Letter

Constraining models of inflationary magnetogenesis with NANOGrav data

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(Received 5 March 2021; accepted 26 January 2022; published 18 February 2022)

Generation of magnetic field during inflation can explain its presence over a wide range of scales in the Universe. In [Sharma *et al* Phys. Rev. D **96**, 083511 (2017)], we proposed a model to generate these fields during inflation. These fields have nonzero anisotropic stress which lead to the generation of a stochastic background of gravitational waves (GW) in the early universe. Here we show that for a scenario of magnetogenesis where reheating takes place around QCD epoch, this stochastic GW background lies in the 95% confidence region of the stochastic common spectrum process probed by NANOGrav collaboration. This is the case when the generated electromagnetic field (EM) energy density is 3%–10% of the background energy density at the end of reheating. For this case, the values of magnetic field strength $B_0 \sim (0.7\text{–}1.4) \times 10^{-11}$ G and its coherence length ∼3 kpc at the present epoch. These values are for the models in which EM fields are of nonhelical nature. For the helical nature of the fields, these values are $B_0 \sim (2.1\text{–}3.8) \times 10^{-10}$ G and its coherence length ∼90 kpc.

DOI: 10.1103/PhysRevD.105.L041302

I. INTRODUCTION

Magnetic fields have been observed over a wide range of scales in the universe from planets, stars to galactic and extragalactic scales [1–5]. These fields are assumed to be generated by the amplification of seed fields via flux freezing evolution followed by a turbulent dynamo mechanism [6]. A number of scenarios of generation of seed magnetic fields have been suggested in literature such as generation during inflation [7–36], phase transitions [37–41], recombination, reionization and structure formation [42–46]. The importance of inflationary scenarios of magnetic field generation as against other mechanisms lies in the fact that the former gives a natural way of generating fields coherent on large length scales. A popular model of inflationary generation involves coupling of a time dependent function with the usual electromagnetic (EM) action. In particular Ratra [8] model takes the Lagarangian density of the form $f^2 F^{\mu\nu} F_{\mu\nu}$ where f is a function of inflaton field and $F_{\mu\nu}$ the electromagnetic field tensor. Although this model generates magnetic fields of sufficient strength to satisfy a number of observational constraints, it potentially suffers from backreaction and strong coupling problems [47]. It is also strongly constrained by the Schwinger effect which leads to the production of charged particles and arrests the growth of magnetic field [48].

In a recent study [49], we have suggested a scenario in which these problems can be circumvented at the cost of having a low scale inflation. In this model, the coupling function f increases during inflation starting from an initial value of unity and becomes very large at the end of inflation. Such an evolution of f is free from the above-mentioned problems. However, the coupling between the charges and EM field becomes very small at the end. To get back the standard EM theory we introduced a transition in the evolution of f immediately after the end of inflation during which time it decreases back to unity at about reheating epoch and after that f becomes a constant. During this postinflationary era both electric and magnetic energy density increase. By demanding that EM energy density should remain below the background energy density, we obtained a bound on reheating and inflationary scales. Our models can generate both nonhelical and helical magnetic fields and satisfy known observational constraints. They predict a blue spectrum for the magnetic field energy density peaked at small length scales, typically a fraction of the Hubble radius at reheating [49,50]. The generated field energy density can also be a significant fraction of the energy density of the Universe at those epochs.

The anisotropic stresses associated with such primordial EM fields lead to the production of a stochastic gravitational wave background. In a recent study, we have estimated the produced GW spectrum in such a scenario of inflationary magnetogenesis. Recently, the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) collaboration has reported evidence for a stochastic common spectrum process in the frequency range $[2.5 \times 10^{-9}\ \mathrm{Hz}, 7.0 \times 10^{-8}\ \mathrm{Hz}]$. Assuming that this signal is due to a stochastic background of GW, there have

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been various suggestions for their origin. These include mergers of supermassive black holes [51–53] or scenarios involving cosmic string [54–58], primordial black holes [59–65], phase transitions [66–68] and magnetohydrodynamics turbulence during the QCD phase transition [69,70] and others [71–76]. In this work, we focus on the GW spectrum produced in our model of inflationary magnetogenesis where reheating takes place around QCD epoch (150 MeV) and compare the predicted signals with those reported by NANOGrav Collaboration.

