Letter

No-go guide for the Hubble tension: Late-time solutions

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The Hubble tension, if not caused by any systematics, could be relieved or even resolved from modifying either the early-time or late-time Universe. The early-time modifications are usually in tension with either galaxy clustering or galaxy lensing constraints. The late-time modifications are also in conflict with the constraint from the inverse distance ladder, which, however, is weakened by the dependence on a sound-horizon prior and some particular approximation for the late-time expansion history. To achieve a more general no-go argument for the late-time scenarios, we propose to use a global parametrization based on the cosmic age to consistently use the cosmic chronometers data beyond the Taylor expansion domain and without the input of a sound-horizon prior. Both the early-time and late-time scenarios are therefore largely ruled out, indicating the possible ways out of the Hubble tension from either exotic modifications of our concordance Universe or some unaccounted for systematics.

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I. INTRODUCTION

The extrapolation from globally fitting the Λ-cold-dark-matter (ΛCDM) model to the cosmic microwave background (CMB) data [1–3] renders the Hubble constant $H_0 = 67.27 \pm 0.60\ \mathrm{kms^{-1}\ Mpc^{-1}}$ to an unprecedented accuracy [2]. On the other hand, the combined big bang nucleosynthesis (BBN) + baryon acoustic oscillation (BAO) constraint [4–12] is independent of the CMB data, yet still shares a similar H_0 value as inferred by CMB data if ΛCDM is assumed throughout the early Universe. Furthermore, the consistency of ΛCDM with respect to the early-Universe observations is also manifested in the consistency tests of the early integrated Sachs-Wolfe effect [13] and the sound horizon measured at the matter-radiation equality, recombination, and the end of the drag epoch [14,15]. Apart from some anomalies that arise in the high-ℓ [1,16,17] or EE-polarized [18,19] CMB data, the early-Universe observations could, at the very least, achieve a consensus on the Hubble constant $H_0 \lesssim 70\ \mathrm{kms^{-1}\ Mpc^{-1}}$.

However, the Hubble constants inferred by ΛCDM from early-Universe observations are systematically lower than those from local measurements either depending on or independent of the distance ladders. The local measurements with distance ladders heavily rely on the calibrations from Cepheid [20–24], tip of the red giant branch (TRGB) [25–29], or Miras [30] for connecting the local geometric measurements with the type Ia supernovae (SNe) in the Hubble flow. Nevertheless, the most recent measurement $H_0 = 69.8 \pm 2.2\ \mathrm{kms^{-1}\ Mpc^{-1}}$ from TRGB calibration [29] is lower than the most recent Cepheid calibration result $H_0 = 73.2 \pm 1.3\ \mathrm{kms^{-1}\ Mpc^{-1}}$ [24], which might be affected by the choice of Cepheid color-luminosity calibration method [31] and other sources of uncertainty in the supernova distance ladder [32]. On the other hand, the local measurements independent of distance ladders vary largely among megamaser [30,33], surface brightness fluctuations [34,35], baryonic Tully-Fisher relation [36,37], gravitational-wave standard sirens [38–42], strong lensing time delay (SLTD) [43–45], parallax measurement of quasar 3C 273 [46], and extragalactic background light γ-ray attenuation [47]. At the very least, the local measurements from late-Universe observations seem to agree on $H_0 \gtrsim 70\ \mathrm{kms^{-1}\ Mpc^{-1}}$.

The $\sim 4\sigma$ Hubble tension [17,48–51] only arises when confronting the Planck measurement [2] with the Cepheid measurement [24], the most precise measurement from each camp. However, the rest of comparisons drawn from early-time and late-time observations are insufficient to claim a significant tension but with a rough compatibility around $H_0 \simeq 70\ \mathrm{kms^{-1}\ Mpc^{-1}}$ [29]. In perspective of future developments, there are two possibilities to pursue:

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(1) If the Hubble tension is not real, then it should be feasible to show the consistency of ΛCDM with late-time data independent of CMB and local H_0 measurements.

