Effective quantum gravity, cosmological constant, and the Standard Model of particle physics

Breno L. Giacchini,[1,§] Tibério de Paula Netto,[1,‡] and Ilya L. Shapiro[2,*,†]

[1]*Department of Physics, Southern University of Science and Technology, Shenzhen 518055, China*
[2]*Departamento de Física, ICE, Universidade Federal de Juiz de Fora, Juiz de Fora 36036-900, Minas Gerais, Brazil*

(Received 19 December 2021; accepted 14 June 2022; published 27 June 2022)

The renormalization group in effective quantum gravity can be consistently formulated using the Vilkovisky and DeWitt version of effective action and assuming a nonzero cosmological constant. Taking into account that the vacuum counterpart of the cosmological constant is dramatically different from the observed energy density of the vacuum, the running of the last quantity in the late cosmology indicates strong constraints on the physics beyond the minimal Standard Model of particle physics.

DOI: 10.1103/PhysRevD.105.126025

I. INTRODUCTION

One of the main open issues in nowadays fundamental physics is the origin of the so-called dark energy. The most likely candidate is the cosmological constant because such a constant is an essential element of the consistent quantum field theory (QFT) in curved space (see e.g., [1] for a recent discussion and further references). On the other hand, there are many theoretically interesting alternative models, assuming that the vacuum energy may vary with time (see, e.g., [2,3] as starting points). The question is how one can expect to falsify these models using the existing observational data. Indeed, the possibilities to do so are restricted to the observation of the time-variable vacuum energy, or the effective equation of state [4].

Does the time dependence of the vacuum energy mean that there is an essence more sophisticated than the cosmological constant, governing the accelerated expansion of the Universe? The answer is not obvious because the proper cosmological constant may be changing for at least two reasons. The observable cosmological constant is a sum of the two contributions, namely, the fundamental constant in the action of gravity and the induced counterpart owing to the phase transition and symmetry breaking in the vacuum state [5] (see also [6] for the QFT aspects of the problem). The induced component could have a different magnitude because of the typical energy scale at which the symmetry breaking occurs or, almost equivalently, to the restoration of the symmetry in the hot vacuum of the early Universe. Furthermore, the observable cosmological constant may change in the late Universe because of the renormalization group running. The running of the vacuum component of the cosmological constant in flat spacetime is a basic example of such a running (see e.g., [7]), that can be extended to semiclassical [8] and quantum [9] gravity.

The running cosmology models are based on the universal form of the scale-dependent density of the energy of vacuum, that can be established using covariance arguments [10] or the assumption of a standard quadratic decoupling of massive degrees of freedom in the IR, in the semiclassical approach [6,11],

$$\rho_\Lambda = \rho_\Lambda^0 + \frac{3\nu}{8\pi G}(H^2 - H_0^2). \tag{1}$$

Here ρ_Λ^0 and H_0 are the vacuum energy density and the Hubble parameter at the reference point, e.g., in the present moment of time. ν is a phenomenological parameter which cannot be calculated with the known theoretical methods [10,12]. Since the particle creation from vacuum in the IR is suppressed [13], the energy conservation leads to the "traditional" logarithmic form of the running for the Newton constant [14],

$$G(\mu) = \frac{G_0}{1 + \nu \log(H^2/H_0^2)}. \tag{2}$$

There is a possibility of a systematic scale-setting procedure in the cosmological context [15], providing the identification of the Hubble parameter H with the scaling parameter μ.

Let us stress that the aforementioned running in (1) and (2) is owing to the quantum effects of *massive* particles in

---

[*]Also at Tomsk State Pedagogical University.
[†]ilyashapiro2003@ufjf.br
[‡]tiberio@sustech.edu.cn
[§]breno@sustech.edu.cn

the IR, according to the corresponding decoupling theorems [16,17]. Usually, massless degrees of freedom do not produce running of the dimensional parameters, such as ρ_Λ and G. On the other hand, (2) looks like the one-loop running of the dimensionless coupling in the minimal subtraction scheme of renormalization, regardless it is derived in a very different framework.

The models of running vacuum based on (1) were extensively explored and became an active field of research (see e.g., [10,12,14,18,19] and further references in the last work). These models give a well-motivated alternative to the theories of modified gravity of all kinds, as in both cases one can describe a slowly varying vacuum energy density.

From the theoretical side, one of the open questions is whether the aforementioned universality of the running of vacuum energy (1) can be extended into the full quantum gravity (QG). The definitive answer to this question is unknown, as there is no completely consistent theory of quantum gravity to derive the running. On the other hand, any kind of a purely metric quantum gravity should have massless and massive degrees of freedom. The typical dimensional constant in quantum gravity is the Planck mass M_P; therefore, we can expect that all massive modes have the mass of this magnitude [20]. For the cosmological applications, what we need is an effective formulation of QG, when the massive modes are assumed to decouple (see, e.g., [21,22] and references therein). In this regime, we expect to meet the universal IR theory of QG based on the quantum general relativity (GR). Therefore, we need to account only for the massless modes [23,24] and it seems this can be a solid basis for deriving the QG-based running of ρ_Λ and G that can be applied in the late running-vacuum cosmology.

