

Violation of custodial symmetry from W -boson mass measurements

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The new measurement of the W -boson mass from the CDF Collaboration shows a significant tension with the Standard Model prediction. We quantify this discrepancy within a state-of-the-art analysis of electroweak precision data and scrutinize the leading deformations of the Standard Model effective field theory arising at dimension six. We find evidence for a nonzero value of the T parameter, i.e., for a novel source of violation of custodial symmetry, pointing to physics beyond the Standard Model at the 4.5σ level. We contextualize the implications of our findings in light of other present anomalies in particle physics.

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I. INTRODUCTION

The CDF Collaboration has recently delivered a very precise measurement of the W -boson mass, with $M_W = 80.434 \pm 0.009$ GeV, reaching the level of 0.01% precision [1] superseding the previous CDF measurement [2]. Such a phenomenal result provides an unprecedented probe of the underlying dynamics in the two-point function of the $SU(2)_L$ gauge-boson field, following up on recent dedicated studies from ATLAS, D0, and LHCb Collaborations [3–5]. At a first glance, the Standard Model (SM) expectation of $M_W = 80.357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}}$ GeV [6] establishes a tension of about 7σ with the new CDF II measurement, with the latter leaving room for a possible indirect imprint of new physics (NP) beyond the SM.

Decades of theoretical development have brought about very precise computations of the electroweak (EW) precision observables (EWPO) which constitute a fundamental test bed for the standard theory, yielding powerful constraints in many scenarios of physics beyond the SM (BSM) [7–37]. Of particular significance, the misalignment between charged and neutral EW boson masses offers a deep insight into the SM theory and its possible extensions, being a remnant of the $SU(2)_L \times SU(2)_R$ custodial

symmetry present in the Higgs sector, but broken in the SM by hypercharge and Yukawa couplings [38].

Putting the above theoretical considerations next to the very accurate measurement of the Z -boson mass of the LEP precision program, it should be clear that any relevant experimental progress on the determination of the W -boson mass may represent a pillar for advancement in the field. In this work, motivated by such an opportunity, we explore some of the most evident implications of the recent CDF II measurement of M_W . We will focus, in particular, on how the prediction of M_W gets affected in SM extensions where a NP mass gap above the EW scale is present, and $SU(2)_L \times U(1)_Y$ symmetry can be linearly realized once heavy new dynamics is integrated out. Any source of BSM physics of this sort should be described by means of the widely studied Standard Model effective field theory (SMEFT); see, e.g., [39–42]. Within the simplifying assumption of $U(3)^5$ flavor universality for the NP effects under scrutiny, in the following, we make an attempt to learn about possible new sources of custodial-symmetry breaking supported by current data [43].

In Sec. II, we characterize in greater detail our study of the EW sector in the SMEFT; in Sec. III, we describe the strategy of our analysis; in Sec. IV, we detail our most important findings, reporting evidence for a new source of custodial-symmetry breaking at the level of 4.5σ ; in Sec. V, we briefly discuss some implications regarding other ongoing anomalies in particle physics and draw our conclusions.

II. W -BOSON MASS AT DIMENSION SIX

It is a well-known fact that the leading deformations of the SM EW sector can be encoded in ten SMEFT operators

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at dimension six. However, only eight independent directions can be constrained using the EWPO; see, e.g., [40,46]. Nevertheless, in the Warsaw basis [47], the independent SMEFT operators that explicitly contribute to the W -boson mass are [48]

$$\begin{aligned}
 \mathcal{O}_{HWB} &= H^\dagger \sigma^a H W_{\mu\nu}^a B^{\mu\nu}, \\
 \mathcal{O}_{HD} &= (H^\dagger D^\mu H)^* (H^\dagger D_\mu H), \\
 \mathcal{O}_{LL} &= (\bar{L} \gamma^\mu L) (\bar{L} \gamma_\mu L), \\
 \mathcal{O}_{HL}^{(3)} &= (H^\dagger i \overleftrightarrow{D}_\mu^a H) (\bar{L} \gamma^\mu \sigma^a L). \quad (1)
 \end{aligned}$$

Indeed, at the linear level in NP effects, the relative shift $\delta M_W^2 / M_W^2$ in the SMEFT at dimension six is proportional to the combination [49]

$$4C_{HWB} + \cot \theta_W C_{HD} + 2 \tan \theta_W (2C_{HL}^{(3)} - C_{LL}), \quad (2)$$

with θ_W being the tree-level weak-mixing angle. Given the focus of the present paper on the effects of the SMEFT operators (described in the Warsaw basis [47]) that enter in Eq. (2), we can restrict here the discussion to four operators of Eq. (1), and their Wilson coefficients, C_{HWB} , C_{HD} , C_{LL} and $C_{HL}^{(3)}$, assuming lepton universality, all of which can be constrained by EWPO data.

