$O(\alpha_s)$ perturbative and nonperturbative corrections to polarized semileptonic Λ_b decay distributions

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In this paper we calculate first order radiative QCD corrections to the decay process $b \to cW^-(\to \ell^-\bar{\nu}_\ell)$ of a polarized bottom quark. Taking into account both the bottom and charm quark masses, the analytical expressions given in this paper allow for a detailed analysis of the differential decay rate of the polarized quark in dependence on the polar and azimuthal angles of the lepton pair and the angle of the polarization vector of the quark relative to the momentum direction of the W boson. The results are given for different polarization states of the charged lepton. In addition, we calculated nonperturbative corrections and estimated their contribution.

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I. INTRODUCTION

The top quark as the heaviest known fermion is an ideal test particle, as it decays before the hadronization process can take place. Therefore, the quantum state of this particle is transferred mostly unperturbed to the decay products. While top quarks and antiquarks produced in pairs are mostly unpolarized [1,2], top quarks produced in single top production are found to be highly polarized due to parity violation [3–7]. As the top quark decays with more than 99% probability via the channel $t \to bW^+$, the question arises whether the polarization of the top quark gained in the production process is passed on to the decay products, i.e., the W boson and the bottom quark.

The Standard Model (SM) of elementary particle physics describes the polarization states of the W boson, measured by the subsequent decay of the W boson into, e.g., a pair of lepton and antineutrino. Any deviation from this prediction opens a window for beyond Standard Model (BSM) physics. Therefore, QCD and electroweak radiative corrections for the process $t \to bW$ have been calculated in order to distinguish BSM effects from perturbative and nonperturbative SM contributions (cf., e.g., Refs. [8–10]). In a couple of publications, we have given our contribution to this analysis, calculating electroweak corrections to next-to-leading (NLO) order [11,12] and QCD corrections to NLO [13,14] and next-to-next-to leading (NNLO) order [15].

Another channel for the production of polarized bottom quarks is the decay of the Z boson. If it is possible to separate the Z boson from the photon with which it is mixed, e.g., on e^+e^- annihilations, the decay of the Z boson to a heavy quark-antiquark pair also shows a significant degree of longitudinal polarization which amount to $\langle P_b\rangle = -0.94$ for bottom quarks and $\langle P_c\rangle = -0.68$ for charm quarks [16]. In a sequence of papers the possibility to measure the polarization of bottom (and charm) quarks at ATLAS and CMS has been analyzed [17–20]. The authors come to an affirmative answer, though the quarks are observed only as a jet of hadrons. As noted already in Ref. [21], at least in the heavy quark limit the polarization transfer from a heavy quark to the heavy hadron approaches 100%, i.e., a hadron like B or Λ_b can be considered to carry the same polarization as the bottom quark. For a less idealistic situation, Falk and Peskin have estimated the reduction factor to be of the order of 75% [22].

In this paper we analyze the polarization of the bottom quark by considering the energy and angular distribution of the leptons in the cascade process $b \to cW^-(\to \ell^-\bar{\nu}_\ell)$, taking into account both QCD corrections and the exact masses of both bottom and charm quark. In Born approximation, the differential rate is given by [23,24]

$$\frac{d\Gamma_b^{(0)}}{dx\,d\cos\theta} = \frac{G_F^2 m_b^5}{32\pi^5}\frac{x^2(1-x-y^2)^2}{6(1-x)^2} \times\left[3-2x+y^2+\frac{2y^2}{1-x}+S\cos\theta\left(1-2x+y^2-\frac{2y^2}{1-x}\right)\right], \tag{1}$$

where $y = m_c/m_b$ is the scaled charm quark mass and $x = 2E_\ell/m_b$ is the scaled energy of the charged lepton, ranging between 0 and $1-y^2$. $S=1$ corresponds to a fully polarized bottom quark, while $S=0$ gives the result for an unpolarized bottom quark.

In Ref. [24] it is emphasized that while QCD corrections modify significantly the decay rate and, therefore, the life time of the heavy quarks [25], this is not true for the energy distribution of the charged lepton [26–31]. However, the parts dependent and independent of the polarization may be affected differently. Therefore, the calculation of QCD corrections turns out to be necessary.

QCD corrections to the energy distribution have been calculated in Refs. [26,28,30] which agree with numerical results translated from top quark decays [32]. QCD corrections to the joint energy and angular distribution have been calculated in Ref. [32]. In Ref. [24] QCD corrections to the angular distribution has been calculated for the charm quark decay where the mass of the quark in the decay channel has been neglected. This, however, is not appropriate for the decay of a bottom quark into a charm quark. For this reason, in this publication we calculate QCD corrections for both energy and angular distributions, taking into account the masses of both quarks. In addition to the dependence on the polar angle θ between the direction of the W boson and the charged lepton, boosted back to the rest frame of the W boson, we also take into account the azimuthal angle ϕ between the plane spanned by these and the plane spanned by the W momentum and the polarization vector of the bottom quark with a relative angle θ_P in the rest frame of the bottom quark. This kinematics is shown in Fig. 1.

The paper is organized as follows. Starting from the hadron and lepton tensor given separately, in Sec. II we calculate the differential decay rate. The hadron tensor, responsible for the inclusive decay $b^\uparrow \to cW^-$, is written in terms of five unpolarized and nine polarized covariants. In Sec. III we calculate the corresponding coefficients, called invariant structure functions, and relate them to another set of structure functions more appropriate for the angular distribution and obtained from the hadron tensor by applying covariant helicity projection operators. In Secs. IV and V we calculate the first order QCD three graph and one loop corrections. In Sec. VI we integrate the former over the energy of the intermediate W boson and add up both contributions. In Sec. VII we give non-perturbative corrections. In Appendix we give explicit results for the integrated helicity rates.

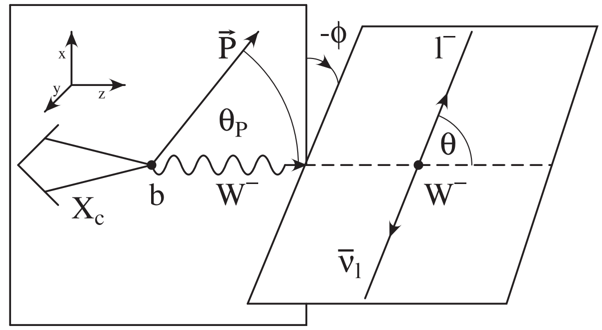

FIG. 1. Definition of the angles θ_P, θ, and ϕ in the decay process $b^\uparrow \to c + \ell^- + \bar\nu_\ell$. θ_P is the polar angle between the direction of the W boson momentum in positive z direction and the polarization vector of the bottom quark in the rest frame of the bottom quark which define the hadron plane. θ is the polar angle between the z direction and the lepton momentum in the rest frame of the W boson which define the lepton plane. The azimuthal angle between hadron and lepton plane is denoted by ϕ.

II. DIFFERENTIAL DECAY RATE

The differential decay rate for the process

$$b^\uparrow(p_b) \to \ell^-(p_\ell) + \bar\nu_\ell(p_\nu) + X_c(p_c) \tag{2}$$

is given by the contraction of the hadron tensor with the lepton tensor,

$$\frac{d\Gamma}{dq^2 dE_\ell} = \frac{G_F^2|V_{bc}|^2}{(2\pi)^3 m_b} W_{\mu\nu}(p_b, s_b, q) L^{\mu\nu}(p_\ell, p_\nu), \tag{3}$$

where $q = p_\ell + p_\nu$ is the four momentum of the W boson and E_ℓ is the charged lepton energy in the rest frame of the decaying bottom quark. In addition to the dependencies on momenta, we consider the dependence on the spin s_b of the bottom quark. For massless neutrinos the lepton tensor is given by

$$L^{\mu\nu} = p_\ell^\mu p_\nu^\nu + p_\ell^\nu p_\nu^\mu - p_\ell \cdot p_\nu g^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma} p_{\ell,\rho} p_{\nu,\sigma} \tag{4}$$

where we use the convention $\epsilon_{0123} = +1$. Based on Lorentz covariance and CP symmetry, the hadron tensor can be written in terms of five unpolarized and nine polarized invariant structure functions,

$$\begin{aligned} W_{\mu\nu}(p_b, s_b, q) = {} & -g_{\mu\nu} W_1(x) + v_\mu v_\nu W_2(x) - i\epsilon_{\mu\nu\rho\sigma} v^\rho x^\sigma W_3(x) + x_\mu x_\nu W_4(x) + (v_\mu x_\nu + x_\mu v_\nu) W_5(x) \\ & - (x\cdot s_b)[-g_{\mu\nu} W_1^P(x) + v_\mu v_\nu W_2^P(x) - i\epsilon_{\mu\nu\rho\sigma} v^\rho x^\sigma W_3^P(x) + x_\mu x_\nu W_4^P(x) + (v_\mu x_\nu + x_\mu v_\nu) W_5^P(x)] \\ & + (s_{b,\mu} v_\nu + s_{b,\nu} v_\mu) W_6^P(x) + (s_{b,\mu} x_\nu + s_{b,\nu} x_\mu) W_7^P(x) + i\epsilon_{\mu\nu\rho\sigma} v^\rho s_b^\sigma W_8^P(x) + i\epsilon_{\mu\nu\rho\sigma} x^\rho s_b^\sigma W_9^P(x), \end{aligned} \tag{5}$$

where $v = p_b/m_b$ and $x = q/m_b$ are momenta normalized to the mass of the bottom quark.[1] Note that instead of the traditional notation we use $W_i^P = G_i$ for the polarized structure functions. This helps to keep track of the different structure functions.

In the rest frame of the b quark and with the z axis along the momentum of the W boson, the parametrization of the momentum and polarization four vectors explicitly reads

$$\begin{aligned} p_b &= (m_b, 0, 0, 0), \qquad s_b = (0, \sin\theta_P, 0, \cos\theta_P), \\ p_\ell &= (E_\ell, -|\vec{p}_\ell|\sin\theta\cos\phi, -|\vec{p}_\ell|\sin\theta\sin\phi, |\vec{p}_\ell|\cos\theta), \qquad q = (q_0, 0, 0, |\vec{q}|), \\ p_c + p_G &= p_b - q, \qquad p_\nu = q - p_\ell. \end{aligned} \tag{6}$$

Inserting lepton and hadron tensors, using $\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\tau\omega} = -2(\delta_\rho^\tau\delta_\sigma^\omega - \delta_\rho^\omega\delta_\sigma^\tau)$ and

$$2p_\ell q = q^2 + p_\ell^2 - p_\nu^2 = q^2 + m_\ell^2, \qquad 2p_\nu q = q^2 - p_\ell^2 + p_\nu^2 = q^2 - m_\ell^2, \tag{7}$$

one obtains for the differential decay rate

$$\begin{aligned} \frac{d\Gamma}{dq^2 dE_\ell} = \frac{G_F^2|V_{bc}|^2}{(2\pi)^3}\Bigg\{&(q^2 - m_\ell^2)W_1 - \frac{1}{2}(4E_\ell^2 - 4E_\ell q_0 + q^2 - m_\ell^2)W_2 - \frac{1}{m_b}(2E_\ell q^2 - q_0(q^2 + m_\ell^2))W_3 \\ &+ \frac{m_\ell^2}{2m_b^2}(q^2 - m_\ell^2)W_4 - \frac{2m_\ell^2}{m_b}(E_\ell - q_0)W_5 + \frac{|\vec{q}|}{m_b}\cos\theta_P\Bigg[(q^2 - m_\ell^2)W_1^P - \frac{1}{2}(4E_\ell^2 - 4E_\ell q_0 + q^2 - m_\ell^2)W_2^P \\ &- \frac{1}{m_b}(2E_\ell q^2 - q_0(q^2 + m_\ell^2))W_3^P + \frac{m_\ell^2}{2m_b^2}(q^2 - m_\ell^2)W_4^P - \frac{2m_\ell^2}{m_b}(E_\ell - q_0)W_5^P\Bigg] \\ &- 2(E_\ell|\vec{q}|\cos\theta_P - (q_0 - 2E_\ell)|\vec{p}_\ell|(\cos\theta_P\cos\theta - \sin\theta_P\sin\theta\cos\phi))W_6^P - 2\frac{m_\ell^2}{m_b}(|\vec{q}|\cos\theta_P - |\vec{p}_\ell|(\cos\theta_P\cos\theta \\ &- \sin\theta_P\sin\theta\cos\phi))W_7^P - 2(E_\ell|\vec{q}|\cos\theta_P - q_0|\vec{p}_\ell|(\cos\theta_P\cos\theta - \sin\theta_P\sin\theta\cos\phi))W_8^P \\ &- \frac{1}{m_b}((q^2 + m_\ell^2)|\vec{q}|\cos\theta_P - 2q^2|\vec{p}_\ell|(\cos\theta_P\cos\theta - \sin\theta_P\sin\theta\cos\phi))W_9^P\Bigg\}. \end{aligned} \tag{8}$$

The angle θ is reexpressed by the energy E_ℓ of the lepton via

$$\cos\theta = \frac{\vec{p}_\ell\cdot\vec{q}}{|\vec{p}_\ell||\vec{q}|} = \frac{2E_\ell q_0 - q^2 - m_\ell^2}{2\sqrt{E_\ell^2 - m_\ell^2}\sqrt{q_0^2 - q^2}}. \tag{9}$$

III. INVARIANT STRUCTURE FUNCTIONS

The Born term results to the unpolarized and polarized invariant structure functions read

$$\begin{aligned} &W_1(\mathrm{Born}, x^2) = \frac{1}{2}(1 - x^2 + y^2), \qquad &&W_2(\mathrm{Born}, x^2) = 2, \qquad &&W_3(\mathrm{Born}, x^2) = 1, \\ &W_4(\mathrm{Born}, x^2) = 0, &&W_5(\mathrm{Born}, x^2) = -1, && \\ &W_1^P(\mathrm{Born}, x^2) = -1, &&W_2^P(\mathrm{Born}, x^2) = 0, &&W_3^P(\mathrm{Born}, x^2) = 0, \\ &W_4^P(\mathrm{Born}, x^2) = 0, &&W_5^P(\mathrm{Born}, x^2) = 0, &&W_6^P(\mathrm{Born}, x^2) = 1, \\ &W_7^P(\mathrm{Born}, x^2) = -1, &&W_8^P(\mathrm{Born}, x^2) = -1, &&W_9^P(\mathrm{Born}, x^2) = 1 \end{aligned} \tag{10}$$

[1] Note that while x is defined as scalar in the Introduction, at this point we use the same symbol for a four-vector. Still, this should not cause problems, as $p_\nu = q - p_\ell$ for vanishing lepton masses leads to $q^2 = m_\nu^2 - m_\ell^2 + 2qp_\ell = 2E_\ell\sqrt{q^2}$, $\sqrt{q^2} = 2E_\ell$ and, therefore, $x_\mu x^\mu = q^2/m_b^2 = 4E_\ell^2/m_b^2 = x^2$.

