Letter

Chiral vortical effect in extended Rarita-Schwinger field theory and chiral anomaly

G. Yu. Prokhorov [*] and O. V. Teryaev [†]

Joint Institute for Nuclear Research, Joliot-Curie street 6, Dubna 141980, Russia and Institute of Theoretical and Experimental Physics, NRC Kurchatov Institute, B. Cheremushkinskaya 25, Moscow 117218, Russia

V. I. Zakharov [‡]

Institute of Theoretical and Experimental Physics, NRC Kurchatov Institute, B. Cheremushkinskaya 25, Moscow 117218, Russia and Pacific Quantum Center, Far Eastern Federal University, 10 Ajax Bay, Russky Island, Vladivostok 690950, Russia

(Received 20 September 2021; accepted 1 December 2021; published 16 February 2022)

We consider the theory of Rarita-Schwinger field interacting with a field with spin $1/2$, in the case of finite temperature, chemical potential and vorticity, and calculate the chiral vortical effect for spin $3/2$. We have clearly demonstrated the role of interaction with the spin $1/2$ field, the contribution of the terms with which to CVE is 6. Since the contribution from the Rarita-Schwinger field is -1, the overall coefficient in CVE is $6-1=5$, which corresponds to the recent prediction of a gauge chiral anomaly for spin $3/2$. The obtained values for the coefficients μ^2 and T^2 are proportional to each other, but not proportional to the spin, which indicates a possible new universality between the temperature-related and the chemical potential-related vortical effects. The results obtained allow us to speculate about the relationship between the gauge and gravitational chiral anomalies.

DOI: 10.1103/PhysRevD.105.L041701

I. INTRODUCTION

The Rarita-Schwinger spin $3/2$ theory is an essential element of supergravity theories [1] and grand unification models [2], in which it is used for anomaly cancellation. Rarita-Schwinger fields are also used to describe hadronic resonances [3] and have applications in solid state physics when describing Rarita-Schwinger-Weyl semimetals [4].

However, the Rarita-Schwinger theory of fields is characterized by a number of pathologies [5–8], in particular, the singular Dirac bracket turns out to be in the weak-field limit and there is the discontinuity in the number of degrees of freedom when an external field is present. These problems were overcome in [7] by introducing a field with spin $1/2$, which ultimately made it possible to construct a consistent quantum field perturbation theory and calculate the chiral quantum anomaly. An interesting observation is that the coefficient in the chiral anomaly turned out to be 5, which is different from the previous calculations for spin $3/2$.

The question that interests us in this work is the manifestation of quantum anomalies in hydrodynamics. In particular, it was shown in a number of works that the chiral vortical effect (CVE) is directly related to the chiral quantum anomaly [9–15]. Namely, the coefficient μ^2 in the mean value of the axial current in a medium with vorticity corresponds to the coefficient in the chiral anomaly

$$\text{CVE:}\ \langle \hat{j}_A^\nu \rangle = (AT^2 + C\mu^2)\omega^\nu,$$
$$\text{Anomaly:}\ \langle \partial_\mu \hat{j}_A^\mu \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{1.1}$$

where $\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$ is the vorticity, u_μ is the 4-velocity of the fluid, μ is the chemical potential and T is the temperature. This relationship has been well studied in the case of spin $1/2$, for which

$$\text{CVE:}\ \langle \hat{j}_A^\nu \rangle = \left(\frac{1}{6}T^2 + \frac{1}{2\pi^2}\mu^2\right)\omega^\nu,$$
$$\text{Anomaly:}\ \langle \partial_\mu \hat{j}_A^\mu \rangle = -\frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{1.2}$$

Recently a test of the connection with the anomaly was carried out for spin $3/2$ for another phenomenon in an

[*] prokhorov@theor.jinr.ru
[†] teryaev@jinr.ru
[‡] vzakharov@itep.ru

external magnetic field, the chiral separation effect (CSE) [16]. The direct verification in the case of higher spins also in the case of CVE (1.1) would be a reliable test of the relationship between the CVE and the chiral anomaly.

In this work, we for the first time obtain the CVE for spin 3/2 within the framework of the theory [7]. We will explicitly demonstrate its correspondence with the quantum anomaly. However, as we will show, this correspondence is achieved in a nontrivial way, in which an essential role is played by the interaction with the additional field with spin 1/2.

We use the notations $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, $\epsilon^{0123} = 1$, in the rest frame $u_\mu = (1,0,0,0)$, and we use the system of units $e = \hbar = c = k_B = 1$.

II. THE THEORY OF RARITA-SCHWINGER FIELD COUPLED TO A FIELD WITH SPIN 1/2

In this section, we present the main relations for the theory of spin 3/2 field, interacting with spin 1/2 field [7] (see also [8]). The action has the form

$$S = \int d^4x(-\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu\partial_\nu\psi_\rho + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - im\bar{\lambda}\gamma^\mu\psi_\mu + im\bar{\psi}_\mu\gamma^\mu\lambda), \tag{2.1}$$

where ψ_μ is the Rarita-Schwinger field, λ is the field with spin 1/2, m is the interaction constant.

