decay channels $X_0^{(\prime)} \to D^-K^+$ and $X_0^{(\prime)} \to \bar{D}^0K^0$. Because aforementioned strong couplings correspond to tetraquark-meson-meson type vertices, the LCSR method should be applied alongside technical tools of the soft-meson approximation.

This work is organized as follows. In Sec. II we calculate the masses and couplings of the ground-state and radially excited tetraquarks $X_0^{(\prime)}$. To this end, we use both the scalar-scalar and axial-axial type interpolating currents. The sum rule computations are carried out by including effects of vacuum condensates up to dimension 15. In Sec. III we compute the strong couplings $g^{(\prime)}$ and $G^{(\prime)}$ that describe the strong interaction of particles at the vertices $X_0^{(\prime)}D^-K^+$ and $X_0^{(\prime)}\bar{D}^0K^0$. Here, we also evaluate the partial widths of the decays $X_0^{(\prime)} \to D^-K^+$ and $X_0^{(\prime)} \to \bar{D}^0K^0$, and find the full widths of the tetraquarks $X_0^{(\prime)}$. Section IV is devoted to discussions and conclusions.

II. MASS AND CURRENT COUPLING OF 1S AND 2S TETRAQUARKS X_0 AND X_0'

The masses and current couplings are important parameters of the tetraquarks X_0 and X_0'. The masses of these states are necessary to compare them with the LHCb data and fix whether one of these particles may be interpreted as the resonance $X_0(2900)$. The current couplings of X_0 and X_0' in conjunction with their masses are required to calculate the partial widths of the decay channels $X_0^{(\prime)} \to D^-K^+$ and $X_0^{(\prime)} \to \bar{D}^0K^0$, and hence to evaluate the full widths of these tetraquarks.

We compute the masses and couplings of X_0 and X_0' in the framework of the QCD two-point sum rule method, which is one of the most effective nonperturbative approaches in high-energy physics [31,32]. It rests on fundamental principles of QCD and leads to reliable predictions, using as input parameters only a few universal vacuum condensates. Remarkably, sum rules derived by means of this method are applicable to investigating both ordinary and multiquark hadrons [33–36].

We start our study by considering the following two-point correlation function:

$$\Pi(p) = i\int d^4x e^{ipx}\langle 0|\mathcal{T}\{J(x)J^\dagger(0)\}|0\rangle, \qquad (1)$$

where $\mathcal{T}$ is the time-ordered product, and $J(x)$ is the interpolating current for the tetraquarks X_0 and X_0'. In general, the tetraquarks X_0 and X_0' with the required quantum numbers $J^{\mathrm{P}}=0^+$ can be built from different diquarks: either a scalar diquark and antidiquark pair $u^T C\gamma_5 d$ and $\bar{c}\gamma_5 C\bar{s}^T$, or an axial-vector diquark $u^T C\gamma_\mu d$ and axial-vector antidiquark $\bar{c}\gamma^\mu C\bar{s}^T$, where C is the charge-conjugation matrix. Interpolating currents that correspond to these structures have the following forms:

$$J_{\mathrm{S}}(x)=\epsilon\tilde{\epsilon}[u_b^T(x)C\gamma_5 d_c(x)][\bar{c}_d(x)\gamma_5 C\bar{s}_e^T(x)], \tag{2}$$

and

$$J(x)=\epsilon\tilde{\epsilon}[u_b^T(x)C\gamma_\mu d_c(x)][\bar{c}_d(x)\gamma^\mu C\bar{s}_e^T(x)], \tag{3}$$

where $\epsilon\tilde{\epsilon}=\epsilon_{abc}\tilde{\epsilon}_{ade}$, and a, b, c, d, and e are color indices. In Eqs. (2) and (3), $c(x)$, $s(x)$, $u(x)$, and $d(x)$ are corresponding quark fields. In what follows, we consider in a detailed manner the interpolating current $J(x)$, and provide only final results obtained while employing $J_{\mathrm{S}}(x)$.

To derive required sum rules, the correlation function $\Pi(p)$ has to be expressed in terms of the X_0 and X_0' tetraquarks' physical parameters. The function $\Pi^{\mathrm{Phys}}(p)$ obtained after relevant manipulations constitutes the physical (phenomenological) side of the sum rules. We analyze ground-state and first radially excited particles, and therefore include contributions of these states to the correlation function explicitly. As a result, we obtain

$$\Pi^{\mathrm{Phys}}(p)=\frac{\langle 0|J|X_0\rangle\langle X_0|J^\dagger|0\rangle}{m^2-p^2}+\frac{\langle 0|J|X_0'\rangle\langle X_0'|J^\dagger|0\rangle}{m'^2-p^2}\cdots, \tag{4}$$

where m and m' are the masses of the tetraquarks X_0 and X_0'. Equation (4) is derived by saturating the correlation function $\Pi(p)$ with a full set of scalar four-quark states and performing an integration over x in Eq. (1). The dots in Eq. (4) stand for effects of higher resonances and continuum states in the X_0 channel.

Equation (4) contains two simple-pole terms, which in the case of multiquark hadrons have to be used with some caution. The reason is that the physical side may also contain two-meson reducible contributions. Indeed, the current $J(x)$ not only couples to the tetraquarks X_0 and X_0', but also interacts with conventional two-meson states [37,38]. These two-meson contributions modify the quark propagator in Eq. (4),

$$\frac{1}{m^2-p^2}\to\frac{1}{m^2-p^2-i\sqrt{p^2}\Gamma(p)}, \tag{5}$$

where $\Gamma(p)$ is the finite width of the tetraquark generated by two-meson effects. They should be subtracted from the sum rules, or taken into account in the parameters of the pole terms. For tetraquarks, the second method was applied Refs. [39–41] and it was demonstrated that these contributions can be absorbed into the current coupling, while at the same time ensuring that the mass of the tetraquark is stable. Detailed analyses proved that two-meson effects are small, and do not exceed theoretical errors of the sum rule method itself [38–41]. Therefore, the physical side of the sum rules is written down above by applying the zero-width single-pole approximation.

Using the matrix elements

$$\langle 0|J|X_0^{(\prime)}\rangle=f^{(\prime)}m^{(\prime)}, \tag{6}$$

it is possible to simplify the function $\Pi^{\mathrm{Phys}}(p)$. Simple operations for $\Pi^{\mathrm{Phys}}(p)$ lead to the expression

$$\Pi^{\mathrm{Phys}}(p)=\frac{f^2m^2}{m^2-p^2}+\frac{f'^2m'^2}{m'^2-p^2}\cdots. \tag{7}$$

The function $\Pi^{\mathrm{Phys}}(p)$ has a simple Lorentz structure $\sim I$ and, depending on the problem under consideration, one or a sum of two terms may form the corresponding invariant amplitude $\Pi^{\mathrm{Phys}}(p^2)$.

The second component of the sum rules $\Pi^{\mathrm{OPE}}(p)$ should be computed in the operator product expansion (OPE) with a certain accuracy. It can be found by employing the expression for the interpolating current $J(x)$, and replacing contracted quark fields with relevant propagators. After these operations, for $\Pi^{\mathrm{OPE}}(p)$ we obtain

$$\Pi^{\mathrm{OPE}}(p)=i\int d^4x e^{ipx}\epsilon\tilde{\epsilon}\epsilon'\tilde{\epsilon}'\mathrm{Tr}[S_s^{e'e}(-x)\gamma^\mu \times\tilde{S}_c^{d'd}(-x)\gamma^\nu]\mathrm{Tr}[S_u^{bb'}(x)\gamma_\nu\tilde{S}_d^{cc'}(x)\gamma_\mu], \tag{8}$$

where

$$\tilde{S}_{c(q)}(x)=CS_{c(q)}^T(x)C. \tag{9}$$

Here, $S_c(x)$ and $S_q(x)$ are the heavy c- and light $q=u(s,d)$-quark propagators, respectively. Their explicit expressions are collected in the Appendix. The correlation function $\Pi^{\mathrm{OPE}}(p)$ also has a trivial Lorentz structure proportional to I. We denote the invariant amplitude corresponding to this structure by $\Pi^{\mathrm{OPE}}(p^2)$.