where $y = m_c/m_b$ is the scaled mass of the c quark. Note that for the Born term results the final state c quark is kinematically connected directly to the vertex with the b quark and the W boson. Because of this, the on-shell condition

$$\hat{u} = (v - x)^2 - y^2 = 1 - 2x_0 + x^2 - y^2 = 0 \quad (11)$$

is satisfied. In this case, the scaled energy x_0 of the W boson is not an independent variable but is fixed by x^2 and y^2 to the value $x_{00} = (1 + x^2 - y^2)/2$. Therefore, Born term results depend only on x^2 as a dynamical variable. This is different from radiative corrections including a gluon as considered in the next section. In this case the hadron tensor explicitly depends on x_0 (if essential, this will be indicated in the argument) and has to be integrated over x_0 and over the energy of the gluon. The invariant structure functions W_i and W_i^P can be written in terms of the helicity structure functions W_X,

$$\begin{aligned}
W_1(x) &= \frac{1}{2} W_U, \\
W_2(x) &= \frac{x^2}{2\hat{x}^2}(2W_L - W_U), \\
W_3(x) &= -\frac{1}{2\hat{x}} W_F, \\
W_4(x) &= \frac{1}{2x^2\hat{x}^2}(-x^2 W_U + 2x_0^2 W_L + 2(3x_0^2 - x^2)W_S - 8x_0\hat{x}W_{SL}), \\
W_5(x) &= \frac{1}{2\hat{x}^2}(x_0(W_U - 2W_L - 2W_S) + 4\hat{x}W_{SL}), \\
W_1^P(x) &= \frac{1}{2\hat{x}} W_{U^P}, \\
W_2^P(x) &= \frac{1}{2\hat{x}^3}(x^2(2W_{L^P} - W_{U^P}) - 4x_0\sqrt{2x^2}W_{I^P}), \\
W_3^P(x) &= \frac{1}{2x^2\hat{x}^2}(-x^2 W_{F^P} + 2x_0\sqrt{2x^2}W_{A^P} + 2\hat{x}\sqrt{2x^2}\,W_{SN^P}), \\
W_4^P(x) &= \frac{1}{2x^2\hat{x}^3}(-x^2 W_{U^P} + 2x_0^2 W_{L^P} + 2(3x_0^2 - x^2)W_{S^P} - 8x_0\hat{x}W_{SL^P} - 4x_0\sqrt{2x^2}W_{I^P} + 4\hat{x}\sqrt{2x^2}\,W_{ST^P}), \\
W_5^P(x) &= \frac{1}{2x^2\hat{x}^3}(x^2 x_0(W_{U^P} - 2W_{L^P} - 2W_{S^P}) + 4x^2\hat{x}W_{SL^P} + 2(x_0^2 + x^2)\sqrt{2x^2}W_{I^P} - 2x_0\hat{x}\sqrt{2x^2}\,W_{ST^P}), \\
W_6^P(x) &= \frac{\sqrt{2x^2}}{\hat{x}} W_{I^P}, \\
W_7^P(x) &= -\frac{\sqrt{2x^2}}{x^2\hat{x}}(x_0 W_{I^P} - \hat{x}W_{ST^P}), \\
W_8^P(x) &= \frac{\sqrt{2x^2}}{\hat{x}} W_{SN^P}, \\
W_9^P(x) &= -\frac{\sqrt{2x^2}}{x^2\hat{x}}(\hat{x}W_{A^P} + x_0 W_{SN^P}),
\end{aligned} \quad (12)$$

where $x_0 = q_0/m_b$,

$$\hat{x} = \frac{|\vec{q}|}{m_b} = \sqrt{(v \cdot x)^2 - x^2}, \quad (13)$$

and $4\hat{x}^2 = \lambda(1, x^2, y^2) = 1 + x^4 + y^4 - 2x^2 - 2y^2 - 2x^2y^2$. The helicity structure functions can be calculated from the hadron tensor by using the covariant helicity projection operators

$$
\begin{aligned}
P^{\mu\nu}_{U+L} &= -g^{\mu\nu} + \frac{x^\mu x^\nu}{x^2}, \\
P^{\mu\nu}_{U} &= -g^{\mu\nu} + \frac{x^\mu x^\nu}{x^2} - \frac{x^2}{\hat{x}^2}\left(v^\mu - \frac{v\cdot x}{x^2}x^\mu\right)\left(v^\nu - \frac{v\cdot x}{x^2}x^\nu\right), \\
P^{\mu\nu}_{L} &= \frac{x^2}{\hat{x}^2}\left(v^\mu - \frac{v\cdot x}{x^2}x^\mu\right)\left(v^\nu - \frac{v\cdot x}{x^2}x^\nu\right), \\
P^{\mu\nu}_{F} &= -\frac{i}{\hat{x}}\epsilon^{\mu\nu\rho\sigma}v_\rho x_\sigma, \\
P^{\mu\nu}_{I} &= \frac{\sqrt{x^2}}{2\sqrt{2}\hat{x}}\left[s^\mu_T\left(v^\nu - \frac{v\cdot x}{x^2}x^\nu\right) + \left(v^\mu - \frac{v\cdot x}{x^2}x^\mu\right)s^\nu_T\right], \\
P^{\mu\nu}_{A} &= -\frac{i\sqrt{x^2}}{2\sqrt{2}\hat{x}^2}\left[\epsilon^{\mu\rho\sigma\tau}\left(v^\nu - \frac{v\cdot x}{x^2}x^\nu\right)\right. \\
&\qquad \left. - \left(v^\mu - \frac{v\cdot x}{x^2}x^\mu\right)\epsilon^{\nu\rho\sigma\tau}\right]v_\rho x_\sigma s_{T,\tau}, \\
P^{\mu\nu}_{S} &= \frac{x^\mu x^\nu}{x^2}, \\
P^{\mu\nu}_{SL} &= -\frac{1}{2\hat{x}}\left(v^\mu x^\nu + x^\mu v^\nu - 2\frac{v\cdot x}{x^2}x^\mu x^\nu\right), \\
P^{\mu\nu}_{ST} &= -\frac{1}{2\sqrt{2x^2}}(s^\mu_T x^\nu + x^\mu s^\nu_T), \\
P^{\mu\nu}_{SN} &= -\frac{i}{2\hat{x}\sqrt{2x^2}}(\epsilon^{\mu\rho\sigma\tau}x^\nu - x^\mu\epsilon^{\nu\rho\sigma\tau})v_\rho x_\sigma s_{T,\tau}
\end{aligned} \tag{14}
$$

and the longitudinal or transverse polarization vectors of the bottom quark,

$$
s^\mu_L = \frac{1}{\hat{x}}(x^\mu - (v\cdot x)v^\mu) = (0,0,0,1), \quad s^\mu_T = (0,1,0,0). \tag{15}
$$

The helicity structure functions W_{X^P} for $X = U, L, F, S, SL$ depend on the longitudinal polarization vector while W_{X^P} for $X = I, A, ST, SN$ depend on the transverse polarization vector. Note that the relations between the helicity structure functions W_X and the invariant unpolarized and polarized structure functions W_i are expressed with coefficients depending on $\hat{x}$ and x_0. This means that after integration over x_0 the relations will mix. However, because the helicity structure functions describe angular distributions with angles being the same for Born term and first order contributions, in the following we will use the helicity structure functions only. With

$$
\begin{aligned}
W_U &= 2W_1, \\
W_L &= W_1 + \frac{\hat{x}^2}{x^2}W_2, \\
W_F &= -2\hat{x}W_3, \\
W_S &= -W_1 + \frac{x_0^2}{x^2}W_2 + x^2W_4 + 2x_0W_5, \\
W_{SL} &= \frac{1}{2\hat{x}}\left(-x_0W_1 + \frac{2x_0^2 - x^2}{x^2}x_0W_2 + x_0x^2W_4\right. \\
&\qquad \left. + (3x_0^2 - x^2)W_5\right), \\
W_{U^P} &= 2\hat{x}W_1^P, \\
W_{L^P} &= \hat{x}\left(W_1^P + \frac{\hat{x}^2}{x^2}W_2^P + \frac{2x_0}{x^2}W_6^P\right), \\
W_{F^P} &= -2(\hat{x}^2W_3^P + W_8^P + x_0W_9^P), \\
W_{I^P} &= \frac{\hat{x}}{\sqrt{2x^2}}W_6^P, \\
W_{A^P} &= \frac{-1}{\sqrt{2x^2}}(x_0W_8^P + x^2W_9^P), \\
W_{S^P} &= \hat{x}\left(-W_1^P + \frac{x_0^2}{x^2}W_2^P + x^2W_4^P + 2x_0W_5^P + \frac{2x_0}{x^2}W_6^P\right. \\
&\qquad \left. + 2W_7^P\right), \\
W_{SL^P} &= \frac{1}{2}\left(-x_0W_1^P + \frac{2x_0 - x^2}{x^2}x_0W_2^P + \frac{x^2x_0}{2}W_4^P\right. \\
&\qquad \left. + (3x_0^2 - x^2)W_5^P + \frac{4x_0^2 - x^2}{x^2}W_6^P + 3x_0W_7^P\right), \\
W_{ST^P} &= \frac{1}{\sqrt{2x^2}}(x_0W_6^P + x^2W_7^P), \\
W_{SN^P} &= \frac{1}{\sqrt{2x^2}}\hat{x}W_8^P
\end{aligned} \tag{16}
$$

we perform the transition from the invariant structure functions to the helicity structure functions which are dealt with in the following.

IV. FIRST ORDER TREE GRAPH CONTRIBUTIONS

We calculate the tree graph contributions with a finite gluon mass m_g to regularize the infrared divergence which arises in the phase space integration. To isolate the divergence it is necessary to write the result as a sum of a singular (or soft) part $W^s(\text{tree})$ which contains the divergence and an IR-finite (regular) part $W^r(\text{tree})$,

$$W(\text{tree}) = W^s(\text{tree}) + W^r(\text{tree}) \tag{17}$$

(W stands for W_i or for W_X). The singular part can be written in the form

$$W^s(\text{tree}, x^2, x_0) = \frac{\alpha_s C_F}{4\pi} W(\text{Born}, x^2, x_0) S_g(x^2, x_0, \Lambda), \tag{18}$$

where the scaled gluon mass $\Lambda = m_g/m_b$ is introduced. The soft gluon factor is defined by

$$S_g(x^2, x_0, \Lambda) = -\left[\frac{1}{\hat{u}_-} - \frac{1}{\hat{u}_+} - 4\frac{1-x_0}{\hat{u}-\Lambda^2}\ln\left(\frac{\hat{u}_+}{\hat{u}_-}\right) + 4\frac{1+x^2-2x_0-\Lambda^2}{(\hat{u}-\Lambda^2)^2}(\hat{u}_+ - \hat{u}_-)\right], \tag{19}$$

where $\hat{u} = 1 + x^2 - y^2 - 2x_0$ is the off-shell parameter and

$$\hat{u}_\pm = \frac{(1-x_0)(\hat{u}+\Lambda^2) \pm \sqrt{x_0^2 - x^2}\sqrt{(\hat{u}-\Lambda^2)^2 - 4y^2\Lambda^2}}{1 + x^2 - 2x_0}. \tag{20}$$

The procedure would be much easier if the IR singular part (the logarithmic singularity residing in $1/\hat{u}$) would be proportional to the proper Born term contribution. However, this is not the case. Instead, $W(\text{Born}, x^2, x_0)$ depends independently on x_0 as well because the three-body kinematics is used. In order to get rid of this problem, we add and subtract the two-body Born term contribution times the soft gluon factor to obtain

$$W^s(\text{tree}, x^2, x_0) = [W^s(\text{tree}, x^2, x_0)]_+ + \frac{\alpha_s C_F}{4\pi} W(\text{Born}, x^2) S_g(x^2, x_0, \Lambda). \tag{21}$$

The term

$$[W^s(\text{tree}, x^2, x_0)]_+ = \frac{\alpha_s C_F}{4\pi}(W(\text{Born}, x^2, x_0) - W(\text{Born}, x^2)) S_g(x^2, x_0, \Lambda) \tag{22}$$

is IR finite, and the replacement $\Lambda \to 0$ allows for an analytical integration of this contribution, while for the second part on the right-hand side of Eq. (21) the Born term factor can be kept out of the integration. This is the benefit of the plus prescription applied in Eq. (21). We are left with the integrated soft gluon factor defined by

$$A_g(x^2, \Lambda) = \int_{x_-}^{x_+} S_g(x^2, x_0, \Lambda) dx_0, \tag{23}$$

where the integration limits are given by

$$x_- = \sqrt{x^2}, \qquad x_+ = \frac{1}{2}(1 + x^2 - (y+\Lambda)^2). \tag{24}$$

In the limit $\Lambda \to 0$ this leads to the analytical result

$$\begin{aligned} A_g(x^2, \Lambda) = 4\Bigg\{&1 + 2\ln\left(\frac{\Lambda y}{(1-x_-)^2 - y^2}\right) - 2\mathrm{Li}_2(x_-) + \mathrm{Li}_2\left(\frac{x_-}{\eta}\right) + \mathrm{Li}_2(\eta x_-)\Bigg\} \\ &+ \frac{2}{\sqrt{\lambda}}(1 - x^2 + y^2)\Bigg\{-\ln(\omega_1) + \frac{1}{2}\ln^2(\omega_1) - 2\ln^2\left(\frac{1-x_-}{1-\eta x_-}\right) + 2\ln(\omega_1)\ln\left(\frac{(\eta+1)\Lambda y}{(\eta-1)\sqrt{\lambda}}\right) + 2\mathrm{Li}_2(1-\omega_1) \\ &- 4\mathrm{Li}_2\left(\frac{(\eta-1)x_-}{1-x_-}\right) - 4\mathrm{Li}_2\left(\frac{(\eta-1)x_-}{\eta - x_-}\right)\Bigg\}, \end{aligned} \tag{25}$$

where we have used the definitions

$$\begin{aligned} &\lambda = \lambda(1, x^2, y^2) = 1 + x^4 + y^4 - 2x^2 - 2y^2 - 2x^2y^2, \qquad \eta = \frac{1 + x^2 - y^2 + \sqrt{\lambda}}{2x} \\ &\omega_1 = \frac{\eta(1-\eta x_-)}{\eta - x_-} = \frac{1 - x^2 + y^2 - \sqrt{\lambda}}{1 - x^2 + y^2 + \sqrt{\lambda}}, \qquad x_- = \sqrt{x^2}. \end{aligned} \tag{26}$$

To describe the spectrum of the charged lepton in the final state we have to change the lower integration limit x_- in the integrated soft gluon factor (23) to $x_0(x_\ell)$ which depends on the scaled lepton energy x_ℓ. $A_g(x^2, \Lambda)$ then changes to the x_ℓ-dependent soft gluon factor

$$A_g(x^2, \Lambda, x_\ell) = \int_{x_0(x_\ell)}^{x_+} S_g(x^2, x_0, \Lambda) dx_0. \tag{27}$$

The change can be performed by calculating

$$A_g(x^2,\Lambda,x_\ell) = \int_{x_-}^{x_+} S_g(x^2,x_0,\Lambda)dx_0 - \int_{x_-}^{x_0(x_\ell)} S_g(x^2,x_0,0)dx_0 \tag{28}$$

because the subtracted integral is IR finite. The limit of integration $x_0(x_\ell)$ for a finite scaled lepton mass $\zeta = m_\ell/m_b$ is determined by the condition

$$-1 \le \cos\theta = \frac{x^2 x_\ell - x_0(x^2+\zeta^2)}{\sqrt{x_0^2 - x^2}(x^2-\zeta^2)} \le 1. \tag{29}$$

This gives two solutions for x_0,

$$x_{0\pm} = \frac{1}{4\zeta^2}[x_\ell(x^2+\zeta^2) \pm \sqrt{x_\ell^2 - 4\zeta^2}(x^2-\zeta^2)], \tag{30}$$

where the solution x_{0-} is the physical one which for vanishing lepton mass $\zeta = 0$ has the limit

$$x_0(x_\ell) = \frac{x^2 + x_\ell^2}{2x_\ell}. \tag{31}$$

The analytical result for the subtrahend in Eq. (28) is given by

$$\begin{aligned}\int_{x_-}^{x_0(x_\ell)} S_g(x^2,x_0,0)dx_0 = \Bigg[&4\ln\left(\frac{(u_2-u)(uu_2-1)}{uu_2}\right) + 2\mathrm{Li}_2\left(\frac{x_-}{u}\right) + 2\mathrm{Li}_2(ux_-)\\
&+ \frac{2}{\sqrt{\lambda}}(1-x^2+y^2)\Bigg\{\mathrm{Li}_2\left(\frac{u}{u_2}\right) - \mathrm{Li}_2\left(\frac{uu_2-1}{uu_2}\right) + \mathrm{Li}_2\left(\frac{uu_2-1}{u_2(u-x_-)}\right)\\
&- \mathrm{Li}_2\left(\frac{u-x_-}{u_2-x_-}\right) - \mathrm{Li}_2\left(\frac{u_2(1-ux_-)}{u_2-x_-}\right) - \mathrm{Li}_2\left(\frac{1-u_2x_-}{1-ux_-}\right) - \frac{1}{2}\ln^2(uu_2)\\
&+ \ln u\ln\left(\frac{u_2-u}{u_2}\right) + \ln(u_2)\ln(uu_2-1) - \ln(u-x_-)\ln\left(\frac{u_2-u}{u_2-x_-}\right)\\
&+ \ln((u_2-u)x_-)\ln(1-ux_-) - \ln\left(\frac{(uu_2-1)x_-}{u_2-x_-}\right)\ln(1-ux_-) - \frac{1}{2}\ln^2(1-ux_-)\\
&- \ln\left(\frac{u_2}{1-u_2x_-}\right)\ln\left(\frac{uu_2-1}{1-u_2x_-}\right) + \frac{1}{2}\ln^2\left(\frac{u_2(u-x_-)}{1-u_2x_-}\right) - \frac{1}{2}\ln^2(1-u_2x_-)\Bigg\}\Bigg]_{u=1}^{u(x_\ell)},\end{aligned} \tag{32}$$

where $u_2 = (1 + x^2 - y^2 + \sqrt{\lambda})/(2x_-)$. The upper limit depends on the scaled lepton energy x_ℓ and is defined by

$$u(x_\ell) = \frac{1}{x_-}(x_0(x_\ell) + \sqrt{x_0(x_\ell)^2 - x^2}), \tag{33}$$

where $x_0(x_\ell)$ is the integration boundary defined in Eq. (31). As the quantity x_- cease from being directly related to the lower boundary, for simplicity in the following we will skip the lower minus sign and use the obvious notation $x = \sqrt{x^2}$ instead. Our results presented in the main text are obtained for zero charged lepton mass, $m_\ell = 0$.

