Letter

Timelike nucleon electromagnetic form factors: All about interference of isospin amplitudes

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A striking feature of the timelike nucleon electromagnetic form factors, investigated in $e^+e^- \to N\bar{N}$ annihilation reactions, is the modulation by local structures of small magnitude and oscillatory form, showing up above the $N\bar{N}$ threshold. Starting from an isospin decomposition of the proton and neutron form factors it is shown that such structures are the natural consequence of the interference of a large and a small amplitude, resulting in a sinusoidal behavior as a function of the "invariant energy" if the relative phase shift varies with energy. Thus, periodic oscillations superimposed on a smooth background will be observed. In this scenario, the equal size of the modulation for neutrons and protons (discovered by recent BESIII data) implies the particular isoscalar or isovector nature of these local structures, or their orthogonal interference. Hence, it specifies their origin as excited vector mesons whose widths are tied to the modulation frequency. We clarify that the phase difference of the modulation between neutrons and protons as BESIII data found, but not the modulation itself, is the evidence of an imaginary part of the timelike nucleon electromagnetic form factors, which is associated with the rescattering processes.

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I. INTRODUCTION

A new era of research on nucleon electromagnetic form factors (EMFFs) started with the advent of the modern electron-nucleon and electron-positron facilities. They allow us, in an unprecedented manner, to investigate fundamental properties of the nucleon in the spacelike and timelike domains, thus giving us access to the static and the dynamical properties [1,2]. Advanced experimental and theoretical methods are allowing investigations of all facets of the tomography of the nucleon with new prospects for extended studies of the hadron structure at future electron-ion colliders [3–5]. In this paper we address the unsolved issue of the recently identified oscillations seen in the timelike EMFFs [6,7]. Various explanations have been proposed [6–8], but a satisfactory solution of that problem is still pending. The role of vector mesons above the $N\bar{N}$ threshold is not fully established though they attracted attention in a microscopic version of the vector meson dominance (VMD) model more than a decade ago [9,10].

The timelike EMFFs have been measured in the annihilation process $e^+e^- \to N\bar{N}$, whose transition amplitude G_N is complex, connected to the spacelike EMFFs by the dispersion relation [11]. The known sources of the imaginary part of amplitudes are multimeson rescattering loops and intermediate vector meson excitations. While the former is incorporated by the dispersion analysis of the nucleon EMFFs including meson continua [11,12], the latter is only phenomenologically investigated in the isoscalar and isovector form factor [13]. Isospin decomposition is in fact far from trivial when isoscalar and isovector components are both involved in $e^+e^- \to N\bar{N}$ reactions. Two independent isospin channels are insufficient to fully disentangle two complex isospin amplitudes. Combining available data with isospin decomposition, we show that the range of isospin-related parameters could be constrained by available data. Then we reconcile to a model-independent separation of the sources of the imaginary part of timelike EMFFs.

The absolute value of G_N is the so-called nucleon effective form factor (EFF),

$$|G_N| = \sqrt{\frac{2\tau|G_M(q^2)|^2 + |G_E(q^2)|^2}{2\tau + 1}}, \qquad (1)$$

which is explicitly related to the Sachs electric and magnetic form factors $G_{E,M}$. Here q^2 is the invariant

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four-momentum transfer squared, and $\tau = q^2/4m_N^2$ with m_N being the nucleon mass. The proton EFF $|G_p(q^2)| = G_p^D(q^2) + G_p^{\rm rsd}(q^2)$ is split into a leading component of modified dipole shape

$$G_p^D(q^2) = \frac{\mathcal{A}_p}{(1+\frac{q^2}{m_a^2})(1-\frac{q^2}{0.71\ \mathrm{GeV}^2})^2}, \tag{2}$$

and a residual part, $G_p^{\rm rsd}$, accounting for deviations from the smooth behavior of G_p^D. $G_p^{D,\rm rsd}$ must be real-valued and G_n is defined accordingly.