It should be noted that C_{HWB} and C_{HD} also modify the Z -boson mass while all four of these coefficients affect $\sin^2 \theta_W$ and the width of the W boson, while gauge couplings are affected by C_{HWB} and $C_{HL}^{(3)}$ [48]. Hence, any modification in the W mass will eventually be correlated with several other EWPO, highlighting the importance of a global analysis. In the following, we will focus our attention on any deviation from the SM sourced only by these four operators and assume that NP will not generate other dimension-six operators that may also characterize the study of precision measurements of Z , W boson couplings to leptons and quarks; see for instance [50].

The focus on the set of operators in Eq. (2) provides a concrete practical ground to explore oblique NP contributions, namely, NP effects affecting the vacuum-polarization diagrams of EW gauge bosons, typically described in terms of S , T and U parameters [45,51] (or, equivalently, in terms of ϵ_1 , ϵ_2 and ϵ_3 [52–54]). Note that the SMEFT encodes the broad class of theories where U receives corrections at dimension eight or higher [55]: To the level of precision of the present analysis, $U = 0$. On the other hand, the commonly noted S , T parameters at dimension six in the SMEFT correspond to

$$\alpha S = \frac{2 \sin 2\theta_W}{\sqrt{2} G_F} C_{HWB}, \quad \alpha T = -\frac{1}{2\sqrt{2} G_F} C_{HD}. \quad (3)$$

Therefore, in our analysis, new sources of custodial-symmetry violation will be potentially spotted by a nonzero

TABLE I. Values of key input parameters used in our EW fits. For M_Z , m_H and m_{top} , PDG values are adopted [6]. Note, however, the inflated uncertainty for m_{top} : It reflects a conservative choice along the lines of what is well motivated, e.g., in Ref. [57]. Values of $\alpha_s(M_Z)$ and $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$, as well as some minor inputs controlling intrinsic theory uncertainties, are gathered from Ref. [57]. Values of $G_F = 1.1663787 \times 10^{-5}$ and $\alpha(0) = 1/137.035999139$ are used as fixed parameters given the precision to which they are measured [6].

Main inputs	Mean \pm std
$\alpha_s(M_Z)$	0.1177 ± 0.0010
$\Delta\alpha_{\text{had}}^{(5)}(M_Z)$	0.02766 ± 0.00010
m_{top} [GeV]	172.58 ± 1.00
m_H [GeV]	125.25 ± 0.17
M_Z [GeV]	91.1876 ± 0.0021

inference of C_{HD} . The considerations above shape the strategy for the fits, we propose in the next section to study the new W -mass measurement.

III. MODUS OPERANDI

The core input parameters of our analysis are reported in Table I. We adopt Gaussian priors for the Z -boson mass M_Z , the top-quark mass m_t , the strong coupling constant $\alpha_s(M_Z)$, and the hadronic-loop contribution to the electromagnetic coupling constant, $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$. In our analysis, we fix the values of G_F and $\alpha_{\text{em}}(0)$ since their measurements are well beyond per-mille-level precision. As for the W -boson mass, we use two different reference values in this work:

$$\begin{aligned}
 M_W^{2021} &= 80.379 \pm 0.012 \text{ GeV}, \\
 M_W^{2022} &= 80.4060 \pm 0.0075 \text{ GeV}, \quad (4)
 \end{aligned}$$

where 2021 refers to the previous global average [6] before the current CDF II measurement and 2022 refers to our naive combination of the 2021 global average and the new CDF II measurement after removing the previous CDF II measurement. This is done since the current CDF II measurement subsumes all previous CDF II measurements [1] as all the data used in the previous analyses [2,56] are also included in the current CDF II measurement.