To calculate the stress-energy tensor, it is necessary to go to a curved space-time with an arbitrary metric $g_{\mu\nu}$ and vary the action with respect to the metric.As a result, we obtain [6,17]

$$\begin{aligned} T^{\mu\nu} = &\frac{1}{2}\varepsilon^{\lambda\alpha\beta\rho}\bar{\psi}_\lambda\gamma_5(\gamma^\mu\delta^\nu_\alpha + \gamma^\nu\delta^\mu_\alpha)\partial_\beta\psi_\rho \\ &+ \frac{1}{8}\partial_\eta(\varepsilon^{\lambda\alpha\beta\rho}\bar{\psi}_\lambda\gamma_5\gamma_\alpha([\gamma^\eta,\gamma^\mu]\delta^\nu_\beta + [\gamma^\eta,\gamma^\nu]\delta^\mu_\beta)\psi_\rho) \\ &+ \frac{i}{4}(\bar{\lambda}\gamma^\nu\partial^\mu\lambda - \partial^\mu\bar{\lambda}\gamma^\nu\lambda + \bar{\lambda}\gamma^\mu\partial^\nu\lambda - \partial^\nu\bar{\lambda}\gamma^\mu\lambda) \\ &+ \frac{i}{2}m(\bar{\psi}^\mu\gamma^\nu\lambda - \bar{\lambda}\gamma^\mu\psi^\nu + \bar{\psi}^\nu\gamma^\mu\lambda - \bar{\lambda}\gamma^\nu\psi^\mu). \end{aligned} \tag{2.2}$$

Currents can be constructed from Noether's theorem

$$\begin{aligned} j^\mu &= i\varepsilon^{\lambda\rho\nu\mu}\bar{\psi}_\lambda\gamma_5\gamma_\nu\psi_\rho + \bar{\lambda}\gamma_\mu\lambda, \\ j^\mu_A &= -i\varepsilon^{\lambda\rho\nu\mu}\bar{\psi}_\lambda\gamma_\nu\psi_\rho + \bar{\lambda}\gamma_\mu\gamma_5\lambda. \end{aligned} \tag{2.3}$$

It is easy to check that

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu j^\mu = 0, \qquad \partial_\mu j^\mu_A = 0. \tag{2.4}$$

The conservation of the axial current is violated by the chiral quantum anomaly, calculated in [7] in the limit $m \to \infty$

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{5}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{2.5}$$

Note that the coefficient 5 recently found in [7] is different from the previous evaluations, and is associated with the interaction with the field λ. However, one can see, that it is equal to the sum of the previously known contributions of a nonghost part of free spin 3/2 and free spin 1/2 fields.

Propagators at finite temperature can be constructed according to the standard procedure [18]. When passing to the finite temperature $T = |\beta|^{-1}$, it is convenient to introduce new notations

$$\begin{aligned} &\mathcal{L}_E(\tau) = -\mathcal{L}_M(t = -i\tau), \qquad \gamma_\mu = i^{\delta_{0\mu}-1}\tilde{\gamma}_\mu, \tilde{\gamma}_5 = \gamma_5, \\ &\partial_\mu = i^{\delta_{0\mu}}\tilde{\partial}_\mu, \qquad \psi_\mu = i^{\delta_{0\mu}}\tilde{\psi}_\mu, P^\pm_\mu = (p^\pm_n, -\mathbf{p}), \\ &p^\pm_n = \pi(2n+1)/|\beta| \pm i\mu \quad (n = 0, \pm 1, \pm 2, \cdots), \\ &X_\mu = (\tau, -\mathbf{x}), \qquad \sum\!\!\!\!\!\!\int_P = \frac{1}{|\beta|}\sum_{n=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}, \\ &\not{P} = P_\mu\tilde{\gamma}_\mu, \qquad (P^+)^2 = P^+_\mu P^+_\mu. \end{aligned} \tag{2.6}$$

The propagators at finite temperature have a form similar to the real-time form in [7]. Substituting the Fourier transforms, we obtain the Euclidean action

$$\begin{aligned} S_E &= \sum\!\!\!\!\!\!\int_P (\,\tilde{\bar{\psi}}_\lambda(P) \quad \tilde{\bar{\lambda}}(P)\,)\mathcal{M}\begin{pmatrix} \tilde{\psi}_\rho(P) \\ \tilde{\lambda}(P) \end{pmatrix}, \\ \mathcal{M} &= \begin{pmatrix} \epsilon^{\lambda\rho\mu\nu}\gamma_5\tilde{\gamma}_\mu iP^+_\nu & m\tilde{\gamma}_\lambda \\ -m\tilde{\gamma}_\lambda & i\not{P}^+ \end{pmatrix}. \end{aligned} \tag{2.7}$$