The residual component is chosen of oscillatory shape as a function of the three-momentum $p = q\sqrt{\tau - 1}$ of N (or $\bar{N}$) in the $\bar{N}$ (or N) frame [6],

$$G_N^{\rm rsd} \sim G_N^{\rm osc} = A_N \exp(-B_N p) \cos(C_N p + D_N), \tag{3}$$

as imposed by *BABAR* proton data [14,15] and also confirmed for the neutron by recent high-precision measurements between 2.0 GeV and 3.08 GeV by the BESIII Collaboration [16]. Fits to the *BABAR* data have led to $\mathcal{A}_p = 7.7 \pm 0.3$ and $m_a^2 = 14.8\ \mathrm{GeV}^2$ [6,7]. From the BESIII data, the normalization of the neutron-dipole form factor was obtained as $\mathcal{A}_n = 4.87 \pm 0.09$, using the same pole mass as for the proton case. The form chosen for $G_N^{\rm rsd}$ is derived on purely phenomenological grounds. The physical origin of the residual component is largely unknown, speculations range from rescattering contributions [6,7] to yet unknown vector-meson resonances and threshold effects [8]. Surprisingly, the microscopic VMD framework incorporating properly the isospin relation predicted the correct magnitude of the neutron EFF [9,10]. The present status for $G_N^{\rm rsd}$ is shown in Fig. 1 with $A_p = 0.07 \pm 0.01$, $A_n = 0.08 \pm 0.03$, and sharing of the common parameters for proton and neutron [16]: $B_N = 1.01 \pm 0.24\ \mathrm{GeV}^{-1}$ and $C_N = 5.28 \pm 0.36\ \mathrm{GeV}^{-1}$. The phases are $D_p = 0.31 \pm 0.17$ and $D_n = -3.77 \pm 0.55$. It is noteworthy that D_p is consistent with zero within large error bars in a fit to all available $p\bar{p}$ data [17].

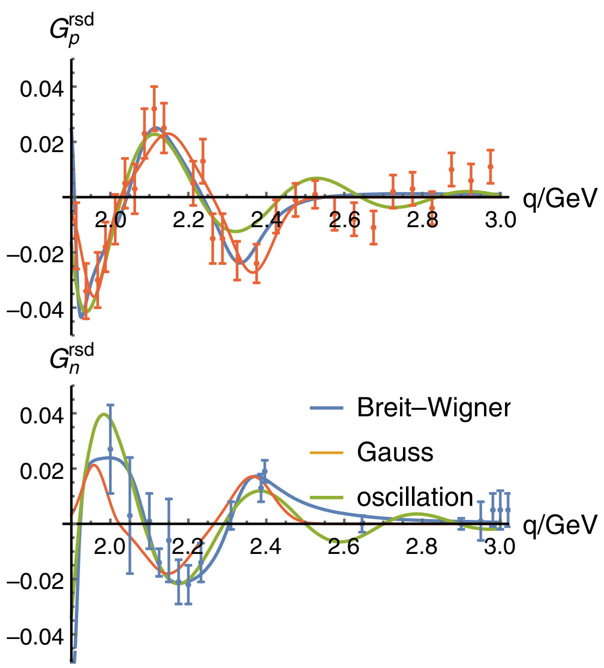

FIG. 1. The fit of the Breit-Wigner distribution and the Gauss distribution to three local structures below 2.5 GeV in comparison with BESIII data [16,18].

II. ISOSPIN DECOMPOSITION AND RESIDUAL FORM FACTORS

The striking similarity of the proton and neutron EFF points to the importance of isospin symmetry—or more precisely, of charge symmetry—in the $e^+e^- \to N\bar{N}$ reactions. Hence, we express the $N\bar{N}$ states in terms of the $|I, I_3\rangle = |0,0\rangle$ and $|1,0\rangle$ isospin components which allows to represent the proton and neutron form factors with regard to isoscalar (I_0) and isovector (I_1) amplitudes $G_{p,n} = (I_1 \pm I_0)/\sqrt{2}$ [19], where the upper (lower) sign applies to the proton (neutron) case. The mixtures of isoscalar and isovector amplitudes of equal relative weight but different sign are imposed by the isospin symmetry as introduced by the underlying Clebsch-Gordan coefficients. Hence, proton and neutron form factors will be different provided both isospin amplitudes are of comparable size.