Now we are coming close to the main subject of this contribution, i.e., the role of the "unique" effective action of Vilkovisky and DeWitt for the IR running in the effective QG. In the framework of usual perturbative field theory, the running of ρ_Λ and G is possible only if the initial gravity theory has a nonzero cosmological constant. However, even in this case, the individual running of both quantities depends on the gauge fixing and on the parametrization of the quantum metric. Only the running of the dimensionless ratio of these parameters is universal [9], but this is insufficient for the cosmological applications. The construction of the Vilkovisky-DeWitt effective action resolves this difficulty at the one-loop [25] and higher-loop [26] levels. This useful feature of the "unique" effective action is owing to its geometric, covariant formulation in the space of the fields. The renormalization group running of ρ_Λ and G, based on Vilkovisky's construction has been explored in [27]. In recent works [28,29] we performed an explicit verification of the gauge- and parametrization-invariance of these renormalization group equations and proved that, in the effective QG framework, these equations are exact, i.e., not restricted by the one-loop approximation, as explained also in Sec. III below.

The status of the Vilkovisky-DeWitt effective action has been previously discussed in the work [30] based on three interesting applications where the gauge fixing and/or parametrization ambiguities in the usual effective action do not enable one to achieve the desired qualitative output, while the geometric methods provide the result. The general conclusion of this consideration is that the Vilkovisky-DeWitt construction is not a panacea, nor a placebo, but rather a useful tool for the calculations. In the present contribution, we discuss the application of the effective QG-based running [29] of ρ_Λ and G in cosmology and show that these effects may provide dramatic consequences for particle physics. This can be viewed as one more application where the Vilkovisky-DeWitt geometric approach yields a nontrivial consequence. Since the technical details of the Vilkovisky formalism are not the main topic of this paper, we postpone a brief review of it and a discussion about its possible limitations to Appendix. However, we would like to stress, from the very beginning, that this approach is an additional independent input and that its consequences may be verified or falsified only by means of experiments or observations.

The manuscript is organized as follows. In Sec. II we briefly review the cosmological constant and the fine-tuning problem, from the standard perspective of [5,6]. Section III discusses the exact effective QG running [29] and explains why it might break down the fine tuning for the cosmological constant. In Sec. IV we perform the numerical estimates of the mentioned breaking for a few models of particle physics and arrive at severe constraints on the physics beyond the standard model. Finally, in Sec. V we draw our conclusions and add some extra discussion.

II. COSMOLOGICAL CONSTANT PROBLEM AND THE RUNNING

As it was already mentioned above, the running of ρ_Λ and G in GR-based QG is possible only because of the nonzero cosmological constant. The higher loop contributions come with growing powers of the dimensionless ratio $\rho_\Lambda G^2 = \rho_\Lambda M_P^{-4}$. It is important to stress that ρ_Λ in this expression is *not* the observable vacuum energy density $\rho_\Lambda^{\rm obs}$. It is well known [5] (see also [6]) that

$$\rho_\Lambda^{\rm obs} = \rho_\Lambda + \rho_\Lambda^{\rm ind}, \tag{3}$$

where ρ_Λ and $\rho_\Lambda^{\rm ind}$ are the vacuum and the induced densities of cosmological constant, respectively. An independent quantity ρ_Λ is a necessary element of the renormalizable semiclassical theory (see, e.g., [1] for an introduction). Loop corrections without external lines of matter fields produce divergences, including those without derivatives of the metric tensor. These divergences require renormalization and, in particular, fixing the renormalization condition.

As usual with independent parameters, this procedure involves a measurement. Thus, the value of ρ_Λ can be defined only from the cosmological observations of $\rho_\Lambda^{\rm obs}$. After that, one has to subtract $\rho_\Lambda^{\rm ind}$ to arrive at the value of $\rho_\Lambda = \rho_\Lambda^{\rm vac}$.

From the theoretical side, the minimal magnitude of ρ_Λ is defined by its running in a semiclassical theory. For example, in the minimal standard model (MSM) this indicates at a value of the order of the fourth power of the Fermi mass, M_F^4. As ρ_Λ is an independent parameter, its value at the reference scale μ_0 can be defined only from the experimental or observational data. On the other hand, $\rho_\Lambda^{\rm ind}$ is, in principle, calculable from the underlying matter fields model. If its origin is the spontaneous symmetry breaking (SSB) in the MSM, we have the well-known relation with the vacuum expectation value (VEV) of the Higgs field, $\rho_\Lambda^{\rm ind} \sim \lambda v^4 \approx 10^8\ {\rm GeV}^4$. As the value of v is defined by the typical (Fermi) energy scale $M_F \approx 293$ GeV, in what follows we shall associate $\rho_\Lambda^{\rm ind}$ with M_F^4. In case there is another, similar, phase transition at a higher energy scale such as M_X, we have to replace M_F by M_X in both ρ_Λ and $\rho_\Lambda^{\rm ind}$.

It is remarkable that the theoretical predictions for the two ingredients in the right-hand side of (3) give the same order of magnitude. At the same time, the relation (3) is famous for the huge amount of fine tuning required for the cancelation in its right-hand side, providing a very small value for the observable sum. In the particle physics units, the value of $\rho_\Lambda^{\rm obs}$, it is about $10^{-47}\ {\rm GeV}^4$, such that even in the MSM we need about 56 orders of the fine tuning in the choice of the renormalization condition $\rho_\Lambda(\mu_0)$.

