Letter

Sum rule for the partial decay rates of bottom hadrons based on the dynamical supersymmetry of the $\bar{s}$ quark and the ud diquark

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(Received 6 December 2021; accepted 16 February 2022; published 14 March 2022)

We investigate the weak decays of $\bar{B}_s^0$ and Λ_b to charm hadrons based on the dynamical supersymmetry between the $\bar{s}$ quark and the ud diquark. We derive a new sum rule relating the decay rates of the processes $\bar{B}_s^0 \to D_s^+ P^-$, $\bar{B}_s^0 \to D_s^{*+} P^-$, and $\Lambda_b \to \Lambda_c P^-$, where P^- is a negatively charged meson, such as π^- and K^-. It is found that the observed decay rates satisfy the sum rule very well. This implies that the supersymmetry between the $\bar{s}$ quark and the ud diquark is also seen in the wave functions of the heavy hadrons and suggests that the ud diquark can be regarded as a valid effective constituent for heavy hadrons.

DOI: 10.1103/PhysRevD.105.L051504

Finding fundamental correlations is a clue to understand the structure of strongly interacting systems. In electron systems the Cooper pair is a key ingredient and its condensation leads to superconductivity [1]. In nuclear physics, the nucleon pair correlation is an important object to describe the nuclear structure in the interacting boson model [2,3]. Also the two-neutron correlation can be a hint to understand the structure of unstable light nuclei [4,5]. In hadron physics, the two-quark correlation called diquark has been already mentioned in Ref. [6] when quarks were proposed, and it can be used as an effective constituent in many-body systems. The importance of the diquark correlation in hadronic systems was discussed phenomenologically in [7,8]. It is also known that diquark condensation induces color superconductivity at high density quark systems [9,10].

The diquark is a colored object that cannot be observed at low energies as an isolated particle due to color confinement. Its existence, however, is expected as a constituent inside hadrons similar to the constituent quark, which is a quasiparticle of the fundamental particles and is regarded as an effective building block of hadrons. The role of the diquark in the baryon structure has been extensively investigated by diquark pictures, in which baryons are composed of a diquark and a constituent quark [11–21]. Light scalar mesons may be described by a configuration of diquark and antidiquark [22–25] and their decay properties are reproduced reasonably well [24,25]. Lattice QCD calculations also have suggested attractive diquark correlations [26,27].

Recently a dynamical supersymmetry between the ud scalar diquark and the $\bar{s}$ constituent quark has been proposed in Ref. [28]. Both objects have the same color charge $\bar{\mathbf{3}}$ and same electric charge. Phenomenologically they are known to have a similar mass around 500 MeV. This is a supersymmetry between a boson and a fermion, but not a symmetry for fundamental particles, rather a dynamical symmetry for quasiparticles which are regarded as effective elements of the dynamics like the constituent quarks. If this supersymmetry is realized universally in hadronic systems, one may conclude the existence of the diquark inside hadrons as seen for the constituent quarks that were established from the symmetry arguments of the light hadrons. Historically such a dynamical supersymmetry was introduced first by Miyazawa for mesons and baryons [29] and later applied to the light hadron spectra [30]. One can also use holographic QCD to motivate a supersymmetry connecting baryons and mesons [31–33].

The supersymmetry among the scalar ud diquark and the $\bar{s}$ quark works rather well for the hadron spectra [28]. For instance, combining a bottom quark b with the ud diquark and the $\bar{s}$ quark, we have three hadrons $(\bar{B}_s^0, \bar{B}_s^{*0}, \Lambda_b)$, which are a spin-0 pseudoscalar meson, a spin-1 vector meson and a spin-1/2 baryon. The observed masses are found as (5367, 5415, 5620) in units of MeV, respectively. Similarly for the charm quark c, we have $(D_s^+, D_s^{*+}, \Lambda_c)$ and these masses are (1968, 2112, 2286) in units of MeV. The symmetry breaking on these hadron masses is about 300 MeV, which is as good as the flavor symmetry breaking

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stemming from the mass difference among the light constituent quarks.