The paper is organized as follows. In Sec. II we summarize the GW background generated in our models of inflationary magnetogenesis. In Sec. III, we compare the results of our model with the reported evidence of a stochastic common spectrum process by NANOGrav. The last section IV contains a discussion of our results and conclusions.

II. GRAVITATIONAL WAVES PRODUCED BY ELECTROMAGENTIC FIELD GENERATED DURING INFLATION

Gravitational waves are represented by the transverse traceless part of the metric perturbations. Any source which has nonzero transverse and traceless part in its energy momentum tensor can lead to the production of gravitational waves in the early universe. In our case such a source is the electromagnetic field, generated during inflation and further during the reheating era. The stochastic GW spectrum results from this source was estimated in our earlier study [77]. Here we give a summary relevant for the current work (see Sharma *et al* [77] for details).

We consider a FLRW spacetime for the background geometry in the early universe. The metric including the tensor perturbations can be expressed as,

$$ds^2 = a^2(\eta)(-d\eta^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j).$$

Here x^i represents the comoving coordinates for the space dimensions and η is the conformal time, $a(\eta)$ is the scale factor and h_{ij} represents transverse trace-less part of the metric perturbations. The spectrum of GW can be expressed in terms of the tensor perturbation as,

$$\left.\frac{d\Omega_{\rm GW}}{d\ln k}\right|_0 = \frac{k^3 a^2}{4(2\pi)^3 G\rho_{c_0}} \sum_{\aleph} \left(\left| \frac{dh^{\aleph}(k,\eta)}{d\eta} \right|^2 \right), \quad (1)$$

Here $d\Omega_{\rm GW}(k)$ is the energy density in GW as a function of the closure density ρ_c, in a logarithmic interval ($d\ln k$) in wave number space. Also, $\aleph$ represents the different polarization state of GW and $\aleph = T, \times$ or $\aleph = +, -$ depending on whether it is linearly or circularly polarized. The evolution of the tensor perturbation (h_{ij}) in presence of a source, is given by linearized Einstein's equation,

$$h''_{\aleph} + \frac{2a'}{a} h'_{\aleph} + k^2 h_{\aleph} = 8\pi G a^2 (\rho + p)\Pi_{\aleph}. \quad (2)$$

where $\Pi_{\aleph}$ is defined as $\Pi_{\aleph} \equiv [1/(\rho + p)]\bar{T}^{TT}_{\aleph}$ and $\bar{T}^{TT}_{\aleph}$ is the transverse traceless part of the energy-momentum tensor of the source.

In our earlier studies Refs. [49,50], to address the strong coupling and backreaction problems in $f^2 F^{\mu\nu} F_{\mu\nu}$ type models of inflationary inflationary magnetogenesis, we have taken a particular evolution of the coupling function, f which evolves with time both during as well as postinflation till reheating. This function increases as $f \propto a^2$ during inflation and transits to a decaying phase ($f \propto a^{-\beta}$) postinflation. We have assumed that Universe evolves as in a matter dominated era between the end of inflation and the beginning of reheating. After this matter dominated era, reheating takes place which is assumed to be instantaneous in our model and standard radiation dominance starts. During inflation the magnetic field spectrum is scale invariant but the strength is very low compared to the background energy density because of the low energy scale of inflation. In the postinflationary era when coupling function, f decreases, the scale invariant contribution to the magnetic spectrum decreases but contribution from the next order gets amplified on the superhorizon scales. This postinflationary era ends when the EM energy density is ϵ times the background energy density and after this reheating takes place and EM energy density evolves like radiation. The magnetic field spectrum generated in our model is a blue spectrum, $d\rho_B(k,\eta)/d\ln k \propto k^4$ and the spectrum peaks at a wave number approximately β times the value of Hubble parameter at reheating; here spectrum is defined such that $\int d\ln k (d\rho_B(k,\eta)/d\ln k)$ gives the total magnetic energy density (ρ_B). Such a model of inflationary magnetogenesis may be realized in a hybrid type of inflationary model by making the coupling to be a function of both the fields in hybrid model [78,79]. It may also be realized in those models where there is a long matter-dominated era due to the dominance of the moduli field after inflation, for example [80,81]. However, this requires a detailed analysis which we leave for future work.