The correlation function $\Pi^{\mathrm{Phys}}(p)$ corresponds to the "ground-state+excited particle+continuum" scheme, and encompasses contributions of two particles. As the first step of our analysis, we employ the familiar "ground-state +continuum" scheme, and find the mass and coupling of the ground-state tetraquark X_0. This means that we include the second term in $\Pi^{\mathrm{Phys}}(p)$ in a list of "higher resonances and continuum states," and get the standard expression for the correlation function. Operations which are necessary to derive sum rule for m and f are well- known and discussed repeatedly in the literature. Therefore, we skip further details and provide final formulas:

$$m^2 = \frac{\Pi'(M^2, s_0)}{\Pi(M^2, s_0)} \tag{10}$$

and

$$f^2 = \frac{e^{m^2/M^2}\Pi(M^2, s_0)}{m^2}, \tag{11}$$

where M^2 and s_0 are the Borel and continuum threshold parameters, respectively. Here, $\Pi(M^2, s_0)$ is the Borel transformed and subtracted invariant amplitude $\Pi^{\rm OPE}(p^2)$, and $\Pi'(M^2, s_0) = d\Pi(M^2, s_0)/d(-1/M^2)$.

At this stage, one should fix the working windows for the parameters M^2 and s_0, which are auxiliary quantities of sum rule computations and should obey some important restrictions. These restrictions are connected with dominance of the pole contribution (PC) to the correlation function $\Pi(M^2, s_0)$, with convergence of OPE and stability of physical quantities against variations of the Borel parameter. Fulfillment of aforementioned constraints can be fixed using the following expressions

$$\mathrm{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}, \tag{12}$$

and

$$R(M^2) = \frac{\Pi^{\rm DimN}(M^2, s_0)}{\Pi(M^2, s_0)}, \tag{13}$$

and numerical limits on PC, $R(M^2)$, as well as fixing acceptable variations of m and f. In Eq. (13) $\Pi^{\rm DimN}(M^2, s_0)$ is the last term or the sum of the last few terms in the correlation function. In the present paper, we employ the last three terms in the OPE, and hence $\Pi^{\rm DimN}(M^2, s_0) = \Pi^{\rm Dim(13+14+15)}(M^2, s_0)$.

Having fixed the working regions for M^2 and s_0, one can extract the mass and coupling of the $1S$ tetraquark X_0. The quantities m and f, strictly speaking, should not depend on the Borel parameter. But real calculations demonstrate that working regions for M^2 and s_0 have an impact on extracted parameters and generate uncertainties, which nevertheless should be kept within acceptable limits. On the contrary, the continuum threshold parameter s_0 bears physical information about the mass of the excited tetraquark X_0'. In fact, the parameter s_0 separates the contribution of the ground-state particle from those due to higher resonances and continuum states. This means that the masses of X_0 and X_0' must obey the restrictions $m < \sqrt{s_0} \leq m'$.

After calculating the mass and coupling of X_0, we can find the parameters of the excited state X_0'. For this purpose, we treat m and f as input parameters and look for new working regions for M^2 and s_0^*, which not only have to satisfy Eqs. (12) and (13), but also have to obey $s_0^* > s_0$. The necessity of the last constraint is evident, because in the "ground-state+excited particle+continuum" scheme the parameter s_0^* separates two states from remaining higher resonances. The mass of the X_0' extracted from the new sum rule is bounded by the conditions $\sqrt{s_0} \leq m' < \sqrt{s_0^*}$. The regions for M^2 and s_0^*, and extracted mass m' should comply with these regulations, then performed analysis can be considered as being selfconsistent and giving reliable predictions.

The sum rules for m' and f' obviously differ from those for m and f. For the mass m' we derive the expression

$$m'^2 = \frac{\Pi'(M^2, s_0^*) - f^2 m^4 e^{-m^2/M^2}}{\Pi(M^2, s_0^*) - f^2 m^2 e^{-m^2/M^2}}, \tag{14}$$

whereas for f' we get

$$f'^2 = \frac{e^{m'^2/M^2}[\Pi(M^2, s_0^*) - f^2 m^2 e^{-m^2/M^2}]}{m'^2}. \tag{15}$$

It is evident that the parameters m' and f' of the excited particle X_0' depend explicitly on the mass and current coupling of the ground-state tetraquark X_0. Such a dependence is natural, because Eq. (7) contains two terms, and m and f appear as inputs when calculating m' and f'. In turn, the excited state X_0' also affects the mass m and coupling f of the ground-state particle, but its effect is implicit and encoded in the choice of the continuum threshold parameter s_0. In fact, the parameters m and f extracted from the sum rules depend on the correlation function $\Pi(M^2, s_0)$ at s_0, which is limited by the mass m' of the excited state $\sqrt{s_0} \leq m'$. Because the two sets (m, f) and (m', f') are determined by the same correlation function at different s_0 and s_0^*, one may consider the difference of $\Pi(M^2, s_0)$ at s_0 and s_0^* as a "measure" of this effect.

The correlation function $\Pi(M^2, s_0)$ has the following form:

$$\Pi(M^2, s_0) = \int_{\mathcal{M}^2}^{s_0} ds\rho^{\rm OPE}(s) e^{-s/M^2} + \Pi(M^2), \tag{16}$$

where $\mathcal{M} = m_c + m_s$. In this work, we neglect the masses of the quarks u and d and terms $\sim m_s^2$, but take into account the contributions from m_s. The spectral density $\rho^{\rm OPE}(s)$ is calculated as an imaginary part of the correlator $\Pi^{\rm OPE}(p)$. The function $\Pi(M^2)$ is the Borel transformation of terms in $\Pi^{\rm OPE}(p)$ derived directly from their expressions. Computations are performed by taking into account vacuum condensates until dimension 15. In the Appendix, for the sake of brevity, we provide analytical expressions for $\rho^{\rm OPE}(s)$ and $\Pi(M^2)$ up to dimension 11.

Our analytical results contain nonperturbative terms up to dimension 15, which makes it necessary to explain the treatment of higher-dimensional vacuum condensates. The propagator $S_q(x)$ contains various quark, gluon, and mixed condensates of different dimensions, and terms proportional

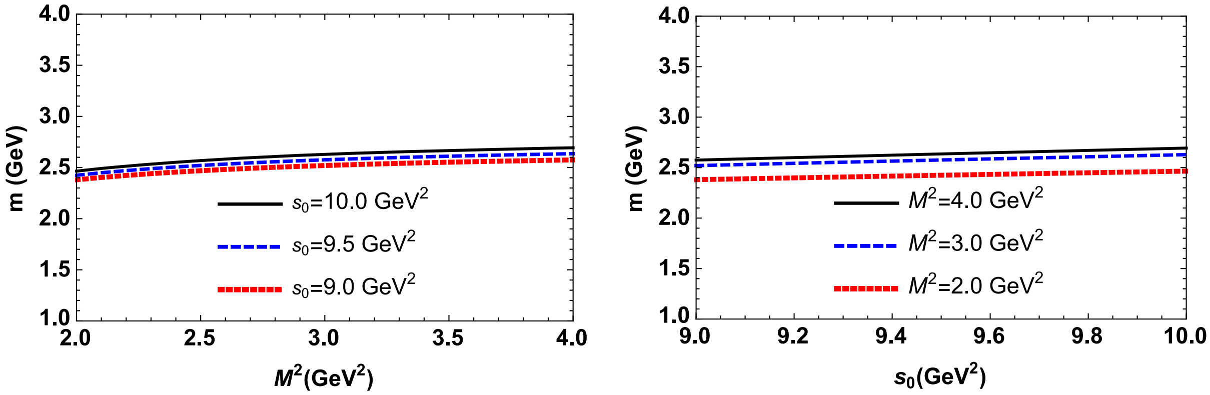

FIG. 1. Mass m of the tetraquark X_0 as a function of the Borel parameter M^2 (left) and as a function of s_0 (right).

to $g_s^2G^2$ and $g_s^3G^3$ are taken into account in $S_c(x)$. Some of the terms in the propagator $S_q(x)$, such as those proportional to $\langle \bar{q}g_s\sigma Gq\rangle$, $\langle \bar{q}q\rangle^2$, and $\langle \bar{q}q\rangle\langle g_s^2G^2\rangle$, are obtained using the factorization hypothesis of higher-dimensional condensates. These terms and their products with condensates from other light quark propagators, as well as with relevant components of $S_c(x)$, enter into $\rho^{\mathrm{OPE}}(s)$ and $\Pi(M^2)$. We carry out computations by taking into account all contributions up to dimension 15 obtained in this way, but the factorization of higher-dimensional condensates is not precise and generates uncertainties [42], which are sometimes difficult to estimate. Because contributions of higher-dimensional terms are numerically very small, we neglect the impact of such uncertainties on extracted quantities.