The symmetry among these hadrons may be based on a similar mass for the ud diquark and the $\bar{s}$ constituent quark. Color electric interactions play the main role for confinement and are mainly determined by the masses and color of the interacting particles. Because ud diquark and $\bar{s}$ quark have same color and a similar mass, the interactions of the heavy quark with the ud diquark and the $\bar{s}$ quark must be very similar. Possible sources of symmetry breaking are the mass difference between the ud diquark and the $\bar{s}$ quark and spin-dependent forces such as the spin-spin interaction between quarks. The former is responsible for the mass difference of the mesons and the baryon, while the latter induces the mass difference between pseudoscalar and vector mesons.

The purpose of this article is to investigate whether this supersymmetry is realized also in the wave functions in heavy hadrons. The symmetry of the wave functions can be seen in the decay of the heavy hadrons, where the decay rates are expressed by the matrix elements of the parent and daughter particles with the wave functions of the initial and final states. For this purpose, we compare the weak decays of $\bar{B}_s^0$ into D_s^+ and D_s^{*+} with those of Λ_b into Λ_c.

From now on, let us call the $\bar{s}$ quark and the ud diquark collectively as $\hat{\psi}$ and consider a spin doublet $\bar{s}$ and a scalar ud to form a triplet $\hat{\psi}$ of the V(3) supersymmetry introduced by Miyazawa [29]. We denote hadrons composed of the triplet $\hat{\psi}$ and a heavy quark h collectively as $\hat{\psi}h$. This yields $\hat{\psi}b = (\bar{B}_s^0, \bar{B}_s^{*0}, \Lambda_b)$ for the bottom hadrons and $\hat{\psi}c = (D_s^+, D_s^{*+}, \Lambda_c)$ for the charm hadrons. Hadrons $\hat{\psi}h$ form a sextet of $\mathrm{V}(3) \otimes \mathrm{SU}(2)$ where SU(2) denotes the spin symmetry of the heavy quark.

We consider several weak decay modes in parallel; pionic decay $\hat{\psi}b \to \pi^-\hat{\psi}c$, kaonic decay $\hat{\psi}b \to K^-\hat{\psi}c$, ρ mesonic decay $\hat{\psi}b \to \rho^-\hat{\psi}c$, D mesonic decay $\hat{\psi}b \to D\hat{\psi}c$, D_s mesonic decay $\hat{\psi}b \to D_s\hat{\psi}c$, D_s^* mesonic decay $\hat{\psi}b \to D_s^*\hat{\psi}c$, and semileptonic decay $\hat{\psi}b \to \ell^-\bar{\nu}_\ell\hat{\psi}c$. We abbreviate these decays as $\hat{\psi}b \to P\hat{\psi}c$, where P stands for the emitted particles; that is, a pion and a kaon for the mesonic decay, and leptons for the leptonic decay.

Let us first consider pionic decay $\hat{\psi}b \to \pi^-\hat{\psi}c$. This decay is induced by transition $b \to cW^-$ and then either the weak boson W^- turns into a pion π^- or W^- is absorbed into $\hat{\psi}$. In the former process, the $\bar{s}$ quark or the ud diquark is a spectator in the weak decay, and thus the weak transition of the b quark commonly contributes to the decays of $\bar{B}_s^0$ and Λ_b and the wave functions of the $\bar{s}$ quark in $\bar{B}_s^0$ and of the ud diquark in Λ_b are responsible for the difference of their decay rates. The latter process involves two particles in the initial state. Because such a two-body process is known to be strongly suppressed compared to one-body processes [34], we can safely neglect it. Therefore, the decay process $\hat{\psi}b \to \pi^-\hat{\psi}c$ is good to investigate the supersymmetry in the $\bar{s}$ and ud wave functions. This situation is also true for kaonic, ρ mesonic, and semileptonic decays.

More systematically, we show the relevant diagrams of the weak decays $\hat{\psi}b \to P\hat{\psi}c$ in Fig. 1 by making use of the topological classification of Ref. [35]. Based on the supersymmetry we extend this classification from mesons to baryons and use it for both. In the "external W-emission" diagram (a), the weak decay is induced by the transition of the b quark to the c quark with emitting a meson P directly from the W boson. The "horizontal W-loop" diagram (b) contains charm quark pair creation and contributes to the D, D_s, and D_s^* mesonic decays. (The D mesonic decay is doubly Cabibbo suppressed.) Also in this diagram, $\hat{\psi}$ is a spectator. Diagrams (c) and (d) contribute differently to the $\bar{B}_s^0$ and Λ_b. These two diagrams, however, contain two-body processes. There are two more diagrams in the classification of Ref. [35]: the internal W-emission and the W-annihilation diagrams. These diagrams are irrelevant for the present calculation, because the former diagram does not contain D_s, D_s^*, nor Λ_c in the final state and the latter is only relevant for a charged meson decay. In order to explore the diquark ansatz, we did not consider in Fig. 1 the processes in which the ud diquark falls apart during the weak decay.

