Electromagnetic transition amplitude for Roper resonance from holographic QCD

Daisuke Fujii,* Akihiro Iwanaka,† and Atsushi Hosaka‡,§

Research Center for Nuclear Physics(RCNP), Osaka University, Ibaraki 567-0048, Japan

(Received 29 March 2022; accepted 27 June 2022; published 18 July 2022)

The Roper resonance, the first excited state of the nucleon, is one of the best established baryon resonances. Yet, its properties have not been consistently explained by effective models of QCD, such as the nonrelativistic quark model. In this paper, we propose an alternative approach in the Sakai-Sugimoto model that is one of the holographic models of QCD. In particular, we analyze the helicity amplitude of the electromagnetic transitions at the leading of 't Hooft coupling $1/\lambda$. The model incorporates baryon structure at short distance by nonlinear mesons surrounded by meson clouds at long distance. We demonstrate that the recently observed data by CLAS are explained in the present approach.

DOI: 10.1103/PhysRevD.106.014010

In this paper, dynamic properties of nucleons are discussed in the holographic model of QCD. So far, static properties have been investigated by this model, and its success has been shown [1–3] On the other hand, it is also essential to elucidate the dynamical properties of resonances and their interactions. In this study, as a milestone of the new development of the study of dynamical properties in the holographic model of QCD, the properties of the Roper resonance, the most difficult and intriguing state to understand, are investigated. This method can be extended to various nucleon resonances, and further developments are expected in the future (In fact, we have recently succeeded in reproducing the decay width of one pion emission of negative parity excitation $N^*(1535)$ using the Sakai-Sugimoto model [4].).

The Roper resonance $N^*(1440)$ is the first excited state of the nucleon with spin-parity $J^P = 1/2^+$. Despite the known spin and parity quantum numbers, its properties have not been explained consistently by the standard model of hadrons, that is the nonrelativistic quark model. Since it was first suggested by L. D. Roper in the 1960s [5], its mass, smaller than the negative parity state $N^*(1535)$, has long been a puzzle because the quark model predicts the mass of the Roper resonance larger than that of $N^*(1535)$.

*daisuke@rcnp.osaka-u.ac.jp
†iwanaka@rcnp.osaka-u.ac.jp
‡hosaka@rcnp.osaka-u.ac.jp
§Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195 Japan.

Turning to the electromagnetic interaction, the photo- and electroproduction experiments revealed a further difficulty of the nonrelativistic quark model [6,7]. In particular, the transverse helicity amplitude $A_{1/2}$ of the Roper resonance at the real photon point cannot be reproduced; $A_{1/2}$ value of the quark model is significantly smaller than the experimental ones even with the wrong sign. A problem is also in the strong interaction process [8]; the experimentally observed large decay width of the one pion emission cannot be explained. Generally, transition amplitudes of various resonances are not easily explained by the nonrelativistic quark model.

It was pointed out that relativistic effects of the confined quarks at short distance and pion cloud effects at long distance are important to improve the above mentioned problems; the mass ordering, the electromagnetic and strong interaction transitions [9–12]. For instance, the reason for the failure of the electromagnetic and strong transitions in the nonrelativistic quark model is that the leading terms are suppressed due to the orthogonality of the wave functions of the nucleon and the Roper resonance in the long wave length limit $q \to 0$, where q is the momentum carried either by the photon or the pion.

In this paper, we propose an alternative but viable approach based on the holographic model for baryons that is the Sakai-Sugimoto model. The model supports instantons for baryons in the four-dimensional space with an extra dimension [13,14], which is known to be reduced to the Skyrme model [15]. In this picture, the pion cloud effect is naturally accommodated at long distance, while the quark dynamics at short distance by the nonlinear structure of an instanton. In this paper it is shown that the electromagnetic transitions of the Roper resonance are well reproduced. Combining the present results with the previous successes in the mass and pion decay, we discuss that the holographic approach provides an effective method for hadrons that incorporates important features of low energy QCD.