The sum rules for $m^{(\prime)}$ and $f^{(\prime)}$ contain universal quark, gluon, and mixed vacuum condensates, which we list below:

$$\langle \bar{q}q\rangle = -(0.24\pm 0.01)^3\ \mathrm{GeV}^3,\quad \langle \bar{s}s\rangle = (0.8\pm 0.1)\langle \bar{q}q\rangle,$$
$$\langle \bar{q}g_s\sigma Gq\rangle = m_0^2\langle \bar{q}q\rangle,\quad \langle \bar{s}g_s\sigma Gs\rangle = m_0^2\langle \bar{s}s\rangle,$$
$$m_0^2 = (0.8\pm 0.2)\ \mathrm{GeV}^2$$
$$\left\langle \frac{\alpha_s G^2}{\pi}\right\rangle = (0.012\pm 0.004)\ \mathrm{GeV}^4,$$
$$m_s = 93^{+11}_{-5}\ \mathrm{MeV},\quad m_c = 1.27\pm 0.2\ \mathrm{GeV}. \tag{17}$$

The masses of c and s quarks are also included in Eq. (17).

We begin from analysis of the ground-state tetraquark X_0, and fix regions M^2 and s_0, where paremeters of X_0can be extracted. To determine the region for M^2, we require fulfillment of the condition $\mathrm{PC}\geq 0.2$ at maximal value of M^2_{max} and convergence of OPE at its minimum, i.e., $R(M^2_{\mathrm{min}})\leq 0.01$. Our calculations demonstrate that the working regions

$$M^2\in [2,4]\ \mathrm{GeV}^2,\qquad s_0\in [9,10]\ \mathrm{GeV}^2 \tag{18}$$

satisfy the aforementioned restrictions. Thus, at $M^2_{\mathrm{max}} = 4\ \mathrm{GeV}^2$ the pole contribution is equal to 0.23, whereas at $M^2_{\mathrm{min}} = 2\ \mathrm{GeV}^2$ it is equal to 0.7. At $M^2_{\mathrm{min}} = 2\ \mathrm{GeV}^2$, we get $R(M^2_{\mathrm{min}}) < 0.01$, and hence the convergence of the sum rules is ensured. The mean values of m and f averaged over the regions (18) read

$$m = (2545\pm 160)\ \mathrm{MeV},$$
$$f = (3.0\pm 0.5)\times 10^{-3}\ \mathrm{GeV}^4. \tag{19}$$

The uncertainties of the results in Eq. (19) are within acceptable limits: for the mass and coupling they form $\pm 6.3\%$ and $\pm 16.7\%$ of the corresponding central values, respectively. Theoretical uncertainties of m are smaller,

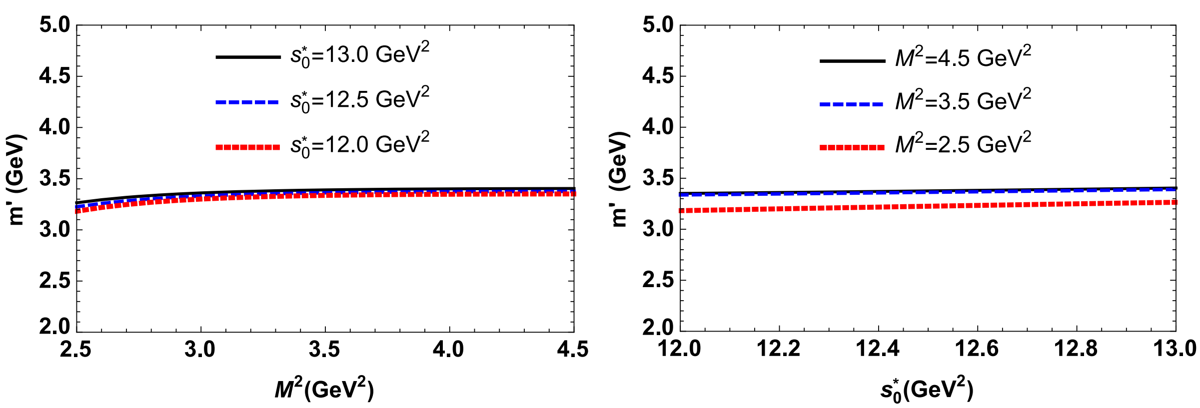

FIG. 2. The same as in Fig. 1, but for the mass m' of the excited tetraquark X_0'.

TABLE I. Mass and current coupling of the tetraquarks X_{S} and X'_{S}, and the regions for the parameters M^2 and s_0 used to calculate them.

Tetraquarks	X_{S}	X'_{S}
M^2 (GeV^2)	2–4	2.5–4.5
$s_0(s_0^\star)$ (GeV^2)	9–10	12–13
m_{S} (MeV)	2663 ± 110	3325 ± 85
$f_{\mathrm{S}} \cdot 10^3$ (GeV^4)	2.2 ± 0.3	2.7 ± 0.4

because the relevant sum rule (10) is given as a ratio of correlation functions, whereas f is determined by the expression with the correlation function in the numerator of Eq. (11). In Fig. 1, we depict the sum rule's predictions for m as functions of M^2 and s_0 in which one can see the dependence of m on the Borel and continuum threshold parameters.

To find the parameters of the first radially excited tetraquark X'_0, we start our analysis from Eqs. (14) and (15) and explore regions of M^2 and s_0^*, bearing in mind that $s_0^* > s_0$. It is not difficult to see that the working windows

$$M^2 \in [2.5, 4.5]\ \mathrm{GeV}^2, \qquad s_0^* \in [12, 13]\ \mathrm{GeV}^2 \tag{20}$$

obey the necessary constraints. In these regions the pole contribution to $\Pi(M^2, s_0^*)$ changes inside of the interval

$$0.75 \geq \mathrm{PC} \geq 0.34. \tag{21}$$

The mass and coupling of the radially excited tetraquark are

$$m' = (3320 \pm 120)\ \mathrm{MeV},$$
$$f' = (3.7 \pm 0.6) \times 10^{-3}\ \mathrm{GeV}^4, \tag{22}$$

respectively. The dependence of m' on the parameters M^2 and s_0^* is shown on Fig. 2. Comparing Figs. 1 and 2, one can see that theoretical ambiguities for the mass of the tetraquark X'_0 are smaller than those for m.

With these final predictions in hand, one can check correctness of performed analysis. Thus, using mean values of the parameters $\sqrt{s_0^*} = 3.54$ GeV and $\sqrt{s_0} = 3.08$ GeV it is easy to be convinced that all regulations discussed above are correct.

The mass and coupling of the ground-state and excited tetraquarks X_{S} and X'_{S} extracted from the sum rules by employing the interpolating current $J_{\mathrm{S}}(x)$ are shown in Table I. We also plot the masses m_{S} and m'_{S} in Figs. 3 and 4

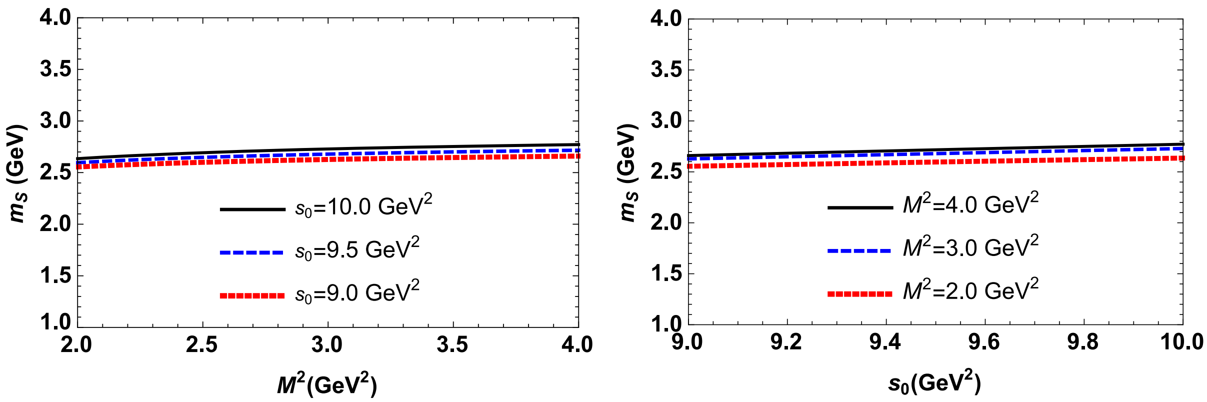

FIG. 3. Dependence of the mass m_{S} on the Borel parameter M^2 at some fixed s_0 (left) and on the continuum threshold parameter s_0 at fixed Borel parameter (right).