The decays of $\hat{\psi}b$ can be calculated from diagrams (a) and (b), in which $\hat{\psi}$ can be regarded as a spectator of the decay process. The effective Hamiltonian that we consider here for the transition b to c reads

$$\mathcal{H}_W^L = \bar{c}\gamma^\mu(A + B\gamma_5)bP_\mu \equiv J_h^\mu P_\mu, \tag{1}$$

where P_μ is the weak current for each weak process, such as $P^\mu = \partial^\mu\pi^\dagger$ for the pionic decay, $P^\mu = \rho^{\mu\dagger}$ for the ρ mesonic decay, and $P^\mu = \bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell$ for the leptonic decay. The effective coupling strengths A and B in the current J_h^μ depend on the weak process specified by P^μ, but do *not* depend on whether the spectator is a ud diquark or an $\bar{s}$ quark. Here the supersymmetry enters.

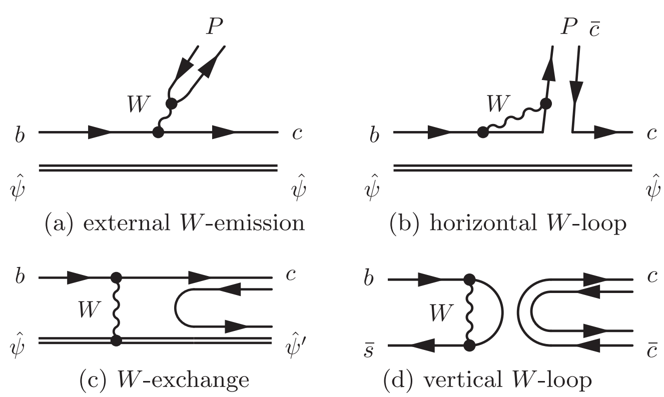

FIG. 1. Relevant diagrams for the weak decay of $\hat{\psi}b$ based on the topological classification given in Ref. [35].

The decay rate of a bottom hadron $\hat{\psi}b$ (with mass $M_{\hat{\psi}b}$) to a charm hadron $\hat{\psi}c$ (mass $M_{\hat{\psi}c}$) by emitting particle(s) P is calculated as

$$\Gamma = \frac{1}{2M_{\hat{\psi}b}} \int \sum_{\text{spin}} |\mathcal{M}_h^\mu \mathcal{M}_{P\mu}|^2 d\Phi_f, \tag{2}$$

with the phase-space element of the final states $d\Phi_f \equiv (2\pi)^4 \delta^4(q - \sum_i p_i) \prod_i \frac{d^3 p_i}{2E_i(2\pi)^3}$. The spin average of the initial state and spin summation of the final states are taken. The matrix elements $\mathcal{M}_h^\mu$ and $\mathcal{M}_P^\mu$ are defined by

$$\mathcal{M}_h^\mu = \langle \hat{\psi}c | J_h^\mu | \hat{\psi}b \rangle, \qquad \mathcal{M}_P^\mu = \langle P | P^\mu | 0 \rangle. \tag{3}$$

The latter matrix element $\mathcal{M}_P^\mu$ describes the particle emission during the transition and is common for the process $\hat{\psi}b \to P\hat{\psi}c$, irrespective of the choice of the triplet member from $\hat{\psi}$ and irrespective of the spin orientations of the heavy quarks. For two-body decays in the rest frame, the decay rate (2) is written as

$$\Gamma = \underbrace{\frac{p_{\text{c.m.}}}{32\pi^2 M_{\hat{\psi}b}^2}}_{\equiv K} \int \sum_{\text{spin}} |\mathcal{M}_h^\mu \mathcal{M}_{P\mu}|^2 d\Omega, \tag{4}$$

with the center of mass momentum of the final states $p_{\text{c.m.}} = \{(M_{\hat{\psi}b}^2 - (M_{\hat{\psi}c} + m)^2)(M_{\hat{\psi}b}^2 - (M_{\hat{\psi}c} - m)^2)\}^{1/2}/(2M_{\hat{\psi}b})$, where m denotes the mass of particle P. Thanks to the symmetry of the masses in the same multiplet, the mass of the decaying hadron and the phase space of the final states are also the same in each decay mode specified by P.