The Sakai-Sugimoto model (SS model) realizes spontaneous breaking of chiral symmetry in terms of brane dynamics and has been very successful in explaining light flavor hadron physics [16,17]. The action in the SS model is composed of the Yang-Mills term $S_{\rm YM}$ and the Chern-Simons term $S_{\rm CS}$,

$$S = S_{\rm YM} + S_{\rm CS} \tag{1}$$

where

$$S_{\rm YM} = -\kappa \int d^4x dz {\rm tr}\left[\frac{1}{2}h(z)\mathcal{F}_{\mu\nu}^2 + k(z)\mathcal{F}_{\mu z}^2\right],$$
$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int_{M^4\times\mathbb{R}} \omega_5(\mathcal{A}), \qquad \kappa = \frac{\lambda N_c}{216\pi^3} = a\lambda N_c. \tag{2}$$

Here, N_c is the number of colors, λ the 't Hooft coupling and the indices $\mu, \nu = 0, 1, 2, 3$ are for the 4-dimensional spacetime. The curvatures along the extra dimension z are derived from the D4-D8 brane construction to $h(z) = (1 + z^2)^{-1/3}$ and $k(z) = 1 + z^2$. The 1-form $\mathcal{A} = A_\alpha dx^\alpha + \hat{A}_\alpha dx^\alpha$ consists of the flavor SU(2) part A_α and the U(1) part $\hat{A}_\alpha$ with $\alpha = 0, 1, 2, 3, z$, and the field strength is $\mathcal{F}_{\alpha\beta} = \partial_\alpha\mathcal{A}_\beta - \partial_\beta\mathcal{A}_\alpha + i[\mathcal{A}_\alpha, \mathcal{A}_\beta]$ with $(-,+,+,+,+)$ convention. The Chern-Simons 5-form is given by $\omega_5(\mathcal{A}) = {\rm tr}(\mathcal{A}\mathcal{F}^2 - i\mathcal{A}^3\mathcal{F}/2 - \mathcal{A}^5/10)$. This term plays an important role in reproducing the chiral anomaly. The hadron effective model in $4+1$ dimension corresponds to the holographic dual of massless QCD, and the gauge field $\mathcal{A}_\alpha$ derived from the open string with both ends on the D8 brane is identified by mode expansion with an infinite number of vector/axial vector mesons, including pions.

The dynamics of the baryons are dominated by the collective motion of the instanton [1], which looks quite different from the quark model where the baryons are described by the single particle motions of the quarks. Notably, in the SS model, the masses of the Roper resonances and the negative parity state are degenerate. This feature is originated from the collective motion of the baryons, and it is suggested that the SS model captures the features of the baryon spectrum better than the quark model.

Motivated by this fact, we have studied the decay width of one pion emission in the SS model and obtained encouraging results [18]. This success is due to the fact that, in contrast to the quark model, the decay width is proportional to nonvanishing matrix element as follows,

$$\langle\psi_{N^*}(\vec{x};\rho,\ldots)|\rho^2 e^{i\vec{q}\cdot\vec{x}}|\psi_N(\vec{x};\rho,\ldots)\rangle, \tag{3}$$

where ρ is the collective coordinate corresponding to the size of the instanton. The presence of ρ^2 evades the forbidden nature of the leading contribution in the nonrelativistic quark model. This is analogous to the effect of relativistic corrections in the quark model.

Electromagnetic excitations of nucleon resonances have long been studied experimentally and theoretically as an important source of information for understanding QCD. Helicity amplitudes extracted from experiments distinguish competing models and provide important features for understanding QCD. In earlier years, experimental data were not sufficiently precise and the number of data points were not enough [19]. However in recent years, mainly with the advent of the Continuous Electron Beam Accelerator Facility (CEBAF) at the Thomas Jefferson National Accelerator Facility (JLab), a large amount of precise data has been obtained [20–24]. Motivated by this, several theoretical studies have also been carried out.

The helicity amplitudes are defined by the electromagnetic current, j^μ_{em} [25],

$$A_{1/2}(Q^2) = \sqrt{\frac{2\pi\alpha}{K}}\int d^3x\left\langle\psi_N, s_3 = \frac{1}{2}\middle|\epsilon_\mu^{(+)} j^\mu_{em}\middle|\psi_{N^*}, s_3 = -\frac{1}{2}\right\rangle e^{i|\vec{k}|x^3}, \tag{4}$$

$$S_{1/2}(Q^2) = \sqrt{\frac{2\pi\alpha}{K}}\int d^3x\left\langle\psi_N, s_3 = \frac{1}{2}\middle|\frac{|\vec{k}|}{Q}\epsilon_\mu^{(0)} j^\mu_{em}\middle|\psi_{N^*}, s_3 = \frac{1}{2}\right\rangle e^{i|\vec{k}|x^3}. \tag{5}$$