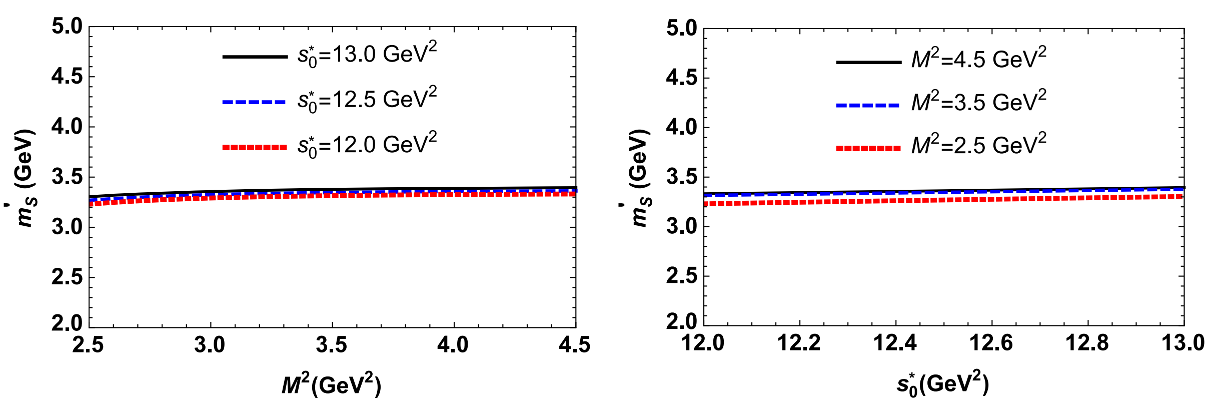

FIG. 4. The same as in Fig. 3, but for the mass m'_{S} of the excited state.

as functions of the Borel and continuum threshold parameters.

Results obtained for the masses of the states X_0 and X_0' are either smaller than the LHCb data for the resonance $X_0(2900)$ (as in the case of the ground-state tetraquark X_0) or exceed it. The same conclusions are valid also for the tetraquarks X_s and X_s'. Even masses $m^{(\prime)}$ and $m_S^{(\prime)}$ in which one takes into account ambiguities of calculations, do not agree with experimental data. It seems the diquark-antidiquark state X_0 and its radial excitation X_0' are exotic mesons not yet seen in experiments. To gain detailed information on their properties, in the next section we consider decays of the tetraquarks X_0 and X_0' and estimate their full widths.

III. PROCESSES $X_0^{(\prime)} \to D^- K^+$ AND $X_0^{(\prime)} \to \bar{D}^0 K^0$

The masses of the tetraquarks $X_0^{(\prime)}$ calculated in the previous section, as well as their quark content, allow us to specify their decay channels. It is not difficult to see that thresholds $\approx$2364 MeV for the production of conventional meson pairs $D^- K^+$ and $\bar{D}^0 K^0$ are smaller than the masses of $X_0^{(\prime)}$. Moreover, the modes $X_0^{(\prime)} \to D^- K^+$ and $X_0^{(\prime)} \to \bar{D}^0 K^0$ are S-wave decay channels for the tetraquarks $X_0^{(\prime)}$, and decay to $D^- K^+$ mesons is the dominant process for the resonance $X_0(2900)$.

In this section, we consider in a rather detailed form the decays $X_0^{(\prime)} \to D^- K^+$, and provide final results for the channels $X_0^{(\prime)} \to \bar{D}^0 K^0$. The partial widths of the processes $X_0 \to D^- K^+$ and $X_0' \to D^- K^+$ are determined by strong couplings at the corresponding tetraquark-meson-meson vertices $X_0 D^- K^+$ and $X_0' D^- K^+$, respectively. We denote strong couplings corresponding to these vertices by g and g' respectively, and use for their calculations the QCD sum rules on the light cone [43,44], and techniques of the soft-meson approximation [45].

The strong couplings g and g' are defined by the on-mass-shell matrix element

$$\langle K(q) D(p) | X_0^{(\prime)}(p') \rangle = g^{(\prime)} p \cdot p'. \tag{23}$$

In the framework of the LCSR method the vertex $X_0 D^- K^+$ can be investigated by means of the correlation function

$$\Pi(p,q) = i \int d^4x e^{ipx} \langle K(q) | \mathcal{T}\{J^D(x) J^\dagger(0)\} | 0 \rangle, \tag{24}$$

where the mesons K^+ and D^- are denoted by K and D, respectively. In Eq. (24), $J(x)$ and $J^D(x)$ are the interpolating currents for the tetraquarks $X_0^{(\prime)}$ and meson D^-. $J(x)$ is defined by Eq. (3), and for $J^D(x)$ we employ

$$J^D(x) = \bar{c}_j(x) i\gamma_5 d_j(x), \tag{25}$$

with j being the color index.

The current $J(x)$ couples to both the ground-state and radially excited tetraquarks X_0 and X_0', and therefore in the function $\Pi^{\rm Phys}(p,q)$ we should take into account the contribution of these particles explicitly. We are interested in terms that have poles at the variables p^2 and p'^2, where p and $p' = p + q$ are the momenta of the D^- meson and tetraquarks $X_0^{(\prime)}$, and q is the momentum of the K^+ meson. The terms in $\Pi^{\rm Phys}(p,q)$ necessary for our analysis have the following forms:

$$\Pi^{\rm Phys}(p,q) = \frac{f_D m_D^2}{m_c(p^2 - m_D^2)} \left[\frac{g f m}{(p'^2 - m^2)} + \frac{g' f' m'}{(p'^2 - m'^2)} \right] \times p \cdot p' + \cdots, \tag{26}$$

where m_D and f_D are the mass and decay constant of the D^- meson. To derive Eq. (26) we use the vertex function given by Eq. (23), the well-known matrix elements of the tetraquarks $X_0^{(\prime)}$ [Eq. (6)], and the new matrix element of the D^- meson,

$$\langle 0 | J^D | D(p) \rangle = \frac{f_D m_D^2}{m_c}. \tag{27}$$

The terms presented explicitly in Eq. (26) correspond to a ground-state meson in the D^- channel, and ground-state and radially excites tetraquarks in the X_0 channel. Contributions of remaining higher resonances and continuum states in the D^- and X_0 channels are denoted by dots.

An expression for the same correlation function obtained using quark-gluon degrees of freedom forms the second component $\Pi^{\rm QCD}(p,q)$ of the sum rule analysis. Calculations carried out using quark propagators give

$$\Pi^{\rm OPE}(p,q) = \int d^4x e^{ipx} \epsilon\tilde{\epsilon} [\gamma^\mu \tilde{S}_d^{jc}(x) \gamma_5 \times \tilde{S}_c^{jd}(-x) \gamma_\mu]_{\alpha\beta} \langle K(q) | \bar{u}_\alpha^b(0) s_\beta^e(0) | 0 \rangle, \tag{28}$$

with α and β being the spinor indices. The correlator $\Pi^{\rm OPE}(p,q)$ contains quark propagators, which determine the hard part of this function, but it also depends on the $\bar{u}s$ operator's local matrix elements; this is the soft factor in $\Pi^{\rm OPE}(p,q)$.