The propagators are defined by the elements of the inverse matrix $\mathcal{N}$, for which $\mathcal{M}\mathcal{N} = \begin{pmatrix} \delta_{\lambda\eta} & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{aligned} &\langle T_\tau \tilde{\psi}_{a\mu}(X_1)\tilde{\bar{\psi}}_{b\nu}(X_2)\rangle_T = \sum\!\!\!\!\!\!\int_P e^{iP^+_\alpha(X_1-X_2)^\alpha}\frac{i}{2(P^+)^2} \\ &\quad\times\left(\tilde{\gamma}_\nu\not{P}^+\tilde{\gamma}_\mu + 2\left[\frac{1}{m^2} - \frac{2}{(P^+)^2}\right]P^+_\mu P^+_\nu\not{P}^+\right)_{ab}, \\ &\langle T_\tau \lambda_a(X_1)\tilde{\bar{\psi}}_{b\mu}(X_2)\rangle_T = \sum\!\!\!\!\!\!\int_P e^{iP^+_\alpha(X_1-X_2)^\alpha}\frac{P^+_\mu\not{P}^+_{ab}}{m(P^+)^2}, \\ &\langle T_\tau \tilde{\psi}_{a\mu}(X_1)\bar{\lambda}_b(X_2)\rangle_T = \sum\!\!\!\!\!\!\int_P e^{iP^+_\alpha(X_1-X_2)^\alpha}\frac{-P^+_\mu\not{P}^+_{ab}}{m(P^+)^2}, \\ &\langle T_\tau \lambda_a(X_1)\bar{\lambda}_b(X_2)\rangle_T = 0. \end{aligned} \tag{2.8}$$

where μ, ν are Lorentz indices and a, b are bispinor indices, and T_τ means ordering by the imaginary time τ. When deriving (2.8), we assumed that the subsystems of the fields with spin 3/2 and 1/2 are in equilibrium and $\mu_\psi = \mu_\lambda = \mu$. Finally, we note that following [7,8], the ghost fields should be considered nonpropagating and noninteracting with the rest of the fields, due to which ghosts do not contribute to the quantities we are considering.

III. CHIRAL VORTICAL EFFECT FOR SPIN 3/2

The properties of the medium in the state of global thermodynamic equilibrium are described by the density operator of Zubarev [19–22]

$$\hat{\rho} = \frac{1}{Z}\exp\left\{-\beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu}_x + \zeta\hat{Q}\right\}, \quad (3.1)$$

where $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$ is the thermal vorticity tensor, $\zeta = \frac{\mu}{T}$, $\hat{P}^\mu$ is the 4-momentum operator, $\hat{Q}$ is the charge operator, and $\hat{J}^{\mu\nu}_x$ are the Lorentz transformation generators shifted by the vector x^μ, which are expressed in terms of the shifted operators of the stress-energy tensor

$$\hat{J}^{\mu\nu}_x = \int d\Sigma_\lambda[y^\mu\hat{T}^{\lambda\nu}_x(y) - y^\nu\hat{T}^{\lambda\mu}_x(y)]. \quad (3.2)$$

where $d\Sigma_\lambda$ is a volume element on an arbitrary spacelike hypersurface (arbitrariness of the hypersurface follows from the conditions of global thermodynamic equilibrium). Density operator (3.1) provides a universal and fundamental approach to the description of effects in a relativistic moving and charged medium [21–27]. In particular, it was used to find a lot of chiral effects [20,28], corrections to them [25], and also to prove the Unruh effect from the point of view of statistics as well as the duality between statistics and field theory in a space with a conical singularity [21,23,29,30].

In a particular case of rigidly rotating medium, the operator (3.1) can be transformed to the more well-known form of the density operator for an equilibrium rotating medium [31–33].

The mean value of the local operator $\hat{O}(x)$ can be obtained using the perturbative expansion for (3.1)

$$\langle\hat{O}(x)\rangle = \langle\hat{O}(0)\rangle_{\beta(x)} + \sum_{N=1}^{\infty}\frac{\varpi^N}{2^N|\beta|^N N!}\int_0^{|\beta|}d\tau_1 d\tau_2 \ldots d\tau_N \times\langle T_\tau\hat{J}_{-i\tau_1 u}\ldots\hat{J}_{-i\tau_N u}\hat{O}(0)\rangle_{\beta(x),c}, \quad (3.3)$$

where each tensor $\hat{J}^{\mu\nu}$ is convolved with one tensor $\varpi_{\mu\nu}$. Connected correlators are taken, this fact is reflected in the subscript c, and the subscript $\beta(x)$ means that the mean values are taken at $\varpi = 0$.