Here, α is the fine structure constant, Q the four-momentum transfer of the photon, and the 3-momentum $\vec{k}$ of the photon is assumed to be directed along the x^3 axis in the N^* rest frame. Due to the energy conservation law, we have the following equation, $k^2 = Q^2 + (Q^2 + m_i^2 - m_f^2)^2/4m_f^2$. In the case of real photons, i.e., $Q^2 = 0$, we find that $|\vec{k}|$ becomes $K = (m_f^2 - m_i^2)/(2m_f)$. The photon polarization vectors are defined by

$$\epsilon_\mu^{(0)} = \frac{1}{Q}(-|\vec{k}|, 0, 0, k_0) \quad \text{(longitudinal mode)} \tag{6}$$

$$\epsilon_\mu^{(\pm)} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \quad \text{(transverse mode)}. \tag{7}$$

Therefore, to calculate this helicity amplitude, we prepare the wave function and electromagnetic current in the SS model.

The classical solution for a four-dimensional gauge field theory is known as the instanton solution, which is usually scale-invariant. In the present case, the extra dimension, labeled by z, is curved, which makes the instanton solution shrink. However, by the repulsive nature of the U(1) term coupled with the Chern-Simons term, the classical solution of this action, the instanton solution, is stabilized. An analytic solution for such a system in a curved space is not known. However, since the size of the instanton is found to be proportional to $\lambda^{-1/2}$ in this model, the effect of this curvature is small in the large λ limit. Therefore, the Belavin, Polyakov, Schwartz, and Tyupkin (BPST) instanton solution [26] is used as an approximate solution of the SS action [1]. Therefore, for $M = 1, 2, 3, z$ and Pauli matrices $\boldsymbol{\tau}$.

$$A_M^{cl}(\mathbf{x}, z) = -if(\xi) g \partial_M g^{-1}, \qquad A_0^{cl} = 0, \tag{8}$$

$$\hat{A}_M^{cl} = 0, \qquad \hat{A}_0^{cl} = \frac{1}{8\pi^2 a} \frac{1}{\xi^2} \left[1 - \frac{\rho^4}{(\xi^2 + \rho^2)^2}\right], \tag{9}$$

where $g(\mathbf{x}, z) = [(z - Z) - i(\mathbf{x} - \mathbf{X}) \cdot \boldsymbol{\tau}]/\xi$, with $(\boldsymbol{X}, Z)$ and ρ the location and size of the instanton, respectively. The profile function $f(\xi)$ is given by $f(\xi) = \xi^2/(\xi^2 + \rho^2)$ with $\xi = \sqrt{(\mathbf{x} - \mathbf{X})^2 + (z - Z)^2}$.

In order to obtain the baryon wave function, we need to quantize the classical instanton solution [1]. For this purpose, we first consider the motion of the instanton in the moduli space and make the collective coordinates time-dependent. Next, we consider a dynamical system in which the collective coordinates of the instanton are the dynamical variables, and perform the quantization. The relevant time dependent dynamical variables in the moduli space are, $\boldsymbol{X}(t), Z(t)$, $\rho(t)$ and the $SU(2)$ orientation $V(t, x^M; a(t))$ with $V(z \to \pm\infty) \to \boldsymbol{a}(t)$ related to the rotational variable $\boldsymbol{a}(t) = a_4(t) + i a_a(t) \tau^a$ in the isospin and spin space. We implement the time dependent collective coordinates in the gauge field as $A_M(t, x^N) = V A_M^{cl}(x^N; X^N, \rho) V^{-1} - iV\partial_M V^{-1}$. By substituting this gauge field for the action (2), integrating over the space of (x^μ, z), and quantizing these collective coordinates, we obtain the baryon wave functions

$$\psi_N \propto e^{i\vec{p}\cdot\vec{X}} \psi^N_{\text{radial}}(\rho) e^{-\frac{M_0}{\sqrt{6}} Z^2} (a_1 + i a_2),$$
$$\psi_{N^*(1440)} \propto e^{i\vec{p}\cdot\vec{X}} \psi^{N^*}_{\text{radial}}(\rho) e^{-\frac{M_0}{\sqrt{6}} Z^2} (a_1 + i a_2), \tag{10}$$

where

$$\psi^N_{\text{radial}}(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}} e^{-\frac{M_0}{\sqrt{6}}\rho^2}, \tag{11}$$