Using HEPfit [58], we perform a Bayesian fit of EWPO data, the values for which can be found in [57], for the SM and in several NP scenarios characterized by different combinations of nonvanishing Wilson coefficients. To perform model comparison of different scenarios, we compute the information criterion (IC) [59]:

$$\text{IC} \equiv -2\overline{\log \mathcal{L}} + 4\sigma_{\log \mathcal{L}}^2, \quad (5)$$

where the first and second terms represent the mean and variance of the log-likelihood posterior distribution,

TABLE II. The ICs of the various fit scenarios. A lower number indicates a better fit to data. The details of the implications of these numbers are discussed in the text. All values for the Wilson coefficients are expressed in TeV^{-2} . A significance of NP greater than 4.5σ is seen in the single-Wilson-coefficient fit with C_{HD} .

Scenario	Mean \pm error 2021	IC ²⁰²¹	Mean \pm error 2022	IC ²⁰²²
SM	...	22	...	52
Single-Wilson-coefficient fits				
C_{HWB}	$(-3.6 \pm 2.6) \times 10^{-3}$	21	$(-8.5 \pm 2.3) \times 10^{-3}$	33
C_{HD}	$(-1.3 \pm 0.7) \times 10^{-2}$	20	$(-2.7 \pm 0.6) \times 10^{-2}$	23
C_{LL}	$(6.6 \pm 6.1) \times 10^{-3}$	22	$(1.5 \pm 0.6) \times 10^{-2}$	46
$C_{HL}^{(3)}$	$(-5.8 \pm 3.6) \times 10^{-3}$	20	$(1.2 \pm 0.3) \times 10^{-2}$	37
Two-Wilson-coefficient fit				
C_{HWB}	$(2.1 \pm 5.9) \times 10^{-3}$	22	$(6.6 \pm 5.8) \times 10^{-3}$	24
C_{HD}	$(-1.8 \pm 1.7) \times 10^{-2}$	22	$(-4.2 \pm 1.5) \times 10^{-2}$	24
Four-Wilson-coefficient fit				
C_{HWB}	$(1.4 \pm 7.5) \times 10^{-3}$		$(1.4 \pm 7.6) \times 10^{-3}$	
C_{HD}	$(-1.6 \pm 1.7) \times 10^{-2}$		$(-3.8 \pm 1.5) \times 10^{-2}$	
C_{LL}	$(-1.0 \pm 1.4) \times 10^{-2}$	27	$(-2.1 \pm 1.4) \times 10^{-2}$	27
$C_{HL}^{(3)}$	$(-6.4 \pm 7.3) \times 10^{-2}$		$(-6.4 \pm 7.3) \times 10^{-3}$	

respectively. The first term measures the quality of the fit, while the second one counts effective degrees of freedom and thus penalizes models with a larger number of parameters. Models with lower IC should be preferred according to the canonical scale of evidence of Ref. [60], related in this context to (positive) IC differences.

We explore several scenarios in this work, each for the two M_W averages quoted in Eq. (4) separately.

- (i) *SM*.—We perform an SM fit of the EWPO measurements and report the corresponding IC in Table II. We also predict the SM expectation of $M_W = 80.355 \pm 0.0008$ GeV that is shown with a black band in Fig. 1.
- (ii) *One-parameter fits*.—We isolate each of the four Wilson coefficients C_{HWB} , C_{HD} , C_{LL} and $C_{HL}^{(3)}$ and study the constraints of the new measurement on each of them separately.
- (iii) *Two-parameter fits*.—We perform two-parameter fits with C_{HWB} and C_{HD} to study the effects of solely oblique contributions reflected through the S and T parameters.
- (iv) *Four-parameter fits*.—We then take a look at four-parameter fits of C_{HWB} , C_{HD} , C_{LL} and $C_{HL}^{(3)}$ to gauge the constraints set by the new measurement on these Wilson coefficients when they are present simultaneously.
- (v) $\Delta\alpha_{had}^{(5)}(M_Z)$ study.—In the spirit of what was originally done in Ref. [61], to investigate any possible interesting effects in relation to the long-standing anomaly $(g-2)_\mu$, we perform fits where, we exclude the current measurements of $\Delta\alpha_{had}^{(5)}(M_Z)$ with