The cosmological constant problem is a real mystery, as the 56-order fine tuning can be violated even by very small changes in the Yukawa couplings that enter the game via the one-loop or higher-loop corrections (up to 21 loops). It is worth noting that even a small mismatch in the choice of the condition for $\rho_\Lambda(\mu_0)$ may lead to either a negative, zero or too big (more than 100 times greater) positive value of $\rho_\Lambda^{\rm obs}$. All three options contradict our own existence through the anthropic arguments [31]. In the next sections we shall see that the well-defined running $\rho_\Lambda(\mu)$ in effective QG imposes strong constraints on the particle physics beyond MSM.

III. ON THE RUNNING IN EFFECTIVE QG

The starting point in the discussion of the running in effective QG is the gravitational action,

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}(R + 2\Lambda), \tag{4}$$

where $\rho_\Lambda = \frac{\Lambda}{8\pi G}$, together with the theory of gauge-invariant renormalization (see e.g., [1] and references therein) and the power counting formula for quantum GR,

$$\omega(\mathcal{G}) + d(\mathcal{G}) = 2 + 2L - 2K_\Lambda. \tag{5}$$

Here L is the number of loops in the given diagram $\mathcal{G}$ and K_Λ is the number of vertices coming from the cosmological constant term. For the logarithmic divergences $\omega(\mathcal{G})$ is zero and then the last formula gives the number of derivatives $d(\mathcal{G})$ in the corresponding counterterm.

It is easy to see that for $K_\Lambda = 0$ we never get the renormalization of the Einstein-Hilbert term, but only higher-derivative counterterms. However, the situation is different in the case $K_\Lambda \neq 0$. One of the main observations of [29] and of the present work is that Λ in (4) does not correspond to $\rho_\Lambda^{\rm obs}$ but, instead, to the $\rho_\Lambda = \frac{\Lambda}{8\pi G}$ in the vacuum part of (3). As we have seen in the previous section, the two quantities $\rho_\Lambda^{\rm obs}$ and ρ_Λ are dramatically different and this may change the game in the effective QG-based running.

The important aspect in the effective QG is whether we are capable of obtaining results which are free of ambiguities. For instance, the known theorems about gauge-fixing and parametrization dependence [32,33] tell us that the one-loop divergences of the effective action,

$$\Gamma^{(1)}_{\rm div} = \frac{1}{\epsilon}\int d^4x\sqrt{-g}\{c_1 R^2_{\mu\nu\alpha\beta} + c_2 R^2_{\alpha\beta} + c_3 R^2 + c_4\Box R + c_5 R + c_6\}, \tag{6}$$

are universal only on the classical mass shell. Here $\epsilon = (4\pi)^2(n-4)$ is the parameter of dimensional regularization. Using the approach of [9,34] we arrive at the relation

$$\Gamma^{(1)}_{\rm div}(\alpha_i) - \Gamma^{(1)}_{\rm div}(\alpha_i^0) = \frac{1}{\epsilon}\int d^4x\sqrt{-g}(b_1 R_{\mu\nu} + b_2 R g_{\mu\nu} + b_3 g_{\mu\nu}\Lambda + b_4 g_{\mu\nu}\Box + b_5\nabla_\mu\nabla_\nu)\varepsilon^{\mu\nu},$$

where $b_k = b_k(\alpha_i)$ and α_i represent the full set of parameters defining an arbitrary gauge fixing and parametrization of the metric, and $\varepsilon^{\mu\nu} = G^{\mu\nu} - \Lambda g^{\mu\nu}$ are the classical equations of motion. The two invariant quantities are

$$c_1 \quad \text{and} \quad c_{\rm inv} = c_6 - 4\Lambda c_5 + 4\Lambda^2 c_2 + 16\Lambda^2 c_3, \tag{7}$$

which means that the invariant running is possible only for the dimensionless combination of G and Λ [9,35].

The Vilkovisky [25] and DeWitt [26] definition of effective action is different, as it is based on the covariant calculus in the space of physical fields. A brief review of the formalism can be found in Appendix, where we also discuss some subtleties involving its application to QG. More detailed considerations can be found, e.g., in [27–30,36–40].

The renormalization group running of G and Λ, in the effective QG based on the Vilkovisky-DeWitt version of effective action, has the form [27,29]

$$G(\mu) = G_0\left[1 + \frac{10}{(4\pi)^2}\gamma_0 \ln\frac{\mu}{\mu_0}\right]^{-4/5}, \tag{8}$$

and

$$\Lambda(\mu) = \Lambda_0\left[1 + \frac{10}{(4\pi)^2}\gamma_0 \ln\frac{\mu}{\mu_0}\right]^{-1/5}, \tag{9}$$

where $\gamma = 16\pi G\Lambda$ and the subscript label zero indicated the value μ_0 of the reference scale, e.g., the present epoch of the Universe. Thus, $\gamma_0 = 16\pi G_0\Lambda_0 = 128\pi^2 G_0^2 \rho_\Lambda^0 = 128\pi^2 \rho_\Lambda^0 M_P^{-4}$.