(2) If the Hubble tension is real, then it is necessary to narrow down the possible models [51,52] either from early-time or late-time scenarios:

(i) For early-time solutions, one can modify either the expansion history or recombination history. The early-time expansion history could be altered by some temporary energy injection around the matter-radiation equality, for example, dark radiation (DR) and early dark energy (EDE) [53–63]. The free-streaming DR is strongly constrained by BAO + BBN [9] before the BBN epoch and disfavored by the absence of the neutrino free-streaming phase shift in CMB [17]. The non-free-streaming DR, for example, strongly self-interacting neutrinos [64], is also disfavored by the high-ℓ polarisation CMB data [65,66]. The EDE models also deteriorate the S_8 tension [67–69] (see, however, Refs. [70–74]) and BBN constraint [75]. On the other hand, changing the recombination history [76–79] (see also Ref. [80]) via primordial magnetic fields [76] found no evidence for the required baryon clumping [81]. Furthermore, a general no-go argument [82] could be put forward as follows: for early-time solutions that solely reduce the cosmic sound horizon, models with lower values of $\Omega_m h^2$ are in tension with galaxy clustering data [83], while models with higher values of $\Omega_m h^2$ are in tension with galaxy weak lensing data [84,85]. This therefore largely rules out early-time solutions.

(ii) For various late-time solutions, the H_0 constraints from the SNe data with their absolute magnitude calibrated by Cepheid variables are quasi-model-independent [86]. However, the situation changes when including BAO data. This is the usual no-go argument for the late-time solutions using the inverse distance ladder [5,87–94], which combines BAO + SNe with a CMB prior on the sound horizon r_s [48,87,89,91,95–98] since BAO can only constrain the combinations $H(z)r_s$ and $D_A(z)/r_s$. Note that r_s is mainly determined by the early-Universe evolution (thus independent of late-time evolution) and therefore unharmful to be used to discriminate the late-time models. To implement the inverse distance ladder [99], one first assumes a Planck's prior on $r_s \simeq 147$ Mpc and some phenomenological parametrization for $H(z)$ at late times and then fits the combined BAO + SNe data with an astrophysical determination on the SNe Ia absolute magnitude M_B [24], leading to a strong constraint on the late-time Universe to be barely deviated from ΛCDM within $0.01 \lesssim z \lesssim 1$. Although a sudden phantom transition below $z \lesssim 0.01$ seems to raise the local H_0 value while still maintaining the phenomenological success of ΛCDM above $z \gtrsim 0.01$ [100], the price to pay is to deviate M_B fitted by CMB + BAO + SNe significantly from the one used to derive a locally higher H_0 [101–103]. This therefore largely rules out phantomlike dark energy models.

A more general no-go argument [104] without a CMB prior on r_s (thus also independent of the early-time cosmology) could be made by combining BAO + SNe with observational $H(z)$ data (OHD) for some late-time $H(z)$ parametrizations from Taylor expansions in z or $(1-a)$ [105]. However, the Taylor expansions of $H(z)$ in z or $(1-a)$ even to the fourth order still fail to cover the OHD redshift with the modest accuracy even for the ΛCDM case. We therefore propose in this paper to use a global parametrization based on the cosmic age (PAge) [106,107] that not only reproduces ΛCDM up to high redshift with high accuracy but also covers a large class of late-time models in a wide redshift range with a high accuracy. Furthermore, it is logically more consistent to use PAge for OHD from the cosmic chronometer (CC), and the cosmic age was recently found to play an important role in the Hubble tension [108–111]. Whether the Hubble tension turns out to be real or not, our work could serve as either a no-go guide beyond or a consistency test for the ΛCDM model, respectively.

II. MODEL

The usual model-independent parametrization for the late-time expansion history adopts a Taylor expansion [105] either in redshift z [112,113] or in y-redshift $y \equiv 1-a = z/(1+z)$ [114] as shown in Appendix A of the Supplemental Material [115] for the dimensionless Hubble expansion rate $E = H/H_0$ and dimensionless luminosity distance $d_L = D_L/(c/H_0)$. Although the Taylor expansion in y-redshift slightly improves the convergence of the Taylor expansion in redshift z, both of them still deviate significantly from the exact formula even for the ΛCDM case as shown in Fig. 1 with blue and green dashed lines. Introducing more terms with higher orders in z or y could certainly improve the convergence behavior but also weaken the constraining power of data fitting due to the presence of more nuisance parameters.