$$\psi^{N^*}_{\text{radial}}(\rho) = \left(\frac{2M_0}{\sqrt{6}}\rho^2 - 1 - 2\sqrt{1 + \frac{N_c^2}{5}}\right) \psi^N_{\text{radial}}, \tag{12}$$

for the spin up proton ($I_3 = 1/2, s_3 = 1/2$) with $M_0 = 8\pi^2\kappa$ and a finite momentum $\vec{p}$. Here, baryon states are labeled by their momentum $\vec{p}$ and quantum numbers $(l, I_3, s_3, n_\rho, n_z)$, where $l/2$ is the equal isospin and spin values; I_3, s_3 are the third components of the isospin and spin; and n_ρ, n_z are the quantum numbers for oscillations along the radial and z-directions. The Roper resonance is the first radial excited state, $(l, I_3, s_3, n_\rho, n_z) = (1, 1/2, 1/2, 1, 0)$.

The electromagnetic current is defined as the Noether current of chiral symmetry as follows. Chiral transformation $(g_L, g_R) \in U(N_f)_L \times U(N_f)_R$ is realized by the flavor $SU(N_f)$ gauge transformation, $\mathcal{A}_M \to g\mathcal{A}_M g^{-1} - ig\partial_M g^{-1}$ with $g(x^\mu, z) \to g_{L/R}$ $(z \to \pm\infty)$ and $g(x^\mu, z) \in SU(N_f)$ and constants (g_L, g_R).

The infinitesimal local gauge transformation of this gauge symmetry $\delta_\xi \mathcal{A}_M(x^\mu, z) = \epsilon(x^\mu, z) \mathcal{D}_M \zeta(x^\mu, z)$, leads to the following five-dimensional Noether current,

$$\begin{aligned}
J_\zeta^M &= J_{\text{YM}\zeta}^M + J_{\text{CS}\zeta}^M, \\
J_{\text{YM}\zeta}^\mu(x, z) &= -2\kappa \text{tr}(h(z)\mathcal{F}^{\mu\nu}\mathcal{D}_\nu\zeta + k(z)\mathcal{F}^{\mu z}\mathcal{D}_z\zeta), \\
J_{\text{YM}\zeta}^z(x, z) &= -2\kappa k(z) \text{tr}(\mathcal{F}^{z\nu}\mathcal{D}_z\zeta), \\
J_{\text{CS}\zeta}^M(x.z) &= -\frac{N_c}{64\pi^2} \epsilon^{MNPQR} \text{tr}(\{\mathcal{F}_{NP}, \mathcal{F}_{QR}\}\zeta),
\end{aligned} \tag{13}$$

where $\zeta(x^\mu, z)$ is a $u(N_f)$ Lie algebra. Since the chiral symmetry of the SS model is related to the $SU(N_f)$ gauge transformation, we define the chiral current as follows [3]:

$$j_\zeta^\mu(x) = \int_{-\infty}^{+\infty} dz J_\zeta^\mu(x, z). \tag{14}$$

Here, to satisfy the 4-dimensional current conservation law, we impose the following boundary condition, $J_\zeta^z(x, z \to \pm\infty) = 0$. With $C = 0$, 1, 2, 3 and

$$\psi_\pm(z) = \frac{1}{2} \pm \frac{1}{\pi} \arctan z \to \begin{cases} 1 & (z \to \pm\infty) \\ 0 & (z \to \mp\infty) \end{cases}, \tag{15}$$

we employ $\zeta(x, z) = \psi_\pm(z) t_C$ with $t_C = (I_2/2, \boldsymbol{\tau}/2)$, then the expression of the current (14) leads to left/right current $j_{L/R}^\mu(x)$. Therefore, by substituting (8) and (9) for (14), the electromagnetic current $j_{em}^\mu = j_V^{\mu, C=3} + j_V^{\mu, C=0}/N_c$ with the vector current $j_V^{\mu, C} = j_L^{\mu, C} + j_R^{\mu, C}$ are given by

$$j_{em}^0(x^\mu) = \frac{3}{4\pi} \frac{\rho^2}{(r^2 + \rho^2)^{5/2}} I_3 + \frac{15}{16\pi} \frac{\rho^4}{(r^2 + \rho^2)^{7/2}} \tag{16}$$