The EM fields generated can lead to the production of GW. The main contribution to the GW energy spectrum is during the end phase of the postinflationary matter dominated era. During this era both electric and magnetic fields contribute to the production of GW. However after reheating, electric fields get shorted out because of the large conductivity of the universe and only magnetic field contributes to the production of GW.

In our case, the GW spectrum depends upon the expectation value of $\bar{T}_{ij}(\eta_1)\bar{T}^{ij}(\eta_2)$ at unequal times and Green function which takes the form $\cos(k(\eta_1 - \eta_2)/(\eta_1\eta_2))$ in the subhorizon limit in a radiation dominated universe. Further, we write the former as the expectation value of $\bar{T}_{ij}\bar{T}^{ij}$ at equal time and an unequal time correlation function of the EM fields. The GW spectrum has been obtained by numerically solving the expressions and the details are provided in Sec. IV in Sharma *et al.* [77] and it can be summarized as follows. The generated GW spectrum rises with wave number k as $d\Omega_{\rm GW}/d\ln(k) \propto k^3$, at low wave numbers. It remains almost k^3 until the wave number $k = k_{\rm peak}$ for $\epsilon = 1$, where ϵ denotes the fraction of EM energy density to the background energy density at reheating. However, for $\epsilon = 10^{-2}$, it changes to a shallower spectrum compared to k^3. The GW spectrum then falls for the modes $k > k_{\rm peak}$ as $d\Omega_{\rm GW}/d\ln(k) \propto k^{-5/3}$ for $\epsilon = 1$ and $d\Omega_{\rm GW}/d\ln(k) \propto k^{-8/3}$ for $\epsilon = 10^{-2}$. This change in the slope of the spectrum for different ϵ could arise due to the turbulence correlation time being longer for a smaller ϵ.

III. COMPARISON WITH THE NANOGRAV SIGNAL

The NANOGrav collaboration has recently reported evidence for a stochastic common spectrum process using 12.5 years of timing residual dataset of pulsars [51]. This result may be interpreted as a stochastic background of GW. The time residual cross power spectral density versus frequency dataset has 30 frequency components in the range $[2.5\times10^{-9}\ \text{Hz}, 7.0\times10^{-8}\ \text{Hz}]$. In the NANOGrav collaboration paper [51], the time residual cross power spectral density has been modeled as a simple power law and a broken power law in frequency and the 68% and 95% confidence regimes for the amplitude and the spectral index using the lowest five frequency components have been determined. For our analysis, it is the broken power law case that is relevant. For this case, we convert the modeled time residual cross power spectral density in terms of GW density fraction ($\Omega_{\rm GW}$) using Eq. (29) in Ref. [82],

$$\Omega_{\rm GW} = \frac{2\pi^2 A_{{\rm GW}\,B}^2 f_{yr}^2}{3H_0^2}\left(\frac{f}{f_{yr}}\right)^{5-\gamma}\left(1+\left(\frac{f}{f_{\rm bend}}\right)^{\frac{1}{\kappa}}\right)^{\kappa(\gamma-5-\delta)}. \tag{3}$$

Here $A_{{\rm GW}\,B}$ is the characteristic strain at $f = f_{yr}$, H_0 is the value of Hubble parameter today, γ and δ are the power law index at frequencies lower and higher than $f_{\rm bend} = 1.2\times10^{-8}$ Hz, respectively, and κ controls the smoothness of the transition. In Ref. [51] δ is taken to be zero and $\kappa = 0.1$. The 95% confidence contour in terms of the frequency spectrum for density fraction and frequency is shown in Fig. 1 in pink color. To determine this contour, we plot all the value of $A_{{\rm GW}\,B}$ and γ for the 95% confidence region and extract the maximum covered area. For 95% confidence region, $\gamma \in (3.1, 6.7)$. As it is evident from the left panel of Fig. 1 in Ref. [4] that the time residual has very large spread for $f > f_{\rm bend}$ compared to the value for $f < f_{\rm bend}$. Therefore, we do not include the confidence contour for $f > f_{\rm bend}$ while comparing with the GW signal obtained in a model. We plot the resulting GW spectrum for our model in the same figure for different values of the ratio of EM energy density to the background energy density (ϵ), and a reheating temperature $T_R = 150$ MeV.