The experimentally observed oscillatory structures are taken into account by decomposing the isospin transition amplitudes explicitly into smooth leading components $I_{0,1}^D$ and residual components $I_{0,1}^{\rm rsd}$,

$$G_{p,n} = \frac{I_{p,n}^D + I_{p,n}^{\rm rsd}}{\sqrt{2}} = \frac{I_1^D \pm I_0^D}{\sqrt{2}} + \frac{I_1^{\rm rsd} \pm I_0^{\rm rsd}}{\sqrt{2}}. \tag{4}$$

In general, the isospin form factors $I_{0,1}^{D,\rm rsd}$ will be complex numbers which is taken into account by using without loss of generality

$$I_{p,n}^D = I_1^D \pm I_0^D = \sqrt{2} G_{p,n}^D e^{i\phi_{p,n}^D}, \tag{5}$$

$$I_{p,n}^{\rm rsd} = I_1^{\rm rsd} \pm I_0^{\rm rsd} = |I_{p,n}^{\rm rsd}| e^{i\phi_{p,n}^{\rm rsd}}, \tag{6}$$

where the phases $\phi_{p,n}^D(q^2)$, $\phi_{p,n}^{\rm rds}(q^2)$, and the amplitudes $|I_{p,n}^{\rm rsd}|(q^2)$ depend on energy and have yet to be determined. We obtain

$$|G_N|^2 - (G_N^D)^2 = G_N^{\rm rsd}(2G_N^D + G_N^{\rm rsd}) = \frac{1}{2}|I_N^{\rm rsd}|^2 + \sqrt{2} G_N^D |I_N^{\rm rsd}| \Re[e^{i(\phi_N^D - \phi_N^{\rm rsd})}].$$

Thus, we have found equations relating $G_{p,n}^{\rm rsd}$ to the isospin amplitudes $I_{p,n}^{\rm rsd}$. The leading-order solutions are

$$G_N^{\rm rsd} \simeq \frac{|I_N^{\rm rsd}|}{\sqrt{2}} \cos(\phi_N^D - \phi_N^{\rm rsd}) \tag{7}$$

where terms of order $|I_{p,n}^{\rm rsd}/I_{p,n}^D| \ll 1$ are neglected. Thus, the form factors $G_{p,n}^{\rm rsd}$ are given in magnitude determined by the corresponding residual isospin form factors, modulated by a sinusoidal energy dependence introduced by the interference with the leading (smooth) components $G_{p,n}^D$.

A closer inspection reveals, that the form factors $G_{p,n}^{\rm rsd}$ are given in fact by the real parts of the interference term of the corresponding leading and the residual nucleon form factors. The imaginary part of interference terms is accessible through polarized asymmetries as pointed out in [20,21]. As addressed in Sec. I the pattern of these interference structures were already highlighted in [6] based on the *BABAR* measurements [22] and confirmed by the high-precision data of the BESIII Collaboration [18,23].

III. VMD MODEL AND ISOSPIN

By Eq. (7) the EMFFs of protons and neutrons should display sinusoidal modulations of a similar pattern, although differences in modulus and phase have to be expected because isoscalar and isovector components are superimposed differently. Surprisingly, the recent BESIII data disclosed that they are of the same magnitude over a wide energy range [16,18] implying

$$\frac{|I_1^{\rm rsd} + I_0^{\rm rsd}|}{|I_1^{\rm rsd} - I_0^{\rm rsd}|} = \frac{A_p}{A_n} = 0.88 \pm 0.35, \tag{8}$$

where the large error is mainly due to the less accurate neutron data. Its consistency with unity means that either $I_0^{\rm rsd} = 0$ or $I_1^{\rm rsd} = 0$ or—as an unlikely third option—a vanishing interference term by a coincidental phase difference of $\pi/2$ or odd-numbered multiples thereof.