Let us stress that, differently from (1) and (2), Eqs. (8) and (9) come from a real calculation, based solely on the assumption of applicability of the Vilkovisky and DeWitt effective action in quantum field theory.

According to the power counting (5), the higher-loop corrections to these equations are proportional to higher powers of γ and, for a sufficiently small Λ, can be regarded as negligible. Starting from this point and taking into account the arguments from Sec. II, we can explore the physical consequences of the running (8) and (9).

IV. COSMOLOGY WITH RUNNING VACUUM IN EFFECTIVE QG

To evaluate ρ_Λ and γ_0 we have to remember that ρ_Λ has approximately—and with a great precision—the same absolute value as $\rho_\Lambda^{\rm ind}$. It is clear that $\rho_\Lambda \gg \rho_\Lambda^{\rm obs}$, but numerically γ_0 is still a small quantity. For instance, in the theory when the MSM is valid until the Planck energy scale, we have $\gamma_0 \sim 10^{-65}$, while for the supersymmetric grand unified theory (SUSY GUT) this coefficient may be $\gamma_0 \sim 10^{-12}$–10^{-8}. Do these small numbers guarantee that the variations caused by (8) and (9) are irrelevant?

To evaluate the consequences of the running, let us consider the strongest option, that is, the SUSY GUT case. Then, the value of ρ_Λ should be of the order of M_X^4, assuming $M_X \sim 10^{16}$ GeV. Accordingly, the parameter γ is of the order of $(M_X/M_P)^4$. In this case, we find the strong inequality $\frac{10\gamma_0}{(4\pi)^2} \ll 1$. After a little algebra, (8) and (9) boil down to

$$G(\mu) = G_0\left[1 - \frac{8}{(4\pi)^2}\gamma_0 \ln\frac{\mu}{\mu_0}\right], \tag{10}$$

and

$$\rho_\Lambda(\mu) = \rho_\Lambda^0\left[1 + \frac{6}{(4\pi)^2}\gamma_0 \ln\frac{\mu}{\mu_0}\right]. \tag{11}$$

The derivation of the last equation requires expanding the right-hand side of both (8) and (9) up to the first order in the small parameter γ_0 and replacing the result into the formula $\rho_\Lambda = \frac{\Lambda}{8\pi G}$.

Remember that the standard identification of scale in cosmology is $\mu \propto H$ [6,10,15]. Taking this identification, the effects of (10) and (11) are dramatically different. Indeed, while for $G(\mu/\mu_0) = G(H/H_0)$ there is a usual slow logarithmic running that is not too relevant in cosmology, the effect of the cosmological constant running (11) may be strong.

In the SUSY GUT case, a simple calculation gives

$$\frac{6}{(4\pi)^2}\gamma_0 \sim 48\left(\frac{M_X}{M_P}\right)^4 \approx 10^{-11}. \tag{12}$$

This is a really huge number, because it has to be multiplied not only by the logarithmic factor but, at the first place, by the ρ_Λ^0, i.e., by the *vacuum* energy density in the right-hand side of the main relation (3). In SUSY GUT the value of ρ_Λ^0 is about 111 orders of magnitude greater than the observed value $\rho_\Lambda^{\rm obs}$. Thus, the running (12) produces a discrepancy with the cosmological observations proportional to 10^{100} (googol) for a change of about one order of magnitude in the parameter H. Needless to say that this result contradicts the anthropic calculations [31]. Thus, we have to give up either on the effective QG based on the Vilkovisky "unique" effective action, or on the SUSY GUT and the corresponding generation of induced vacuum energy in the right-hand side of (3). As the present report is devoted to the effective QG-based running, we conclude that the SUSY GUT hypothesis fails in this framework.

Let us consider another extreme of the energy scale and assume that the MSM is valid up to the Planck scale. In this case, instead of (12) we meet

$$\frac{6}{(4\pi)^2}\gamma_0 \sim 48\left(\frac{M_F}{M_P}\right)^4 \approx 10^{-65}. \tag{13}$$

When multiplied by $\rho_\Lambda^0 \sim M_F^4$, we find the variation of the observed cosmological constant given by $\delta\rho_\Lambda^{\rm obs} \approx 10^{-55}\ln(H/H_0)$ GeV^4.

Taking the range of change of H between the inflationary epoch with $H_{\rm infl} \le 10^{15}$ GeV and the present-day Universe with $H_0 \approx 10^{-42}$ GeV, the logarithmic factor is $\ln(H/H_0) \approx 131$. Then the numerical estimate based on (13) gives $\delta\rho_\Lambda^{\rm obs} \approx 10^{-53}$ GeV^4, that is just six orders of magnitude smaller than the observed value $\rho_\Lambda^{\rm obs}$. At this point we can make two observations:

(1) Our model of the effective QG running is lucky enough to pass the test related to MSM. This means, e.g., that the experimentally confirmed model of particle physics does not contradict the anthropic arguments. The opposite output would mean the disproval of the Vilkovisky and DeWitt approach.

(2) Since the result is proportional to M_F^8, we can state that the existence of new physics based on the symmetry breaking beyond the energy scale of $10M_F$ contradicts the effective running of the

cosmological constant. It is remarkable that the effective QG provides such a relation between the cosmological constant problem and the particle physics.