The matrix elements $\langle K | \bar{u}s | 0 \rangle$ bear spinor and color indices, and are inconvenient for further usage. To recast them into color-singlet form and factor out spinor indices, we expand $\bar{u}s$ over the full set of Dirac matrices Γ^J,

$$\Gamma^J = \mathbf{1}, \quad \gamma_5, \quad \gamma_\mu, \quad i\gamma_5\gamma_\mu, \quad \sigma_{\mu\nu}/\sqrt{2}, \tag{29}$$

and project them onto the colorless states

$$\bar{u}_\alpha^b(0) s_\beta^a(0) \to \frac{1}{12} \delta^{ba} \Gamma_{\beta\alpha}^J [\bar{u}(0) \Gamma^J s(0)]. \tag{30}$$

New operators $\bar{u}(0)\Gamma^J s(0)$ sandwiched between the K mason and vacuum states give rise to local matrix elements of the K meson.

When considering the tetraquark-meson-meson vertices $X_0^{(\prime)} D^- K^+$, we encounter the correlation function containing only local matrix elements of quark operators. Let us note that such behavior of $\Pi^{\mathrm{OPE}}(p,q)$ is typical for all one tetraquark-two conventional mesons's vertices. The reason is actually very simple: the tetraquark current $J(0)$ is composed of four quark fields at the same space-time position. Contractions of relevant fields from the interpolating currents $J^D(x)$ and $J^\dagger(0)$ leave two free quark fields at the space-time point $x=0$. As a result, local matrix elements of the K meson appear in the correlation function as overall normalization factors.

It is instructive to compare this situation with three-meson vertices, in which contractions of quark fields with different space-time coordinates generate $\Pi^{\mathrm{OPE}}(p,q)$ containing nonlocal operators. Then, manipulations performed in accordance with Eqs. (29) and (30) lead to operators, the matrix elements of which are distribution amplitudes (DAs) of a final-state meson. In other words, for a three-meson vertex a correlation function depends on integrals over DAs of a meson. The situation described above in the LCSR method emerges in the kinematical limit $q \to 0$, which is known as the soft-meson approximation [44]. In this approximation, instead of a light-cone expansion, one gets an expansion in terms of local matrix elements of a final meson. Because in the soft limit the phenomenological and QCD sides of the light-cone sum rules acquire distinctive features, they have to be treated in accordance with elaborate methods [44,45]. It is important that strong couplings at three-meson vertices calculated using the full version of the LCSR method and soft-meson approximation lead to predictions that are numerically very similar [44].

The soft-meson approximation was applied to explore tetraquark-meson-meson vertices in Ref. [46] and later used in numerous similar studies [36]. It is worth emphasizing, that correlation functions of tetraquark-tetraquark- meson vertices contain integrals over DAs of a meson, and their treatment does not differ from standard LCSR analysis [47].

Here we employ this technique to analyze the vertices $X_0^{(\prime)} \to D^- K^+$. As is seen from Eq. (28), the soft-meson approximation considerably simplifies the QCD side of the sum rule: there are only local matrix elements of the K meson in $\Pi^{\mathrm{OPE}}(p^2)$, and only a few of them contribute in the limit $q=0$. On the contrary, the physical side of the sum rule has a more complicated structure than in the case of the full version of the LCSR method. The soft limit implies the fulfillment of the equality $p=p'$, and hence in the limit $q \to 0$ the invariant amplitudes $\Pi^{\mathrm{Phys}}(p^2,p'^2)$ and $\Pi^{\mathrm{OPE}}(p^2,p'^2)$ are functions of the variable p^2. Therefore, in Eq. (26) one should take into account that $p^2=p'^2$, and we get

$$\Pi^{\mathrm{Phys}}(p^2) = \frac{f_D m_D^2}{m_c}\left[g f m \frac{\tilde{m}^2}{(p^2-\tilde{m}^2)^2} + g' f' m' \frac{\tilde{m}'^2}{(p^2-\tilde{m}'^2)^2}\right] + \cdots, \tag{31}$$

where $\tilde{m}^2 = (m^2+m_D^2)/2$ and $\tilde{m}'^2 = (m'^2+m_D^2)/2$. The remaining problems are connected to the Borel transform of the amplitude $\Pi^{\mathrm{Phys}}(p^2)$ which, due to double poles at $p^2=\tilde{m}^2$ and $p^2=\tilde{m}'^2$, has the following form:

$$\Pi^{\mathrm{Phys}}(p^2) = \frac{f_D m_D^2}{m_c}\left[g f m \frac{\tilde{m}^2 e^{-\tilde{m}^2/M^2}}{M^2}, + g' f' m' \frac{\tilde{m}'^2 e^{-\tilde{m}'^2/M^2}}{M^2}\right] + \cdots. \tag{32}$$

In general, the Borel transformation applied to a correlation function suppresses contributions of higher resonances and continuum states. This allows one to subtract these terms from the QCD side of the sum rule using an assumption about quark-hadron duality. In the soft approximation, after the Borel transformation there are still unsuppressed terms on the physical side of the sum rule, which contribute to $\Pi^{\mathrm{Phys}}(p^2)$ on an equal footing with the ground-state term. Because we are interested in analyzing both the ground-state X_0 and excited X_0' particles, it is necessary to clarify the nature of these unsuppressed terms. The main contribution to $\Pi^{\mathrm{Phys}}(p^2)$ comes from the vertex $X_0 D^- K^+$, where the tetraquark and mesons are ground-state particles. Unsuppressed terms correspond to vertices, in which X_0 is in its excited state. When considering the vertex $X_0 D^- K^+$, such contributions should be treated as contaminations and subtracted applying some procedures. Such prescriptions are well known and were described in Refs. [44,45]: to eliminate contaminations from $\Pi^{\mathrm{Phys}}(p^2)$, one has to apply the operator

$$\mathcal{P}(M^2,m^2) = \left(1 - M^2 \frac{d}{dM^2}\right) M^2 e^{m^2/M^2} \tag{33}$$

to both sides of the sum rule equality, and subtract the remaining conventional terms in a standard manner.

But, we are also interested in extracting of the strong coupling g' which corresponds to the vertex $X_0' D^- K^+$. Therefore, we use the following strategy: we determine the strong coupling g utilizing the "ground-state+continuum" scheme and the first term in $\Pi^{\mathrm{Phys}}(p^2)$. At this stage we apply the operator $\mathcal{P}(M^2,\tilde{m}^2)$ that singles out the ground-state term. Afterwards, we use g as an input parameter in the "ground-state+excited-state+continuum" scheme, and by employing the full expression for $\Pi^{\mathrm{Phys}}(p^2)$ determine the strong coupling g'.

Then, the sum rule for g reads

$$g = \frac{m_c}{f m f_D m_D^2 \tilde{m}^2} \mathcal{P}(M^2,m^2) \Pi^{\mathrm{OPE}}(M^2,s_0), \tag{34}$$

whereas for g' we obtain

$$g' = \frac{e^{\tilde{m}'^2/M^2}}{f'm'\tilde{m}'^2}\left[\frac{M^2 m_c}{f_D m_D^2}\Pi^{\rm OPE}(M^2, s_0^*) - gfm\tilde{m}^2 e^{-\tilde{m}^2/M^2}\right]. \tag{35}$$

The K meson is characterized by some local matrix elements of different quark-gluon contents and twists. Having performed our numerical computations, we see that the correlator $\Pi^{\rm OPE}(p,q)$ receives a contribution from the two-particle twist-3 element

$$\langle 0|\bar{u}i\gamma_5 s|K\rangle = \frac{f_K m_K^2}{m_s}. \tag{36}$$

The technical aspects of the required calculations of the $\Pi^{\rm OPE}(p,q)$ in the soft limit were described in Ref. [46], and hence we omit further details and write down the final formula for the Borel-transformed and subtracted invariant amplitude, which is computed with dimension-nine accuracy:

$$\Pi^{\rm OPE}(M^2, s_0) = -\frac{\mu_K}{4\pi^2}\int_{\mathcal{M}^2}^{s_0}\frac{ds(m_c^2 - s)^2}{s}e^{-s/M^2} + \mu_K m_c \Pi_{\rm NP}(M^2). \tag{37}$$