However, a decisive role is played by the production mechanism hidden in the photon-nucleon-antinucleon vertex. If the $N\bar{N}$ channel is populated by the decay of an intermediate (off–shell) isoscalar mesons as e.g., the $\phi(2170)$ [24] isospin conservation in strong interaction processes restricts the production process to the $I = 0$ component. If an intermediate isovector meson is involved, e.g., $\rho(2150)$ [9,10,24] of a width comparable to the one of $\phi(2170)$, the $I = 1$ components of the $N\bar{N}$ channels are populated exclusively. In either of these isospin-clean VMD scenarios, the states will display distributions of the same spectral shapes in $p\bar{p}$ and $n\bar{n}$ channels, except for minor distortion due to isospin symmetry breaking by electromagnetic (and weak) interactions. Hence, the observed similarity of EMFFs in $e^+e^- \to p\bar{p}$ and $e^+e^- \to n\bar{n}$ reactions leads to the conclusion that the final channel is populated in an isospin-clean production process.

The general case of physical relevance will be given by isoscalar and isovector form factors composed of superposition of the various isospin-clean VMD process, accessible by the available energy. In particular, the residual amplitudes become $I_{0,1}^{\rm rsd} = \sum_k \delta_{0,1}^k e^{i\phi_{0,1}^k} f_{0,1}^k$, with energy-independent relative phases $\phi_{0,1}^k$, amplitudes $\delta_{0,1}^k$ of magnitudes much lesser than $|I_{0,1}^D|$, and spectral distributions $f_{0,1}^k$. The $\delta_{0,1}^k$ and $f_{0,1}^k$ are the same for $p\bar{p}$ and $n\bar{n}$ channels, which is a direct consequence of the above isospin relation. Within each isospin channel, the overlapping spectral distributions may eventually lead to destructive interference at certain energies and phase-coherent enhancements in other energy regions. However, unfortunately the present data do not yet allow us to identify the isospin character underlying the production process. Herein, a single f^k is used to describe one local structure for simplicity. This hypothesis does not affect the following discussion and conclusion. Specific experiments and theory input are indispensable in order to explore the multicomponent spectral structure of the whole set of isospin amplitudes.

IV. A PRACTICAL APPROACH TO DATA ANALYSIS

A physically well-motivated ansatz for a spectral distribution mimicking the line shape of the residual component is the Breit-Wigner (BW) distribution,

$$f_{\rm BW}^k(q^2) = \frac{e^{-i\phi_{\rm BW}^k} M_k \Gamma_k}{|q^2 - M_k^2 + iM_k\Gamma_k|}, \tag{9}$$

with $\phi_{\rm BW}^k = {\rm arccot}\frac{q^2 - M_k^2}{-M_k\Gamma_k}$, which is consistent with the VMD-inspired model with proper analytic continuation [9,10,13]. Within the VMD interpretation, the local structures are coherent superpositions of vector mesons of isoscalar ϕ/ω or isovector ρ character, located above the $N\bar{N}$ threshold and with a width Γ_k of around 100 MeV, found in [24] and used in [8,12]. Thus, in the BW–approach the residual components carry the energy-dependent phases $\phi_N^D - \phi_N^{\rm rsd} = \phi_N^D - \phi_N^k - \phi_{\rm BW}^k$ where the BW widths provide the scale for the momentum frequency $C_N \sim \frac{1}{\Gamma_k}$ in Eq. (3).

Another frequently used alternative, also considered in the fits to data, is to approximate the spectral functions by Gauss (GS) functions $f_{\rm GS}^k(q^2) = \exp(-(\sqrt{q^2} - M_k)^2/2\Gamma_k^2)$ together with $\phi_{p,n}^k = {\rm const}$. With a proper choice of parameters the Gaussians lead to distributions of shapes closely resembling the Breit-Wigner distributions. In this case, the phase differences $\phi_N^D - \phi_N^{\rm rsd} = \phi_N^D - \phi_N^k$ are constants and can be chosen to be zero. That choice is supported by the fitted values in Table I, which are consistent with zero within large error bars.