V. CONCLUSIONS AND DISCUSSIONS

The running derived in the effective QG based on the Vilkovisky-DeWitt effective action enables one to formulate the link between particle physics and cosmology. In particular, we find that the MSM with the corresponding SSB leads to the running of the vacuum cosmological constant that does not contradict the cosmological observations and, in particular, the anthropic restrictions derived by Weinberg [31].

On the other hand, the mentioned running of the cosmological constant imposes severe restrictions on the SSB and the generation of induced cosmological constant in the physics beyond the MSM. Even assuming the symmetry breaking at the 10 TeV scale means we may run out of the scope with the cosmological constant violating the fine tuning in (3). The energy scale below 10 TeV is explored in LHC, but this does not mean that new physics beyond MSM is "forbidden" by the effective QG and the corresponding running. The obtained restrictions leave a lot of space for constructing particle physics models beyond MSM. Thus, it would be interesting to explore this possibility in more details. The corresponding analysis is beyond the scope of the present work and will be presented as a separate publication. Let us just note that the limitation concerns the value of the new Higgs-like VEV and not the masses of the particles. Anyway, the preliminary result is that many (albeit not all) GUT models and supersymmetric extensions of the MSM may be ruled out by the new criterion based on quantum gravity. On the other hand, there are theories, e.g., based on the technicolor approach, which may have rather large masses of the particles beyond the standard model and still escape the restrictions discussed in the present work.

Finally, the well-defined gauge and parametrization independent running (9), originally described by Taylor and Veneziano [27] and explored in the effective framework in [29], provides interesting hints concerning the connection between different branches of Physics and also opens new horizons for further work.

ACKNOWLEDGMENTS

I. Sh. is grateful to A. Belyaev for useful discussions of the particle physics aspects of the restrictions on the SSB. The work of I. Sh. is partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico—CNPq under the Grant No. 303635/2018-5, by Fundação de Amparo à Pesquisa de Minas Gerais—FAPEMIG under the Grant No. PPM-00604-18, and by Ministry of Education of Russian Federation under the Project No. FEWF-2020-0003.

APPENDIX: BRIEF REVIEW AND DISCUSSION OF THE VILKOVISKY EFFECTIVE ACTION FOR QG

In this appendix we make a short presentation of the Vilkovisky effective action and comment on the possible limitations of the formalism, with a focus on the choice of metric in the space of fields in QG. This discussion is done in more detail than in our previous publications [28,29].

The Lagrangian quantization of gauge theories, including QG, involves fixing a gauge which breaks the classical action's gauge invariance. In DeWitt's background field method (see [41] for review and references) the resultant effective action is covariant with respect to gauge transformations of the background field, but it still depends on the choice of gauge fixing for the quantum field. The difference between these two situations is sometimes referred to as "gauge invariance" and "gauge-fixing dependence" [30].

In quantum GR, the background field method allows the evaluation of the divergent part of the effective action. Owing to locality, the divergences are scalars constructed with the curvature tensors and their covariant derivatives [see Eq. (6)]. As discussed in Sec. III, the expression (6) is *invariant* under spacetime diffeomorphisms, although some of the coefficients c_i depend on the gauge chosen for the quantized field, i.e., they are *gauge-fixing dependent*.

The gauge-fixing dependence of the effective action can be regarded as part of the more fundamental dependence on the field parametrization, which also affects nongauge theories [25,33]. At one-loop level this can be understood by recalling that the Hessian of the classical action, $\frac{\delta^2 S}{\delta\varphi^i \delta\varphi^j}$, does not behave as a tensor under redefinitions of the field φ^i. In the same spirit as in Riemannian geometry, Vilkovisky [25] introduced an affine structure on the configuration space $\mathcal{M}_\perp$ of physical fields, with a metric $\bar{G}_{ij}$ and a connection $\mathcal{T}^k_{ij}$, and modified the definition of the effective action such that it transforms in a covariant manner under diffeomorphism; namely,

$$\exp i\Gamma(\varphi) = \int \mathcal{D}\varphi' \mu(\varphi') \exp\{i[S(\varphi') + \sigma^i(\varphi, \varphi')\Gamma_{,i}(\varphi)]\}, \tag{A1}$$

where $\mu(\varphi')$ is an invariant functional measure and $\sigma_i(\varphi, \varphi')$ is the derivative with respect to φ^i of the world function $\sigma(\varphi, \varphi')$ [42,43]. Note that the affine structure is defined on the space of physical fields, which means that if $\mathcal{G}$ is a gauge group acting on a space $\mathcal{M}$ of fields φ^i, then $\mathcal{M}_\perp = \mathcal{M}/\mathcal{G}$. Therefore, since $\sigma^i(\varphi, \varphi')$ behaves as a vector with respect to φ^i and as a scalar with regard to φ'^i, the effective action $\Gamma(\varphi)$ defined by (A1) is gauge invariant and it is independent of the parametrization and gauge fixing choices. Because of this, the covariant effective action $\Gamma(\varphi)$ is also known as "unique effective action".