The infrared finite regular part of the tree graph contribution to the unpolarized and polarized invariant structure functions of the decay $b^\uparrow \to c + \ell^- + \bar{\nu}_\ell$ can be calculated directly with zero gluon mass. Employing an operational notation

$$\begin{aligned}f(x_0)[S_g(x^2,x_0,\Lambda)]_+ = {}&(f(x_0) - f(x_{00}))S_g(x^2,x_0,0)\\ &+ f(x_{00})S_g(x^2,x_{00},\Lambda)\end{aligned} \tag{34}$$

for the plus prescription where $f(x_0)$ is regular in $x_0 = x_{00}$, the tree graph contributions read explicitly

$$
\begin{aligned}
W_1(\mathrm{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\bigg\{&\frac{1}{2}(1-x^2+y^2)[S_g(x^2,x_0,\Lambda)]_+\\
&+\frac{1+x^2-y^2-2x_0}{2(x_0^2-x^2)}\bigg[9-2x^2+y^2-x_0+\frac{1}{2}\frac{(1-x^2)(1-5x^2+2y^2)}{1+x^2-2x_0}+\frac{1}{2}\frac{(1-x^2)^3}{(1+x^2-2x_0)^2}\\
&+\frac{5+x^2+y^2-2x_0(2+x_0)}{\sqrt{x_0^2-x^2}}\ln(\tau)\bigg]\bigg\},\\
W_2(\mathrm{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\bigg\{&4[S_g(x^2,x_0,\Lambda)]_++\frac{1}{x_0^2-x^2}\bigg[-11-9x^2+5y^2-16x_0\\
&+\frac{3}{2}x\frac{(1+x)^2-y^2}{(1+x)(x_0+x)}(5+4x-x^2+y^2)-\frac{3}{2}x\frac{(1-x)^2-y^2}{(1-x)(x_0-x)}(5-4x-x^2+y^2)\\
&+\frac{(1-x^2)^3+2(2+x^2-3x^4)y^2+3(1+x^2)y^4}{(1-x^2)(1+x^2-2x_0)}-\frac{y^2(1-x^2)(1-x^2+y^2)}{(1+x^2-2x_0)^2}\bigg]\\
&+\frac{1}{4\sqrt{x_0^2-x^2}}\bigg[32-\frac{(1-x^2)(5+x^2)-2(2-x^2)y^2-y^4+8(1-x^2+y^2)x_0}{x_0^2-x^2}\\
&-\frac{3}{2}\frac{(1-x)^2-y^2}{(x_0-x)^2}(5-4x-x^2+y^2)-\frac{3}{2}\frac{(1+x)^2-y^2}{(x_0+x)^2}(5+4x-x^2+y^2)\bigg]\ln(\tau)\bigg\},\\
W_3(\mathrm{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\bigg\{&-2[S_g(x^2,x_0,\Lambda)]_++\frac{1}{2(x_0^2-x^2)}\bigg[1+5x^2-y^2+14x_0+\frac{(1-x^2)^2y^2}{(1+x^2-2x_0)^2}\\
&-\frac{(1-x^2)^2+4x^2y^2}{(1+x^2-2x_0)}+2x_0\frac{3-x^2+y^2-2x_0}{\sqrt{x_0^2-x^2}}\ln(\tau)\bigg]\bigg\},\\
W_4(\mathrm{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\bigg\{&\frac{1}{2(x_0^2-x^2)}\bigg[-28-4x_0\\
&-2\frac{(1-x^2)y^4}{(1+x^2-2x_0)^2}+2\frac{y^2}{1-x^2}\frac{6(1-x^2)+(5+x^2)y^2}{1+x^2-2x_0}\\
&-3\frac{(1-x)^2-y^2}{x(1-x)}\frac{5-4x-x^2+y^2}{x_0-x}+3\frac{(1+x)^2-y^2}{x(1+x)}\frac{5+4x-x^2+y^2}{x_0+x}\bigg]\\
&+\frac{1}{4\sqrt{x_0^2-x^2}}\bigg[\frac{15-36x^2+5x^4-2(6+x^2)y^2-3y^4+16x^2x_0}{x^2(x_0^2-x^2)}-\frac{3}{2}\frac{(1-x)^2-y^2}{x^2}\frac{5-4x-x^2+y^2}{(x_0-x)^2}\\
&-\frac{3}{2}\frac{(1+x)^2-y^2}{x^2}\frac{5+4x-x^2+y^2}{(x_0+x)^2}\bigg]\ln(\tau)\bigg\},\\
W_5(\mathrm{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\bigg\{&-2[S_g(x^2,x_0,\Lambda)]_++\frac{1}{2(x_0^2-x^2)}\bigg[-7(3-x^2)-3y^2+18x_0+\frac{(1-x^2)y^2(1-x^2+2y^2)}{(1+x^2-2x_0)^2}\\
&-\frac{(1-x^2)^3+2(1-x^2)(5+3x^2)y^2+4(2+x^2)y^4}{(1-x^2)(1+x^2-2x_0)}\\
&+3\frac{(1-x)^2-y^2}{1-x}\frac{5-4x-x^2+y^2}{x_0-x}+3\frac{(1+x)^2-y^2}{1+x}\frac{5+4x-x^2+y^2}{x_0+x}\bigg]\\
&+\frac{1}{4\sqrt{x_0^2-x^2}}\bigg[-8+\frac{3}{2}\frac{(1-x)^2-y^2}{x}\frac{5-4x-x^2+y^2}{(x_0-x)^2}\\
&-\frac{5-14x+7x^2+2x^3-(3+2x)y^2}{x(x_0-x)}-\frac{3}{2}\frac{(1+x)^2-y^2}{x}\frac{5+4x-x^2+y^2}{(x_0+x)^2}\\
&+\frac{5+14x+7x^2-2x^3-(3-2x)y^2}{x(x_0+x)}\bigg]\ln(\tau)\bigg\},
\end{aligned}
$$

$$W_1^P(\text{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\left\{-2[S_g(x^2,x_0,\Lambda)]_+ +\frac{1}{2(x_0^2-x^2)}\left[27+7x^2-5y^2+10x_0-\frac{(1-x^2)^2+4x^2y^2-2y^4}{1+x^2-2x_0}\right.\right.$$
$$\left.+\frac{(1-x^2)^2y^2}{(1+x^2-2x_0)^2}+\frac{3}{x}\frac{((1-x)^2-y^2)^2}{x_0-x}-\frac{3}{x}\frac{((1+x)^2-y^2)^2}{x_0+x}\right]$$
$$+\frac{1}{8\sqrt{x_0^2-x^2}}\left[-16+\frac{3(1-x)((1-x)^2-y^2)^2}{x^2(x_0-x)^2}+\frac{3(1+x)((1+x)^2-y^2)^2}{x^2(x_0+x)^2}\right.$$
$$\left.\left.-\frac{2}{x^2}\frac{3-18x^2-x^4-6(1-x^2)y^2+3y^4+4x^2(3+x^2-y^2)x_0}{x_0^2-x^2}\right]\ln(\tau)\right\},$$

$$W_2^P(\text{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\left\{\frac{1}{2(x_0^2-x^2)}\left[\frac{4y^2}{(1-x^2)^2}\frac{3(1-x^2)^2-2(3+2x^2)y^2}{1+x^2-2x_0}-\frac{9-6x+7x^2}{2x(1-x)^2}\frac{((1-x)^2-y^2)^2}{x_0-x}\right.\right.$$
$$\left.+\frac{9+6x+7x^2}{2x(1+x)^2}\frac{((1+x)^2-y^2)^2}{x_0+x}+\frac{4y^4}{(1+x^2-2x_0)^2}-\frac{15}{2}\frac{((1-x)^2-y^2)^2}{(x_0-x)^2}-\frac{15}{2}\frac{((1+x)^2-y^2)^2}{(x_0+x)^2}\right]$$
$$+\frac{1}{4\sqrt{x_0^2-x^2}}\left[\frac{(1-x^2)(3-7x^2)-2(3+5x^2)y^2+3y^4}{4x^2(x_0^2-x^2)}-\frac{3}{8}\frac{(1-3x)((1-x)^2-y^2)^2}{x^2(x_0-x)^2}-\frac{3}{8}\frac{(1+3x)((1+x)^2-y^2)^2}{x^2(x_0+x)^2}\right.$$
$$\left.\left.-\frac{15}{4}\frac{(1-x)((1-x)^2-y^2)^2}{x(x_0-x)^3}+\frac{15}{4}\frac{(1+x)((1+x)^2-y^2)^2}{x(x_0+x)^3}\right]\ln(\tau)\right\},$$

$$W_3^P(\text{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\left\{\frac{1}{(x_0^2-x^2)}\left[-1-\frac{3y^2}{1+x^2-2x_0}+\frac{3}{2}\frac{1+2x}{x}\frac{(1-x)^2-y^2}{x_0-x}-\frac{3}{2}\frac{1-2x}{x}\frac{(1+x)^2-y^2}{x_0+x}\right]\right.$$
$$+\frac{1}{(x_0^2-x^2)^{3/2}}\left[-1-2x^2+2y^2+\frac{3}{4}\frac{(1-x)(1+2x)}{x}\frac{(1-x)^2-y^2}{x_0-x}\right.$$
$$\left.\left.-\frac{3}{4}\frac{(1+x)(1-2x)}{x}\frac{(1+x)^2-y^2}{x_0+x}\right]\ln(\tau)\right\},$$

$$W_4^P(\text{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\left\{\frac{1+x^2-y^2-2x_0}{4(x_0^2-x^2)}\left[-8\frac{(1-x^2)^2-10y^2}{(1-x^2)^2(1+x^2-2x_0)}-8\frac{y^2}{(1+x^2-2x_0)^2}-15\frac{(1-x)^2-y^2}{x^2(x_0-x)^2}\right.\right.$$
$$-15\frac{(1+x)^2-y^2}{x^2(x_0+x)^2}+2\frac{(1-x^2)^2(15-19x^2)-5(3-8x^2+x^4)y^2+20x^2y^2x_0}{x^2(1-x^2)^2(x_0^2-x^2)}$$
$$\left.\left.-6\frac{(1-x_0)(5+x^2-5y^2-10x_0+4x_0^2)}{(x_0^2-x^2)^{5/2}}\ln(\tau)\right]\right\},$$

$$W_5^P(\text{tree},x^2,x_0)=\frac{\alpha_s C_F}{4\pi}\left\{\frac{1}{2(x_0^2-x^2)}\left[-6-\frac{2y^2}{(1-x^2)^2}\frac{3(1-x^2)^2-4(4+x^2)y^2}{1+x^2-2x_0}-\frac{4y^4}{(1+x^2-2x_0)^2}+\frac{15}{2}\frac{((1-x)^2-y^2)^2}{x(x_0-x)^2}\right.\right.$$
$$-\frac{15}{2}\frac{((1+x)^2-y^2)^2}{x(x_0+x)^2}-\frac{(1-x)^2-y^2}{x(1-x)^2}\frac{4(1-x)^2(3-2x)+(3+2x)y^2}{x_0-x}$$
$$\left.+\frac{(1+x)^2-y^2}{x(1+x)^2}\frac{4(1+x)^2(3+2x)+(3-2x)y^2}{x_0+x}\right]$$
$$+\frac{1}{16\sqrt{x_0^2-x^2}}\left[\frac{(1-x^2)(3+5x^2)-2(3-x^2)y^2+3y^4}{x^2(x_0^2-x^2)}-\frac{3}{2}\frac{(1-x)^2-y^2}{x^3}\frac{(1-x)(5+16x-9x^2)-(5+x)y^2}{(x_0-x)^2}\right.$$
$$+\frac{3}{2}\frac{(1+x)^2-y^2}{x^3}\frac{(1+x)(5-16x-9x^2)-(5-x)y^2}{(x_0+x)^2}$$
$$\left.\left.+\frac{15(1-x)((1-x)^2-y^2)^2}{x^2(x_0-x)^3}+\frac{15(1+x)((1+x)^2-y^2)^2}{x^2(x_0+x)^3}\right]\ln(\tau)\right\},$$

$$
\begin{aligned}
W_6^P(\mathrm{tree},x^2,x_0) = \frac{\alpha_s C_F}{4\pi}\bigg\{ & -2[S_g(x^2,x_0,\Lambda)]_+ + \frac{1}{2(x_0^2-x^2)}\bigg[13+15x^2-10y^2+16x_0-\frac{(1-x^2)^2+(1+3x^2)y^2+2y^4}{1+x^2-2x_0} \\
& +\frac{(1-x^2)^2y^2}{(1+x^2-2x_0)^2}-3\frac{((1-x)^2-y^2)^2}{x_0-x}-3\frac{((1+x)^2-y^2)^2}{x_0+x}\bigg] \\
& +\frac{1}{8\sqrt{x_0^2-x^2}}\bigg[-32-\frac{3(1-x)((1-x)^2-y^2)^2}{x(x_0-x)^2}+\frac{3(1+x)((1+x)^2-y^2)^2}{x(x_0+x)^2} \\
& +2\frac{3+x^4-2(2+x^2)y^2+y^4+4(1-2x^2+2y^2)x_0}{x_0^2-x^2}\bigg]\ln(\tau)\bigg\}, \\
W_7^P(\mathrm{tree},x^2,x_0) = \frac{\alpha_s C_F}{4\pi}\bigg\{ & 2[S_g(x^2,x_0,\Lambda)]_+ + \frac{1}{(x_0^2-x^2)}\bigg[5(3+x^2-2y^2)-16x_0+\frac{(1-x^2)^2+(1+3x^2)y^2+2y^4}{1+x^2-2x_0} \\
& -\frac{(1-x^2)^2y^2}{(1+x^2-2x_0)^2}+\frac{3((1-x)^2-y^2)^2}{x(x_0-x)}-\frac{3((1+x)^2-y^2)^2}{x(x_0+x)}\bigg] \\
& +\frac{1}{4\sqrt{x_0^2-x^2}}\bigg[8-\frac{3-8x^2-23x^4-6(1-2x^2)y^2+3y^4+4x^2(6+x^2-y^2)x_0}{x^2(x_0^2-x^2)} \\
& +\frac{3}{2}\frac{(1-x)((1-x)^2-y^2)^2}{x^2(x_0-x)^2}+\frac{3}{2}\frac{(1+x)((1+x)^2-y^2)^2}{x^2(x_0+x)^2}\bigg]\ln(\tau)\bigg\}, \\
W_8^P(\mathrm{tree},x^2,x_0) = \frac{\alpha_s C_F}{4\pi}\bigg\{ & -2[S_g(x^2,x_0,\Lambda)]_+ + \frac{1}{2(x_0^2-x^2)}\bigg[3+x^2+16x_0-\frac{(1-x^2)^2+(3+x^2)y^2}{1+x^2-2x_0}+\frac{(1-x^2)^2y^2}{(1+x^2-2x_0)^2}\bigg] \\
& +\frac{1}{2(x_0^2-x^2)^{3/2}}[1+x^2-y^2+2(3x_0-4x_0^2)]\ln(\tau)\bigg\}, \\
W_9^P(\mathrm{tree},x^2,x_0) = \frac{\alpha_s C_F}{4\pi}\bigg\{ & 2[S_g(x^2,x_0,\Lambda)]_+ + \frac{1}{2(x_0^2-x^2)}\bigg[-15+3x^2-4y^2-8x_0-\frac{(1-x^2)^2y^2}{(1+x^2-2x_0)^2}+\frac{(1-x^2)^2+(3+x^2)y^2}{1+x^2-2x_0}\bigg] \\
& +\frac{1}{4\sqrt{x_0^2-x^2}}\bigg[8-\frac{1}{x}\frac{(1-x)(7+3x-2x^2)+(1-2x)y^2}{x_0-x} \\
& +\frac{1}{x}\frac{(1+x)(7-3x-2x^2)+(1+2x)y^2}{x_0+x}\bigg]\ln(\tau)\bigg\}, \qquad (35)
\end{aligned}
$$