Because the $\bar{s}$ quark and the ud diquark can be regarded as spectators in the transition, the hadronic matrix element $\mathcal{M}_h^\mu$ can be evaluated in terms of the heavy quark states. Specifying the quark spins, we write the matrix element of the hadronic current for the bottom and charm quarks with spin α and β as

$$\mathcal{M}_{\alpha\beta}^\mu = \langle c^{(\beta)} | J_h^\mu | b^{(\alpha)} \rangle = \bar{u}_c^{(\beta)} \gamma^\mu (A + B\gamma_5) u_b^{(\alpha)}. \tag{5}$$

Under the assumption that the wave functions are the same due to the supersymmetry, the matrix elements $\mathcal{M}_{\alpha\beta}^\mu$ appear commonly in the calculations of each decay mode.

The spin of the heavy baryon Λ_h and the heavy quark coincide thanks to the spinless diquark. Thus the spin wave functions of the heavy baryon spin doublet $\Lambda_h^{(1)}$ and $\Lambda_h^{(2)}$ are given by $(ud)h^{(1)}$ and $(ud)h^{(2)}$, respectively. For the decay rate of an unpolarized Λ_b to Λ_c, we take a spin average of the initial Λ_b and sum up all of the spin states of the final Λ_c,

$$\sum_{\text{spin}} (\mathcal{M}_{\Lambda_c}^\mu)^* \mathcal{M}_{\Lambda_c}^\nu = \frac{1}{2} (\mathcal{M}_{11}^{\mu*} \mathcal{M}_{11}^\nu + \mathcal{M}_{22}^{\mu*} \mathcal{M}_{22}^\nu + \mathcal{M}_{12}^{\mu*} \mathcal{M}_{12}^\nu + \mathcal{M}_{21}^{\mu*} \mathcal{M}_{21}^\nu). \tag{6}$$

The spin configuration of a pseudoscalar meson composed of a heavy quark h and an $\bar{s}$ quark is given by $\frac{1}{\sqrt{2}}(\bar{s}^{(1)} h^{(1)} + \bar{s}^{(2)} h^{(2)})$. For the weak decay of the pseudoscalar $\bar{B}_s^0$, the spin of the $\bar{s}$ quark does not change in the decay as it is a spectator. The hadronic part of the decay amplitude of $\bar{B}_s^0$ to the pseudoscalar D_s^+ is calculated as

$$\begin{aligned}\mathcal{M}_{D_s^+}^\mu &= \langle D_s^+ | J_h^\mu | B_s^0 \rangle \\ &= \left\langle \frac{1}{\sqrt{2}} (\bar{s}^{(1)} c^{(1)} + \bar{s}^{(2)} c^{(2)}) \middle| J_h^\mu \middle| \frac{1}{\sqrt{2}} (\bar{s}^{(1)} b^{(1)} + \bar{s}^{(2)} b^{(2)}) \right\rangle \\ &= \frac{1}{2} (\mathcal{M}_{11}^\mu + \mathcal{M}_{22}^\mu),\end{aligned} \tag{7}$$

where we have used the orthogonality of the states having different spin for the $\bar{s}$ quark. This implies that the weak decay of $\bar{B}_s^0$ to D_s^+ has only the spin nonflip amplitude A. The square of the amplitude is given by

$$(\mathcal{M}_{D_s^+}^\mu)^* \mathcal{M}_{D_s^+}^\nu = \frac{1}{4} (\mathcal{M}_{11}^{\mu*} \mathcal{M}_{11}^\nu + \mathcal{M}_{22}^{\mu*} \mathcal{M}_{22}^\nu + \mathcal{M}_{11}^{\mu*} \mathcal{M}_{22}^\nu + \mathcal{M}_{22}^{\mu*} \mathcal{M}_{11}^\nu). \tag{8}$$