The metric G_{ij} in the full configuration space $\mathcal{M}$ of fields is obtained through the following criteria: (i) It should be an ultralocal quantity and do not contain derivatives of the fields in order to not violate the S-matrix theory, (ii) For quadratic noninteracting field theories, G_{ij} should provide a flat field space, and (iii) It must be uniquely determined by the classical action $S(\varphi)$ of the theory; namely, it should be chosen as the local metric contained in the highest-derivative term of the classical action after projecting out the gauge-dependent degrees of freedom. This is known as Vilkovisky's prescription for the choice of metric in the space of fields [25].

The projection onto the space $\mathcal{M}_\perp$ is performed by the operator [25,37,38]

$$\Pi^i{}_j = \delta^i{}_j - R^i_\alpha (N^{-1})^{\alpha\beta} R^k_\beta G_{kj}, \tag{A2}$$

where R^i_α are the generators of gauge transformations and $(N^{-1})^{\alpha\beta}$ is the inverse of the metric $N_{\alpha\beta}$ on the gauge group,

$$N_{\alpha\beta} = R^i_\alpha G_{ij} R^j_\beta. \tag{A3}$$

Therefore, the projection of the metric on $\mathcal{M}_\perp$ is

$$\bar{G}_{ij} = \Pi^k{}_i G_{k\ell} \Pi^\ell{}_j = G_{ij} - G_{ik} R^k_\alpha (N^{-1})^{\alpha\beta} R^\ell_\beta G_{\ell j}. \tag{A4}$$

The affine connection $\mathcal{T}^k_{ij}$ can then be obtained by requiring its compatibility with the physical field-space metric, $\bar{\nabla}_k \bar{G}_{ij} = 0$ (see, e.g., [25,37,44] for further comments and explicit formulas).

Notice, however, that by projecting out the gauge-dependent part of the field we obtain

$$\varphi^i_\perp \equiv \Pi^i{}_j \varphi^j = \varphi^i - R^i_\alpha (N^{-1})^{\alpha\beta} R^k_\beta G_{kj} \varphi^j. \tag{A5}$$

Thus, in the so-called Landau-DeWitt gauge, defined by

$$R^k_\beta G_{kj} \varphi^j = 0, \tag{A6}$$

we have

$$\varphi^i_\perp = \varphi^i. \tag{A7}$$

This means that Vilkovisky's prescription for G_{ij} is equivalent to getting the metric which follows from the theory's action using the Landau-DeWitt gauge [38]. We stress that this is only due to the fact that in the gauge (A6) the identity (A7) holds, and that at this stage one is not choosing a specific gauge which could, potentially, simplify calculations. Thus, the aforementioned detail does not reduce the generality of the scheme.

Let us discuss in more detail the choice of the metric G_{ij} in the case of QG. In metric theories of gravity, $\varphi^i = g_{\mu\nu}$ and there is a one-parameter family of metrics on $\mathcal{M}$ in accordance with (i) and (ii),

$$G_{ij} = G^{\mu\nu,\alpha\beta} = \frac{1}{2}(\delta^{\mu\nu,\alpha\beta} + a g^{\mu\nu} g^{\alpha\beta}),$$
$$\delta^{\mu\nu,\alpha\beta} = \frac{1}{2}(g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}), \tag{A8}$$

where $a \neq -1/4$ is a constant. The value $a = -1/4$ is discarded, as otherwise the metric (A8) has no inverse. In order to fix the parameter a, we shall follow Vilkovisky's prescription.

Particularizing the discussion for quantum GR, the expansion of the Einstein-Hilbert action (4) in the background field method splitting

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + \kappa h_{\mu\nu}, \qquad \kappa^2 = 16\pi G, \tag{A9}$$

where $g_{\mu\nu}$ is the background field and $h_{\mu\nu}$ is the quantum field, gives the quadratic form

$$S^{(2)} = -\frac{1}{2}\int d^4x \sqrt{-g}\left\{ h_{\mu\nu}\left[\frac{1}{2}\left(\delta^{\mu\nu,\alpha\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta}\right)\Box\right] h_{\alpha\beta} + (\nabla_\lambda h^{\mu\lambda} - \frac{1}{2}\nabla^\mu h)^2 \right\} + \cdots. \tag{A10}$$

Here the ellipsis stands for terms without derivatives of the quantum field, which are unimportant for the present discussion. Since the generators of the spacetime diffeomorphisms are

$$R^i_\alpha \equiv R_{\mu\nu,\alpha} = -(g_{\mu\alpha}\nabla_\nu + g_{\nu\alpha}\nabla_\mu), \tag{A11}$$

a simple calculation shows that the Landau–DeWitt gauge condition (A6) reads

$$R^k_\alpha G_{kj} \varphi^j = \nabla_\mu h^\mu_\alpha + a \nabla_\alpha h = 0 \Rightarrow \nabla_\mu h^{\alpha\mu} = -a \nabla^\alpha h. \tag{A12}$$

Substituting this condition into (A10) and recalling (A7), we find the bilinear projected action on $\mathcal{M}_\perp$,

$$S^{(2)}_\perp = -\frac{1}{2}\int d^4x \sqrt{-g} h^\perp_{\mu\nu} \times \left\{\frac{1}{2}[\delta^{\mu\nu,\alpha\beta} - (2a^2 + 2a + 1) g^{\mu\nu} g^{\alpha\beta}]\Box\right\} h^\perp_{\alpha\beta} + \cdots. \tag{A13}$$