PAge is introduced as a global approximation of the cosmic expansion history [106]. Assuming our Universe is dominated by the matter component at high redshift $z \gg 1$ and ignoring the radiation component and the very short

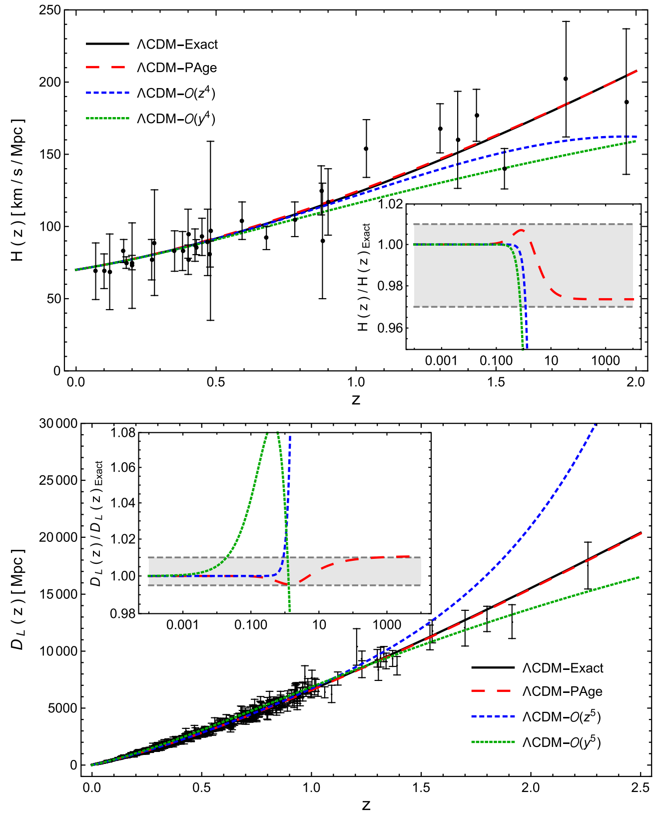

FIG. 1. The comparison between the PAge model (red dashed) and Taylor expansions in redshift z (blue dashed) and y-redshift (green dashed) compared to the exact expression from ΛCDM (black) for the Hubble expansion rate $H(z)$ and luminosity distance $D_L(z)$ with fiducial cosmology $\Omega_m = 0.3$, $H_0 = 70\ \mathrm{km s^{-1}\ Mpc^{-1}}$. The $H(z)$ data and Pantheon data (converted into luminosity distances with fiducial value $M_B = -19.34$) are shown for illustration. The relative errors are shown in the inserts for a larger redshift range up to $z \sim 10^4$.

period of radiation dominance before matter dominance, one could approximate the product of the Hubble expansion rate H and the cosmological time t as a quadratic function of t, namely,

$$\frac{H}{H_0} = 1 + \frac{2}{3}\left(1 - \eta\frac{H_0 t}{p_{\mathrm{age}}}\right)\left(\frac{1}{H_0 t} - \frac{1}{p_{\mathrm{age}}}\right), \tag{1}$$

where the parameter η could be evaluated as

$$\eta = 1 - \frac{3}{2}p_{\mathrm{age}}^2(1 + q_0) \tag{2}$$

by taking a time derivative of H in (1) followed by a replacement of the deceleration parameter $q(t) = -\ddot{a}a/\dot{a}^2$. $p_{\mathrm{age}} \equiv H_0 t_0$ is the product of $H_0 \equiv 100h\ \mathrm{km s^{-1}\ Mpc^{-1}}$ and the current age of Universe t_0.