For the axial current $\hat{j}^\mu_A$ in the first order of the perturbation theory formulas (3.1)–(3.3) give (see [20,25] for details)

$$\langle\hat{j}^\mu_A\rangle^{(1)} = W\omega^\mu, W = C^{023|1} - C^{013|2},$$
$$C^{\alpha\beta\gamma|i} = \int_0^{|\beta|}d\tau\int d^3x x^i\langle T_\tau\hat{T}^{\alpha\beta}(-i\tau,\mathbf{x})\hat{j}^\gamma_A(0)\rangle_{T,c}, \quad (3.4)$$

where the scalar coefficient W can be evaluated in the rest frame $\beta_\mu = (T^{-1},0,0,0)$, which is expressed by the subscript T, and in the following we denote $\hat{T}^{\alpha\beta}(-i\tau,\mathbf{x}) \to \hat{T}^{\alpha\beta}(\tau,\mathbf{x})$. Now the main goal is to find the correlators of the form $C^{\alpha\beta\gamma|i}$. To do this, let us first split $\hat{T}^{\mu\nu}$ and $\hat{j}^\mu_A$ into terms with a different set of fields

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\bar{\psi}\psi} + \hat{T}^{\mu\nu}_{\bar{\lambda}\lambda} + \hat{T}^{\mu\nu}_{\bar{\psi}\lambda} + \hat{T}^{\mu\nu}_{\bar{\lambda}\psi},$$
$$\hat{j}^\mu_A = \hat{j}^\mu_{A\bar{\psi}\psi} + \hat{j}^\mu_{A\bar{\lambda}\lambda}, \quad (3.5)$$

where the notation is obvious. Then we get that W is split into 8 terms depending on the set of the fields

$$W = W_{\bar{\psi}\psi\bar{\psi}\psi} + W_{\bar{\psi}\psi\bar{\lambda}\lambda} + W_{\bar{\lambda}\lambda\bar{\psi}\psi} + W_{\bar{\lambda}\lambda\bar{\lambda}\lambda} + W_{\bar{\psi}\lambda\bar{\psi}\psi} + W_{\bar{\psi}\lambda\bar{\lambda}\lambda} + W_{\bar{\lambda}\psi\bar{\psi}\psi} + W_{\bar{\lambda}\psi\bar{\lambda}\lambda}, \quad (3.6)$$

where the first two indices denote fields in $\hat{T}^{\mu\nu}$, and the second two—in $\hat{j}^\mu_A$. From the equality $\langle\lambda\bar{\lambda}\rangle = 0$ it is obvious that $W_{\bar{\lambda}\lambda\bar{\lambda}\lambda} = W_{\bar{\lambda}\psi\bar{\lambda}\lambda} = W_{\bar{\psi}\lambda\bar{\lambda}\lambda} = 0$. Since we are interested in the limit of $m\to\infty$, it is also clear in advance that $W_{\bar{\psi}\psi\bar{\lambda}\lambda}, W_{\bar{\lambda}\lambda\bar{\psi}\psi} \to 0$ at $m\to\infty$. Thus, only three terms remain $W = W_{\bar{\psi}\psi\bar{\psi}\psi} + W_{\bar{\psi}\lambda\bar{\psi}\psi} + W_{\bar{\lambda}\psi\bar{\psi}\psi}$.

All operators are to be presented in split form. The term $W_{\bar{\psi}\psi\bar{\psi}\psi}$ is expressed through the operators

$$\hat{T}^{\sigma\tau}_{\bar{\psi}\psi}(X) = \lim_{X_1,X_2\to X} i^{\delta_{0\eta}+\delta_{0\xi}}\mathcal{D}^{\sigma\tau\eta\xi}_{(\bar{\psi}\psi)ab}\tilde{\bar{\psi}}_{\eta a}(X_1)\tilde{\psi}_{\xi b}(X_2),$$
$$\mathcal{D}^{\sigma\tau\eta\xi}_{(\bar{\psi}\psi)}(\partial_{X_1},\partial_{X_2}) = \frac{1}{2}i^{1-\delta_{0\sigma}+\delta_{0\beta}}\varepsilon^{\eta\xi\tau\beta}\bigg(\gamma_5\tilde{\gamma}_\sigma\tilde{\partial}^{X_2}_\beta - \frac{1}{4}\gamma_5\tilde{\gamma}_\beta[\tilde{\gamma}_\vartheta,\tilde{\gamma}_\sigma](\tilde{\partial}^{X_1}_\vartheta + \tilde{\partial}^{X_2}_\vartheta)\bigg) + (\sigma\leftrightarrow\tau),$$
$$\hat{j}^\sigma_{A\bar{\psi}\psi}(X) = \lim_{X_1,X_2\to X} i^{\delta_{0\eta}+\delta_{0\xi}}\mathcal{J}^{\sigma\eta\xi}_{A(\bar{\psi}\psi)ab}\tilde{\bar{\psi}}_{\eta a}(X_1)\tilde{\psi}_{\xi b}(X_2). \quad (3.7)$$