From Fig. 1, we conclude that the GW produced in our model, for a scenario in which reheating temperature is

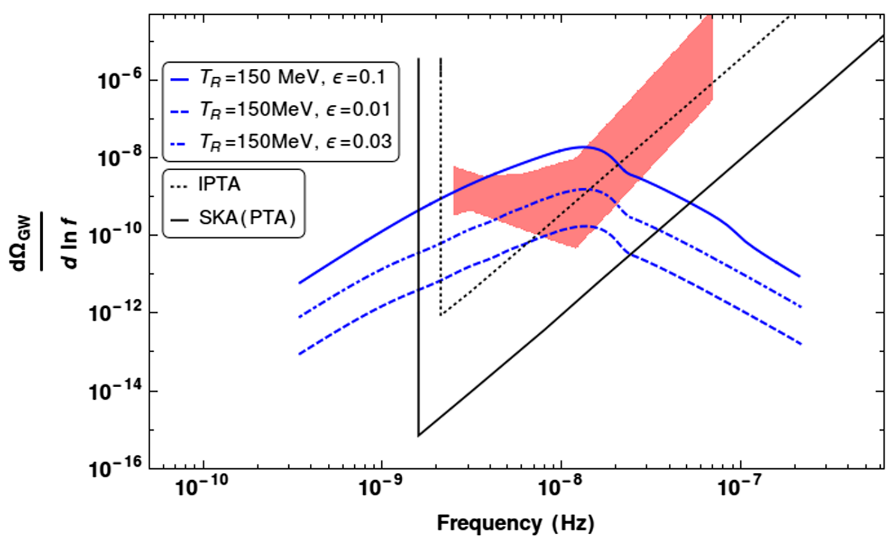

FIG. 1. In this figure, density fraction of gravitational waves in logarithmic frequency interval with frequency is shown. The blue curves are the GW spectrum obtained in our model of inflationary magnetogenesis for a scenario where reheating temperature $T_R = 150$ MeV for different value of the ratio of EM and background energy density (ϵ). Pink color region is 95% confidence region of the parameter space of the GW signal modeled as broken power law by NANOGrav collaboration [51]. Solid and dotted black curves represent the sensitivity curve of the international pulsar timing array and pulsar timing array with the upcoming mission square kilometer array (SKA), respectively Moore *et al.* [82].

$T_R \sim 150$ MeV, lies within the 95% confidence regime of parameter space of the signal, modeled as a broken power law, for $\epsilon = (0.03, 0.1)$.

Further, we estimate the magnetic field strength today consistent with the signal seen by the NANOGrav collaboration in [51]. After the generation, apart from the dilution due to the adiabatic expansion, magnetic fields also undergo turbulent decay due to the nonlinear processing. Including these effects, the magnetic field and its coherence length at the present epoch can be related to its strength and coherence length at the epoch of generation as follows,

$$B_0 \approx B_r \left(\frac{a_0}{a_r}\right)^{-2} \left(\frac{a_{\rm eq}}{a_r}\right)^{-p}, \qquad L_{c0} = L_c \left(\frac{a_0}{a_r}\right) \left(\frac{a_{\rm eq}}{a_r}\right)^{q} \tag{4}$$

Here a_0, $a_{\rm eq}$ and a_r represent the value of the scale factor at the present epoch, matter-radiation equality, and end of reheating, respectively and $L_c = (\beta H_r)^{-1}$ (where β is power law index in the functional form of the coupling function, $f \propto a^{-\beta}$ in the postinflationary matter dominated era and H_r the value of Hubble parameter at the end of reheating) is the coherence length of the magnetic field at the end of reheating. The procedure to determine the value of β is given in Appendix A of [83] and its value is approximately 2.7 for our case of interest. In the above expression, $B_r = \sqrt{2\rho_{\rm Br}}$ ($\rho_{\rm Br}$ is the magnetic energy density at the end of reheating) is the value of the magnetic field at the end of reheating. To estimate $\rho_{\rm Br}$, we calculate the ratio of the electric to magnetic energy density which depends on the value of β. This ratio is determined by integrating Eq. (41) and Eq. (42) of [49] from k_i to $k_r \approx \beta H_r$ and it turns out to be,

$$\frac{\rho_{\rm Br}}{\rho_{\rm Er}} \approx \frac{\beta^2}{2(2\beta + \frac{1}{2})^2}. \tag{5}$$