For ΛCDM with late-time parametrization $H(a) = H_0\sqrt{\Omega_m a^{-3} + (1 - \Omega_m)}$, one has $q_0 = -1 + \frac{3}{2}\Omega_m$, and the current age of our Universe reads

$$t_0 = \int_0^1 \frac{\mathrm{d}a}{aH(a)} = \frac{9.77788\ \mathrm{Gyr}}{3h\sqrt{1-\Omega_m}}\ln\frac{1+\sqrt{1-\Omega_m}}{1-\sqrt{1-\Omega_m}}. \tag{3}$$

Therefore, $\eta = 0.3726$ and $p_{\mathrm{age}} = 0.9641$ for fiducial ΛCDM with $\Omega_m = 0.3$ and $H_0 = 70\ \mathrm{km s^{-1}\ Mpc^{-1}}$. The corresponding $H(z)$ and $D_L(z)$ are shown in Fig. 1 with red dashed lines, which differ from the exact ΛCDM expressions below 3% and 1%, respectively, over the whole redshift range up to $z \sim 10^4$ as shown in the inserts.

For models beyond ΛCDM, both parameters η and p_{age} should be treated as free parameters and the only two free parameters in H/H_0 of (1). To see this, we can directly solve (1) for the combination $H_0 t$ after replacing H with $-\mathrm{d}z/\mathrm{d}t/(1+z)$, namely,

$$1 + z = \left(\frac{p_{\mathrm{age}}}{H_0 t}\right)^{\frac{2}{3}} e^{\frac{1}{3}\left(1-\frac{H_0 t}{p_{\mathrm{age}}}\right)\left(3p_{\mathrm{age}}+\eta\frac{H_0 t}{p_{\mathrm{age}}}-\eta-2\right)}; \tag{4}$$

then, $H_0 t$ in (1) is a function of z, leaving only two free parameters η and p_{age} in H/H_0 of (1). For a specific physical model, η and p_{age} could be expressed by the model parameters. Mapping a specific model in the PAge parameter space requires matching $q(t)$ at some characteristic time, for example, at redshift zero, as done in Table I of Ref. [107] for a large class of illustrative models, where the relative error for the PAge representation of the ow_{CPL}CDM model [116,117] is less than 1% over $0 < z < 2.5$. See Appendix B in the Supplemental Material [115] for more details on model matching. Note that the focus of Ref. [106] for proposing the PAge parametrization is to reconfirm the late-time acceleration from a lower bound on $t_0 > 12$ Gyr, where SNe data with a H_0 prior and a CMB distance prior are used for data analysis. This is totally different from the purpose of this paper and the data analysis strategy as presented below.

III. DATA ANALYSIS

The data we use includes SNe Ia (standard candle), BAO (standard ruler), and OHD (standard clock), which is independent of either local H_0 measurements or the early-Universe observations like CMB and BBN.

For SNe Ia data, we use the Pantheon sample [118] containing 1048 SNe Ia within $0.01 < z < 2.3$. The SNe data directly measure the apparent magnitude $m_B(z)$, which could be computed theoretically from a model by

$$m_B(z) = M_B + 5\lg\frac{D_L(z)}{10\ \mathrm{pc}} = a_B + 5\lg d_L(z). \tag{5}$$

For a given model with dimensionless Hubble expansion rate $E = H/H_0$, the dimensionless luminosity distance is known as

$$d_L(z) \equiv \frac{D_L(z)}{c/H_0} = (1+z)\int_0^z \frac{\mathrm{d}z'}{E(z')}. \tag{6}$$

What the SNe magnitude-redshift relation actually constrains is $a_B \equiv M_B + 42.3841 - 5\lg h$. With a M_B prior from local distance ladders, one infers the value of H_0. However, as pointed out in Refs. [101–103], this inferred H_0 might not be consistent with the constraint on H_0 if one sets both H_0 and M_B free in the inverse distance ladder. In other words, if one adopts the SH0ES's prior on H_0 for a certain model, the inferred M_B from a_B might not be consistent with the constraint on M_B if both H_0 and M_B are free in the inverse distance ladder. As a result, both H_0 and M_B will be regarded as free parameters in the inverse distance ladder.