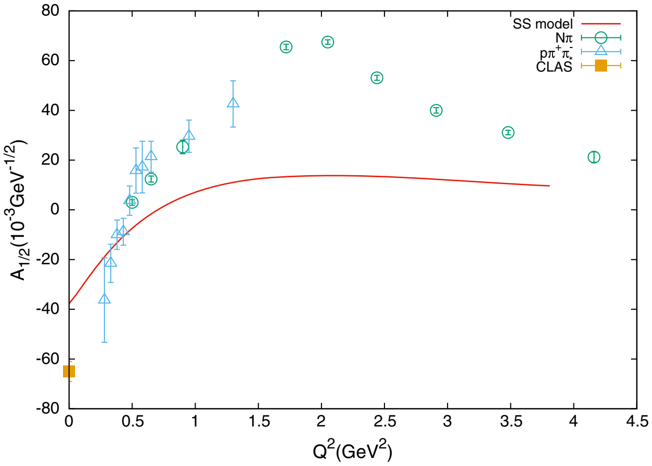

FIG. 1. The transverse helicity amplitude $A_{1/2}$ in units of 10^{-3} $\mathrm{GeV}^{-1/2}$ as function of the four-momentum transfer Q^2. The red line is result of the present study, and the sources of experimental data are shown in the panel [20–24].

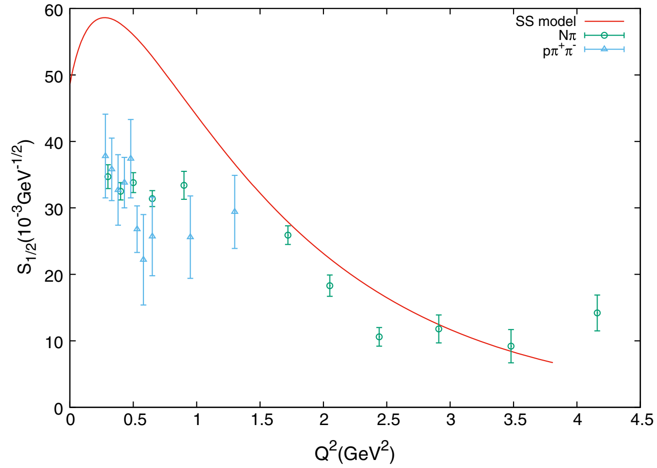

FIG. 2. The longitudinal helicity amplitude $S_{1/2}$ in units of 10^{-3} $\mathrm{GeV}^{-1/2}$. The same conventions are used as in Fig. 1.

$$j^i_{em}(x^\mu) = \frac{4\pi\kappa}{\rho^2}\left(\frac{8}{r} - \frac{8r^4 + 20\rho^2 r^2 + 15\rho^4}{(r^2+\rho^2)^{5/2}}\right)\epsilon_{ijk}x^j \mathrm{tr}(t_k \boldsymbol{a}^{-1} t_3 \boldsymbol{a}) + \frac{15}{32\pi}\frac{\rho^2}{(r^2+\rho^2)^{7/2}}\left(-\epsilon_{ija}x^j\chi^a + 2x^i\frac{d}{dt}\ln\rho\right) \quad (17)$$

where $\chi^a = -2i\mathrm{tr}(t^a \boldsymbol{a}^{-1}\dot{\boldsymbol{a}})$ and I_a is isospin operator $I_a = 8\pi^2\kappa\rho^2 \mathrm{tr}(i\dot{\boldsymbol{a}}\boldsymbol{a}^{-1}t_a)$ $(i, a = 1, 2, 3)$.

Although this current is not gauge invariant, the validity of the results of our calculations are supported by the facts that the currents reduce to the chiral current of the Skyrme model in low energy limit and reproduce various static properties of nucleon [3].

This completes the preparation for calculating the helicity amplitude of the electromagnetic transition.

There are two parameters in this model, Kaluza-Klein Mass M_{KK} and κ, which we determine according to Ref. [3] as follows. For M_{KK}, we determine it to reproduce the N-Δ mass difference, $(1232 - 939)$ MeV. The currents used in this paper are derived by taking into account the leading order of $1/\lambda$.

Therefore, in the Ref. [3], they identify the nucleon mass with the leading term $8\pi^2\kappa$ of the mass formula. The parameter κ is then determined to reproduce the mass of the nucleon 939 MeV at the classical level. As a consequence, the two parameters are $M_{KK} = 488$ MeV and $\kappa = 0.0243$. However, in this paper, we choose a slightly smaller kappa value of 0.0225 to take into account the rotation energy for the mass of the nucleon.