Using Wick's theorem, the mean value of four fields can be transformed into a product of two propagators Using the propagators (2.8), we obtain

$$C^{\alpha\beta\gamma|i}_{(\bar{\psi}\psi\bar{\psi}\psi)} = \int_0^{|\beta|}d\tau\int\frac{d^3x d^3p d^3q}{(2\pi)^6}x^i\frac{1}{|\beta|^2}\sum_{\substack{p_n=\pi(2n+1)\\ q_l=\pi(2l+1)}}\frac{-1}{4(P^+)^2(Q^-)^2}i^{\delta_{0\eta}+\delta_{0\xi}+\delta_{0\rho}+\delta_{0\lambda}}e^{i(p_n+q_l)\tau}e^{-i(\mathbf{p}+\mathbf{q})x}$$
$$\times\operatorname{tr}\left[\mathcal{D}^{\alpha\beta\eta\xi}_{(\bar{\psi}\psi)}(iQ^-,iP^+)\left\{\tilde{\gamma}_\rho\not{P}^+\tilde{\gamma}_\xi + 2\left[\frac{1}{m^2} - \frac{2}{(P^+)^2}\right]P^+_\rho P^+_\xi\not{P}^+\right\}\mathcal{J}^{\gamma\rho\lambda}_{A(\bar{\psi}\psi)}\left\{\tilde{\gamma}_\eta\not{Q}^-\tilde{\gamma}_\lambda + 2\left[\frac{1}{m^2} - \frac{2}{(Q^-)^2}\right]Q^-_\eta Q^-_\lambda\not{Q}^-\right\}\right]. \quad (3.8)$$

Summation over the Matsubara frequencies should be made taking into account the poles $((p_n \pm i\mu)^2 + E^2)^{-r}$, where $r = 1,\ 2$, according to the formulas from Appendix A.4 of [22],[1] in particular, for the pole $r = 1$

$$\frac{1}{|\beta|}\sum_{p_n}\frac{(p_n \pm i\mu)^k e^{i(p_n\pm i\mu)\tau}}{(p_n \pm i\mu)^2 + E^2} = \frac{1}{2E}\sum_{s=\pm 1}(-isE)^k e^{\tau s E}[\theta(-s\tau) - n_F(E \pm s\mu)], \qquad (3.9)$$

where $n_F(E) = (1 + e^{E/T})^{-1}$ is the Fermi-Dirac distribution. The explicit dependence on the coordinate x^i can be absorbed into the derivative of the exponent. After that, the integration and summation over one of the momenta is removed by the delta function. Finally, integration over the angles at $d^3p = \sin(\vartheta)p^2 dp d\phi d\vartheta$ and integration over τ and differentiation $\frac{\partial}{\partial q^i}$ can be done directly. As a result, we get that each of the coefficients is expressed as a combination of an infinite and a finite momentum integrals, for example

$$W_{\bar\psi\psi\bar\psi\psi} = -\frac{2}{3\pi^2}\int_0^\infty p dp + \int_0^\infty \frac{dp}{\pi^2 T} \times \left(-\frac{2p^3}{3T}[n_F(x)^3 + n_F(y)^3] + \left(\frac{p^3}{T} - \frac{p^2}{6}\right)[n_F(x)^2 + n_F(y)^2] + \left(-\frac{p^3}{3T} + \frac{p^2}{6} + \frac{2pT}{3}\right)[n_F(x) + n_F(y)]\right), \qquad (3.10)$$

where $x = p + \mu$, $y = p - \mu$. The finite parts can be found analytically as they are expressed in terms of polynomial combinations of polylogarithms [29,34]

$$W_{\bar\psi\psi\bar\psi\psi} = -\frac{2}{3\pi^2}\int_0^\infty p dp - \frac{T^2}{6} - \frac{\mu^2}{2\pi^2},$$
$$W_{\bar\psi\lambda\bar\psi\psi} = -\frac{1}{3\pi^2}\int_0^\infty p dp + \frac{T^2}{2} + \frac{3\mu^2}{2\pi^2},$$
$$W_{\bar\lambda\psi\bar\psi\psi} = \frac{1}{\pi^2}\int_0^\infty p dp + \frac{T^2}{2} + \frac{3\mu^2}{2\pi^2}. \qquad (3.11)$$