where the argument τ of the logarithm is given by

$$
\tau = \frac{1-x_0-\sqrt{x_0^2-x^2}}{1-x_0+\sqrt{x_0^2-x^2}}. \qquad (36)
$$

V. ONE LOOP CONTRIBUTIONS

Finally, the contributions to one-loop corrections of unpolarized and polarized invariant structure functions are given by

$$
\begin{aligned}
W_1(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{8\pi}\left\{-A_0(1-x^2+y^2)-\frac{2}{x^2}(1-y^2)(1-x^2+y^2)\ln(y)+\frac{\sqrt{\lambda}}{x^2}(1-3x^2+y^2)\ln(\omega_1)\right\}, \\
W_2(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{2\pi}\left\{-A_0-\frac{3}{\sqrt{\lambda}}(1-x^2+y^2)\ln(\omega_1)\right\}, \\
W_3(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{4\pi}\left\{A_0+\frac{2}{x^2}(1-y^2)\ln(y)-\left[\frac{1}{x^2}\sqrt{\lambda}-\frac{2}{\sqrt{\lambda}}(1-x^2+y^2)\right]\ln(\omega_1)\right\},
\end{aligned}
$$

$$
\begin{aligned}
W_4(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{2\pi x^2}\left\{-2-\frac{2}{x^2}(1-2x^2-y^2)\ln(y)+\left[\frac{\sqrt{\lambda}}{x^2}-\frac{1}{\sqrt{\lambda}}(1-x^2-3y^2)\right]\ln(\omega_1)\right\},\\
W_5(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{4\pi}\left\{A_0+\frac{2}{x^2}(1-y^2)+2\left(\frac{\lambda}{x^4}-1\right)\ln(y)-\frac{1}{x^2}(1+x^2-y^2)\left[\frac{\sqrt{\lambda}}{x^2}-\frac{2}{\sqrt{\lambda}}(1-x^2)\right]\ln(\omega_1)\right\},\\
W_1^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{4\pi}\left\{A_0+\frac{2}{x^2}(1-y^2)\ln(y)-\left[\frac{\sqrt{\lambda}}{x^2}-\frac{2}{\sqrt{\lambda}}(1-x^2+y^2)\right]\ln(\omega_1)\right\},\\
W_2^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{2\pi}\left\{\frac{2}{x^2}\ln(y)-\frac{1}{\sqrt{\lambda}x^2}(1-x^2-y^2)\ln(\omega_1)\right\},\\
W_3^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{4\pi x^2}\left\{2+\frac{2}{x^2}(1-2x^2-y^2)\ln(y)-\left[\frac{\sqrt{\lambda}}{x^2}-\frac{1-x^2-3y^2}{\sqrt{\lambda}}\right]\ln(\omega_1)\right\},\\
W_4^P(\mathrm{loop},x^2) &= 0,\\
W_5^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{4\pi x^2}\left\{2+\frac{2}{x^2}(1-2x^2-y^2)\ln(y)-\left[\frac{\sqrt{\lambda}}{x^2}-\frac{1}{\sqrt{\lambda}}(1-x^2-3y^2)\right]\ln(\omega_1)\right\},\\
W_6^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{8\pi}\left\{2A_0+\frac{2}{x^2}(1+x^2-y^2)\ln(y)-\left[\frac{\sqrt{\lambda}}{x^2}-\frac{4}{\sqrt{\lambda}}(1-x^2+y^2)\right]\ln(\omega_1)\right\},\\
W_7^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{8\pi}\left\{-2A_0-\frac{2}{x^2}(1-x^2-y^2)-\frac{2}{x^2}\left(\frac{\lambda}{x^2}+1+x^2-3y^2\right)\ln(y)\right.\\
&\quad\left.+\frac{1}{x^2}\left[\frac{\sqrt{\lambda}}{x^2}(1+2x^2-y^2)-\frac{2}{\sqrt{\lambda}}(1-x^2+y^2)(1+x^2-y^2)\right]\ln(\omega_1)\right\},\\
W_8^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{8\pi}\left\{2A_0-4-\frac{2}{x^2}(1-3x^2-y^2)\ln(y)+\left[\frac{\sqrt{\lambda}}{x^2}+\frac{4}{\sqrt{\lambda}}(1-x^2+y^2)\right]\ln(\omega_1)\right\},\\
W_9^P(\mathrm{loop},x^2) &= \frac{\alpha_s C_F}{8\pi}\left\{-2A_0+\frac{2}{x^2}(1+x^2-y^2)+\frac{2}{x^2}\left[\frac{\lambda}{x^2}+1-3x^2+5y^2\right]\ln(y)\right.\\
&\quad\left.-\frac{1}{\sqrt{\lambda}x^4}\left[(1-x^2)(1-x^2+4x^4)-(1+x^2)(3-7x^2)y^2+(3-2x^2)y^4-y^6\right]\ln(\omega_1)\right\}. \qquad (37)
\end{aligned}
$$

The IR divergent part is proportional to the product of the Born result times the factor

$$
\begin{aligned}
A_0(x^2,\Lambda) &= \frac{2}{\sqrt{\lambda}}(1-x^2+y^2)\left\{-2\mathrm{Li}_2(1-\omega_2)+2\mathrm{Li}_2(1-\omega_3)-\ln(\omega_1)\ln\left(\frac{y}{\Lambda^2}\right)\right.\\
&\quad\left.-\ln\left(\frac{1}{2}(1-x^2+y^2+\sqrt{\lambda})\right)\ln(\omega_2\omega_3)\right\}-4\left[\ln\left(\frac{y}{\Lambda^2}\right)-2\right], \qquad (38)
\end{aligned}
$$

where

$$
\omega_2=\frac{1+x^2-y^2-\sqrt{\lambda}}{1+x^2-y^2+\sqrt{\lambda}}, \qquad \omega_3=\frac{1-x^2-y^2-\sqrt{\lambda}}{1-x^2-y^2+\sqrt{\lambda}}. \qquad (39)
$$

VI. INTEGRATED HELICITY STRUCTURE FUNCTIONS

In this section we integrate the invariant structure functions $W_X(\mathrm{tree},x^2,x_0)$ over the scaled energy x_0 of the W boson and combine it with the Born term and loop results,

$$
\begin{aligned}
W_X(\mathrm{incl},x^2) &= W_X(\mathrm{Born},x^2)\left(1+\frac{\alpha_s C_F}{4\pi}A(x^2)\right)\\
&\quad+W_X^f(\mathrm{loop},x^2)+\int W_X^f(\mathrm{tree},x^2,x_0)dx_0 \qquad (40)
\end{aligned}
$$

($X \in \{U, L, F, S, SL, U^P, L^P, F^P, I^P, A^P, S^P, SL^P, ST^P, SN^P\}$), where $W_X^f(\text{loop}, x^2)$ and $W_X^f(\text{tree}, x^2, x_0)$ contain only finite parts where the contributions of $A_g(x^2,\Lambda)$ and $A_0(x^2,\Lambda)$ are skipped. In calculating the difference $A_g - A_0$, the IR singularities cancel according to the Kinoshita–Lee–Nauenberg theorem, and the IR-finite factor A is defined by

$$A(x^2) = \lim_{\Lambda\to 0}[A_g(x^2,\Lambda) - A_0(x^2,\Lambda)]$$

$$= 4\left\{\ln(y) - 1 + 2\ln\left[\frac{y}{(1-x)^2-y^2}\right] - 2\mathrm{Li}_2(x) + \mathrm{Li}_2\left(\frac{x}{\eta}\right) + \mathrm{Li}_2(\eta x)\right\} \tag{41}$$

$$+\frac{2}{\sqrt{\lambda}}(1-x^2+y^2)\left\{-\ln(\omega_1) + \frac{1}{2}\ln^2(\omega_1) - 2\ln^2\left(\frac{1-x}{1-\eta x}\right)\right.$$
$$+\ln(\omega_1)\left[2\ln\left(\frac{\eta+1}{\eta-1}y\right) + \ln\left(\frac{y}{\lambda}\right)\right] + \ln\left[\frac{1}{2}(1-x^2+y^2+\sqrt{\lambda})\right]\ln(\omega_2\omega_3)$$
$$+4\ln(\eta)\ln\left(\frac{\eta-x}{\eta}\right) + 2\mathrm{Li}_2(\eta x) - 2\mathrm{Li}_2\left(\frac{x}{\eta}\right)$$
$$\left.-4\mathrm{Li}_2\left[\frac{(\eta-1)x}{1-x}\right] - 4\mathrm{LiLi}_2\left[\frac{(\eta-1)x}{\eta-x}\right]\right\}, \tag{42}$$

where the dilogarithmic identity

$$\mathrm{Li}_2(1-\omega_1) + \mathrm{Li}_2(1-\omega_2) - \mathrm{Li}_2(1-\omega_3) = \mathrm{Li}_2(\eta x) - \mathrm{Li}_2\left(\frac{x}{\eta}\right) + 2\ln(\eta)\ln\left(\frac{\eta-x}{\eta}\right) \tag{43}$$

is employed. According to Eqs. (16), the unintegrated inclusive helicity structure functions $W_X(\text{tree}, x^2, x_0)$ are linear combinations of the invariant structure functions $W_i(\text{tree}, x^2, x_0)$. In the following we discuss the results for the differential decay rates with respect to x^2 and three angles θ_P, θ and ϕ which are defined in Fig. 1. The helicity structure functions are the coefficients of the angular dependence in the angular decay distribution of the decay process.

The angular decay distributions of the process $b^\uparrow \to c + \ell^- + \bar{\nu}_\ell$ into leptons with negative or positive helicity, respectively, are given by

$$\frac{d\Gamma^-}{dx^2 d\cos\theta d\cos\theta_P d\phi} = \frac{\Gamma_b}{4\pi}\left[\frac{3}{8}\left(\frac{d\hat{\Gamma}_U^-}{dx^2} + \frac{d\hat{\Gamma}_{U^P}^-}{dx^2}P\cos\theta_P\right)(1+\cos^2\theta) + \frac{3}{4}\left(\frac{d\hat{\Gamma}_L^-}{dx^2} + \frac{d\hat{\Gamma}_{L^P}^-}{dx^2}P\cos\theta_P\right)\sin^2\theta\right.$$
$$+\frac{3}{4}\left(\frac{d\hat{\Gamma}_F^-}{dx^2} + \frac{d\hat{\Gamma}_{F^P}^-}{dx^2}P\cos\theta_P\right)\cos\theta$$
$$\left.+\frac{3}{\sqrt{2}}\frac{d\hat{\Gamma}_{I^P}^-}{dx^2}P\sin\theta_P\sin\theta\cos\theta\cos\phi + \frac{3}{\sqrt{2}}\frac{d\hat{\Gamma}_{A^P}^-}{dx^2}P\sin\theta_P\sin\theta\cos\phi\right]$$

$$\frac{d\Gamma^+}{dx^2 d\cos\theta d\cos\theta_P d\phi} = \frac{\Gamma_b}{4\pi}\left[\frac{3}{4}\left(\frac{d\hat{\Gamma}_U^+}{dx^2} + \frac{d\hat{\Gamma}_{U^P}^+}{dx^2}P\cos\theta_P\right)\sin^2\theta + \frac{3}{2}\left(\frac{d\hat{\Gamma}_L^+}{dx^2} + \frac{d\hat{\Gamma}_{L^P}^+}{dx^2}P\cos\theta_P\right)\cos^2\theta\right.$$
$$+\frac{3}{2}\left(\frac{d\hat{\Gamma}_S^+}{dx^2} + \frac{d\hat{\Gamma}_{S^P}^+}{dx^2}P\cos\theta_P\right) + 3\left(\frac{d\hat{\Gamma}_{SL}^+}{dx^2} + \frac{d\hat{\Gamma}_{SL^P}^+}{dx^2}P\cos\theta_P\right)\cos\theta$$
$$\left.+3\sqrt{2}\frac{d\hat{\Gamma}_{ST^P}^+}{dx^2}P\sin\theta_P\sin\theta\cos\phi + 3\sqrt{2}\frac{d\hat{\Gamma}_{I^P}^+}{dx^2}P\sin\theta_P\sin\theta\cos\theta\cos\phi\right] \tag{44}$$

where

$$\Gamma_b = \frac{G_F^2 m_b^5 |V_{bc}|^2}{192\pi^3} \tag{45}$$

is the total decay rate of the bottom quark in the limit $m_c \to 0$. The angular decay distributions for the transverse components of the spin for the charged lepton read

$$
\begin{aligned}
\frac{d\Gamma^x}{dx^2 d\cos\theta d\cos\theta_P d\phi} &= \frac{\Gamma_b}{4\pi}\left\{\frac{3}{2\sqrt{2}}\left(\frac{d\hat{\Gamma}^x_F}{dx^2}+\frac{d\hat{\Gamma}^x_{F^P}}{dx^2}P\cos\theta_P\right)\sin\theta\right.\\
&\quad -\frac{3}{2\sqrt{2}}\left[\frac{d\hat{\Gamma}^x_U}{dx^2}+\frac{d\hat{\Gamma}^x_{U^P}}{dx^2}P\cos\theta_P-2\left(\frac{d\hat{\Gamma}^x_L}{dx^2}+\frac{d\hat{\Gamma}^x_{L^P}}{dx^2}P\cos\theta_P\right)\right]\sin\theta\cos\theta\\
&\quad +\frac{3}{\sqrt{2}}\left(\frac{d\hat{\Gamma}^x_{SL}}{dx^2}+\frac{d\hat{\Gamma}^x_{SL^P}}{dx^2}P\cos\theta_P\right)\sin\theta-3\frac{d\hat{\Gamma}^x_{I^P}}{dx^2}P\sin\theta_P\cos(2\theta)\cos\phi\\
&\quad \left.-3\left(\frac{d\hat{\Gamma}^x_{A^P}}{dx^2}+\frac{d\hat{\Gamma}^x_{ST^P}}{dx^2}\right)P\sin\theta_P\cos\theta\cos\phi-3\frac{d\hat{\Gamma}^x_{SN^P}}{dx^2}P\sin\theta_P\cos\phi\right\}\\
\frac{d\Gamma^y}{dx^2 d\cos\theta d\cos\theta_P d\phi} &= \frac{\Gamma_b}{4\pi}\left[3\left(\frac{d\hat{\Gamma}^y_{A^P}}{dx^2}+\frac{d\hat{\Gamma}^y_{ST^P}}{dx^2}\right)P\sin\theta_P\sin\phi+3\left(\frac{d\hat{\Gamma}^y_{I^P}}{dx^2}+\frac{d\hat{\Gamma}^y_{SN^P}}{dx^2}\right)P\sin\theta_P\cos\theta\sin\phi\right]. \qquad (46)
\end{aligned}
$$