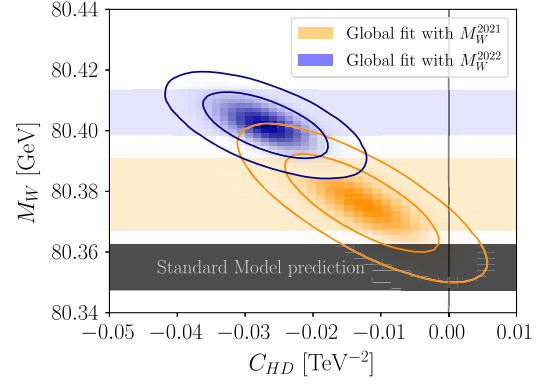


FIG. 1. A “before and after” plot showing the constraints from EWPO in the M_W vs C_{HD} plane. The orange and blue horizontal bands correspond to the global fits for M_W without the recent CDF II measurement (assuming the absence of NP contributions) and the current global fit including the recent measurement, respectively. The 68% (shaded region) and 95% probability contours highlight the NP contribution to the EWPO from \mathcal{O}_{HD} only. Remarkably, the new CDF II measurement yields a deviation of this NP Wilson coefficient from 0 at 4.5σ . The black band is the SM prediction obtained by using all other EWPO data in the fit except M_W .

an aim to compare the SM prediction for the hadron vacuum polarization contribution to α against the one predicted in the best-fit NP scenario found.

IV. LESSONS FROM THE SMEFT

We report the IC for several cases in Table II from which, we get a clear picture of what the effects of the new CDF II measurement are on the SM and NP scenarios.

- (i) *SM*.—The recent measurement of M_W has increased the tension between the data and the SM prediction. $\text{IC}_{\text{SM}}^{2022}$ is much higher than $\text{IC}_{\text{SM}}^{2021}$ showcasing this increase in discrepancy between the data and the model.
- (ii) *2021*.—If, we focus on $\text{IC}_{\text{NP}}^{2021}$, the addition of single NP Wilson coefficients did not improve the fit (barring small differences that are not significant) over a SM fit with M_W^{2021} . The same can be said for the two-parameter fit where, we see no improvement over the SM fit. The four-parameter fit with all four Wilson coefficients is actually worse than any of the fits because the fit does not improve and it gets penalized for having a larger number of parameters. All in all, the fits with M_W^{2021} do not show any hints of NP contributions within the framework, we consider.
- (iii) *2022*.—Now taking a look at the $\text{IC}_{\text{NP}}^{2022}$, we see a very different pattern. The fit with only C_{HD} performs much better than any other fits including NP contributions. In fact, it alleviates almost all the tension that has been generated by the new CDF II

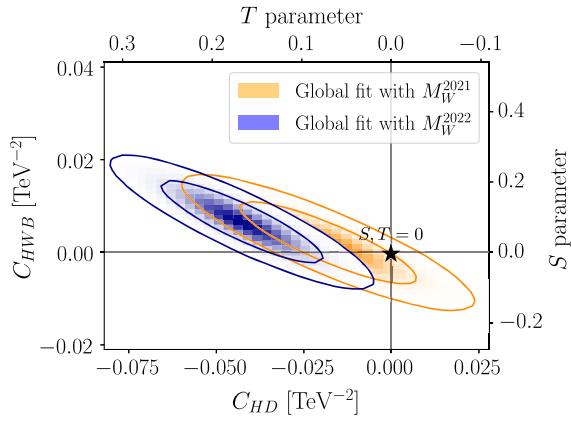


FIG. 2. The results of a two-parameter fit with the Wilson coefficients C_{HWB} and C_{HD} allowing for the encapsulation of NP effects that are purely oblique, namely, captured by the Peskin-Takeuchi parameters S and T [51] which are linearly dependent on C_{HWB} and C_{HD} at dimension six in the SMEFT. The 68% (shaded region) and 95% probability contours in orange and blue, respectively, correspond to fits excluding and including the recent CDF II measurement of the M_W mass, respectively. A clear hint of custodial-symmetry breaking can be seen from this plot with $T \neq 0$ at more than 3σ from the SM limit, marked with the dark star.

measurement of the W mass yielding an IC very close to IC_{SM}^{2021} . The other single-parameter fits perform much worse than they do with M_W^{2021} . The IC_{NP}^{2022} of the two-parameter fit with C_{HWB} and C_{HD} performs slightly worse than the single-parameter C_{HD} fit, presumably because of no improvement in the goodness of fit while being penalized for the increase in the number of parameters. The IC_{NP}^{2022} of the four-parameter fit is the same as the IC_{NP}^{2021} for the same fit showing no marked improvement.