Despite the fact that each of the terms has an ultraviolet divergence, the sum is finite. Thus, ultraviolet divergences appear at intermediate stages of calculations, but mutually cancel out between different contributions in the final formula for the physical effect

$$W_{\bar\psi\psi\bar\psi\psi} + W_{\bar\psi\lambda\bar\psi\psi} + W_{\bar\lambda\psi\bar\psi\psi} = \frac{5T^2}{6} + \frac{5\mu^2}{2\pi^2}, \qquad (3.12)$$

where $n_F(E)' = \frac{d}{dE}n_F(E)$. As a result, we obtain the following expression for the axial current, which corresponds exactly to the chiral anomaly (2.5)

$$\langle \hat{j}^\nu_A \rangle^{(1)} = \left(\frac{5T^2}{6} + \frac{5\mu^2}{2\pi^2}\right)\omega^\nu. \qquad (3.13)$$

In this case, the coefficients 5 in the terms T^2 and μ^2 were obtained as a result of summation $6 - 1 = 5$, where 6 is the contribution of the interaction terms, and -1 is the contribution of the pure Rarita-Schwinger field. This distinguishes the above calculation from the calculation of the chiral anomaly and CSE in [7,16], where the additional field did not contribute. Thus, the anomaly is reconstructed, but in a qualitatively different way.

IV. DISCUSSION: QUANTUM ANOMALIES IN HYDRODYNAMICS

We have shown an exact correspondence between hydrodynamics and quantum field theory: the coefficient in front of the chiral anomaly (2.5) corresponds to the coefficient in CVE (3.13). Such a correspondence of the two theories is not accidental and was predicted in a number of papers [9,10]. Our result confirms the accuracy of the predictions made not only in the case of spin $1/2$, but also for higher spins and demonstrates how this correspondence is realized at the level of microscopic theory.

In particular, it was shown in [10,35] that hydrodynamics can be considered as an effective field theory with additional interaction corresponding to the substitution

$$A_\nu \to A_\nu + \mu \cdot u_\nu. \qquad (4.1)$$

Using the well-known expression for the chiral anomaly, but now for the effective field (4.1), one can clearly obtain a number of chiral phenomena. It is necessary to use the substitution (4.1) in the anomaly (1.1) and collect all the additional terms into the divergence of the effective current

$$\partial_\nu(n_5 u^\nu + C\mu^2\omega^\nu + C\mu B^\nu) = -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \qquad (4.2)$$

The second term on the left-hand side of (4.2) corresponds to CVE, and the third—to the CSE.

The same relationship was substantiated from the point of view of the equations of relativistic hydrodynamics and the second law of thermodynamics in [9]. Equation (2.5) is to be included into the system of equations of hydrodynamics. From the condition of non-negativity of the divergence of the entropy current it follows that the currents arise, directly related to the anomaly.

[1] G.P. thanks M. Buzzegoli for discussing this issue.

Both [9,10] approaches deal with the μ^2 term, while the temperature term T^2 and the coefficient A in (1.1) are assumed to be associated with either the gravitational anomaly [11–13,34,36] (see also [37,38] on transport phenomena related to the gravitational anomaly) or the global one [39]. In particular, [34] considers radiation from an analogue of a rotating black hole, a quantum anomaly on the horizon of which serves as a pump that creates an anomalous axial current. Such a relationship between the T^2 term and the gravitational anomaly has been verified for the case of spin $1/2$ [34], and recently for spin 1 [13].

Now we could hypothesize that the gravitational chiral anomaly is also 5 times higher for system of interacting spin $3/2$ and spin $1/2$ fields than for spin $1/2$ and, moreover, there is a sign of a possible relationship between different types of chiral anomalies. An indication of the possible existence of such a relationship between gravitational and gauge anomalies was recently found in a completely different context in [40]. However, the peculiarities of the theory of higher spins may lead to a very nontrivial realization of the gravitational anomaly in hydrodynamics, and it is possible, that there is no such a direct relationship between the coefficients.

On the other hand, the comparison of (3.13) and (1.2) indicates the existence of possible universality $A \sim C$ of the vortical effects associated with temperature $\langle \hat{j}_A^\nu \rangle = A \cdot T^2 \omega^\nu$ and with chemical potential $\langle \hat{j}_A^\nu \rangle = C \cdot \mu^2 \omega^\nu$. A similar proportionality of the two coefficients also follows from the analysis of the semiclassical equations of motion for free particles in a rotating frame and the chiral kinetic theory [41]. In this approach the relation $A \sim C \sim S$ follows from the spin-vorticity effective coupling $\vec{S} \cdot \vec{\Omega}$. Now we see that the proportionality of the coefficients $A \sim C$ also arises in a different context for a system of two interacting quantum fields. Thus, the universality $A \sim C$ is probably a more general phenomenon than $A \sim C \sim S$.

We also note that the result [41] for the CVE for arbitrary spin is also in agreement with the quantum anomaly for free fermions obtained in [42], where the coefficient in the anomaly is also proportional to the spin.