The spin configurations for the vector mesons D_s^{*+} with $s_z = +1, 0, -1$ are given by $\bar{s}^{(2)} h^{(1)}$, $\frac{1}{\sqrt{2}}(\bar{s}^{(1)} h^{(1)} - \bar{s}^{(2)} h^{(2)})$, and $\bar{s}^{(1)} h^{(2)}$, respectively. In analogy to Eq. (7), we calculate the decay amplitudes of the pseudoscalar $\bar{B}_s^0$ to the vector D_s^{*+}. Summing up the spin of D_s^{*+} in the final state, we obtain

$$\sum_{\text{spin}} (\mathcal{M}_{D_s^{*+}}^\mu)^* \mathcal{M}_{D_s^{*+}}^\nu = \frac{1}{2} \mathcal{M}_{21}^{\mu*} \mathcal{M}_{21}^\nu + \frac{1}{2} \mathcal{M}_{12}^{\mu*} \mathcal{M}_{12}^\nu + \frac{1}{4} (\mathcal{M}_{11}^{\mu*} \mathcal{M}_{11}^\nu + \mathcal{M}_{22}^{\mu*} \mathcal{M}_{22}^\nu - \mathcal{M}_{11}^{\mu*} \mathcal{M}_{22}^\nu - \mathcal{M}_{22}^{\mu*} \mathcal{M}_{11}^\nu). \tag{9}$$

The heavy hadrons $(\bar{B}_s^0, \Lambda_b)$ and $(D_s^+, D_s^{*+}, \Lambda_c)$ are in the same multiplets, respectively, and the V(3) supersymmetry demands the kinematical factors of these decays to be the same. In addition, if the wave functions of the heavy hadrons are the same in each multiplet, the hadronic matrix elements can be calculated commonly using the amplitude (5). Under these conditions, we find that the sum of Eqs. (8) and (9) coincides with Eq. (6). This implies that we have a sum rule for the decay probabilities of $\bar{B}_s^0$ and Λ_b as

$$\Gamma_{\bar{B}_s^0 \to D_s^+} + \Gamma_{\bar{B}_s^0 \to D_s^{*+}} = \Gamma_{\Lambda_b \to \Lambda_c}. \quad (10)$$

With this sum rule, we can check the symmetry of the wave functions for the heavy hadrons $\hat{\psi}h$.

We will examine whether the sum rule (10) agrees with experimental observations and we will derive predictions for partial decay rates that have not been measured yet. The experimental data collected by the Particle Data Group (PDG) [36] are summarized in Table I, where the partial decay rates are evaluated in units of 10^9/s using the central values of the mean life of the decaying particle and the branching fraction of the corresponding decay mode. For $\bar{B}_s^0$, we use the average of the mean lives of the heavy and light CP eigenstates [37]. Although the branching fractions for B_s^0 are provided by the PDG, we use them for $\bar{B}_s^0$ since the CP violation is very small.

First of all, it is very interesting to note that for each decay mode the partial decay rates of the $\bar{B}_s^0$ meson and the Λ_b baryon have the same order of magnitude. This can be interpreted already as a consequence of the supersymmetry between the $\bar{s}$ quark and the ud diquark.

For the decays $\hat{\psi}b \to \pi^- \hat{\psi}c$, the sum of decay rates of $\bar{B}_s^0 \to \pi^- D_s^+$ and $\to \pi^- D_s^{*+}$ yields $(3.3 \pm 0.4) \times 10^9$/s, while the decay rate $\Lambda_b \to \pi^- \Lambda_c$ is $(3.3 \pm 0.3) \times 10^9$/s. Thus, the sum rule (10) is satisfied extremely well.

Next, we discuss the sum rule (10) for $\hat{\psi}b \to K^- \hat{\psi}c$. Unfortunately, present experiments provide only the branching fraction of the $B_s^0 \to K^\pm D_s^{(*)\mp}$ decay, i.e., one cannot discriminate between $B_s^0 \to K^+ D_s^{(*)-}$ and $B_s^0 \to K^- D_s^{(*)+}$. Therefore we consider the kaonic decay fractions for $\bar{B}_s^0$ as upper limits. The sum of the decay rates of $\bar{B}_s^0$ to $K^\mp D_s^\pm$ and $K^\mp D_s^{*\pm}$ is found to be $(2.37 \pm 0.26) \times 10^8$/s, while the decay rate of Λ_b to $K^- \Lambda_c$ is observed as $(2.44 \pm 0.020) \times 10^8$/s. The sum rule may work well.