Application of the BESIII neutron data [16] reveals that $\phi_n^D - \phi_p^D + \phi_p^{\rm rsd} - \phi_n^{\rm rsd} = 4.08 \pm 0.58$ rad is a constant deviating from $3\pi/2$ by 0.63 ± 0.58 rad $= 36.2° \pm 33.2°$, where in fact the part π of the phase trivially reflects the

TABLE I. Parameters of three local structures below 2.5 GeV in the fit of the Breit-Wigner distribution (or Gauss distribution in the parentheses), together with the extracted $\Gamma_{ee}\Gamma_{N\bar{N}}$ for the Breit-Wigner distribution. The χ^2/d.o.f is 0.7 (1.2).

k/BW(GS)	1	2	3
M_k (MeV)	1910 ± 10 (1958 ± 10)	2083 ± 27 (2148 ± 16)	2328 ± 22 (2365 ± 13)
Γ_k (MeV)	32 ± 32 (30 ± 8)	162 ± 55 (60 ± 17)	162 ± 57 (52 ± 14)
δ^k	-0.072 ± 0.048 (-0.036 ± 0.010)	0.041 ± 0.007 (0.024 ± 0.005)	-0.032 ± 0.005 (-0.027 ± 0.007)
$\phi_p^D - \phi^k$	0	0.764 ± 0.373 (0.235 ± 0.307)	1.558 ± 0.310 (0.046 ± 0.241)
$\Gamma_{ee}\Gamma_{N\bar{N}}$ (keV^2)	4 ± 4	80 ± 47	62 ± 37

overall relative sign between $I_p^{\rm rsd}$ and $I_n^{\rm rsd}$. The phases $\phi_N^{\rm rsd}$ of the residual components fully account for the energy dependence as imposed by the data. The constant value of the total phase difference implies that $\phi_n^D - \phi_p^D$ behaves as a function of energy—up to a constant—complementary to the difference of residual phases. Assuming that at least one of the phases $\phi_{p,n}^D$ is nonzero, we have direct evidence for the in general complex-valued character of the form factors of Eqs. (5) and (6), respectively. That result is not unexpected, but here probably for the first time confirmed by data.

It is tempting to interpret the results shown in Fig. 1 in terms of physical processes. Only three clearly developed local structures in the region below $q = 2.5$ GeV are included in our fit, as shown in Fig. 1. The phase difference between neutrons and protons is fixed at the central value of BESIII in order to reduce the number of free parameters. Surprisingly, the parameters in Table I are indeed showing a compatibility with the vector-meson spectrum.

The first structure (just below 2 GeV) could be a threshold effect due to the opening of the $N\bar{N}$ channel [25]. However, isovector processes involving $\rho(1900)$ [24] are favored by the specified isospin of the structure. The second structure (just below 2.2 GeV) corresponds energetically to an isoscalar process running through $\phi(2170)$, whose properties are still a matter of intense investigation [26], competing with an isovector process driven by $\rho(2150)$ [9,10]. The third structure (close to 2.4 GeV) is possibly produced by an isovector $\rho(2350)$ [24]. The state may also be strongly coupled to $\Lambda\bar{\Lambda}$ as found in $p\bar{p} \to \Lambda\bar{\Lambda}$ [27] and $e^+e^- \to \Lambda\bar{\Lambda}$ [28]. However, these interpretations suffer from large uncertainties and must be considered with care, especially while waiting for safe confirmation of the vector-meson spectrum above 2.0 GeV [29].

The product of partial widths is determined by

$$\Gamma_{ee}\Gamma_{N\bar{N}} = \frac{\alpha_{em}^2\beta}{18}\left(1 + \frac{1}{2\tau}\right)(\delta^k\Gamma_k)^2, \qquad (10)$$

where α_{em} is the electromagnetic fine structure constant, β is the center-of-mass velocity of the final state (anti)nucleon. The extracted values of $\Gamma_{ee}\Gamma_{N\bar{N}}$ are shown in Table I and should be compared to data for J/ψ and $\psi(3686)$ amounting to (1.09 ± 0.05) keV^2 and (2.02 ± 0.10) keV^2, respectively.