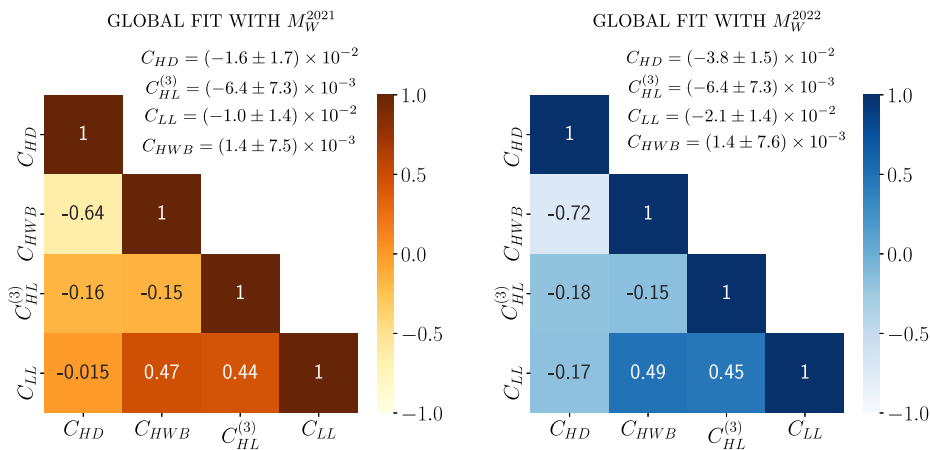


FIG. 3. Outcome of the four-parameter fit using C_{HWB} , C_{HD} , C_{LL} and $C_{HL}^{(3)}$ showing the mean and error of each coefficient from the fit along with the correlation matrix. The panel on the left corresponds to the global fit using the M_W value before the recent CDF II measurement and the panel on the right represents the global fit including the recent CDF II measurement. The Wilson coefficient most affected by the recent measurement is C_{HD} , which even in this multidimensional NP case remains nonzero at the 2.5σ level, with a higher central value and reduced error than before.

The most important message that, we wish to highlight in this work is the fact that the new CDF II measurement of W mass distinctly points at a hint of NP and preferably from a UV model that generates C_{HD} with a significance of greater than 4.5σ . The presence of the other Wilson coefficients does not significantly worsen the fit and an NP model that generates those can also be accommodated by the current EWPO data.

In Fig. 1, we show the constraints on C_{HD} and their correlation with the W -mass measurements. The orange band and curves correspond to global fits using M_W^{2021} in the SM and assuming the presence of NP manifested through C_{HD} , respectively. The blue band and curves are the same for the global fit using M_W^{2022} . While there are no significant hints of NP contribution when using M_W^{2021} , the significance of the same increases drastically with M_W^{2022} . The SM prediction is marked in dark.

In Fig. 2, we show the results of the two-parameter fit to motivate a discussion of hints of custodial-symmetry breaking, represented by a nonzero T parameter. The orange curves show the fit including M_W^{2021} and the blue ones the fit including M_W^{2022} . In the former case the point $S = T = 0$ is included in the 1σ (shaded) region while in the latter case it is significantly outside the 2σ region. The S and T values in the scenario with M_W^{2021} ,

$$S = 0.027 \pm 0.076, \quad T = 0.070 \pm 0.066 \quad (\rho = 0.898),$$

can be compared with the new ones adopting M_W^{2022} :

$$S = 0.086 \pm 0.076, \quad T = 0.167 \pm 0.059 \quad (\rho = 0.916),$$

with the latter set showing significant hints for sources of custodial-symmetry violation beyond the SM.