The reduced rates are given by

$$
\begin{aligned}
\frac{d\hat{\Gamma}^-_X}{dx^2} &= 2\frac{(x^2-\zeta^2)^2}{x^2}T_X(x^2), \qquad \frac{d\hat{\Gamma}^x_X}{dx^2} = \frac{2\zeta}{\sqrt{2x^2}}\frac{(x^2-\zeta^2)^2}{x^2}T_X(x^2),\\
\frac{d\hat{\Gamma}^+_X}{dx^2} &= \frac{\zeta^2}{x^2}\frac{(x^2-\zeta^2)^2}{x^2}T_X(x^2), \qquad \frac{d\hat{\Gamma}^y_X}{dx^2} = \frac{2\zeta}{\sqrt{2x^2}}\frac{(x^2-\zeta^2)^2}{x^2}T_X(x^2) \qquad (47)
\end{aligned}
$$

with

$$
\begin{aligned}
T_U &= 2(1-x^2+y^2)\sqrt{\lambda}-\frac{\alpha_s C_F}{4\pi}\times 2\left\{4(1-x^2+y^2)^2\mathcal{N}_1+\frac{1}{x}[(1-x)^2-y^2][(1-x)(5+x)+y^2]\mathcal{N}_2\right.\\
&\quad -\frac{1}{x}[(1+x)^2-y^2][(1+x)(5-x)+y^2]\mathcal{N}_3+\frac{2}{x^2}\sqrt{\lambda}(1-x^2+y^2)(1-6x^2-y^2)\ln(y)-8\sqrt{\lambda}(1-x^2+y^2)\ln\left(\frac{x}{\lambda}\right)\\
&\quad -\frac{1}{x^2}[(1-x^2)^2(1-6x^2)-(1+4x^2-3x^4)y^2-(1+2x^2)y^4+y^6]\ln(\omega_1)\\
&\quad \left.-4[7+3x^2-(4-5x^2)y^2-3y^4]\ln(\eta)+\sqrt{\lambda}(19+x^2-5y^2)\right\} \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
T_{U^P} &= -2\lambda+\frac{\alpha_s C_F}{4\pi}\times 2\left\{4\sqrt{\lambda}(1-x^2+y^2)\mathcal{N}_4-4[11+3x^2+x^4-2(3+x^2)y^2+y^4]\mathcal{N}_5\right.\\
&\quad +\frac{2}{x^2}\lambda(1-6x^2-y^2)\ln(y)+8\lambda\ln[(1-x)^2-y^2]-\frac{\sqrt{\lambda}}{x^2}[7+21x^2+2x^4-(8+3x^2)y^2+y^4]\ln(\omega_1)\\
&\quad -\frac{4}{x^2}[(1-x^2)(3+14x^2-2x^4)-(6-7x^2-x^4)y^2+(3-x^2)y^4]\ln\left(\frac{1-x}{y}\right)\\
&\quad \left.-\frac{1}{x}[(1-x)^2-y^2][12-55x+6x^2-x^3-3(4+x)y^2]\right\} \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
T_L &= \frac{1}{x^2}[\lambda+x^2(1-x^2+y^2)]\sqrt{\lambda}-\frac{\alpha_s C_F}{4\pi}\frac{1}{x^2}\left\{4(1-x^2+y^2)[\lambda+x^2(1-x^2+y^2)]\mathcal{N}_1\right.\\
&\quad -2x[(1-x)^2-y^2][(1-x)(5+x)+y^2]\mathcal{N}_2+2x[(1+x)^2-y^2][(1+x)(5-x)+y^2]\mathcal{N}_3\\
&\quad -2\sqrt{\lambda}[5(1-x^2)-(12+7x^2)y^2+7y^4]\ln(y)-8\sqrt{\lambda}[1-x^2-(2+x^2)y^2+y^4]\ln\left(\frac{x}{\lambda}\right)\\
&\quad +[5(1-x^2)^2-(3+20x^2-x^4)y^2+(9-2x^2)y^4+y^6]\ln(\omega_1)-8(1+x^2-y^2)[1-7x^2-(2+x^2)y^2+y^4]\ln(\eta)\\
&\quad \left.-\sqrt{\lambda}[5+47x^2-4x^4-(22+x^2)y^2+5y^4]\right\} \qquad (50)
\end{aligned}
$$

$$T_{L^P}=\frac{1}{x^2}(1-y^2)\lambda-\frac{\alpha_sC_F}{4\pi}\times\frac{1}{x^2}\Big\{4\sqrt{\lambda}(1-y^2)(1-x^2+y^2)\mathcal{N}_4-4[2+22x^2+11x^4-(5+12x^2+x^4)y^2+2(2+x^2)y^4-y^6]\mathcal{N}_5-2\lambda(5-7y^2)\ln(y)+8\lambda(1-y^2)\ln[(1-x)^2-y^2]-\sqrt{\lambda}[17+53x^2-(18+x^2)y^2+y^4]\ln(\omega_1)-4[(1-x^2)(11+24x^2)-(13-15x^2)y^2+2y^4]\ln\left(\frac{1-x}{y}\right)+[(1-x)^2-y^2][15-22x+105x^2-24x^3+4x^4-(12-22x+x^2)y^2-3y^4]\Big\} \tag{51}$$

$$T_S=\frac{1}{x^2}[\lambda+x^2(1-x^2+y^2)]\sqrt{\lambda}-\frac{\alpha_sC_F}{4\pi}\frac{1}{x^2}\Big\{4(1-x^2+y^2)[\lambda+x^2(1-x^2+y^2)]\mathcal{N}_1-\frac{2}{x^2}\sqrt{\lambda}[(1-x^2)(2+3x^2)-3(2+4x^2+3x^4)y^2+(6+11x^2)y^4-2y^6]\ln(y)-8\sqrt{\lambda}[1-x^2-(2+x^2)y^2+y^4]\ln\left(\frac{x}{\lambda}\right)+\frac{1}{x^2}[(1-x^2)^2(2+3x^2)-(8-3x^2+4x^4-3x^6)y^2+3(4+5x^2)y^4-(8+5x^2)y^6+2y^8]\ln(\omega_1)-8(1-y^2)[1-x^2-(2+x^2)y^2+y^4]\ln(\eta)-3\sqrt{\lambda}[3(1-x^2)-(10+3x^2)y^2+3y^4]\Big\} \tag{52}$$

$$T_{S^P}=\frac{1}{x^2}(1-y^2)\lambda-\frac{\alpha_sC_F}{4\pi}\times\frac{1}{x^2}(1-y^2)\Big\{4\sqrt{\lambda}(1-x^2+y^2)\mathcal{N}_4-4[2+x^4-(3+2x^2)y^2+y^4]\mathcal{N}_5-\frac{2}{x^2}\frac{1}{1-y^2}\lambda[2+3x^2-(4+9x^2)y^2+2y^4]\ln(y)+8\lambda\ln[(1-x)^2-y^2]+\frac{\sqrt{\lambda}}{x^2}[2-9x^2+x^4-(4+3x^2)y^2+2y^4]\ln(\omega_1)-4[(1-x^2)(5-2x^2)+2(2-x^2)y^2]\ln\left(\frac{1-x}{y}\right)+[(1-x)^2-y^2](11-6x-7x^2+7y^2)\Big\} \tag{53}$$

$$T_F=-2\lambda+\frac{\alpha_sC_F}{4\pi}\times2\Big\{4\sqrt{\lambda}(1-x^2+y^2)\mathcal{N}_4+4(1+3x^2-x^4+2x^2y^2-y^4)\mathcal{N}_5+\frac{2}{x^2}\lambda(1-6x^2-y^2)\ln(y)+8\lambda\ln[(1-x)^2-y^2]-\frac{\sqrt{\lambda}}{x^2}[1-9x^2+2x^4-(2+3x^2)y^2+y^4]\ln(\omega_1)+4[(1-x^2)(1+2x^2)-(1+x^2)y^2]\ln\left(\frac{1-x}{y}\right)-2[(1-x)^2-y^2](3-4x-3y^2)\Big\} \tag{54}$$

$$T_{F^P}=2\sqrt{\lambda}(1-x^2+y^2)-\frac{\alpha_sC_F}{4\pi}\times2\Big\{4(1-x^2+y^2)^2\mathcal{N}_1+\frac{2}{x}(1-x)(1+2x)[(1-x)^2-y^2]\mathcal{N}_2-\frac{2}{x}(1+x)(1-2x)[(1+x)^2-y^2]\mathcal{N}_3+\frac{2}{x^2}\sqrt{\lambda}(1-x^2+y^2)(1-6x^2-y^2)\ln(y)-8\sqrt{\lambda}(1-x^2+y^2)\ln\left(\frac{x}{\lambda}\right)-\frac{1}{x^2}[(1-x^2)^2(1-6x^2)-(1-8x^2-3x^4)y^2-(1+4x^2)y^4+y^6]\ln(\omega_1)-4[4-9x^2-(2-5x^2)y^2-2y^4]\ln(\eta)-2\sqrt{\lambda}(4+x^2+2y^2)\Big\} \tag{55}$$

$$T_{I^P} = -\frac{1}{\sqrt{2}x}\lambda + \frac{\alpha_s C_F}{4\pi}\times\frac{1}{\sqrt{2}x}\Big\{4\sqrt{\lambda}(1-x^2+y^2)\mathcal{N}_4 - 2[7+15x^2+4x^4-(11+8x^2)y^2+4y^4]\mathcal{N}_5$$
$$+\frac{1}{x^2}\lambda(1-11x^2-y^2)\ln(y) + 8\lambda\ln[(1-x)^2-y^2] - \frac{\sqrt{\lambda}}{2x^2}[1+30x^2+21x^4-2(1+11x^2)y^2+y^4]\ln(\omega_1)$$
$$-2[(1-x^2)(21+5x^2)-(11-15x^2)y^2-4y^4]\ln\left(\frac{1-x}{y}\right) + 2[(1-x)^2-y^2](12-7x+12x^2-9y^2)\Big\} \quad (56)$$

$$T_{A^P} = \frac{1}{\sqrt{2}x}\sqrt{\lambda}(1-x^2-y^2) - \frac{\alpha_s C_F}{4\pi}\times\frac{1}{\sqrt{2}x}\Big\{4(1-x^2+y^2)(1-x^2-y^2)\mathcal{N}_1 - (1-x)(1+2x)[(1-x)^2-y^2]\mathcal{N}_2$$
$$-(1+x)(1-2x)[(1+x)^2-y^2]\mathcal{N}_3 + \frac{1}{x^2}\sqrt{\lambda}[(1-x^2)(1-11x^2)-2(1-8x^2)y^2+y^4]\ln(y)$$
$$-8\sqrt{\lambda}(1-x^2-y^2)\ln\left(\frac{x}{\lambda}\right) - \frac{1}{2x^2}[(1-x^2)^2(1-11x^2)-(1+x^2)(3-11x^2)y^2+(3-7x^2)y^4-y^6]\ln(\omega_1)$$
$$-2[(1+x^2)(4-7x^2)-(8-7x^2)y^2+4y^4]\ln(\eta) - 2\sqrt{\lambda}(1+2x^2-4y^2)\Big\}. \quad (57)$$

$$T_{SL} = \frac{1}{x^2}(1-y^2)\lambda - \frac{\alpha_s C_F}{4\pi}\times\frac{1}{x^2}\Big\{4\sqrt{\lambda}(1-y^2)(1-x^2+y^2)\mathcal{N}_4 + 4[1+5x^2-x^4-(1+x^2-x^4)y^2-(1+2x^2)y^4+y^6]\mathcal{N}_5$$
$$-\frac{2}{x^2}\lambda[1+4x^2-2(1+4x^2)y^2+y^4]\ln(y) + \frac{\sqrt{\lambda}}{x^2}[1+7x^2+2x^4-(3+8x^2)y^2+(3+x^2)y^4-y^6]\ln(\omega_1)$$
$$+8(1-y^2)\lambda\ln[(1-x)^2-y^2] + 4[(1-x^2)(2+3x^2)-(7-4x^2-x^4)y^2-(1+x^2)y^4]\ln\left(\frac{1-x}{y}\right)$$
$$-[(1-x)^2-y^2][7-10x+13x^2-(26+2x+7x^2)y^2+7y^4]\Big\} \quad (58)$$

$$T_{SL^P} = \frac{\sqrt{\lambda}}{x^2}[\lambda+x^2(1-x^2+y^2)] - \frac{\alpha_s C_F}{4\pi}\frac{1}{x^2}\Big\{4(1-x^2+y^2)[\lambda+x^2(1-x^2+y^2)]\mathcal{N}_1$$
$$-2[(1-x)^2-y^2][3(1-x)-(3-2x)y^2]\mathcal{N}_2 - 2[(1+x)^2-y^2][3(1+x)-(3+2x)y^2]\mathcal{N}_3$$
$$-\frac{2}{x^2}\sqrt{\lambda}[(1-x^2)(1+4x^2)-(3+12x^2+8x^4)y^2+3(1+3x^2)y^4-y^6]\ln(y)$$
$$-8\sqrt{\lambda}[1-x^2-(2+x^2)y^2+y^4]\ln\left(\frac{x}{\lambda}\right) + \frac{1}{x^2}[(1-x^2)^2(1+4x^2)-2(2-6x^2+2x^4-x^6)y^2+(6-12x^2-x^4)y^4$$
$$-2(2+x^2)y^6+y^8]\ln(\omega_1) - 4(1-y^2)[2+13x^2+x^4-(4+3x^2)y^2+2y^4]\ln(\eta)$$
$$+\sqrt{\lambda}[13+19x^2-(8-5x^2)y^2-5y^4]\Big\} \quad (59)$$

$$T_{ST^P} = -\frac{1}{\sqrt{2}x}\sqrt{\lambda}(1-x^2-y^2) + \frac{\alpha_s C_F}{4\pi}\times\frac{1}{\sqrt{2}x}\Big\{4(1-x^2+y^2)(1-x^2-y^2)\mathcal{N}_1 + \frac{1}{x}[(1-x)^2-y^2][3(1-x)-(3-2x)y^2]\mathcal{N}_2$$
$$-\frac{1}{x}[(1+x)^2-y^2][3(1+x)-(3+2x)y^2]\mathcal{N}_3 - \frac{\sqrt{\lambda}}{x^2}[(1-x^2)(1+9x^2)-2(1+10x^2)y^2+y^4]\ln(y)$$
$$-8\sqrt{\lambda}(1-x^2-y^2)\ln\left(\frac{x}{\lambda}\right) + \frac{1}{2x^2}[(1-x^2)^2(1+9x^2)-3(1-x^4)y^2+(3+5x^2)y^4-y^6]\ln(\omega_1)$$
$$-2[(2+x^2)(5+x^2)-(20+3x^2)y^2+10y^4]\ln(\eta) + 2\sqrt{\lambda}(5+4x^2-2y^2)\Big\} \quad (60)$$

$$T_{SN^P} = \frac{\lambda}{\sqrt{2}x} + \frac{\alpha_s C_F}{4\pi} \times \frac{1}{\sqrt{2}x} \Big\{ 4\sqrt{\lambda}(1-x^2+y^2)\mathcal{N}_4 + 2[1+7x^2-2x^4+(1+4x^2)y^2-2y^4]\mathcal{N}_5 - \frac{1}{x^2}\lambda(1+9x^2-y^2)\ln(y) + \frac{1}{2x^2}\sqrt{\lambda}[1+10x^2+x^4-2(1+x^2)y^2+y^4]\ln(\omega_1) + 8\lambda\ln[(1-x)^2-y^2] + 2[(1-x^2)(1+5x^2)-(7-3x^2)y^2]\ln\left(\frac{1-x}{y}\right) - 2[(1-x)^2-y^2](2-3x+4x^2-5y^2) \Big\} \tag{61}$$

The dilogarithmic decay rate terms occurring in these expressions are

$$\begin{aligned}
\mathcal{N}_1 &= 2\left[2\mathrm{Li}_2(1-\omega_1) - \mathrm{Li}_2(\eta x) + \mathrm{Li}_2\left(\frac{x}{\eta}\right)\right] + 2\ln(\omega_1)\ln(1-\omega_1) - \ln(\omega_1)\ln(x) - 2\ln(\eta)\ln(y) \\
\mathcal{N}_2 &= \left[2\mathrm{Li}_2(1-\omega_1) - \mathrm{Li}_2(\eta x) + \mathrm{Li}_2\left(\frac{x}{\eta}\right)\right] + 2\left[\mathrm{Li}_2\left(-\frac{\omega_1}{\eta}\right) - \mathrm{Li}_2\left(-\frac{1}{\eta}\right)\right] - 2\ln(\eta)\ln(1-x) + 2\ln(\omega_1)\ln\left(1+\frac{\omega_1}{\eta}\right) \\
\mathcal{N}_3 &= \left[2\mathrm{Li}_2(1-\omega_1) - \mathrm{Li}_2(\eta x) + \mathrm{Li}_2\left(\frac{x}{\eta}\right)\right] + 2\left[\mathrm{Li}_2\left(\frac{\omega_1}{\eta}\right) - \mathrm{Li}_2\left(\frac{1}{\eta}\right)\right] - 2\ln(\eta)\ln(1+x) + 2\ln(\omega_1)\ln\left(1-\frac{\omega_1}{\eta}\right) \\
\mathcal{N}_4 &= \left[2\mathrm{Li}_2(1-\omega_1) - \mathrm{Li}_2(\eta x) + \mathrm{Li}_2\left(\frac{x}{\eta}\right)\right] + 2\left[\mathrm{Li}_2\left(-\frac{\omega_1}{\eta}\right) - \mathrm{Li}_2\left(-\frac{1}{\eta}\right)\right] + 2\ln(\omega_1)\ln(1-\omega_1) - \ln(\omega_1)\ln(\eta x) \\
\mathcal{N}_5 &= \mathrm{Li}_2(\eta x) + \mathrm{Li}_2\left(\frac{x}{\eta}\right) - 2\mathrm{Li}_2(x).
\end{aligned} \tag{62}$$

As an illustration of our results, in Fig. 2 we plot the dependence of the normalized decay rate $\hat{\Gamma}^-$ in dependence on the cosine of the polar angle θ for different azimuthal angles ϕ. The polarization angle is chosen to be $\theta_P = \pi/2$, i.e., orthogonal to the momentum of the W boson. The plots show the typical enhancement in the backwards direction of the charged lepton for $\phi = 0$ while for $\phi = \pi$ the rate in this direction is nearly extinguished. The $O(\alpha_s)$ corrections reduce the rate uniformly by about 13%.