The nonperturbative component of the correlation function $\Pi_{\rm NP}(M^2)$ is determined by the expression

$$\begin{aligned}\Pi_{\rm NP}(M^2) = {}& \frac{2\langle\bar{d}d\rangle}{3}e^{-m_c^2/M^2} + \left\langle\frac{\alpha_s G^2}{\pi}\right\rangle\frac{m_c^3}{36M^4} \\ &\times\int_0^1 \frac{dx e^{-m_c^2/[M^2 x(1-x)]}}{x^3(1-x)^3} - \frac{\langle\bar{d}g\sigma G d\rangle m_c^2}{3M^4}e^{-m_c^2/M^2} \\ &+ \left\langle\frac{\alpha_s G^2}{\pi}\right\rangle\langle\bar{d}d\rangle\frac{(m_c^2+3M^2)\pi^2}{27M^6}e^{-m_c^2/M^2} \\ &- \left\langle\frac{\alpha_s G^2}{\pi}\right\rangle\langle\bar{d}g\sigma G d\rangle\frac{(m_c^4+6M^2m_c^2+6M^4)\pi^2}{108M^{10}} \\ &\times e^{-m_c^2/M^2},\end{aligned} \tag{38}$$

where $\mu_K = f_K m_K^2/m_s$.

It is worth noting that the limit $q \to 0$ is performed in a hard component of the amplitude. As a result, it does not contain terms $\sim m_K^2$ which nevertheless would be small due to m_K^2/m^2, m_K^2/m'^2, $m_K^2/m_D^2 \ll 1$. In the soft approximation the mass and decay constant of the K^+ meson through μ_K form the nonperturbative soft factor in $\Pi^{\rm OPE}(M^2, s_0)$.

The parameters of the mesons D^- and K^+ that are necessary to calculate g are shown in Table II. The values of the masses and decay constants of these particles are taken from Ref. [48]. In numerical computations of the strong couplings g and g', the Borel and continuum subtraction parameters are chosen as in the corresponding mass analysis. Numerical computations yield

TABLE II. Masses and decay constants of the D and K mesons required for numerical computations.

Quantity	Value (in MeV)
m_D	1869.65 ± 0.05
m_{D^0}	1864.83 ± 0.05
m_K	493.677 ± 0.016
m_{K^0}	497.611 ± 0.013
$f_D = f_{D^0}$	212.6 ± 0.7
$f_K = f_{K^0}$	155.7 ± 0.3

$$g = (1.06 \pm 0.27)\ \mathrm{GeV}^{-1} \tag{39}$$

and

$$g' = (0.52 \pm 0.13)\ \mathrm{GeV}^{-1}. \tag{40}$$

The partial widths of the processes $X_0^{(\prime)} \to D^- K^+$ can be found by means of the expression

$$\Gamma[X_0^{(\prime)} \to D^- K^+] = \frac{g^{(\prime)2} m_D^2 \lambda^{(\prime)}}{8\pi}\left(1 + \frac{\lambda^{(\prime)2}}{m_D^2}\right), \tag{41}$$

where $\lambda^{(\prime)} = \lambda(m^{(\prime)}, m_D, m_K)$ and

$$\lambda(a,b,c) = \frac{1}{2a}[a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)]^{1/2}. \tag{42}$$

Now it is easy to get

$$\begin{aligned}\Gamma[X_0 \to D^- K^+] &= (64.7 \pm 23.3)\ \mathrm{MeV}, \\ \Gamma[X_0' \to D^- K^+] &= (53.3 \pm 18.8)\ \mathrm{MeV}.\end{aligned} \tag{43}$$

The partial widths of the decays $X_0 \to \bar{D}^0 K^0$ and $X_0' \to \bar{D}^0 K^0$ also contribute to the full widths of the tetraquarks X_0 and X_0'. Investigation of these channels is performed in accordance with the scheme described above, therefore we write down only the final results. For the strong couplings G and G' corresponding to the vertices $X_0\bar{D}^0K^0$ and $X_0'\bar{D}^0K^0$, we find

$$G = (1.14 \pm 0.18)\ \mathrm{GeV}^{-1} \tag{44}$$

and

$$G' = (0.54 \pm 0.11)\ \mathrm{GeV}^{-1}. \tag{45}$$

For the partial widths of these decays, we get

$$\begin{aligned}\Gamma[X_0 \to \bar{D}^0 K^0] &= (74.8 \pm 16.7)\ \mathrm{MeV}, \\ \Gamma[X_0' \to \bar{D}^0 K^0] &= (56.3 \pm 16.2)\ \mathrm{MeV}.\end{aligned} \tag{46}$$

Then, the full widths of the particles X_0 and X_0' are

$$\Gamma_0 = (140 \pm 29)\ \text{MeV},$$
$$\Gamma_0' = (110 \pm 25)\ \text{MeV}, \tag{47}$$

respectively.

As is seen, the parameters of the tetraquarks X_0 and X_0' differ considerably from the mass and width of the resonance $X_0(2900)$ measured by the LHCb Collaboration.

IV. DISCUSSION AND CONCLUSIONS

In the present paper, we examined the tetraquark X_0 and its radial excitation X_0' by calculating their masses and widths. The masses of X_0 and X_0' were computed using the axial-axial and scalar-scalar type interpolating currents $J(s)$ and $J_S(x)$. The widths of these particles were estimated for the axial-axial structure.

The diquark-antidiquark state X_0 consists of four quarks of different flavors, $X_0 = [ud][\bar{c}\,\bar{s}]$. The properties of the ground-state scalar tetraquark with similar content $X_c = [su][\bar{c}\,\bar{d}]$ were investigated in Refs. [29,30]. The mass of X_c found in Ref. [29] using axial-axial and scalar-scalar structures is

$$m_{X_c} = (2590 \pm 60)\ \text{MeV}, \qquad \Gamma_{X_c} = (63.4 \pm 14.2)\ \text{MeV}, \tag{48}$$

and

$$\tilde{m}_{X_c} = (2634 \pm 62)\ \text{MeV}, \qquad \tilde{\Gamma}_{X_c} = (57.7 \pm 11.6)\ \text{MeV}, \tag{49}$$

respectively. The prediction

$$m_{X_c} = (2550 \pm 90)\ \text{MeV} \tag{50}$$

for the mass of the state X_c was also made in Ref. [30]. It is worth emphasizing that all of these results were extracted using the QCD two-point sum rule method, and predictions for the mass of X_c from Refs. [29,30] are almost the same. It is also evident that m and m_S in Eq. (19) and Table I are comparable with predictions for m_{X_c} and $\tilde{m}_{X_c}$ within uncertainties of computations. Stated differently, the masses of the ground-state tetraquarks with different internal organizations, but composed of c, s, u, d quarks, vary approximately in the range 2550–2660 MeV.

The parameters of the tetraquark $[cs][\bar{u}\,\bar{d}]$ in the context of the sum rule approach were also recently calculated in Ref. [5]. There, the mass of this particle with either scalar-scalar (SS) or axial-axial (AA) structure was found to be

$$M_{SS} = (3050 \pm 100)\ \text{MeV}, \qquad M_{AA} = (2910 \pm 120)\ \text{MeV}. \tag{51}$$

Because M_{AA} is compatible with the LHCb data, the resonance $X_0(2900)$ was interpreted there as a ground-state tetraquark $[cs][\bar{u}\,\bar{d}]$. The results of that work differ considerably from our findings, as well as from predictions made in Ref. [30]. The $X_0(2900)$ was considered as a radially excited state $\tilde{X}_c(2S)$, with $\tilde{X}_c$ being the tetraquark $[ud][\bar{c}\,\bar{s}]$ [6]. The mass of $\tilde{X}_c(2S)$ was estimated in Ref. [6] around of 2860 MeV, which is lower than our results for X_0' and X_S'.