For BAO data, we use the state-of-the-art datasets [119–132] as listed in the Appendix C of the Supplemental Material [115]. The BAO measurements are summarized at some effective redshifts $z_{\rm eff}$ for the BAO feature in both line-of-sight and transverse directions. Along the line-of-sight direction, BAO directly measures $D_H(z)/r_d$ with respect to some fiducial cosmology, where the Hubble distance is defined as

$$D_H(z) = \frac{c}{H(z)}. \tag{7}$$

Along the transverse direction, BAO directly measures $D_M(z)/r_d$ or $D_A(z)/r_d$ with respect to the same fiducial cosmology, where the (comoving) angular diameter distances are defined via

$$D_M(z) = \frac{D_L(z)}{1+z} = (1+z)D_A(z). \tag{8}$$

For historical reason, BAO measurements could also be summarized by $D_V(z)/r_d$, where the spherically averaged distance is defined as

$$D_V(z) = [zD_M(z)^2 D_H(z)]^{1/3}. \tag{9}$$

To detach the model dependence on the early-Universe cosmology and observations, the sound horizon at drag epoch r_d will be treated as a free parameter.

For OHD from the differential age method [133], the Hubble parameter could be directly measured by

$$H(z) = -\frac{1}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t} \tag{10}$$

from the age difference Δt between two passively evolving galaxies that formed at the same time but are separated by a small redshift interval Δz. This method is independent of any cosmological models but the age estimation on the evolutionary stellar population synthesis (EPS) models. We use OHD [134–139] as listed in the Appendix C of the Supplemental Material [115] from two different EPS models: Bruzual and Charlot (2003) [140] (BC03 hereafter) and Maraston and Strömbäck (2011) [141] (MS11 hereafter). Note that the OHD points at $z = 1.363$ and $z = 1.965$ from [139] have adopted both EPS models of BC03 and MS11, which will not be included in the results presented below. Nevertheless, we have checked that the naive inclusion of these two OHD points in both datasets has little impact on our results and conclusions.

Fitting above SNe + BAO + OHD(BC03/MS11) to the PAge model with $\{\eta, p_{\rm age}, M_B, H_0, r_d\}$ as the free parameters with flat priors as listed in Table I, we then use the Markov chain Monte Carlo code EMCEE [142] to constrain the parameter space with the best-fit χ^2 test, where the likelihood function $\mathcal{L}$ is estimated via $-2\ln\mathcal{L} = \chi^2 = \chi^2_{\rm SNe} + \chi^2_{\rm BAO} + \chi^2_{\rm OHD(BC03/MS11)}$. For comparison, the ΛCDM model is also fitted to the same datasets with free parameters $\{\Omega_m, M_B, H_0, r_d\}$.

IV. RESULTS

The cosmological constraints from fitting the two different datasets, SNe+BAO+OHD(BC03) and SNe + BAO + OHD(MS11), to the ΛCDM and PAge models are summarized in Table I and Fig. 2. For both ΛCDM and PAge models, the results from SNe + BAO + BC03 generally predict a lower H_0, a lower M_B, and a higher r_d than those from SNe + BAO + MS11. The results from SNe + BAO + BC03 are closer to the usual constraints from CMB data, while the results from SNe + BAO + MS11 are closer to the local direct measurements. This systematic shift might be caused by the different EPS models based on different

TABLE I. The cosmological constraints from fitting the datasets SNe + BAO + OHD(BC03) and SNe + BAO + OHD(MS11) to the ΛCDM and PAge models with free parameters $\{\Omega_m, M_B, H_0, r_d\}$ and $\{\eta, p_{\rm age}, M_B, H_0, r_d\}$, respectively.