Finally, in Fig. 3, we show the results from varying all four Wilson coefficients simultaneously in the fit. The left panel is the global fit including M_W^{2021} and the right panel is the global fit including M_W^{2022} . The current measurement increases the correlation ρ between C_{HWB} and C_{HD} , and between C_{HD} , and C_{LL} , reshuffling the significance for NP also in the latter, but leaving the general structure of the correlation pattern almost unchanged.

V. DISCUSSION

The recent W -mass measurement by CDF can have far-reaching consequences on our understanding of the dynamics that govern our Universe and could pave a path to the discovery of possible BSM physics. Here, we would like to conclude by giving a brief overview of further implications that this measurement can actually have.

A. Notes on implications for $(g-2)_\mu$

The recent measurement of the muon anomalous magnetic moment at the Fermilab [62] adds to the discrepancy that already existed from an earlier BNL E821 measurement [63] bringing up the tension with the SM estimate to about 4.2σ [62,64]. It should be noted that the recent lattice results from the BMW Collaboration [65] have not been included in the world average and considering them as a realistic contribution [66–68] to the hadronic vacuum polarization (HVP) of the photon reduces the tension with the SM estimate of $(g-2)_\mu$. On a different note, the same $(g-2)_e$ may represent a puzzle within the SM, given the opposite sign observed with respect to the muon counterpart [69].

The computation of $a_\mu \equiv (g-2)_\mu/2$, is quite sensitive to the estimated value of $\Delta\alpha_{had}^{(5)}(M_Z)$ where an increase in the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ may reduce the discrepancy of a_μ from its experimental measurement if naively translated into a rescaling of the low-energy $e^+e^- \rightarrow$ hadrons cross section. Given the new measurement from CDF II some exploration in this direction is warranted.

In Fig. 4, we see that there is a strong tension between the measurement of $\Delta\alpha_{had}^{(5)}(M_Z)$ from e^+e^- data and the estimate from $(g-2)_\mu$ measurement. If, we assume the absence of NP contributions and perform an SM fit with M_W^{2022} , we find that the discrepancy gets even worse with the mean of $\Delta\alpha_{had}^{(5)}(M_Z)$ getting even lower. However, if, we allow for the presence of NP through a nonzero C_{HD} , the discrepancy between the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ estimated from the global fit and that from the $(g-2)_\mu$ measurement can be completely alleviated. It is to be noted that to arrive at the estimate of $\Delta\alpha_{had}^{(5)}(M_Z)$ from the EWPO data, we did not use the measured value of $\Delta\alpha_{had}^{(5)}(M_Z)$ from the e^+e^- data but rather determined it indirectly from the combination of all other

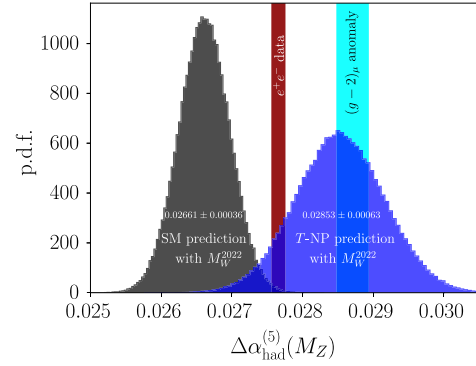


FIG. 4. Given its intimate relation with $(g-2)_\mu$ [69], we show how $\Delta\alpha_{had}^{(5)}(M_Z)$ is affected by the new measurement of M_W . A clear tension lies in the measurement of $\Delta\alpha_{had}^{(5)}(M_Z)$ from e^+e^- data represented by the dark red band and the value expected from the measurement of $(g-2)_\mu$ as estimated in Ref. [70] and represented here by the light blue band. The black distribution represents the SM prediction using all current measurements of EWPO except the $\Delta\alpha_{had}^{(5)}(M_Z)$ measurement which shows an increased tension. The dark blue distribution corresponds to the same in case of NP from C_{HD} where one can see that the tension is resolved.

measurements. We have labeled in Fig. 4 such a prediction as T -NP prediction, given the underlying custodial-symmetry violation implied in the scenario.