In [16] another phenomenon, the CSE, was calculated in the framework of the theory [7], and the result was

$$\mathrm{CSE}: \langle \hat{j}_A^\nu \rangle = \frac{5\mu}{2\pi^2} B^\nu, \tag{4.3}$$

where B^μ is the magnetic field. Thus, the CSE also satisfies the chiral anomaly (2.5). Technically the correspondence between the CSE and the anomaly is clear, since both of them can be described by the same diagrams, but with the replacement of one of the fields in one of the vertices by the chemical potential in the case of CSE. In the case of CVE, similar reasoning cannot be used, since the operators of the stress-energy tensor are located in the vertices instead of the current operators.

The relationship between CSE and anomalies (2.5) follows from [9,10] and also from the recent analysis in [22]. The conditions of global thermodynamic equilibrium fix the chemical potential

$$\zeta(x) = \zeta_0 - \beta_\sigma F^{\lambda\sigma} x_\lambda + \frac{1}{2} \varpi_{\sigma\rho} x^\rho F^{\lambda\sigma} x_\lambda, \tag{4.4}$$

where $\zeta = \frac{\mu}{T}$. Using (4.4) for differentiating (4.3) results in

$$\langle \partial_\nu \hat{j}_A^\nu \rangle = \frac{5}{2\pi^2} T(\partial_\nu \zeta(x)) B^\nu = -\frac{5}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}. \tag{4.5}$$

V. CONCLUSION

We consider the extended Rarita-Schwinger-Adler theory [7] at finite temperature, finite chemical potential, and nonzero vorticity. We calculated the chiral vortical effect in this theory and verified that the coefficient in front of the μ^2 term corresponds to the coefficient 5 in the chiral anomaly. This is achieved by summing of the contribution of the interaction terms equal to 6, and the contribution from only the Rarita-Schwinger field equal to -1. There is a cancellation of ultraviolet divergences between different contributions to CVE, each of which diverges separately. We discussed the possible consequences for the gravitational anomaly.

Comparison of the obtained formulas for CVE with the case of spin $1/2$ suggests the existence of a new universality between the coefficients $A \sim C$ of the vortical effects associated with temperature and with chemical potential.

ACKNOWLEDGMENTS

The authors are thankful to A. I. Vainshtein and M. Buzzegoli for valuable discussions. The work was supported by Russian Science Foundation Grant No. 21-12-00237, the work of V. I. Z. is partially supported by Grant No. 0657-2020-0015 of the Ministry of Science and Higher Education of Russia.