		BC03		MS11	
Parameter	Prior range	ΛCDM	PAge model	ΛCDM	PAge model
Ω_m	0.15–0.5	0.288 ± 0.011	⋯	0.282 ± 0.011	⋯
η	−2–2	⋯	$0.334^{+0.067}_{-0.057}$	⋯	$0.341^{+0.059}_{-0.063}$
$p_{\rm age}$	0.15–2.0	⋯	$0.975^{+0.012}_{-0.011}$	⋯	$0.978^{+0.010}_{-0.009}$
M_B	−20– − 19	-19.374 ± 0.047	$-19.379^{+0.051}_{-0.052}$	$-19.309^{+0.059}_{-0.053}$	-19.322 ± 0.063
H_0	60–80	$69.389^{+1.547}_{-1.474}$	$68.958^{+1.779}_{-1.826}$	$71.591^{+1.950}_{-1.780}$	$70.799^{+2.220}_{-2.118}$
r_d	120–160	$146.563^{+3.293}_{-2.894}$	$146.466^{+3.448}_{-3.302}$	$142.573^{+3.754}_{-3.820}$	$142.885^{+4.287}_{-4.062}$
$\chi^2_{\rm min}$/d.o.f	⋯	0.9724	0.9708	0.9803	0.9789

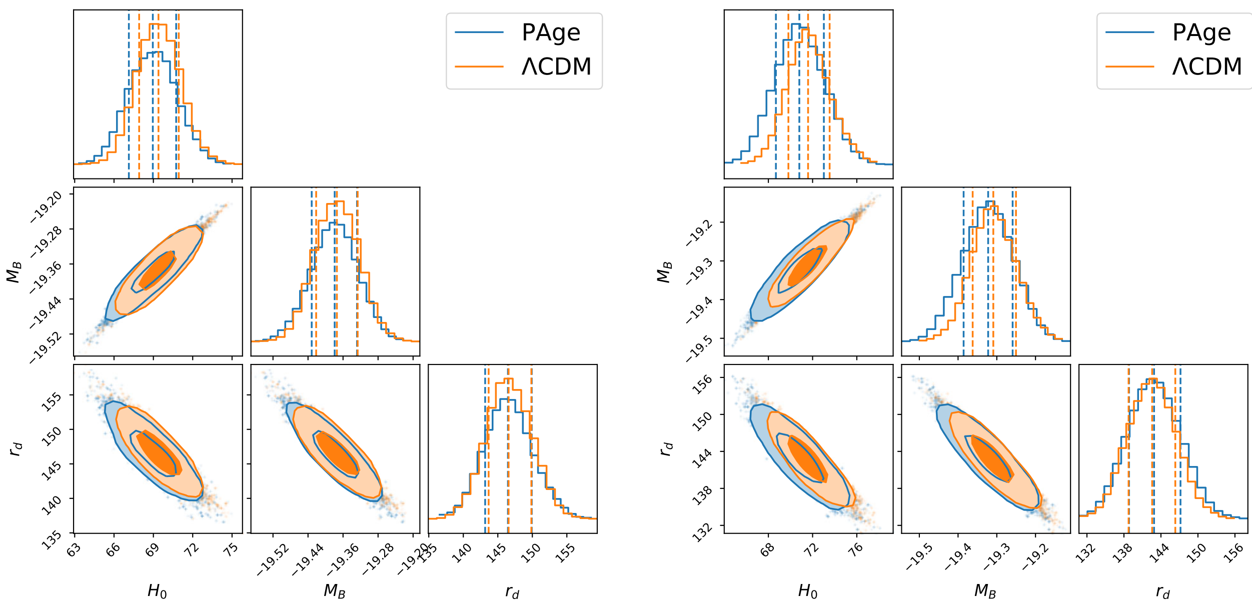

FIG. 2. 1σ and 2σ constraints from fitting the datasets SNe + BAO + OHD(BC03) (left panel) and SNe + BAO + OHD(MS11) (right panel) to the ΛCDM and PAge models with free parameters $\{\Omega_m, M_B, H_0, r_d\}$ and $\{\eta, p_{\rm age}, M_B, H_0, r_d\}$, respectively. The fitting results involving with η, $p_{\rm age}$, and Ω_m are not shown here.