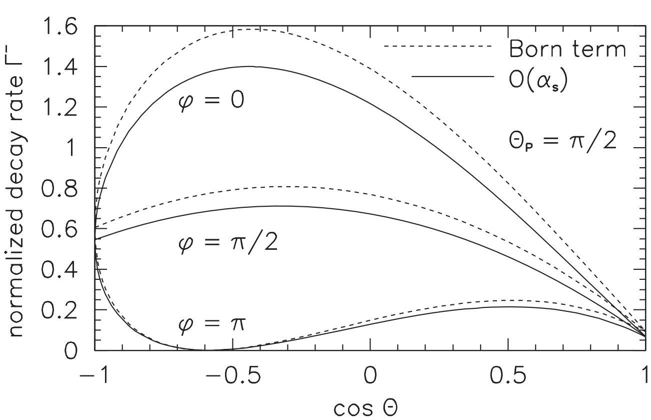

FIG. 2. Dependence of the normalized decay rate $\hat{\Gamma}^-$ on the cosine of the polar angle θ for azimuthal angles $\phi = 0$, $\phi = \pi/2$ and $\phi = \pi$. The polarization of the b quark is orthogonal to the momentum of the W boson, $\theta_P = \pi/2$.

VII. NONPERTURBATIVE CORRECTIONS

Both bottom and charm quarks are enclosed in heavy hadrons. This is accounted for by adding nonperturbative corrections to the decay width. The analysis done here is based on Ref. [33] and a series of publications of our working group on semileptonic decays of B mesons [34–36] and Λ_b baryons [37–44]. Note that the results of Ref. [33] have been independently checked by the authors of Refs. [45,46]. The calculations are done in the framework of the heavy quark effective theory (HQET) and the method of operator product expansion (OPE) as applied to heavy hadron decays. As before, the dynamics of the hadron-side transitions is embodied in the hadron tensor $W^{\mu\nu}$. However, in this case this tensor is given by the absorptive part of the current-current correlator,

$$\tilde{W}^{\mu\nu} = -\frac{1}{\pi}\mathrm{Im}(\Pi^{\mu\nu}), \tag{63}$$

where the correlator

$$\Pi^{\mu\nu}(q^2, q_0) = -i\langle\Lambda_b(p,s)| \times \int d^4x e^{-iqx}\mathcal{T}\{J^{\mu\dagger}(x)J^\nu(0)\}|\Lambda_b(p,s)\rangle \tag{64}$$

can be written again in terms of five unpolarized and nine polarized invariant structure functions. These can be calculated in HQET and read up to order $O(1/m_b^2)$ [33]

$$\begin{aligned}
\Pi_1 &= \frac{1}{2\Delta_0}(m_b - v\cdot q)(1+X_b) + \frac{2m_b}{3}(K_b+G_b)\left(\frac{-1}{2\Delta_0} + \frac{q^2-(v\cdot q)^2}{\Delta_0^2}\right) + \frac{m_b(K_b+G_b)}{2\Delta_0} - \frac{m_b^2 G_b}{3\Delta_0^2}(m_b - v\cdot q)\\
\Pi_2 &= \frac{m_b}{\Delta_0}(1+X_b) + \frac{2m_b}{3}(K_b+G_b)\left(\frac{1}{\Delta_0} + \frac{2m_b(v\cdot q)}{\Delta_0^2}\right) + \frac{m_b(K_b+G_b)}{\Delta_0} + \frac{4m_b^2K_b(v\cdot q)}{3\Delta_0^2} + \frac{2m_b^3G_b}{3\Delta_0^2}\\
\Pi_3 &= \frac{1}{2\Delta_0}(1+X_b) - \frac{2m_b}{3}(K_b+G_b)\frac{m_b - v\cdot q}{\Delta_0^2} + \frac{2m_b^2K_b}{3\Delta_0^2} - \frac{m_b^2G_b}{3\Delta_0^2}\\
\Pi_4 &= \frac{4m_b}{3\Delta_0^2}(K_b+G_b)\\
\Pi_5 &= \frac{-1}{2\Delta_0}(1+X_b) - \frac{2m_b}{3}(K_b+G_b)\frac{2m_b + v\cdot q}{\Delta_0^2} + \frac{m_b^2G_b}{3\Delta_0^2}\\
\Pi_1^P &= -\frac{1+\epsilon_b}{2\Delta_0} - \frac{5m_b}{3\Delta_0^2}(v\cdot q)K_b + \frac{4m_b^2K_b}{3\Delta_0^3}(q^2-(v\cdot q)^2)\\
\Pi_2^P &= \frac{4m_b^2K_b}{3\Delta_0^2}\\
\Pi_3^P &= \frac{2m_bK_b}{3\Delta_0^2}\\
\Pi_4^P &= 0\\
\Pi_5^P &= -\frac{2m_bK_b}{3\Delta_0^2}\\
\Pi_6^P &= -\frac{m_b(1+\epsilon_b)}{2\Delta_0} - \frac{5m_bK_b}{6\Delta_0} - \frac{5m_b^2}{3\Delta_0^2}(v\cdot q)K_b + \frac{4m_b^3K_b}{3\Delta_0^3}(q^2-(v\cdot q)^2)\\
\Pi_7^P &= \frac{1+\epsilon_b}{2\Delta_0} + \frac{(2m_b+3v\cdot q)m_bK_b}{3\Delta_0^2} - \frac{4m_b^2K_b}{3\Delta_0^3}(q^2-(v\cdot q)^2)\\
\Pi_8^P &= \frac{m_b(1+\epsilon_b)}{2\Delta_0} + \frac{m_bK_b}{6\Delta_0} + \frac{5m_b^2}{3\Delta_0^2}(v\cdot q)K_b - \frac{4m_b^3K_b}{3\Delta_0^3}(q^2-(v\cdot q)^2)\\
\Pi_9^P &= -\frac{1+\epsilon_b}{2\Delta_0} - \frac{(2m_b+3v\cdot q)m_bK_b}{3\Delta_0^2} + \frac{4m_b^2K_b}{3\Delta_0^3}(q^2-(v\cdot q)^2) \qquad (65)
\end{aligned}$$

where

$$X_b = -\frac{2(m_b - v\cdot q)m_b(K_b+G_b)}{\Delta_0} - \frac{8m_b^2K_b}{3\Delta_0^2}(q^2-(v\cdot q)^2) + \frac{2m_b^2K_b}{\Delta_0} \qquad (66)$$

and the denominator factor Δ_0 is given by

$$\Delta_0 = (m_b v - q)^2 - m_c^2 + i\epsilon. \qquad (67)$$

K_b is related to the mean kinetic energy of the heavy bottom quark

$$K_b = -\sum_s \langle\Lambda_b(p,s)|\bar{b}_v(x_l)\frac{(iD)^2}{2m_b^2}b_v(x_l)|\Lambda_b(p,s)\rangle = \frac{\mu_\pi^2}{2m_b^2}, \qquad (68)$$

where we can use $\mu_\pi^2 \approx 0.6\ \mathrm{GeV}^2$ [40]. The spin dependent contribution ϵ_b is defined by

$$\langle\Lambda_b(p,s)|\bar{b}\gamma^\lambda\gamma_5 b|\Lambda_b(p,s)\rangle = (1+\epsilon_b)s^\lambda \qquad (69)$$

and is of the order $\Lambda_{\mathrm{QCD}}^2/m_b^2$. Finally, the chromomagnetic contribution G_b is given by

$$\begin{aligned}
G_b &= \sum_s \langle\Lambda_b(p,s)|\bar{b}_v(x_l)\left(\frac{-gF_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b^2}\right)b_v(x_l)|\Lambda_b(p,s)\rangle\\
&= \frac{\mu_G^2}{2m_b^2} \qquad (70)
\end{aligned}$$

of the same order. Invariant structure functions W_i are defined accordingly by

$$\tilde{W}_i = -\frac{1}{\pi}\mathrm{Im}(\Pi_i). \tag{71}$$

The imaginary parts of the inverse powers of Δ_0, which are needed for obtaining W_i, can be calculated with the help of

$$\mathrm{Im}\left(\frac{1}{\Delta_0}\right) = \frac{-\pi}{2m_b}\delta\left[q_0 - \left(\frac{m_b^2 - m_c^2 + q^2}{2m_b}\right)\right]$$
$$\mathrm{Im}\left(\frac{1}{\Delta_0^2}\right) = \frac{-\pi}{4m_b^2}\frac{d}{dq_0}\delta\left[q_0 - \left(\frac{m_b^2 - m_c^2 + q^2}{2m_b}\right)\right]$$
$$\mathrm{Im}\left(\frac{1}{\Delta_0^3}\right) = \frac{-\pi}{16m_b^3}\frac{d^2}{dq_0^2}\delta\left[q_0 - \left(\frac{m_b^2 - m_c^2 + q^2}{2m_b}\right)\right]. \tag{72}$$

Accordingly, the invariant structure functions are given by

$$\tilde{W}_i = \frac{1}{2m_b}\Pi_i\bigg|\left\{\frac{1}{\Delta_0} \to \delta(q_0 - E_q), \frac{1}{\Delta_0^2} \to \frac{1}{2m_b}\delta'(q_0 - E_q),\right.$$
$$\left.\frac{1}{\Delta_0^3} \to \frac{1}{8m_b^2}\delta''(q_0 - E_q)\right\} \tag{73}$$

where $E_q = (m_b^2 - m_c^2 + q^2)/(2m_b)$.

The helicity structure functions $\tilde{W}_X$ can be obtained by linear combinations of the invariant structure functions $\tilde{W}_i^{(P)}$ with the help of Eqs. (16). By integrating over the W energy scale x_0, one obtains the integrated structure functions

$$T_X(x^2) \coloneqq 8m_b \int \sqrt{x_0^2 - x^2}\,\tilde{W}_X(x^2, x_0)dx_0 \tag{74}$$

which are given by

$$T_U = 2(1-K_b)\sqrt{\lambda}(1-x^2+y^2) + \frac{16}{3}K_b\sqrt{\lambda} + \frac{G_b}{3\sqrt{\lambda}}\{2\lambda[15(x^2-y^2)-11] + 8x^2(3(x^2-y^2)-7) + 32(1-y^2)\}$$
$$T_{U^P} = -2(1+\epsilon_b)\lambda + \frac{2}{3}K_b(3\lambda+8x^2)$$
$$T_L = (1-K_b)\frac{\sqrt{\lambda}}{x^2}[\lambda + x^2(1-x^2+y^2)] - \frac{16}{3}K_b\sqrt{\lambda} + \frac{G_b}{3\sqrt{\lambda}x^2}\{\lambda[15(-\lambda+x^4-x^2y^2) - 59x^2 + 12(1-y^2)]$$
$$+ 4x^2[x^2(3(x^2-y^2)-7) + 4(1-y^2)]\}$$
$$T_{L^P} = (1+\epsilon_b)\frac{\lambda}{x^2}(1-y^2) - K_b\frac{1-y^2}{3x^2}(3\lambda+8x^2)$$
$$T_S = (1-K_b)\frac{\sqrt{\lambda}}{x^2}[\lambda + x^2(1-x^2+y^2)] + \frac{G_b}{\sqrt{\lambda}x^2}\{\lambda[-5(\lambda - x^4 + x^2y^2) - 9x^2 + 4(1-y^2)] - 4x^4(1-x^2+y^2)\}$$
$$T_{S^P} = (1+\epsilon_b)\frac{\lambda}{x^2}(1-y^2) - K_b\frac{1-y^2}{3x^2}(3\lambda+8x^2)$$
$$T_F = -2\lambda + \frac{2}{3}K_b(3\lambda+8x^2) + \frac{2}{3}G_b[15\lambda + 16x^2 - 24(1-y^2)]$$
$$T_{F^P} = 2(1+\epsilon_b)\sqrt{\lambda}(1-x^2+y^2) + \frac{2}{3}K_b\sqrt{\lambda}(3(x^2-y^2)+5)$$
$$T_{I^P} = -(1+\epsilon_b)\frac{\lambda}{\sqrt{2}x} - 2K_b\frac{\lambda - 4x^2}{3\sqrt{2}x}$$
$$T_{A^P} = (1+\epsilon_b)\frac{\sqrt{\lambda}}{\sqrt{2}x}(1-x^2-y^2) - 2K_b\frac{\sqrt{\lambda}}{3\sqrt{2}x}(1+x^2-y^2)$$
$$T_{SL} = \frac{\lambda}{x^2}(1-y^2) - K_b\frac{1-y^2}{3x^2}(3\lambda+8x^2) - G_b\frac{1-5y^2}{3x^2}(3\lambda+8x^2)$$
$$T_{SL^P} = (1+\epsilon_b)\frac{\sqrt{\lambda}}{x^2}[\lambda + x^2(1-x^2+y^2)] - K_b\frac{\sqrt{\lambda}}{3x^2}[3\lambda - x^2(3(x^2-y^2)-11)]$$
$$T_{ST^P} = -(1+\epsilon_b)\frac{\sqrt{\lambda}}{\sqrt{2}x}(1-x^2-y^2) - 2K_b\frac{\sqrt{\lambda}}{3\sqrt{2}x}(1+x^2-y^2)$$
$$T_{SN^P} = (1+\epsilon_b)\frac{\lambda}{\sqrt{2}x} - 2K_b\frac{\lambda+4x^2}{3\sqrt{2}x}, \tag{75}$$

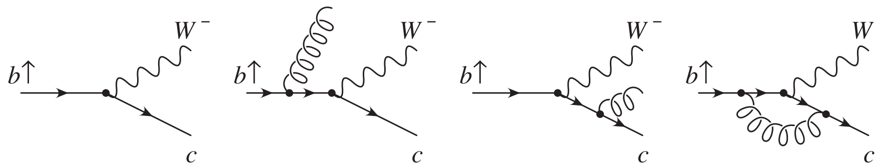

FIG. 3. Feynman diagrams for Born term, QCD first order tree and loop contributions.

where K_b, ϵ_b, and G_b are defined in Eqs. (68), (69), and (70), respectively. Taking typical values for these parameters advised for instance in Refs. [33,40], the corrections to Fig. 2 are below the 1% level and, therefore, outplayed by the first order radiative QCD corrections.

VIII. SUMMARY

In this paper we have given analytical expressions for first order radiative QCD corrections (cf. Fig. 3) to the ten helicity structure functions that determine the angular decay distribution of the semileptonic decay of a polarized bottom quark. We have shown that the radiative corrections change the Born term result significantly. At the same time, we have found that nonperturbative corrections calculated in some detail in the last part of this paper are subdominant.

ACKNOWLEDGMENTS

We gratefully remember our deceased collaborator Jürgen G. Körner who was one of the initiators of this project. The research was supported in part by the European Regional Development Fund under Grant No. TK133. S. G. acknowledges continuing support by the PRISMA/PRISMA+ Cluster of Excellence at the Johannes Gutenberg University in Mainz which allowed to finish this project during his stays in Mainz.