Thus, the requirement that the term between curly brackets in the last expression equals the metric (A8) leads to the algebraic equation [25,38]

$$-(2a^2 + 2a + 1) = a, \tag{A14}$$

which has the solutions

$$a = -\frac{1}{2} \quad \text{and} \quad a = -1. \tag{A15}$$

The value $a = -1$ is discarded because the metric on the gauge group

$$N_{\alpha\beta} = R^i_\alpha G_{ij} R^j_\beta = -[g_{\alpha\beta}\Box + (1+2a)\nabla_\alpha\nabla_\beta + R_{\alpha\beta}] \tag{A16}$$

becomes degenerate. Thus, for the QG based on GR, in the simplest parametrization (A9), the Vilkovisky's prescription for the choice of metric gives $a = -1/2$ in an unambiguous way.

Furthermore, since the metric on $\mathcal{M}$ transforms as a tensor under field reparametrizations, Eq. (A14) holds even for quantum metric parametrizations more general than (A9). Thus, the ambiguity represented by the coefficient a is fixed in the same way, as explicitly shown in [28]. In this paper (as well as in our work [29]) we assume this choice of the metric.

This elementary exposition of the Vilkovisky formalism enables us to discuss some of the usual criticism about it. First of all, we notice that if the conditions (i) and (ii) for the field-space metric are satisfied, the Vilkovisky effective action produces the same result as the ordinary definition of the effective action at on-shell level. In particular, both formalisms generate the same elements for the S-matrix [39]. Therefore, one might argue that if the S-matrix contains all the information about physical observables, then the Vilkovisky effective action gives no new predictions. As shown in [30], however, there are physical quantities (e.g., critical temperatures) that in principle cannot be directly obtained from scattering amplitudes and they depend on the choice of gauge and parametrization of the quantum fields, if calculated using the standard effective action. In this respect, the Vilkovisky effective action can give unambiguous results which may be verified by experiments. The examples shown in the main part of this work represent another application in which this formalism can be useful.

Secondly, there are other ways of constructing covariant effective actions besides (A1). In particular, a generalization of the unique effective action was presented by DeWitt [26], which is usually called "Vilkovisky–DeWitt effective action". This more intricate definition coincides with (A1) in the one-loop approximation, and has the advantage of yielding a perturbative expansion in terms of one-particle irreducible diagrams (which was a major problem in Vilkovisky's original proposal) [39,40,45]. Since the considerations in the present work are based on the one-loop renormalization group equations (and in Sec. III we argued why higher-loop effects are suppressed [29]), the definition (A1) is sufficient for our purposes, and our results also hold within the more elaborate construction by DeWitt.

Last but not least, a common reservation about the use of the Vilkovisky–DeWitt effective action when applied specifically to gravitational theories is because the metric in the space of fields (A8) is not uniquely defined without the prescription (iii). Notice that (iii) is not needed for the invariance of the Vilkovisky–DeWitt effective action. Indeed, by using just (A8) it is possible to construct a whole family of effective actions parametrized by the arbitrary constant a, each of them being gauge and parametrization invariant. Vilkovisky's prescription fixes a in an uniquely way which only depends on the gravitational theory in question. Whether it is the correct prescription is still an open question that might be decided only at the experimental level (in this regard, see also [30,46]).

[1] I. L. Buchbinder and I. L. Shapiro, *Introduction to Quantum Field Theory with Applications to Quantum Gravity* (Oxford University Press, New York, 2021).

[2] P. J. E. Peebles and B. Ratra, The cosmological constant and dark energy, Rev. Mod. Phys. **75**, 559 (2003).

[3] E. J. Copeland, M. Sami, and S. Tsujikawa, Dynamics of dark energy, Int. J. Mod. Phys. D **15**, 1753 (2006).

[4] V. Sahni, T. D. Saini, A. A. Starobinsky, and U. Alam, Statefinder: A new geometrical diagnostic of dark energy, JETP Lett. **77**, 201 (2003).

[5] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. **61**, 1 (1989).

[6] I. L. Shapiro and J. Solà, The scaling evolution of the cosmological constant, J. High Energy Phys. 02 (2002) 006.

[7] L. S. Brown, *Quantum Field Theory* (Cambridge University Press, Cambridge, England, 1994).

[8] B. L. Nelson and P. Panangaden, Scaling behavior of interacting quantum fields in curved space-time, Phys. Rev. D **25**, 1019 (1982).

[9] E. S. Fradkin and A. A. Tseytlin, Renormalizable asymptotically free quantum theory of gravity, Nucl. Phys. **B201**, 469 (1982).

[10] I. L. Shapiro, Effective action of vacuum: Semiclassical approach, Classical Quantum Gravity **25**, 103001 (2008).

[11] A. Babic, B. Guberina, R. Horvat, and H. Stefancic, Renormalization group running of the cosmological constant and its implication for the Higgs boson mass in the standard model, Phys. Rev. D **65**, 085002 (2002).

[12] I. L. Shapiro and J. Solà, On the possible running of the cosmological 'constant', Phys. Lett. B **682**, 105 (2009).

[13] R. Opher and A. Pelinson, Studying the decay of the vacuum energy with the observed density fluctuation spectrum, Phys. Rev. D **70**, 063529 (2004).