[1] J. A. Strathdee, Extended Poincare supersymmetry, Int. J. Mod. Phys. A **02**, 273 (1987).
[2] S. L. Adler, SU(8) family unification with boson-fermion balance, Int. J. Mod. Phys. A **29**, 1450130 (2014).
[3] V. Pascalutsa and R. Timmermans, Field theory of nucleon to higher spin baryon transitions, Phys. Rev. C **60**, 042201 (R) (1999).
[4] I. Boettcher, Interplay of Topology and Electron-Electron Interactions in Rarita-Schwinger-Weyl Semimetals, Phys. Rev. Lett. **124**, 127602 (2020).
[5] G. Velo and D. Zwanziger, Propagation and quantization of Rarita-Schwinger waves in an external electromagnetic potential, Phys. Rev. **186**, 1337 (1969).
[6] S. L. Adler, Classical gauged massless Rarita-Schwinger fields, Phys. Rev. D **92**, 085022 (2015).
[7] S. L. Adler, Analysis of a gauged model with a spin-$\frac{1}{2}$ field directly coupled to a Rarita-Schwinger spin-$\frac{3}{2}$ field, Phys. Rev. D **97**, 045014 (2018).
[8] S. L. Adler and P. Pais, Chiral anomaly calculation in the extended coupled Rarita-Schwinger model, Phys. Rev. D **99**, 095037 (2019).
[9] D. T. Son and P. Surowka, Hydrodynamics with Triangle Anomalies, Phys. Rev. Lett. **103**, 191601 (2009).
[10] A. V. Sadofyev, V. I. Shevchenko, and V. I. Zakharov, Notes on chiral hydrodynamics within effective theory approach, Phys. Rev. D **83**, 105025 (2011).
[11] K. Landsteiner, E. Megias, and F. Pena-Benitez, Gravitational Anomaly and Transport, Phys. Rev. Lett. **107**, 021601 (2011).
[12] G. Yu. Prokhorov, O. V. Teryaev, and V. I. Zakharov, Chiral vortical effect: Black-hole versus flat-space derivation, Phys. Rev. D **102**, 121702(R) (2020).
[13] G. Yu Prokhorov, O. V. Teryaev, and V. I. Zakharov, Chiral vortical effect for vector fields, Phys. Rev. D **103**, 085003 (2021).
[14] V. E. Ambrus and M. N. Chernodub, Helical vortical effects, helical waves, and anomalies of Dirac fermions, **12** (2019).
[15] P. G. Mitkin and V. I. Zakharov, Chiral vortical effect in accelerated matter, Phys. Rev. D **104**, 125011 (2021).
[16] Z. V. Khaidukov and R. A. Abramchuk, Chiral separation effect for spin 3/2 fermions, J. High Energy Phys. 07 (2021) 183.
[17] A. K. Das, On the stress tensor in a class of gauge theories, Phys. Rev. D **18**, 2065 (1978).
[18] M. Laine and A. Vuorinen, Basics of thermal field theory, Lect. Notes Phys. **925**, 1 (2016).
[19] D. N. Zubarev, A. V. Prozorkevich, and S. A. Smolyanskii, Derivation of nonlinear generalized equations of quantum relativistic hydrodynamics, Teor. Mat. Fiz. **40**, 394 (1979).
[20] M. Buzzegoli, E. Grossi, and F. Becattini, General equilibrium second-order hydrodynamic coefficients for free quantum fields, J. High Energy Phys. 10 (2017) 091; Erratum, J. High Energy Phys. 07 (2018) 119.
[21] G. Y. Prokhorov, O. V. Teryaev, and V. I. Zakharov, Unruh effect for fermions from the Zubarev density operator, Phys. Rev. D **99**, 071901(R) (2019).
[22] M. Buzzegoli, Thermodynamic equilibrium of massless fermions with vorticity, chirality and magnetic field, Other thesis, **4** (2020).
[23] G. Y. Prokhorov, O. V. Teryaev, and V. I. Zakharov, Unruh effect universality: Emergent conical geometry from density operator, J. High Energy Phys. 03 (2020) 137.
[24] F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Local Polarization and Isothermal Local Equilibrium in Relativistic Heavy Ion Collisions, Phys. Rev. Lett. **127**, 272302 (2021).
[25] G. Y. Prokhorov, O. V. Teryaev, and V. I. Zakharov, Effects of rotation and acceleration in the axial current: Density operator vs Wigner function, J. High Energy Phys. 02 (2019) 146.
[26] F. Becattini, M. Buzzegoli, and E. Grossi, Reworking the Zubarev's approach to non-equilibrium quantum statistical mechanics, Particles **2**, 197 (2019).
[27] A. Palermo, M. Buzzegoli, and F. Becattini, Exact equilibrium distributions in statistical quantum field theory with rotation and acceleration: Dirac field, J. High Energy Phys. 10 (2021) 077.
[28] M. Buzzegoli and F. Becattini, General thermodynamic equilibrium with axial chemical potential for the free Dirac field, J. High Energy Phys. 12 (2018) 002.
[29] G. Y. Prokhorov, O. V. Teryaev, and V. I. Zakharov, Thermodynamics of accelerated fermion gases and their instability at the Unruh temperature, Phys. Rev. D **100**, 125009 (2019).
[30] F. Becattini, Thermodynamic equilibrium with acceleration and the Unruh effect, Phys. Rev. D **97**, 085013 (2018).
[31] A. Vilenkin, Macroscopic parity violating effects: Neutrino fluxes from rotating black holes and in rotating thermal radiation, Phys. Rev. D **20**, 1807 (1979).
[32] A. Vilenkin, Quantum field theory at finite temperature in a rotating system, Phys. Rev. D **21**, 2260 (1980).
[33] V. E. Ambrus and E. Winstanley, Vortical effects for free fermions on anti-de Sitter space-time, **7** (2021).
[34] M. Stone and J. Kim, Mixed anomalies: Chiral vortical effect and the sommerfeld expansion, Phys. Rev. D **98**, 025012 (2018).
[35] V. I. Zakharov, Chiral magnetic effect in hydrodynamic approximation, Lect. Notes Phys. **871**, 295 (2013).
[36] S. Golkar and D. T. Son, (Non)-renormalization of the chiral vortical effect coefficient, J. High Energy Phys. 02 (2015) 169.
[37] A. Avdoshkin and R. Sharipov, Chiral transport in curved spacetime via holography, J. High Energy Phys. 08 (2021) 007.
[38] K. Kamada, J. Kume, and Y. Yamada, Chiral gravitational effect in time-dependent backgrounds, J. High Energy Phys. 05 (2021) 292.
[39] S. Golkar and S. Sethi, Global anomalies and effective field theory, J. High Energy Phys. 05 (2016) 105.
[40] G. E. Volovik, Analog of gravitational anomaly in topological chiral superconductors, JETP Lett. **113**, 546 (2021).
[41] X.-G. Huang and A. V. Sadofyev, Chiral vortical effect for an arbitrary spin, J. High Energy Phys. 03 (2019) 084.
[42] M. J. Duff, Ultraviolet divergences in extended supergravity, in *First School on Supergravity*, 1 (1982).