APPENDIX: INTEGRATED HELICITY RATES

We integrate the analytical results for the helicity rates $d\hat{\Gamma}_i^{(incl)}/dx^2$ defined in (47) over the scaled W boson momentum squared x^2. The limits of integration in case of massive leptons are $\zeta^2 \le x^2 \le (1-y)^2$, and the analytical results for the integrated helicity rates are given by

$$\hat{\Gamma} = \int_{\zeta^2}^{(1-y)^2} \frac{d\hat{\Gamma}}{dx^2} dx^2. \tag{A1}$$

With the definitions

$$u_1 = \frac{1-\zeta^2+y^2-\sqrt{R}}{2y}, \qquad u_2 = \frac{1+\zeta^2-y^2-\sqrt{R}}{2\zeta},$$
$$u_3 = \frac{(1-y)^2}{\zeta^2} \tag{A2}$$

we obtain the integrated rates into negative helicity leptons

$$\begin{aligned}
\hat{\Gamma}_U^- = {}& \frac{1}{3}\sqrt{R}[(1+y^2)(1-8y^2+y^4)-(7-12y^2+7y^4)\zeta^2-7(1+y^2)\zeta^4+\zeta^6]-8y^4(1-\zeta^4)\ln(u_1)-8(1-y^4)\zeta^4\ln(u_2) \\
& +K_b\left\{\frac{1}{9}\sqrt{R}[13+181y^2+37y^4-3y^6-(59+116y^2-21y^4)\zeta^2-(11-21y^2)\zeta^4-3\zeta^6]\right. \\
& \left.+\frac{8}{3}y^2[8+11y^2-16\zeta^2+(8-3y^2)\zeta^4]\ln(u_1)-\frac{8}{3}(1-y^2)(5-3y^2)\zeta^4\ln(u_2)\right\} \\
& +G_b\left\{\frac{1}{9}\sqrt{R}[41+185y^2+65y^4-15y^6-(127+124y^2-105y^4)\zeta^2+(41+105y^2)\zeta^4-15\zeta^6]\right. \\
& \left.+\frac{8}{3}y^2[12+11y^2-16\zeta^2+(4-15y^2)\zeta^4]\ln(u_1)-\frac{8}{3}(5-4y^2+15y^4)\zeta^4\ln(u_2)\right\}
\end{aligned}$$

$$\begin{aligned}
\hat{\Gamma}_{U^P}^- = {}& (1+\epsilon_b)\left\{\frac{1}{3}[\zeta^2-(1-y)^2][(1-y)^4(1+6y+y^2)-(1-y)^2(7+26y+7y^2)\zeta^2-(7+2y+7y^2)\zeta^4+\zeta^6]-4(1-y^2)^2\zeta^4\ln(u_3)\right\} \\
& -K_b\left\{\frac{1}{9}[\zeta^2-(1-y)^2][(1-y)^4(35+18y+3y^2)-(1-y)^2(85+78y+21y^2)\zeta^2+(11-6y-21y^2)\zeta^4+3\zeta^6]\right. \\
& \left.-4(1-y^2)^2\zeta^4\ln(u_3)\right\}
\end{aligned}$$

$$\hat{\Gamma}_L^- = \frac{2}{3}\sqrt{R}[(1+y^2)(1-8y^2+y^4)+10(1-y^2+y^4)\zeta^2+(1+y^2)\zeta^4]-8y^4[2-(3-y^2)\zeta^2+\zeta^4]\ln(u_1)$$
$$+8\zeta^2(1-y^2)[(1-y^2)^2+(1+y^2)\zeta^2]\ln(u_2)$$
$$+K_b\Big\{-\frac{2}{9}\sqrt{R}[11+59y^2-13y^4+3y^6-10(1+7y^2-3y^4)\zeta^2-(13-3y^2)\zeta^4]$$
$$-\frac{8}{3}y^2[2(4+y^2)-(16-9y^2+3y^4)\zeta^2+(8-3y^2)\zeta^4]\ln(u_1)-\frac{8}{3}\zeta^2(1-y^2)[3(1-y^2)^2-(5-3y^2)\zeta^2]\ln(u_2)\Big\}$$
$$+G_b\Big\{-\frac{2}{9}\sqrt{R}[7+115y^2-53y^4+15y^6-2(1+97y^2-75y^4)\zeta^2-(17-15y^2)\zeta^4]-\frac{8}{3}y^2[2(6+y^2)$$
$$-(28-33y^2+15y^4)\zeta^2+(16-15y^2)\zeta^4]\ln(u_1)-\frac{8}{3}\zeta^2[3(1-y^2)^2(1-5y^2)-(5-16y^2+15y^4)\zeta^2]\ln(u_2)\Big\}$$

$$\hat{\Gamma}_{L^P}^- = (1+\epsilon_b)\Big\{\frac{2}{3}(1-y^2)[(1-y)^2-\zeta^2][(1-y)^2(1+4y+y^2)+10(1+y+y^2)\zeta^2+\zeta^4]$$
$$-4(1-y^2)\zeta^2[(1-y^2)^2+(1+y^2)\zeta^2]\ln(u_3)\Big\}-K_b\{\frac{2}{3}(1-y^2)[(1-y)^2-\zeta^2]$$
$$\times[(1-y)^2(5+4y+y^2)-2(1-5y-5y^2)\zeta^2+\zeta^4]-\frac{4}{3}(1-y^2)\zeta^2[3(1-y^2)^2-(1-3y^2)\zeta^2]\ln(u_3)\Big\}$$

$$\hat{\Gamma}_F^- = -\frac{1}{3}[(1-y)^2-\zeta^2][(1-y)^4(1+6y+y^2)$$
$$-(1-y)^2(7+26y+7y^2)\zeta^2-(7+2y+7y^2)\zeta^4+\zeta^6]-4(1-y^2)^2\zeta^4\ln(u_3)$$
$$+K_b\Big\{\frac{1}{9}[(1-y)^2-\zeta^2][(1-y)^4(35+18y+3y^2)-(1-y)^2(85+78y+21y^2)\zeta^2+(11-6y-21y^2)\zeta^4+3\zeta^6]$$
$$+4(1-y^2)^2\zeta^4\ln(u_3)\Big\}$$
$$+G_b\Big\{-\frac{1}{9}[(1-y)^2-\zeta^2][(1-y)^3(65+133y+75y^2+15y^3)-(1-y)(199+275y+285y^2+105y^3)\zeta^2$$
$$+(41+30y+105y^2)\zeta^4-15\zeta^6]-4(1-y^2)(3+5y^2)\zeta^4\ln(u_3)\Big\}$$

$$\hat{\Gamma}_{F^P}^- = (1+\epsilon_b)\Big\{\frac{1}{3}\sqrt{R}[(1+y^2)(1-8y^2+y^4)-(7-12y^2+7y^4)\zeta^2-7(1+y^2)\zeta^4+\zeta^6]-8y^4(1-\zeta^4)\ln(u_1)$$
$$-8\zeta^4(1-y^4)\ln(u_2)\Big\}$$
$$+K_b\Big\{\frac{1}{9}\sqrt{R}[13+181y^2+37y^4-3y^6-(59+116y^2-21y^4)\zeta^2-(11-21y^2)\zeta^4-3\zeta^6]$$
$$+\frac{8}{3}y^2[8+11y^2-16\zeta^2+(8-3y^2)\zeta^4]\ln(u_1)-\frac{8}{3}\zeta^4(1-y^2)(5-3y^2)\ln(u_2)\Big\}$$

$$\hat{\Gamma}_{I^P}^- = -(1+\epsilon_b)\frac{16\sqrt{2}}{105}(1-y-\zeta)^4[(1-y)(1+5y+y^2)+4(1+5y+y^2)\zeta-4(1-y)\zeta^2-\zeta^3]$$
$$+K_b\frac{16\sqrt{2}}{315}(1-y-\zeta)^3[(1-y)^2(19-10y-2y^2)+3(1-y)(19-10y-2y^2)\zeta+8(9+3y+2y^2)\zeta^2-6(1-y)\zeta^3-2\zeta^4]$$

$$\hat{\Gamma}^{-}_{A^P} = (1+\epsilon_b)\left\{\frac{8\sqrt{2}}{105}\zeta\sqrt{R}[2-13y^2-5y^4+11(4-3y^2)\zeta^2+2\zeta^4]+\frac{8\sqrt{2}}{105}(1+y)[2-17y^2-108y^4-5y^6\right.$$
$$-14(2-7y^2-3y^4)\zeta^2-35(2-y^2)\zeta^4][\mathrm{E}(k^2)-\mathrm{E}(\varphi,k^2)]-\frac{16\sqrt{2}}{105}y[(1+y)(2-3y-13y^2-45y^3-5y^4)$$
$$\left.-14(1+y)(2-3y-3y^2)\zeta^2-35(2-y)\zeta^4][\mathrm{K}(k^2)-\mathrm{F}(\varphi,k^2)]\right\}$$
$$-K_b\left\{\frac{16\sqrt{2}}{315}\zeta\sqrt{R}[5+13y^2-2y^4+11(3-4y^2)\zeta^2-2\zeta^4]+\frac{16\sqrt{2}}{315}(1+y)[5+108y^2+17y^4-2y^6\right.$$
$$-14(3+7y^2-2y^4)\zeta^2-35(1-2y^2)\zeta^4][\mathrm{E}(k^2)-\mathrm{E}(\varphi,k^2)]-\frac{32\sqrt{2}}{315}y[(1+y)(5+45y+13y^2+3y^3-2y^4)$$
$$\left.-14(1+y)(3+3y-2y^2)\zeta^2-35(1-2y)\zeta^4][\mathrm{K}(k^2)-\mathrm{F}(\varphi,k^2)]\right\}. \tag{A3}$$

The integrated rates into positive helicity leptons read

$$\hat{\Gamma}^{+}_{U}=\frac{2}{3}\sqrt{R}\zeta^2[(1-y^2)^2+10(1+y^2)\zeta^2+\zeta^4]-8\frac{y^4\zeta^4}{1-y^2}(1-y^2-\zeta^2)\ln(u_1)+8\frac{\zeta^4}{1-y^2}[(1-y^2)^2(1+y^2)+(1+y^4)\zeta^2]\ln(u_2)$$
$$+K_b\left\{\frac{2}{3}\sqrt{R}\zeta^2[3+6y^2-y^4+10(1-y^2)\zeta^2-\zeta^4]+\frac{8}{3}\frac{y^2\zeta^2}{1-y^2}[4(1-y^2)-(1-y^2)(8-3y^2)\zeta^2+(4-3y^2)\zeta^4]\ln(u_1)\right.$$
$$\left.+\frac{8}{3}\frac{\zeta^4}{1-y^2}[(1-y^2)^2(5-3y^2)+(1+4y^2-3y^4)\zeta^2]\ln(u_2)\right\}$$
$$+G_b\left\{\frac{2}{3}\sqrt{R}\frac{\zeta^2}{1-y^2}[(1-y^2)(7+6y^2-5y^4)-2(3+14y^2-25y^4)\zeta^2-5(1-y^2)\zeta^4]\right.$$
$$+\frac{8}{3}\frac{y^2\zeta^2}{(1-y^2)^2}[4(1-y^2)^2-(1-y^2)^2(4-15y^2)\zeta^2-y^2(23-15y^2)\zeta^4]\ln(u_1)$$
$$\left.+\frac{8}{3}\frac{\zeta^4}{(1-y^2)^2}[(1-y^2)^2(5-4y^2+15y^4)-(7+y^2+23y^4-15y^6)\zeta^2]\ln(u_2)\right\}$$

$$\hat{\Gamma}^{+}_{U^P}=(1+\epsilon_b)\left\{\frac{2}{3}\zeta^2[\zeta^2-(1-y)^2][(1-y)^2(1+4y+y^2)+10(1+y+y^2)\zeta^2+\zeta^4]+4\zeta^4[(1-y^2)^2+(1+y^2)\zeta^2]\ln(u_3)\right\}$$
$$-K_b\left\{\frac{2}{3}\zeta^2[\zeta^2-(1-y)^2][(1-y)^2(5+4y+y^2)-2(1-5y-5y^2)\zeta^2+\zeta^4]\right.$$
$$\left.+\frac{4}{3}\zeta^4[3(1-y^2)^2-(1-3y^2)\zeta^2]\ln(u_3)\right\}$$

$$\hat{\Gamma}^{+}_{L}=-\sqrt{R}\zeta^2[3-4y^2+3y^4+3(1+y^2)\zeta^2]-2\frac{y^4\zeta^2}{1-y^2}[(1-y^2)(3-y^2)-4(1-y^2)\zeta^2+\zeta^4]\ln(u_1)$$
$$-2\frac{\zeta^2}{1-y^2}[(1-y^2)^4+4(1-y^2)^2(1+y^2)\zeta^2+(1+y^4)\zeta^4]\ln(u_2)$$
$$+K_b\left\{\frac{1}{3}\sqrt{R}\zeta^2[1-20y^2+9y^4-(31-9y^2)\zeta^2]-\frac{2}{3}\frac{y^2\zeta^2}{1-y^2}[(1-y^2)(16-9y^2+3y^4)-4(1-y^2)(8-3y^2)\zeta^2\right.$$
$$\left.+(16-3y^2)\zeta^4]\ln(u_1)+\frac{2}{3}\frac{\zeta^2}{1-y^2}[3(1-y^2)^4-4(1-y^2)^2(5-3y^2)\zeta^2-(13+16y^2-3y^4)\zeta^4]\ln(u_2)\right\}$$
$$+G_b\left\{\frac{1}{3}\sqrt{R}\frac{\zeta^2}{1-y^2}[(1-y^2)(3-58y^2+45y^4)-(37-86y^2+45y^4)\zeta^2]\right.$$

$$-\frac{2}{3}\frac{y^2\zeta^2}{(1-y^2)^2}[(1-y^2)^2(28-33y^2+15y^4)-4(1-y^2)^2(16-15y^2)\zeta^2+(36-47y^2+15y^4)\zeta^4]\ln(u_1)$$

$$+\frac{2}{3}\frac{\zeta^2}{(1-y^2)^2}[3(1-y^2)^4(1-5y^2)-4(1-y^2)^2(5-16y^2+15y^4)\zeta^2-(17+23y^2-47y^4+15y^6)\zeta^4]\ln(u_2)\bigg\}$$

$$\hat{\Gamma}^+_{L^P}=(1+\epsilon_b)\bigg\{\zeta^2\frac{1+y}{1-y}[\zeta^2-(1-y)^2][(1-y)^2(3+4y+3y^2)+(3-4y+3y^2)\zeta^2]$$

$$+(1-y^2)\zeta^2[(1-y^2)^2+4(1+y^2)\zeta^2+\zeta^4]\ln(u_3)\bigg\}$$

$$-K_b\bigg\{\frac{1}{3}\zeta^2\frac{1+y}{1-y}[\zeta^2-(1-y)^2][(1-y)^2(1+12y+9y^2)+(1-12y+9y^2)\zeta^2]$$

$$+\frac{1}{3}(1-y^2)\zeta^2[3(1-y^2)^2-4(1-3y^2)\zeta^2+3\zeta^4]\ln(u_3)\bigg\}$$

$$\hat{\Gamma}^+_S=-\sqrt{R}\zeta^2[3-4y^2+3y^4+3(1+y^2)\zeta^2]-2\frac{y^4\zeta^2}{1-y^2}[(1-y^2)(3-y^2)-4(1-y^2)\zeta^2+\zeta^4]\ln(u_1)$$

$$-2\frac{\zeta^2}{1-y^2}[(1-y^2)^4+4(1-y^2)^2(1+y^2)\zeta^2+(1+y^4)\zeta^4]\ln(u_2)$$

$$+K_b\bigg\{\sqrt{R}\zeta^2[3-4y^2+3y^4+3(1+y^2)\zeta^2]+2\frac{y^4\zeta^2}{1-y^2}[(1-y^2)(3-y^2)-4(1-y^2)\zeta^2+\zeta^4]\ln(u_1)$$

$$+2\frac{\zeta^2}{1-y^2}[(1-y^2)^4+4(1-y^2)^2(1+y^2)\zeta^2+(1+y^4)\zeta^4]\ln(u_2)\bigg\}$$