Analysis performed in the present work, demonstrates that tetraquarks $X_0^{(\prime)}$ and $X_S^{(\prime)}$ built of the axial-vector and scalar diquarks (antidiquarks), are states that differ from the resonance $X_0(2900)$ observed by the LHCb Collaboration. Therefore, the parameters calculated in the present work are all the more important in searches for the tetraquarks $X_0^{(\prime)}$ and $X_S^{(\prime)}$ in various processes. The masses of the states $X_0^{(\prime)}$ and $X_S^{(\prime)}$ have been extracted with high enough accuracy. Although $m^{(\prime)}$ and $m_S^{(\prime)}$ contain uncertainties that are typical for all sum-rule computations, they provide valuable information on these exotic mesons. We also evaluated the full widths of the tetraquarks X_0 and X_0' by considering their decays to pairs of conventional mesons D^-K^+ and $\bar{D}^0K^0$. For the particles X_0 and X_0', these two processes are their only S-wave decay channels. Other possible modes of the tetraquarks $X_0^{(\prime)}$, such as S-wave decays $X_0^{(\prime)} \to \bar{D}_0^*(2400)^0 \times K^*(1430)$, are kinematically forbidden processes. Hence, estimates for the full widths $\Gamma_0^{(\prime)}$ of the four-quark mesons X_0 and X_0' are rather credible.

Our results imply that $X_0(2900)$ cannot be identified with a ground-state or radially excited scalar tetraquark $[ud][\bar{c}\,\bar{s}]$. It seems interpretation of the resonance $X_0(2900)$ as hadronic molecules $\bar{D}^{*0}K^{*0}$ and $D^{*-}K^{*+}$, or their admixture is correct and overcomes successfully present examination.

APPENDIX: QUARK PROPAGATORS AND INVARIANT AMPLITUDE $\Pi(M^2, s_0)$

In the current article, for the light-quark propagator $S_q^{ab}(x)$ we employ the following expression:

$$\begin{aligned} S_q^{ab}(x) &= i\delta_{ab}\frac{\not{x}}{2\pi^2x^4} - \delta_{ab}\frac{m_q}{4\pi^2x^2} - \delta_{ab}\frac{\langle\bar{q}q\rangle}{12} + i\delta_{ab}\frac{\not{x}m_q\langle\bar{q}q\rangle}{48} - \delta_{ab}\frac{x^2}{192}\langle\bar{q}g_s\sigma Gq\rangle \\ &+ i\delta_{ab}\frac{x^2\not{x}m_q}{1152}\langle\bar{q}g_s\sigma Gq\rangle - i\frac{g_sG_{ab}^{\alpha\beta}}{32\pi^2x^2}[\not{x}\sigma_{\alpha\beta} + \sigma_{\alpha\beta}\not{x}] - i\delta_{ab}\frac{x^2\not{x}g_s^2\langle\bar{q}q\rangle^2}{7776} - \delta_{ab}\frac{x^4\langle\bar{q}q\rangle\langle g_s^2G^2\rangle}{27648} + \cdots. \end{aligned} \tag{A1}$$

For the heavy quark $Q = c$, we use the propagator $S_Q^{ab}(x)$,

$$S_Q^{ab}(x) = i\int \frac{d^4k}{(2\pi)^4} e^{-ikx}\left\{\frac{\delta_{ab}(\not{k}+m_Q)}{k^2-m_Q^2} - \frac{g_s G_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma_{\alpha\beta}}{(k^2-m_Q^2)^2}\right.$$
$$\left. + \frac{g_s^2 G^2}{12}\delta_{ab} m_Q \frac{k^2+m_Q\not{k}}{(k^2-m_Q^2)^4} + \frac{g_s^3 G^3}{48}\delta_{ab}\frac{(\not{k}+m_Q)}{(k^2-m_Q^2)^6}[\not{k}(k^2-3m_Q^2)+2m_Q(2k^2-m_Q^2)](\not{k}+m_Q)+\cdots\right\}. \quad \text{(A2)}$$

Here, we have used the shorthand notation

$$G_{ab}^{\alpha\beta} \equiv G_A^{\alpha\beta}\lambda_{ab}^A/2, \qquad G^2 = G_{\alpha\beta}^A G_A^{\alpha\beta}, \qquad G^3 = f^{ABC} G_{\alpha\beta}^A G^{B\beta\delta} G_\delta^{C\alpha}, \quad \text{(A3)}$$

where $G_A^{\alpha\beta}$ is the gluon field-strength tensor, and λ^A and f^{ABC} are the Gell-Mann matrices and structure constants of the color group $SU_c(3)$, respectively. The indices A, B, C run in the range $1, 2, \ldots 8$.

The invariant amplitude $\Pi(M^2, s_0)$, obtained using the interpolating current $J(x)$ from Eq. (3), after the Borel transformation and subtraction procedures is given by the expression

$$\Pi(M^2, s_0) = \int_{\mathcal{M}^2}^{s_0} ds \rho^{\mathrm{OPE}}(s) e^{-s/M^2} + \Pi(M^2),$$

where the spectral density $\rho^{\mathrm{OPE}}(s)$ and the function $\Pi(M^2)$ are determined by the formulas

$$\rho^{\mathrm{OPE}}(s) = \rho^{\mathrm{pert}}(s) + \sum_{N=3}^{8}\rho^{\mathrm{DimN}}(s), \qquad \Pi(M^2) = \sum_{N=6}^{15}\Pi^{\mathrm{DimN}}(M^2), \quad \text{(A4)}$$

respectively. The components of $\rho^{\mathrm{OPE}}(s)$ and $\Pi(M^2)$ are given by the expressions

$$\rho^{\mathrm{DimN}}(s) = \int_0^1 d\alpha \rho^{\mathrm{DimN}}(s,\alpha), \qquad \Pi^{\mathrm{DimN}}(M^2) = \int_0^1 d\alpha \Pi^{\mathrm{DimN}}(M^2,\alpha). \quad \text{(A5)}$$

In Eq. (A5) the variable α is the Feynman parameter.

The perturbative and nonperturbative components of the spectral density $\rho^{\mathrm{pert}}(s,\alpha)$ and $\rho^{\mathrm{Dim3(4,5,6,7,8)}}(s,\alpha)$ are given by the following expressions:

$$\rho^{\mathrm{pert}}(s,\alpha) = \frac{[m_c^2 - s(1-\alpha)]^3\alpha^3\Theta(L)}{1536\pi^6(\alpha-1)^3}[4m_c m_s + m_c^2\alpha + 3s\alpha(1-\alpha)], \quad \text{(A6)}$$

$$\rho^{\mathrm{Dim3}}(s,\alpha) = -\frac{\langle\bar{s}s\rangle\Theta(L)}{36\pi^2(\alpha-1)^2}\alpha^2[m_c^2 - s(1-\alpha)][m_c^3 + 2m_c^2 m_s(\alpha-1) + m_c s(\alpha-1) + 4m_s s(\alpha-1)^2], \quad \text{(A7)}$$

$$\rho^{\mathrm{Dim4}}(s,\alpha) = -\frac{\langle\alpha_s G^2/\pi\rangle\Theta(L)}{9\cdot 2^9\pi^4(\alpha-1)^3}\alpha^2[6s^2(\alpha-1)^3(5\alpha-6) + m_c^3 m_s(-9+9\alpha-8\alpha^2) + m_c^4(18-33\alpha$$
$$+19\alpha^2) + m_c m_s s(9-22\alpha+25\alpha^2-12\alpha^3) + 3m_c^2 s(-18+51\alpha-50\alpha^2+17\alpha^3)], \quad \text{(A8)}$$

$$\rho^{\mathrm{Dim5}}(s,\alpha) = \frac{\langle\bar{s}g_s\sigma G s\rangle\Theta(L)}{96\pi^4(\alpha-1)}\alpha[3m_c^3 + 4m_c^2 m_s(\alpha-1) + 3m_c s(\alpha-1) + 6s m_s(\alpha-1)^2], \quad \text{(A9)}$$

$$\rho^{\mathrm{Dim6}}(M^2,\alpha) = -\frac{\Theta(L)}{405\cdot 2^9\pi^6(\alpha-1)^3}\{27\langle g_s^3 G^3\rangle m_c^2\alpha^5 + 34560\langle\bar{d}d\rangle\langle\bar{u}u\rangle\pi^4(\alpha-1)^3[-2m_c m_s + 2m_c^2\alpha$$
$$+3s\alpha(\alpha-1)] + 320 g_s^2\langle\bar{d}d\rangle^2\pi^2(\alpha-1)^3[-m_c m_s + 4m_c^2\alpha + 6s\alpha(\alpha-1)] + 320 g_s^2\pi^2(\alpha-1)^3$$
$$\times[-m_c m_s\langle\bar{u}u\rangle^2 + (\langle\bar{s}s\rangle^2 + \langle\bar{u}u\rangle^2)(4m_c^2\alpha + 6s\alpha(\alpha-1))]\}, \quad \text{(A10)}$$

$$\rho^{\mathrm{Dim7}}(M^2,\alpha) = \frac{\langle\alpha_s G^2/\pi\rangle\langle\bar{s}s\rangle\Theta(L)}{288\pi^2(\alpha-1)^2}[3m_s\alpha(\alpha-1)^2 + m_c(2-7\alpha+5\alpha^2-2\alpha^3)], \quad \text{(A11)}$$

$$\rho^{\mathrm{Dim8}}(M^2,\alpha)=\frac{\langle\alpha_s G^2/\pi\rangle^2\alpha+96\langle\bar{d}g_s\sigma Gd\rangle\langle\bar{u}u\rangle(\alpha-1)}{1152\pi^2}\Theta(L). \tag{A12}$$