empirical stellar libraries, for example, the MILES library [143] for MS11 is slightly bluer (thus older age) than the STELIB library [144] for BC03. Nevertheless, this situation is similar to the inverse distance ladder constraint [145] calibrated by SLTD from the H0LiCOW measurement [43] on $H_0 = 73.3^{+1.7}_{-1.8}$ km/s/Mpc, which results in $r_d = (137 \pm 3^{\rm stat} \pm 2^{\rm syst})$ Mpc in tension with the CMB r_d prior. This r_d tension could be relaxed by calibrating the inverse distance ladder with the most recent SLTD measurement [45] on $H_0 = 67.4^{+4.1}_{-3.2}$ km/s/Mpc from TDCOSMO + SLACS samples. Similar to the EPS-model dependence of the CC calibrator to the inverse distance ladder, the SLTD calibrator to the inverse distance ladder also admits an astrophysical dependence on the mass profile of lens galaxies.

However, the key point is that, although the use of the different EPS models directly affects the cosmological constraints for the same model from different CC data, it affects identically both the PAge and ΛCDM models since CC data are independent of any cosmological models. The difference between the ΛCDM and PAge models fitted by the same datasets is negligibly small. The reduced minimal χ^2 differs by 0.0016 (0.0014) between the ΛCDM and PAge models for SNe + BAO + BC03 (MS11). This could be made more quantitatively from the Bayesian information criterion (BIC) [146] $\mathrm{BIC} = k \ln n - 2 \ln \mathcal{L}$, where k is the number of the model parameters, and n is the number of the data points. For SNe + BAO + BC03 (MS11), the BIC difference of the PAge model with respect to the ΛCDM model is $\Delta\mathrm{BIC} = 4.3(4.5) > 2$. Therefore, there is positive evidence against the PAge model over the ΛCDM model.

Our PAge model is regarded here as a representative collection of various late-time models beyond ΛCDM. Different points in the $\eta - p_{\rm age}$ plane generally represent different models, and different models might also be degenerated at the same point in the $\eta - p_{\rm age}$ plane. Matching the deceleration parameter of a specific model at different redshifts also results in different PAge representations. Therefore, our PAge model could cover a large number of late-time models. Furthermore, since both r_d and M_B are set as free fitting parameters in our data analysis, we also effectively cover those early-time models reducing to different values of r_d and those astrophysical models with local calibrators to different values of M_B. Our final results then imply that there is a very little room for new physics beyond ΛCDM.

V. CONCLUSIONS

Despite the $\sim 4\sigma$ tension in H_0 found between the global fitting result from the CMB data and that from the local distance ladder calibrated by Cepheids, the Hubble tension has been called into question for the potential unaccounted for systematics [29,31,32]. Even if the Hubble tension turns out to be real, most of the early-time solutions run into tension with large-scale structure data [82], while most of the late-time homogeneous solutions develop tension with the inverse distance ladder constraints [101–103], and the cosmic void as a late-time inhomogeneous solution [147–149] is also disfavored by the SNe data [150–155].

In this paper, we aim to generalize the late-time no-go argument with a global parametrization for the cosmic expansion history. Our final results slightly go against the representative PAge models over the ΛCDM model, which could be made tighter with inclusions of $f\sigma_8$ data [156] reserved for future work. No matter whether the Hubble tension turns out to be real or not, our work could be regarded as a no-go guide for the Hubble solutions or a consistency test for the ΛCDM model.

If the Hubble tension persists to exist, then our work indicates that the Hubble solutions might come from some exotic modifications for our concordance Universe. For example, the early-time no-go argument [82] could be escaped from some EDE models [72–74,157–159] that could reduce the matter clumping. The late-time no-go arguments [101–103] could be avoided by some inhomogeneous or anisotropic modifications [160–162] for our local Universe or some modified gravity effects [163–169] for the magnitude-redshift relation. Some other hybrid models modifying both the early-time and late-time Universe might still stand a chance in these H_0 Olympic-like games [170].

If the Hubble tension disappears with improving calibration systematics, our work could be regarded as a consistency test for the ΛCDM model independent of early-Universe data and local H_0 measurements. The simple extensions of the ΛCDM model has already been tested with Refs. [171,172] or without the CMB data [173] from early-Universe observations and local H_0 measurements [174]. Our work simply adds another layer of support for the ΛCDM model.

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