For $\beta > 1$ which is indeed the case, $\rho_{\rm Br}/\rho_{\rm Er} \approx 1/8$. As mentioned above, in our model, the total EM energy density is ϵ times the background energy density at reheating; $\rho_{\rm Er} + \rho_{\rm Br} = \epsilon g_r (\pi^2/30) T_r^4$. These imply $B_r \approx \sqrt{(2\epsilon/9) g_r (\pi^2/30) T_r^4}$. To determine $a_{\rm eq}/a_r$ and $a_{\rm eq}/a_r$, we use the relation $g^{1/3} a T = \text{const}$. This relation comes from the conservation of entropy due to the adiabatic evolution of the Universe. Using the above expression and taking $g_r = 62$, $T_r = 150$ MeV, $g_{\rm eq} = 3.96$, $T_{\rm eq} = 0.8$ ev, $g_0 = 3.96$ and $T_0 = 2.73K$, we get $B_0 \sim (0.7\text{–}1.4) \times 10^{-11}$ G for and its coherence length $\sim$3 kpc for the case when the GW spectrum from EM fields anisotropic stresses is consistent with the signal found by NANOGrav collaboration and $\beta \approx 3$. To calculate these numbers, we take $p = 0.5$ and $q = 0.5$ suggested by numerical simulation for the evolution of magnetic field in the early universe [84–86].

The strength of the generated GW spectrum in the case of helical EM fields is similar to the nonhelical case [77]. However, in the case of helical EM fields, the generated GW spectrum is circularly polarized while it is unpolarized when the EM fields are nonhelical. Assuming that fully helical EM fields explain the NANOGrav signal, we get $B_0 \sim (2.1\text{–}3.8) \times 10^{-10}$ G and its coherence length $\sim$(90) kpc. Here we use $p = 1/3$ and $q = 2/3$ for the evolution of helical magnetic field in the early universe [87–89]. Observational limits on CMB non-Gaussianity from the Planck mission set an upper limit of $B_0 \leq 0.6$ nG on the present value of the primordial cosmic magnetic field [90] and this limit has been obtained for a scale invariant spectrum of magnetic field.

IV. DISCUSSION AND CONCLUSION

The origin of magnetic fields observed in large scale structures is a question of astrophysical interest. Generation of these fields during inflation is an interesting possibility. These fields can be generated in a scenario suggested in Sharma *et al.* [49] which is free from the difficulties raised in the literature. The generated magnetic fields have non zero anisotopic stresses which lead to the production of stochastic background of gravitational waves. If reheating in our scenario takes place around the QCD epoch, the resulting GW background can be interpreted as leading to the signal inferred in NANOGrav 12.5 year data. This requires the EM field energy density to be in the range of 3% to 10% of the background energy density at generation. The magnetic fields consistent with the NANOGrav signal has a present day strength $B_0 \sim (0.7\text{–}1.4) \times 10^{-11}$ G and their coherence length $\sim$3 kpc when the EM fields are of nonhelical nature. These values change to $B_0 \sim (2.1\text{–}3.8) \times 10^{-10}$ G and $\sim$90 kpc for the helical case.

In this work, we only consider the scenario where reheating temperature, $T_R = 150$ MeV. However, scenarios where reheating temperature ranges above 10's of MeV (as required to obtain standard big bang nucleosynthesis [91]) to GeV range can also be constrained by the pulsar timing arrays. For these reheating temperature scales, the nature of the GW spectrum remains the same, except that the signal shown in Fig. 1 shifts toward left for $T_R <$ 150 MeV and toward the right side for $T_R > 150$ MeV. A future dataset for timing residuals of pulsars with observation for more years, may help in better constraining such models of inflationary magnetogenesis. If the dataset turns out to favor a broken power law and follows the power-law slopes consistent with our model's prediction, then it uniquely fixes the scale of reheating in our model with the help of the peak frequency of the spectrum. The nature of resulting GW signal in our model is non-Gaussian; this property may help in distinguish these models from other models of GW generation in the early Universe [92].

ACKNOWLEDGMENTS

The author would like to thank Prof. Kandaswamy Subramanian and Prof. T. R. Seshadri for several useful discussions and for providing valuable suggestions on the manuscript. The author is thankful to Prof. Bhal Chandra Joshi for the discussion and helpful insights on NANOGrav results. The author also thanks Dr. Rajeev Kumar Jain for stimulating discussion.

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