With electromagnetic currents (16) and (17), the helicity amplitudes from the nucleon to the Roper resonance (4) and (5) are obtained by using the wave functions (11), (12). The results are shown in Figs. 1 and 2 as functions of Q^2. It can be seen that the helicity amplitudes obtained from our model with least parameters achieves global agreement with experimental data. The results have some remarkable properties, as follows.

First, we note that our $A_{1/2}$ at the photon point $Q^2 = 0$ takes a finite negative value. The nonrelativistic quark model fails to explain this property because of the orthogonality of the radial wave functions of the Roper resonance and the ground state nucleon. Several theoretical studies have been done to solve this problem, and it has been argued that relativistic corrections and the effect of the meson cloud are important [10,11]. In the present approach, when the current is expanded in powers of ρ, the leading term starts at the order ρ^2. The matrix element survives because of the ρ^2 term. This is the unique feature of the solitonic description of baryons where the collective dynamics of the meson field plays an important role.

Second, the experimental data for $A_{1/2}$ flips its sign at around $Q^2 = 0.5$ GeV^2.

Our result captures this behavior qualitatively well at around $Q^2 \sim 0.7$ GeV^2.

However, our model calculation underestimates the experimental data of $A_{1/2}$ at $Q^2 \gtrsim 1$ GeV^2. This is because our results are calculated up to order $1/\lambda$. The first term in (17) is the order of λ, and the second term is the order of $1/\lambda$. Moreover, the model by meson fields should be applied to the low energy region $Q^2 \leq 1$ GeV^2.

Third, our prediction reproduces qualitatively well the experimental data for $S_{1/2}$ but with some overestimate.

In the calculation of $S_{1/2}$, we can use only the time component of the current (16) because of the current conservation law. We consider that the reason that our prediction of $S_{1/2}$ with sufficient strength is that it is dictated by the conserved charge $\int d^3x j^0_{em} = I_3 + \frac{1}{2}$.

Finally, we emphasize that there are only two parameters in this model, κ and M_{KK}. We have determined these parameters from the masses of the nucleon and the delta.

With a similar set of parameters, static properties of nucleons have been studied with good agreement with experimental data [3]. It is emphasized that by tuning only two parameters the present approach explains well not only static properties but also the dynamical properties of baryons.

Furthermore, we expect that the present results can be further improved by adding corrections to the black brane of the SS model, The curvature of the SS model is known to determine the coupling of mesons and nucleons when projected to 4-dimensions, In order to explain the excited states of nucleons and their transition processes, including the Roper resonance, it is known that it is important to describe them in terms of resonate states considering the continuum states of hadrons [11]. They confirmed that the mass and decay width of the Roper resonance can be reproduced by strengthening the coupling between the meson and the nucleon. As noted above, in the SS model, this corresponds to tuning the curvature. In fact, the masses of the Roper and negative parity resonance are degenerate. Although this property captures the experimental facts better than the quark model, it does not completely explain the mass spectrum of these resonances. Therefore, further improvement of the curvature of the SS model should be attempted. It is hoped that this will result in an improvement the present calculations.

In the present analysis, the Roper resonance has been described as radial density modulation. Hence the results provide information on the stiffness or compressibility of hadronic matter, which is an essential input for the study of high density matter.

This work was supported in part by Grants-in-Aid for Scientific Research [Grant No. 21H04478(A)] and the one on Innovative Areas (Grant No. 18H05407).

[1] H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, Baryons from instantons in holographic QCD, Prog. Theor. Phys. **117**, 1157 (2007).

[2] K. Hashimoto, T. Sakai, and S. Sugimoto, Holographic baryons: Static properties and form factors from gauge/string duality, Prog. Theor. Phys. **120**, 1093 (2008).

[3] H. Hata, M. Murata, and S. Yamato, Chiral currents and static properties of nucleons in holographic QCD, Phys. Rev. D **78**, 086006 (2008).

[4] A. Iwanaka, D. Fujii, and A. Hosaka, Decay properties of $N(1535)$ in the holographic QCD, Phys. Rev. D **105**, 114057 (2022).

[5] L. D. Roper, Evidence for a P_{11} Pion-Nucleon Resonance at 556 MeV, Phys. Rev. Lett. **12**, 340 (1964).