A ratio which contains no residual component and a Coulomb correction is obtained by

$$R_N^D = \frac{\sigma_n^D}{\sigma_p^D/C} = \left(\frac{G_n^D}{G_p^D}\right)^2 = 0.40 \pm 0.03, \qquad (11)$$

where $C(q^2)$ is the S-wave Sommerfeld-Gamow Coulomb-correction factor [30], which is equal to 1 for neutrons. Surprisingly, R_N^D is constant over a wide range of energies. This value is close to $3/7$; a prediction of SU(6) symmetric-nucleon wave function [31]. It lies between the naive $e_u^2/e_d^2 = 1/4$ from the quark-charge ratio [32] and $2/3$ obtained from the constituent quark model [31]. In any case, the significant deviation from unity is a clear signature of its difference from the residual component, see Eq. (8).

An isospin relevant ratio is

$$R_I^D = \frac{(G_p^D)^2 - (G_n^D)^2}{(G_p^D)^2 + (G_n^D)^2} = \frac{2\Re(I_0^D I_1^{D\dagger})}{|I_0^D|^2 + |I_1^D|^2} = 0.43 \pm 0.03, \qquad (12)$$

which means that isovector I_1^D amplitudes are of comparable magnitude. Hence, we may relate the two isospin amplitudes by $I_1 = I_0\delta_I e^{i\phi_I}$, rendering Eq. (12) to

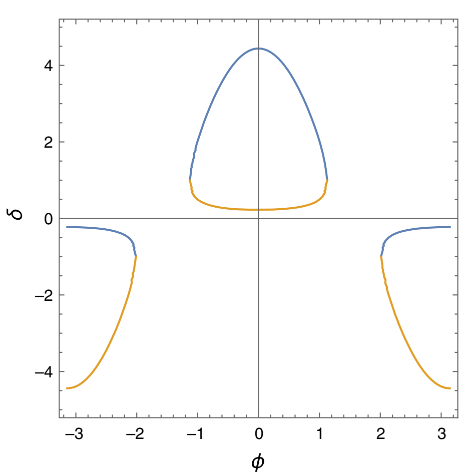

FIG. 2. Present data constraints for the relative moduli $\delta = \delta_I$ (or δ_M) and relative phase $\phi = \phi_I$ (or ϕ_M) of nucleon isoscalar and isovector amplitudes (or form factors).

$$R_I^D = \frac{2\delta_I \cos\phi_I}{1+\delta_I^2} \leq |\cos\phi_I|, \tag{13}$$

which allows us to derive constraints on the allowed (δ_I, ϕ_I) combinations. The correlation diagram, obtained by imposing the value for R_I^D derived above is shown in Fig. 2. The results are easily converted to a constraint on $\phi_p^D - \phi_n^D$ introduced in Eq. (5).

V. FLAVOR DECOMPOSITION

In the isospin limit, when the masses of the up and down quarks are degenerate, the isospin relation is a realization of the flavor decomposition of nucleon EMFFs [33],

$$G_{E,M}^{p,n} = \frac{2}{3}G_{E,M}^{u,d} - \frac{1}{3}G_{E,M}^{d,u} = \frac{1}{2}\left(\frac{G_{E,M}^{u+d}}{3} \pm G_{E,M}^{u-d}\right), \tag{14}$$

if the isospin (or charge) symmetry $G^{u/n} = G^{d/p}$ and $G^{d/n} = G^{u/p}$ is given. We define the effective isovector $(u+d)$ and isoscalar $(u-d)$ form factors as

$$|G_{\text{eff}}^{u\pm d}| = \sqrt{\frac{2\tau|G_M^{u\pm d}(q^2)|^2 + |G_E^{u\pm d}(q^2)|^2}{2\tau+1}}. \tag{15}$$

Then the sum of the proton and neutron EMFFs could be expressed in a compact way,

$$(G_p^D)^2 + (G_n^D)^2 = \frac{1}{2}\left(\left|\frac{G_{\text{eff}}^{u+d}}{3}\right|^2 + |G_{\text{eff}}^{u-d}|^2\right), \tag{16}$$

as a measure of incoherent sum of effective isovector and isoscalar form factors. The nonzero difference between them

$$(G_p^D)^2 - (G_n^D)^2 = \frac{1}{3}\Re\left[\frac{2\tau G_M^{u+d}G_M^{u-d\dagger} + G_E^{u+d}G_E^{u-d\dagger}}{2\tau+1}\right], \tag{17}$$

shows that the interference between these complex form factors is large, arriving at the same conclusion above. For further demonstration we need to assume $G_E = G_M$, so $|\cos\phi_M| \geq R_I^D$ in terms of the notation $G_M^{u+d} = G_M^{u-d}\delta_M e^{i\phi_M}$. Then the range of parameters is the same as in Fig. 2. In the SU(3) Nambu-Jona-Lasinio model, the difference between the isoscalar and isovector form factors is shown to be quite small [34].