For the ρ, D^- and D_s mesonic decays, one of the branching fractions has not been measured yet. Assuming the sum rule (10), we can predict the partial decay rates of these missing decay modes. The predicted values are shown as values in square brackets in Table I. It will be very interesting to see if future measurements of the branching rates of presently missing decays will confirm the validity of our sum rule (10). The decay branching fraction of $\bar{B}_s^0 \to D_s^- D_s^{*+} + D_s^{*-} D_s^+$ has been observed as $(1.39 \pm 0.17) \times 10^{-2}$, which corresponds to $(9.17 \pm 1.12) \times 10^9$/s for the partial decay rate. Using the partial decay rate of $\bar{B}_s^0 \to D_s^- D_s^{*+}$ obtained from the sum rule, we estimate the partial decay rate of $\bar{B}_s^0 \to D_s^- D_s^{*+}$ as $(4.6 \pm 0.8) \times 10^9$/s. Using the sum rule again, we can predict the partial decay rate of $\Lambda_b \to D_s^- \Lambda_c$ as $(14.1 \pm 1.9) \times 10^9$/s.

TABLE I. Weak decay modes of $\bar{B}_s^0$ and Λ_b and the corresponding branching fractions and rates. The partial decay rates Γ_i are shown in units of 10^9/s and are evaluated using the central values of the observed mean life and branching fraction. The values of the partial decay rates in square brackets are predictions based on the sum rule (10). The value of the observed mean life of $\bar{B}_s^0$ is $(1.515 \pm 0.004) \times 10^{-12}$ s, which is the average mean life of the heavy and light CP eigenstates, and that of Λ_b is $(1.471 \pm 0.009) \times 10^{-12}$ s. The charge of the kaonic decay of $\bar{B}_s^0$ cannot be discriminated due to the $\bar{B}_s^0 - B_s^0$ mixing. The data are taken from [36]. The original experiments are found in Refs. [38–56].

	Branching fraction	Γ_i [10^9/s]
Pionic decay		
$\bar{B}_s^0 \to \pi^- D_s^+$	$(3.00 \pm 0.23) \times 10^{-3}$	1.98 ± 0.15
$\bar{B}_s^0 \to \pi^- D_s^{*+}$	$(2.0 \pm 0.5) \times 10^{-3}$	1.3 ± 0.3
Sum		$[3.3 \pm 0.4]$
$\Lambda_b \to \pi^- \Lambda_c$	$(4.9 \pm 0.4) \times 10^{-3}$	3.3 ± 0.3
Kaonic decay		
$\bar{B}_s^0 \to K^- D_s^+$	$< (2.27 \pm 0.19) \times 10^{-4}$	$< (0.150 \pm 0.013)$
$\bar{B}_s^0 \to K^- D_s^{*+}$	$< (1.33 \pm 0.35) \times 10^{-4}$	$< (0.088 \pm 0.023)$
Sum		$[< (0.237 \pm 0.026)]$
$\Lambda_b \to K^- \Lambda_c$	$(3.59 \pm 0.30) \times 10^{-4}$	0.244 ± 0.020
ρ mesonic decay		
$\bar{B}_s^0 \to \rho^- D_s^+$	$(6.9 \pm 1.4) \times 10^{-3}$	4.6 ± 0.9
$\bar{B}_s^0 \to \rho^- D_s^{*+}$	$(9.6 \pm 2.1) \times 10^{-3}$	6.3 ± 1.4
$\Lambda_b \to \rho^- \Lambda_c$	$[(16.0 \pm 2.4) \times 10^{-3}]$	$[10.9 \pm 1.7]$
D mesonic decay		
$\bar{B}_s^0 \to D^- D_s^+$	$(2.8 \pm 0.5) \times 10^{-4}$	0.18 ± 0.03
$\bar{B}_s^0 \to D^- D_s^{*+}$	$[(1.9 \pm 0.8) \times 10^{-4}]$	$[0.13 \pm 0.05]$
$\Lambda_b \to D^- \Lambda_c$	$(4.6 \pm 0.6) \times 10^{-4}$	0.31 ± 0.04
D_s mesonic decay		
$\bar{B}_s^0 \to D_s^- D_s^+$	$(4.4 \pm 0.5) \times 10^{-3}$	2.9 ± 0.3
$\bar{B}_s^0 \to D_s^- D_s^{*+}$	$[(6.9 \pm 1.1) \times 10^{-3}]$	$[4.6 \pm 0.8]$
$\Lambda_b \to D_s^- \Lambda_c$	$(1.10 \pm 0.10) \times 10^{-2}$	7.5 ± 0.7
D_s^* mesonic decay		
$\bar{B}_s^0 \to D_s^{*-} D_s^+$	$[(0.70 \pm 0.20) \times 10^{-2}]$	$[4.6 \pm 1.4]$
$\bar{B}_s^0 \to D_s^{*-} D_s^{*+}$	$(1.44 \pm 0.21) \times 10^{-2}$	9.5 ± 1.4
$\Lambda_b \to D_s^{*-} \Lambda_c$	$[(2.07 \pm 0.28) \times 10^{-2}]$	$[14.1 \pm 1.9]$
Semileptonic decay		
$\bar{B}_s^0 \to \ell^- \bar{\nu}_\ell D_s^+ + X$	$(8.1 \pm 1.3) \times 10^{-2}$	53 ± 9
$\bar{B}_s^0 \to \ell^- \bar{\nu}_\ell D_s^{*+} + X$	$(5.4 \pm 1.1) \times 10^{-2}$	36 ± 7
$\Lambda_b \to \ell^- \bar{\nu}_\ell \Lambda_c + X$	$(10.9 \pm 2.2) \times 10^{-2}$	74 ± 15
$\Lambda_b \to \ell^- \bar{\nu}_\ell \Lambda_c$	$(6.2^{+1.4}_{-1.3}) \times 10^{-2}$	42^{+10}_{-9}