$$+G_b\bigg\{\sqrt{R}\frac{\zeta^2}{1-y^2}[(1-y^2)(1-14y^2+15y^4)+(9+2y^2-15y^4)\zeta^2]-2\frac{y^2\zeta^2}{(1-y^2)^2}[(1-y^2)^2(4-11y^2+5y^4)$$

$$+20y^2(1-y^2)^2\zeta^2-(4+5y^2-5y^4)\zeta^4]\ln(u_1)+2\frac{\zeta^2}{(1-y^2)^2}[(1-y^2)^4(1-5y^2)$$

$$+4(1-y^2)^2(1-5y^4)\zeta^2+(5+3y^2+5y^4-5y^6)\zeta^4]\ln(u_2)\bigg\}$$

$$\hat{\Gamma}^+_{S^P}=(1+\epsilon_b)\bigg\{\zeta^2\frac{1+y}{1-y}[\zeta^2-(1-y)^2][(1-y)^2(3+4y+3y^2)+(3-4y+3y^2)\zeta^2]+\zeta^2(1-y^2)[(1-y^2)^2$$

$$+4(1+y^2)\zeta^2+\zeta^4]\ln(u_3)\bigg\}-K_b\bigg\{\frac{1}{3}\zeta^2\frac{1+y}{1-y}[\zeta^2-(1-y)^2][(1-y)^2(1+12y+9y^2)+(1-12y+9y^2)\zeta^2]$$

$$+\frac{1}{3}\zeta^2(1-y^2)[3(1-y^2)^2-4(1-3y^2)\zeta^2+3\zeta^4]\ln(u_3)\bigg\}$$

$$\hat{\Gamma}^+_{SL}=\zeta^2\frac{1+y}{1-y}[\zeta^2-(1-y)^2][(1-y)^2(3+4y+3y^2)+(3-4y+3y^2)\zeta^2]+\zeta^2(1-y^2)[(1-y^2)^2+4(1+y^2)\zeta^2+\zeta^4]\ln(u_3)$$

$$+K_b\bigg\{-\frac{1}{3}\zeta^2\frac{1+y}{1-y}[\zeta^2-(1-y)^2][(1-y)^2(1+12y+9y^2)+(1-12y+9y^2)\zeta^2]$$

$$-\frac{1}{3}\zeta^2(1-y^2)[3(1-y^2)^2-4(1-3y^2)\zeta^2+3\zeta^4]\ln(u_3)\bigg\}$$

$$+G_b\bigg\{-\frac{1}{3}\zeta^2\frac{1-5y^2}{(1-y)^2}[\zeta^2-(1-y)^2][(1-y)^2(1+12y+9y^2)+(1-12y+9y^2)\zeta^2]$$

$$-\frac{1}{3}\zeta^2(1-5y^2)[3(1-y^2)^2-4(1-3y^2)\zeta^2+3\zeta^4]\ln(u_3)\bigg\}$$

$$\hat{\Gamma}^{+}_{SL^P} = -(1+\epsilon_b)\Big\{\sqrt{R}\zeta^2[3-4y^2+3y^4+3(1+y^2)\zeta^2] + 2\frac{y^4\zeta^2}{1-y^2}[(1-y^2)(3-y^2)-4(1-y^2)\zeta^2+\zeta^4]\ln(u_1)$$
$$+ 2\frac{\zeta^2}{1-y^2}[(1-y^2)^4+4(1-y^2)^2(1+y^2)\zeta^2+(1+y^4)\zeta^4]\ln(u_2)\Big\}$$
$$+ K_b\Big\{\frac{1}{3}\sqrt{R}\zeta^2[5-16y^2+9y^4-(11-9y^2)\zeta^2] - \frac{2}{3}\frac{y^2\zeta^2}{1-y^2}[(1-y^2)(8-9y^2+3y^4)$$
$$-4(4-7y^2+3y^4)\zeta^2+(8-3y^2)\zeta^4]\ln(u_1 + \frac{2}{3}\frac{\zeta^2}{1-y^2}[3(1-y^2)^4-4(1-y^2)^2(1-3y^2)\zeta^2$$
$$-(5+8y^2-3y^4)\zeta^4]\ln(u_2)\Big\}$$
$$\hat{\Gamma}^{+}_{ST^P} = (1+\epsilon_b)\Big\{\frac{4\sqrt{2}}{15}\sqrt{R}\frac{\zeta^3}{1-y^2}[(1-y^2)(8-7y^2)+(8-3y^2)\zeta^2] - \frac{4\sqrt{2}}{15}\frac{\zeta^2}{1-y}[(1-y^2)(2-7y^2-3y^4)$$
$$+10(2-3y^2+y^4)\zeta^2+5(2-y^2)\zeta^4][\mathrm{E}(k^2)-\mathrm{E}(\varphi,k^2)] + \frac{8\sqrt{2}}{15}\frac{y\zeta^2}{1-y^2}[(1-y^2)(1+y)(2-3y-3y^2)$$
$$+10(1-y^2)(2-y)\zeta^2+5(2-y)\zeta^4][\mathrm{K}(k^2)-\mathrm{F}(\varphi,k^2)]\Big\}$$
$$+ K_b\Big\{\frac{8\sqrt{2}}{45}\sqrt{R}\frac{\zeta^3}{1-y^2}[(1-y^2)(7-8y^2)-(3-8y^2)\zeta^2]$$
$$-\frac{8\sqrt{2}}{45}\frac{\zeta^2}{1-y}[(1-y^2)(3+7y^2-2y^4)+10(1-y^2)(1-2y^2)\zeta^2-5(1-2y^2)\zeta^4][\mathrm{E}(k^2)-\mathrm{E}(\varphi,k^2)]$$
$$+\frac{16\sqrt{2}}{45}\frac{y\zeta^2}{1-y^2}[(1-y^2)(1+y)(3+3y-2y^2)+10(1-y^2)(1-2y)\zeta^2-5(1-2y)\zeta^4][\mathrm{K}(k^2)-\mathrm{F}(\varphi,k^2)]\Big\}$$
$$\hat{\Gamma}^{+}_{I^P} = (1+\epsilon_b)\frac{8\sqrt{2}}{15}\frac{\zeta^2}{1-y}(1-y-\zeta)^4[1+3y+y^2-(1-y)\zeta] - K_b\frac{8\sqrt{2}}{45}\frac{\zeta^2}{1-y}(1-y-\zeta)^3$$
$$\times[(1-y)(3-6y-2y^2)+(19+2y+4y^2)\zeta-2(1-y)\zeta^2] \tag{A4}$$

[for K_b, ϵ_b, and G_b cf. again Eqs. (68)–(70)]. The function R is defined by

$$R = \lambda(1, y^2, \zeta^2) = 1 + y^4 + \zeta^4 - 2(y^2 + \zeta^2 + y^2\zeta^2). \tag{A5}$$

In the analytical expressions for the polarized rates $\hat{\Gamma}^{-}_{I^P}$ and $\hat{\Gamma}^{+}_{ST^P}$ occur the elliptical integrals E, F, and K which are defined by

$$\mathrm{E}(\varphi,k^2) = \int_0^{\varphi}\sqrt{1-k^2\sin^2 t}\,dt, \qquad \mathrm{E}(k^2) = \mathrm{E}\left(\frac{\pi}{2},k^2\right)$$
$$\mathrm{F}(\varphi,k^2) = \int_0^{\varphi}\frac{dt}{\sqrt{1-k^2\sin^2 t}}, \qquad \mathrm{K}(k^2) = \mathrm{F}\left(\frac{\pi}{2},k^2\right). \tag{A6}$$

The argument φ and the parameter k of the elliptical integrals read

$$k = \frac{1-y}{1+y}, \qquad \varphi = \arcsin\left(\frac{\zeta}{1-y}\right). \tag{A7}$$

[1] S. Chatrchyan *et al.* (CMS Collaboration), Measurements of $t\bar{t}$ Spin Correlations and Top-Quark Polarization Using Dilepton Final States in pp Collisions at $\sqrt{s} = 7$ TeV, Phys. Rev. Lett. **112**, 182001 (2014).

[2] M. Aaboud *et al.* (ATLAS Collaboration), Measurements of top quark spin observables in $t\bar{t}$ events using dilepton final states in $\sqrt{s} = 8$ TeV pp collisions with the ATLAS detector, J. High Energy Phys. 03 (2017) 113.

[3] G. Mahlon, Spin polarization in single top events, arXiv: hep-ph/9811219.

[4] D. Espriu and J. Manzano, A study of top polarization in single top production at the CERN LHC, Phys. Rev. D **66**, 114009 (2002).

[5] S. D. Rindani, P. Sharma, and A. W. Thomas, Polarization of the top quark as a probe of its chromomagnetic and chromoelectric couplings in single-top production at the Large Hadron Collider, *Proc. Sci.*, TOP2015 (2016) 063 [arXiv:1510.08959].

[6] V. Khachatryan *et al.* (CMS Collaboration), Measurement of Top Quark Polarisation in T-Channel Single Top Quark Production, J. High Energy Phys. 04 (2016) 073.

[7] M. Aaboud *et al.* (ATLAS Collaboration), Probing the W tb vertex structure in t-channel single-top-quark production and decay in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, J. High Energy Phys. 04 (2017) 124.

[8] J. Drobnak, S. Fajfer, and J. F. Kamenik, New physics in $t \to bW$ decay at next-to-leading order in QCD, Phys. Rev. D **82**, 114008 (2010).

[9] S. D. Rindani and P. Sharma, Probing anomalous tbW couplings in single-top production using top polarization at the Large Hadron Collider, J. High Energy Phys. 11 (2011) 082.

[10] Z. J. Ajaltouni and E. Di Salvo, Polarization in (quasi-)two-body decays and new physics, Eur. Phys. J. C **79**, 989 (2019); Erratum, Eur. Phys. J. C **81**, 487 (2021).

[11] H. S. Do, S. Groote, J. G. Körner, and M. C. Mauser, Electroweak and finite width corrections to top quark decays into transverse and longitudinal W bosons, Phys. Rev. D **67**, 091501(R) (2003).

[12] S. Groote and J. G. Körner, Bread and butter physics: NLO electroweak corrections to polarized top quark decays, J. Phys. Conf. Ser. **938**, 012053 (2017).

[13] M. Fischer, S. Groote, J. G. Körner, M. C. Mauser, and B. Lampe, Polarized top decay into polarized W: $t(\uparrow) \to W(\uparrow) + b$ at $O(\alpha_s)$, Phys. Lett. B **451**, 406 (1999).

[14] M. Fischer, S. Groote, J. G. Körner, and M. C. Mauser, Complete angular analysis of polarized top decay at $O(\alpha_s)$, Phys. Rev. D **65**, 054036 (2002).

[15] A. Czarnecki, S. Groote, J. G. Körner, and J. H. Piclum, NNLO QCD corrections to the polarized top quark decay $t(\uparrow) \to X_b + W^+$, Phys. Rev. D **97**, 094008 (2018).

[16] J. H. Kühn and P. M. Zerwas, Heavy flavors in high-energy electron positron collisions, Adv. Ser. Dir. High Energy Phys. **10**, 434 (1992).

[17] M. Galanti, A. Giammanco, Y. Grossman, Y. Kats, E. Stamou, and J. Zupan, Heavy baryons as polarimeters at colliders, J. High Energy Phys. 11 (2015) 067.

[18] Y. Kats, Measuring polarization of light quarks at ATLAS and CMS, Phys. Rev. D **92**, 071503(R) (2015).

[19] Y. Kats, Measuring c-quark polarization in $\mathrm{W} + \mathrm{c}$ samples at ATLAS and CMS, J. High Energy Phys. 11 (2016) 011.

[20] Y. Kats, Measuring quark polarizations at ATLAS and CMS, Frascati Phys. Ser. **65**, 120 (2017), http://www.lnf.infn.it/sis/frascatiseries/Volume65/Volume65.pdf.

[21] F. E. Close, J. G. Körner, R. J. N. Phillips, and D. J. Summers, Heavy quark theory and b polarization at LEP, Report No. RAL–92-016; A. H. Ball *et al.*, Report of the b Fragmentation Working Group, J. Phys. G **18**, 1703 (1992).

[22] A. F. Falk and M. E. Peskin, Production, decay, and polarization of excited heavy hadrons, Phys. Rev. D **49**, 3320 (1994).

[23] Y. S. Tsai, Leptonic decay of heavy leptons, Phys. Rev. D **19**, 2809 (1979).

[24] A. Czarnecki, M. Ježabek, J. G. Körner, and J. H. Kühn, QCD Corrections to Decays of Polarized Charm and Bottom Quarks, Phys. Rev. Lett. **73**, 384 (1994).

[25] N. Cabibbo and L. Maiani, The lifetime of charmed particles, Phys. Lett. **79B**, 109 (1978).

[26] A. Ali and E. Pietarinen, Semileptonic decays of heavy quarks in quantum chromodynamics, Nucl. Phys. **B154**, 519 (1979).

[27] N. Cabibbo, G. Corbo, and L. Maiani, Lepton spectrum in semileptonic charm decay, Nucl. Phys. **B155**, 93 (1979).

[28] G. Corbo, Lepton spectrum in semileptonic b decay, Phys. Lett. B **116B**, 298 (1982).

[29] G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani, and G. Martinelli, Leptonic decay of heavy flavors: A theoretical update, Nucl. Phys. **B208**, 365 (1982).

[30] G. Corbo, Semileptonic b quark decay in quantum chromodynamics, Nucl. Phys. **B212**, 99 (1983).

[31] M. Ježabek and J. H. Kühn, Lepton spectra from heavy quark decay, Nucl. Phys. **B320**, 20 (1989).

[32] M. Ježabek and J. H. Kühn, Semileptonic decays of top quarks, Phys. Lett. B **207**, 91 (1988).

[33] A. V. Manohar and M. B. Wise, Inclusive semileptonic B and polarized Λ_b decays from QCD, Phys. Rev. D **49**, 1310 (1994).

[34] A. Ali, J. G. Körner, G. Kramer, and J. Willrodt, Final states in nonleptonic bottom meson decays, Z. Phys. C **2**, 33 (1979).

[35] J. G. Körner and G. A. Schuler, Exclusive semileptonic heavy meson decays including lepton mass effects, Z. Phys. C **46**, 93 (1990).

[36] S. Balk, J. G. Körner, D. Pirjol, and K. Schilcher, Inclusive semileptonic B decays in QCD including lepton mass effects, Z. Phys. C **64**, 37 (1994).

[37] B. König, J. G. Körner, M. Krämer, and P. Kroll, Infinite momentum frame calculation of semileptonic heavy $\Lambda_b \to \Lambda_c$ transitions including HQET improvements, Phys. Rev. D **56**, 4282 (1997).

[38] J. G. Körner and D. Pirjol, On the $1/m^2$ corrections to the form-factors in $\Lambda_b \to \Lambda_c e \bar{\nu}_e$ decays, Phys. Lett. B **334**, 399 (1994).

[39] J. G. Körner, Λ_b polarization from its inclusive semileptonic decay, Nucl. Phys. B, Proc. Suppl. **50**, 130 (1996).

[40] C. Diaconu, M. Talby, J. G. Körner, and D. Pirjol, Improved variables for measuring the Λ_b polarization, Phys. Rev. D **53**, 6186 (1996).

[41] S. Balk, J. G. Körner, and D. Pirjol, Inclusive semileptonic decays of polarized Λ_b baryons into polarized tau leptons, Eur. Phys. J. C **1**, 221 (1998).
[42] J. G. Körner and D. Pirjol, Spin momentum correlations in inclusive semileptonic decays of polarized Λ_b baryons, Phys. Rev. D **60**, 014021 (1999).
[43] J. G. Körner and B. Melić, Exclusive inclusive ratio of semileptonic Λ_b decays, Phys. Rev. D **62**, 074008 (2000).
[44] J. G. Körner and B. Melić, Exclusive and inclusive semileptonic Λ_b decays, Nucl. Phys. B, Proc. Suppl. **93**, 13 (2001).
[45] B. Blok, L. Koyrakh, M. Shifman, and A. I. Vainshtein, Differential distributions in semileptonic decays of the heavy flavors in QCD, Phys. Rev. D **49**, 3356 (1994).
[46] P. Colangelo, F. De Fazio, and F. Loparco, Inclusive semileptonic Λ_b decays in the Standard Model and beyond, J. High Energy Phys. 11 (2020) 032.