VI. SUMMARY AND PERSPECTIVES

The structure of the nucleon form factors in the timelike region has been investigated in an isospin formalism. In a first application to BESIII data we could indeed explain the observed oscillatory structures to a VMD model.

Although neither wide or narrow vector mesons of light quarks are found to couple strongly to the $N\bar{N}$ channels, they still would be weakly present in timelike nucleon EMFFs. The well-known interference effects in unpolarized cross sections were exploited for extracting quantitative results on the dynamical contributions to the timelike nucleon form factors. Since these residual contributions are of much smaller magnitude than the bulk dipole components, the resulting spectral shapes can be expanded order by order into harmonics of the energy dependent phase. The oscillatory behavior observed in the e^+e^- data below 3.0 GeV is a realization of this phenomenon, described already convincingly well by the leading order harmonics.

An appealing aspect of the combination of the isospin formalism with the VMD model is that it leads to a simple explanation of the—up to an overall sign—almost identical oscillation pattern for protons and neutrons, found in the BESIII data. Within our formalism, the similarity of the periodic structures is a signature of the production process, imprinting the isospin of the virtual meson state into the final nucleon-antinucleon configurations which, as a result, are produced selectively via their isospin-clean $I = 0$ or $I = 1$ components.

Modeling the mesonic spectral functions by a Breit-Wigner distribution, we have shown that the oscillations are in fact a signal of the imaginary parts of the amplitudes. The finite widths—or likewise the finite lifetimes—of the vector mesons alone are already sufficient to explain the observed structures. Hence, we conclude that the frequently discussed multimeson rescattering processes seem to be of minor importance for these structures. However, traces of such interactions are contained in the decay properties and resulting self-energies of the involved heavy mesons. Moreover, the phase difference between the proton and neutron EFF, found by the BESIII Collaboration, may be a signature of a nontrivial imaginary part of a rescattering amplitude contributing to the smooth dipole component.

Hence, two sources can be identified for an imaginary part in the timelike nucleon EMFFs [6,8] based on available data. We emphasize that after subtraction of the residual component the ratio of the proton to the neutron EFF is surprisingly constant over a wide range of energies, indicating a balanced isospin content since neither the isoscalar or isovector component is dominant. Within error bars, the magnitude of that ratio is consistent with SU(6) predictions. For future work it is tempting to investigate the timelike EMFFs of other octet baryons, where the Σ and Λ hyperons are of special interest using the same approach.

Last but not least we address the question of higher-order expansions involving higher-order harmonics $\sim\cos[n(\phi_N^D - \phi_N^{\text{rsd}})]$ of rank $n > 1$. However, the currently achievable accuracies of the measurements does not allow safe conclusions on such contributions, mainly because of the smallness of the residual amplitude. For such purposes deep virtual Compton scattering, which interferes with the Bethe-Heitler process [35] is much more suitable for specific investigations of cross sections in terms of angular harmonics, where induced higher harmonics have already been observed.

In our approach, the oscillations are induced by the broad vector mesons which, however, are coupled weakly to the $N\bar{N}$ channels. A somewhat more favorable situation is encountered in the charmonium region. The J/ψ and $\psi(2S)$ states are very narrow and they decay significantly into $p\bar{p}$ and $n\bar{n}$ with equal strength [36], indicating an isospin-clean population in these cases via the $I=0$ components of $N\bar{N}$ channels. Experimental studies in the vicinity of $\psi(3770)$, where a deep dip is clearly visible in $p\bar{p}$ channel, would be the ideal tool for investigating higher harmonics [37], especially if data for the $n\bar{n}$ channel above $D\bar{D}$ threshold were also available with high precision.

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