For the semileptonic decays, exclusive measurements exist only for the baryon case. For the $\bar{B}_s^0$ decays they have not been performed yet. But the three inclusive decay modes collected in Table I have a similar magnitude to the baryon decay rate. This may be a consequence of the supersymmetry between $\bar{s}$ and ud. In order to confirm the sum rule for the semileptonic decays, exclusive observations are strongly desired.

It is interesting to estimate the magnitude of symmetry breaking of the sum rule (10) coming from the kinematical factor K of Eq. (4). This factor is a function of $M_{\hat{\psi}b}$, $M_{\hat{\psi}c}$ and m. The observed heavy hadron masses deviate from the symmetric mass M_h. The latter is given by a spin average $M_b = (M_{\bar{B}_s^0} + 3M_{B_s^{*0}} + 2M_{\Lambda_b})/6$ and similar for the charm sector. Numerically one obtains $M_b = 5475$ MeV and $M_c = 2146$ MeV. The deviation of the kinematical factor K from the symmetry limit can be evaluated as

$$\frac{K(M_{\hat{\psi}b}, M_{\hat{\psi}c}, m)}{K(M_b, M_c, m)} \approx 1 + \frac{\partial \log K}{\partial M_{\hat{\psi}b}} \delta m_b + \frac{\partial \log K}{\partial M_{\hat{\psi}c}} \delta m_c, \quad (11)$$

where δm_b and δm_c are the deviations of the bottom and charm hadron masses from their symmetric mass, respectively. Evaluating Eq. (11) using the observed masses, we find that the deviation of the kinematical factor from the symmetric limit is 5% at most for these decay modes. Therefore, the fact that the sum rule works very well for the observed weak decay processes implies that the wave functions for the heavy hadrons $\hat{\psi}h$ have also good symmetry stemming from the supersymmetry between the $\bar{s}$ constituent quark and the ud diquark.

In conclusion, based on the supersymmetry between the $\bar{s}$ quark and the ud scalar diquark, we have derived a sum rule for the weak transition rates of the bottom $\bar{B}_s^0$ meson and Λ_b baryon to charm hadrons. The sum rule is well satisfied by the observed weak decays for pionic and kaonic decay modes. This implies that the ud scalar diquark behaves as a quasiparticles inside of the Λ_b and Λ_c baryons like the $\bar{s}$ quark in heavy mesons and can be a clue for the nature of the diquark. We have also predicted from the sum rule several weak decay rates of $\bar{B}_s^0$ and Λ_b that have not been observed yet. If these missing decay modes are observed in future experiments, they can give us further support for the importance of the diquark correlation.

This work was partially supported by the Grant-in-Aid for Scientific Research (Grants No. JP17K05449 and No. JP21K03530) from the Japan Society for the Promotion of Science and by the Swedish Research Council (Vetenskapsrådet) (Grant No. 2019-04303).

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