[6] S. Capstick, Photoproduction amplitudes of P_{11} and P_{33} baryon resonances in the quark model, Phys. Rev. D **46**, 1965 (1992).

[7] S. Capstick and B. D. Keister, Baryon current matrix elements in a light front framework, Phys. Rev. D **51**, 3598 (1995).

[8] A. J. Arifi, H. Nagahiro, A. Hosaka, and K. Tanida, Roper-like resonances with various flavor contents and their two-pion emission decays, Phys. Rev. D **101**, 111502 (2020).

[9] T. Kubota and K. Ohta, Relativistic corrections to the baryon resonance photoexcitation amplitudes in the quark model, Phys. Lett. **65B**, 374 (1976).

[10] V. D. Burkert and C. D. Roberts, Colloquium: Roper resonance: Toward a solution to the fifty year puzzle, Rev. Mod. Phys. **91**, 011003 (2019).

[11] N. Suzuki, B. Julia-Diaz, H. Kamano, T. S. H. Lee, A. Matsuyama, and T. Sato, Disentangling the Dynamical Origin of P_{11} Nucleon Resonances, Phys. Rev. Lett. **104**, 042302 (2010).

[12] A. J. Arifi, D. Suenaga, and A. Hosaka, Relativistic corrections to decays of heavy baryons in the quark model, Phys. Rev. D **103**, 094003 (2021).

[13] M. R. Douglas, Branes within branes, NATO Sci. Ser. C **520**, 267 (1999).

[14] E. Witten, Baryons and branes in anti-de Sitter space, J. High Energy Phys. 07 (1998) 006.

[15] T. H. R. Skyrme, A unified field theory of mesons and baryons, Nucl. Phys. **31**, 556 (1962).

[16] T. Sakai and S. Sugimoto, Low energy hadron physics in holographic QCD, Prog. Theor. Phys. **113**, 843 (2005).

[17] T. Sakai and S. Sugimoto, More on a holographic dual of QCD, Prog. Theor. Phys. **114**, 1083 (2005).

[18] D. Fujii and A. Hosaka, Decay properties of Roper resonance in the holographic QCD, Phys. Rev. D **104**, 014022 (2021).

[19] I. G. Aznauryan, V. D. Burkert, H. Egiyan, K. Joo, R. Minehart, and L. C. Smith, Electroexcitation of the $P_{33}(1232), P_{11}(1440), D_{13}(1520), S_{11}(1535)$ at $Q^2 = 0.4$ and $0.65\ (\mathrm{Gev}/c)^2$, Phys. Rev. C **71**, 015201 (2005).

[20] I. G. Aznauryan *et al.* (CLAS Collaboration), Electroexcitation of the Roper resonance for $1.7 < Q^2 < 4.5\,\mathrm{GeV}^2$ in $\vec{e}p \to en\pi^+$, Phys. Rev. C **78**, 045209 (2008).

[21] I. G. Aznauryan *et al.* (CLAS Collaboration), Electroexcitation of nucleon resonances from CLAS data on single pion electroproduction, Phys. Rev. C **80**, 055203 (2009).

[22] M. Dugger *et al.* (CLAS Collaboration), π^+ photoproduction on the proton for photon energies from 0.725 to 2.875-GeV, Phys. Rev. C **79**, 065206 (2009).

[23] V. I. Mokeev *et al.* (CLAS Collaboration), Experimental study of the $P_{11}(1440)$ and $D_{13}(1520)$ resonances from CLAS data on $ep \to e'\pi^+\pi^- p'$, Phys. Rev. C **86**, 035203 (2012).

[24] V. I. Mokeev *et al.*, New results from the studies of the $N(1440)1/2^+$, $N(1520)3/2^-$, and $\Delta(1620)1/2^-$ resonances in exclusive $ep \to e'p'\pi^+\pi^-$ electroproduction with the CLAS detector, Phys. Rev. C **93**, 025206 (2016).

[25] I. G. Aznauryan and V. D. Burkert, Electroexcitation of nucleon resonances, Prog. Part. Nucl. Phys. **67**, 1 (2012).

[26] A. Belavin, A. M. Polyakov, A. Schwartz, and Y. Tyupkin, Pseudoparticle solutions of the Yang-Mills equations, Phys. Lett. **59B**, 85 (1975).