B. Notes on implications for B anomalies

Hints for lepton universality violation (LUV), stemming in particular from the study of B -meson decays via flavor-changing neutral-current processes, are close to the level of 5σ significance in favor of BSM physics [71,72]. A joint broad analysis of EWPO, LUV observables R_K and R_K^* , angular observables in $b \rightarrow s\ell^+\ell^-$ decays and $B_{s,d} \rightarrow \ell^+\ell^-$ was initiated in Ref. [46]. There, it was shown that in a flavor nonuniversal scenario the constraints from EWPO and the hints for NP from flavor physics can be simultaneously accommodated within a new Z' gauge boson at the scale of Λ_{NP} of a few TeV. It is known that models with a new Z' can generate custodial-symmetry breaking [73] opening a path to tying together the flavor anomalies with the hint of custodial-symmetry breaking. Note that the inference of C_{HD} highlighted in bold in Table II underlies naively an NP scale of about 6 TeV assuming tree-level new dynamics with $\mathcal{O}(1)$ couplings. It would be, therefore, interesting to combine the hint for LUV with the violation of custodial symmetry inferred in this work. A systematic study that would take into account also effects from the SM renormalization group at the one-loop level would be warranted in the spirit of what was carried out in Ref. [46] and is left for future work.

C. Notes on model realizations

The preservation of custodial symmetry has been a guiding principle for the construction of many realistic models of NP. Hence, the hint of breaking of custodial symmetry will have strong implications on BSM model building. We would like to leave a comment on the implication of the new world average of the W -boson mass on a general 2HDM model. Extensive studies of contributions to custodial-symmetry breaking in the 2HDM have been conducted over the years [74–77]. The imposition of custodial symmetry implies that the mass of the charged Higgs should be degenerate with the mass of the pseudoscalar Higgs, i.e., $m_{H^\pm} = m_{A^0}$ in the extended Higgs sector when imposing the symmetry through an additional constraint on the CP -conserving scalar potential. It is also possible to impose custodial symmetry by asserting $m_{H^\pm} = m_H$ or $m_{H^\pm} = m_{H^0}$, where H is the light Higgs identified with the 125 GeV Higgs and H^0 is the heavy Higgs scalar [75,76]. The former condition is already ruled out due to constraints on the charged Higgs from $b \rightarrow s\gamma$ inclusive decay.

An observation of custodial-symmetry breaking would imply that the Higgs mass spectrum in a 2HDM will have bounds on possible degeneracies in the physical states of the scalar and pseudoscalar bosons requiring a splitting between these states depending on the model construction. This numerically and conceptually changes the primary impetus of imposing custodial symmetry on the 2HDM models and moves toward a necessity to generate the breaking of the symmetry.

In general, models with additional scalar multiplets of $SU(2)_L$ have the potential to generate O_{HD} which will contribute to the breaking of custodial symmetry as can be seen from Eq. (3). In Ref. [78] it was shown that while real singlets with explicit or spontaneous Z_2 breaking cannot generate O_{HD} , real and complex triplets and quartets with hypercharge 3/2 or 1/2 and Z_2 symmetry breaking can generate O_{HD} , leading to possible new sources of custodial-symmetry violation. A more detailed investigation of these models is desirable but outside the scope of this work.

D. Final notes

The measurement of the W -boson mass is notoriously difficult and experimentally challenging, requiring a perfect understanding of detector properties and simulations, as well as a nontrivial crosstalk among theory and experimental communities. While, we look forward to new updates on this matter, as well as decisive steps in the future experimental EW precision program, we point out that at present a naive global combination of W -boson mass, namely, $M_W = 80.4060 \pm 0.0075$ GeV, provides robust support, at up to 4.5σ , for tree-level (relatively strongly coupled) new dynamics at the scale of few TeV, entering in the Peskin-Takeuchi T parameter, signaling a new source of violation of custodial symmetry that, we can hope to discover in the near future. The most viable physics scenario in the SMEFT framework that, we consider also leaves open the possibility of alleviating another discrepancy that surfaced in the recent past in the lepton sector, namely, the measurement of the anomalous magnetic moment of the muon. The preferred NP scenario also holds potential implications for the B -physics anomalies that have driven a lot of recent work and theoretical developments in the recent past. Hence, we see a true opportunity to tie together different sectors of the standard theory in order to learn more about its UV completion and carry out pivotal progress in the fundamental understanding of nature.

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