The components of the function $\Pi(M^2)$ are

$$\Pi^{\mathrm{Dim6}}(M^2,\alpha)=-\frac{\langle g_s^3G^3\rangle m_c^3\alpha^3}{45\cdot 2^{10}M^2\pi^6(\alpha-1)^5}\exp\left[-\frac{m_c^2}{M^2(1-\alpha)}\right][m_c^3\alpha(2+\alpha)+4m_c^2m_s(2\alpha-1)-8m_sM^2(\alpha^2-1) -m_cM^2\alpha(\alpha^2+\alpha-2)], \tag{A13}$$

$$\Pi^{\mathrm{Dim7}}(M^2,\alpha)=\frac{\langle\alpha_sG^2/\pi\rangle\langle\bar{s}s\rangle m_c^2\alpha^2}{288M^2\pi^2(\alpha-1)^3}\exp\left[-\frac{m_c^2}{M^2(1-\alpha)}\right][m_c^2m_s+(m_c-m_s)M^2(\alpha-1)], \tag{A14}$$

$$\Pi_1^{\mathrm{Dim8}}(M^2,\alpha)=\frac{\langle\alpha_sG^2/\pi\rangle^2m_c\alpha}{9\cdot 2^9M^2\pi^2(\alpha-1)^3}\exp\left[-\frac{m_c^2}{M^2(1-\alpha)}\right][2m_c^2m_s(\alpha-1)+m_c^3\alpha-m_cM^2\alpha(\alpha-1) -2m_sM^2(\alpha^2-1)], \tag{A15}$$

$$\Pi_2^{\mathrm{Dim8}}(M^2)=\frac{\langle\bar{d}g_s\sigma Gd\rangle\langle\bar{u}u\rangle m_cm_s}{12\pi^2}\exp\left[-\frac{m_c^2}{M^2}\right], \tag{A16}$$

$$\Pi_1^{\mathrm{Dim9}}(M^2,\alpha)=-\frac{1}{135\cdot 2^7M^6\pi^4(\alpha-1)^5}\exp\left[-\frac{m_c^2}{M^2(1-\alpha)}\right]\{3\langle g_s^3G^3\rangle\langle\bar{s}s\rangle m_c^3\alpha^2[m_c^2M^2(2-4\alpha)+8M^4(\alpha-1) +m_c^3m_s(2+\alpha)]+5\langle\alpha_sG^2/\pi\rangle\langle\bar{s}g_s\sigma Gs\rangle M^2\pi^2(\alpha-1)^2[-3m_c^2m_sM^2(\alpha-1)^2+3m_sM^4(\alpha-1)^3+4m_c^4m_s\alpha +6m_c^3M^2\alpha(\alpha-1)+3m_cM^4(3-4\alpha+3\alpha^2-2\alpha^3)]\}, \tag{A17}$$

$$\Pi_2^{\mathrm{Dim9}}(M^2)=\frac{\langle\bar{s}s\rangle}{486M^2\pi^2}\exp\left[-\frac{m_c^2}{M^2}\right]\{g_s^2[\langle\bar{d}d\rangle^2+\langle\bar{u}u\rangle^2][m_c^2m_s+(m_c-m_s)M^2]+54\langle\bar{u}u\rangle\langle\bar{d}d\rangle\pi^2 \times[4m_cM^2+m_s(m_c^2-M^2)]\}, \tag{A18}$$

$$\Pi_1^{\mathrm{Dim10}}(M^2,\alpha)=\frac{g_s^2\langle\alpha_sG^2/\pi\rangle}{729\cdot 2^6M^4\pi^2(\alpha-1)^3}\exp\left[-\frac{m_c^2}{M^2(1-\alpha)}\right]\{-4\langle\bar{u}u\rangle^2m_c^3m_s(\alpha-1)+8\langle\bar{u}u\rangle^2m_cm_sM^2(\alpha-1) +[2\langle\bar{s}s\rangle^2+\langle\bar{u}u\rangle^2][-3m_c^2M^2(\alpha-1)^2+3M^4(\alpha-1)^3]+8m_c^4\alpha[\langle\bar{s}s\rangle^2+\langle\bar{u}u\rangle^2]+\langle\bar{d}d\rangle^2[-4m_c^3m_s(\alpha-1) +8m_cm_sM^2(\alpha-1)-3m_c^2M^2(\alpha-1)^2+3M^4(\alpha-1)^3+8m_c^4\alpha]\}, \tag{A19}$$

$$\Pi_2^{\mathrm{Dim10}}(M^2)=\frac{1}{243\cdot 2^6M^4\pi^2}\exp\left[-\frac{m_c^2}{M^2}\right]\{162\langle\bar{u}g_s\sigma Gu\rangle\langle\bar{d}g_s\sigma Gd\rangle(2m_c^3m_s+m_c^2M^2-M^4) +\langle\alpha_sG^2/\pi\rangle[g_s^2(\langle\bar{d}d\rangle^2+\langle\bar{u}u\rangle^2)M^2(m_c^2+m_cm_s-M^2)+144\pi^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle(2m_c^3m_s+m_c^2M^2-M^4)]\}, \tag{A20}$$

and

$$\Pi_1^{\mathrm{Dim11}}(M^2,\alpha)=-\frac{m_c}{405\cdot 2^8M^8\pi^4(\alpha-1)^5}\exp\left[-\frac{m_c^2}{M^2(1-\alpha)}\right]\{-20\langle\alpha_sG^2/\pi\rangle^2\langle\bar{s}s\rangle(m_c^2-2M^2)M^4\pi^4(\alpha-1)^3 +3\langle g_s^3G^3\rangle\langle\bar{s}g_s\sigma Gs\rangle m_c\alpha[2m_c^4m_s(2+\alpha)+6m_c^3M^2(2\alpha-1)-3m_sM^4(\alpha-1)^2(5\alpha-7) +12m_cM^4(3-4\alpha+\alpha^2)-3m_c^2m_sM^2(1-4\alpha+3\alpha^2)]\}, \tag{A21}$$

$$\Pi_2^{\mathrm{Dim11}}(M^2) = -\frac{m_c}{729\cdot 2^7 M^6\pi^2}\exp\left[-\frac{m_c^2}{M^2}\right]\{16g_s^2\langle\bar{s}g_s\sigma Gs\rangle[\langle\bar{d}d\rangle^2+\langle\bar{u}u\rangle^2]m_c(2m_c^2m_s+3m_cM^2+6m_sM^2)$$
$$+27\langle\alpha_sG^2/\pi\rangle^2\langle\bar{s}s\rangle M^4\pi^2+3456\langle\bar{s}g_s\sigma Gs\rangle\langle\bar{d}d\rangle\langle\bar{u}u\rangle\pi^2m_c(2m_c^2m_s+9m_cM^2+6m_sM^2)\}. \quad \text{(A22)}$$

In the above expressions, $\Theta(z)$ is the Unit Step function. We have also used the following shorthand notation:

$$L\equiv L(s,\alpha)=s\alpha(1-\alpha)-m_c^2\alpha. \quad \text{(A23)}$$

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