[14] I. L. Shapiro, J. Solà, and H. Stefancic, Running G and Λ at low energies from physics at M_X: Possible cosmological and astrophysical implications, J. Cosmol. Astropart. Phys. 01 (2005) 012.

[15] A. Babic, B. Guberina, R. Horvat, and H. Stefancic, Renormalization-group running cosmologies. A scale-setting procedure, Phys. Rev. D **71**, 124041 (2005).

[16] T. Appelquist and J. Carazzone, Infrared singularities and massive fields, Phys. Rev. D **11**, 2856 (1975).

[17] E. V. Gorbar and I. L. Shapiro, Renormalization group and decoupling in curved space, J. High Energy Phys. 02 (2003) 021; Renormalization group and decoupling in curved space: II. The standard model and beyond, J. High Energy Phys. 06 (2003) 004.

[18] J. Grande, J. Solà, J. C. Fabris, and I. L. Shapiro, Cosmic perturbations with running G and Lambda, Classical Quantum Gravity **27**, 105004 (2010).

[19] J. A. Agudelo Ruiz, J. C. Fabris, A. M. Velasquez-Toribio, and I. L. Shapiro, Constraints from observational data for a running cosmological constant and warm dark matter with curvature, Gravitation Cosmol. **26**, 316 (2020).

[20] A. Accioly, B. L. Giacchini, and I. L. Shapiro, On the gravitational seesaw in higher-derivative gravity, Eur. Phys. J. C **77**, 540 (2017).

[21] J. F. Donoghue, The effective field theory treatment of quantum gravity, AIP Conf. Proc. **1483**, 73 (2012).

[22] C. P. Burgess, Quantum gravity in everyday life: General relativity as an effective field theory, Living Rev. Relativity **7**, 5 (2004).

[23] D. A. R. Dalvit and F. D. Mazzitelli, Geodesics, gravitons and the gauge fixing problem, Phys. Rev. D **56**, 7779 (1997).

[24] T. de Paula Netto, I. L. Shapiro, and L. Modesto, Universal leading quantum correction to the Newton potential, Eur. Phys. J. C **82**, 160 (2022).

[25] G. A. Vilkovisky, The unique effective action in quantum field theory, Nucl. Phys. **B234**, 125 (1984).

[26] B. S. DeWitt, The effective action, in *Quantum Field Theory and Quantum Statistics*, Essays in honor of the sixtieth birthday of E. S. Fradkin, edited by C. J. Isham, I. A. Batalin, and G. A. Vilkovisky (Hilger, Bristol, 1987).

[27] T. Taylor and G. Veneziano, Quantum gravity at large distances and the cosmological constant, Nucl. Phys. **B345**, 210 (1990).

[28] B. L. Giacchini, T. de Paula Netto, and I. L. Shapiro, Vilkovisky unique effective action in quantum gravity, Phys. Rev. D **102**, 106006 (2020).

[29] B. L. Giacchini, T. de Paula Netto, and I. L. Shapiro, On the Vilkovisky-DeWitt approach and renormalization group in effective quantum gravity, J. High Energy Phys. 10 (2020) 011.

[30] K. Kobes, G. Kunstatter, and D. J. Toms, The Vilkovisky-DeWitt effective action: Panacea or Placebo?, in *TeV Physics: 12th Johns Hopkins Workshop on Current Problems in Particle Theory*, edited by G. Domokos and S. Kovesi-Domokos (World Scientific, Singapore, 1988).

[31] S. Weinberg, Anthropic Bound on the Cosmological Constant, Phys. Rev. Lett. **59**, 2607 (1987).

[32] I. Ya. Arefeva, A. A. Slavnov, and L. D. Faddeev, Generating functional for the S-matrix in gauge-invariant theories, Teor. Mat. Fiz. **21**, 311 (1974) [Theor. Math. Phys. **21**, 1165 (1974)].

[33] B. L. Voronov, P. M. Lavrov, and I. V. Tyutin, Canonical transformations and the gauge dependence in general gauge theories, Sov. J. Nucl. Phys. **36**, 498 (1982), https://inspirehep.net/literature/183748.

[34] I. L. Shapiro and A. G. Jacksenaev, Gauge dependence in higher derivative quantum gravity and the conformal anomaly problem, Phys. Lett. B **324**, 286 (1994).

[35] J. D. Gonçalves, T. de Paula Netto, and I. L. Shapiro, Gauge and parametrization ambiguity in quantum gravity, Phys. Rev. D **97**, 026015 (2018).

[36] A. O. Barvinsky and G. A. Vilkovisky, The generalized Schwinger-DeWitt technique in gauge theories and quantum gravity, Phys. Rep. **119**, 1 (1985).

[37] S. R. Huggins, G. Kunstatter, H. P. Leivo, and D. J. Toms, The Vilkovisky-DeWitt effective action for quantum gravity, Nucl. Phys. **B301**, 627 (1988).

[38] E. S. Fradkin and A. A. Tseytlin, On the new definition of off-shell effective action, Nucl. Phys. **B234**, 509 (1984).

[39] A. Rebhan, Feynman rules and S-matrix equivalence of the Vilkovisky-DeWitt effective action, Nucl. Phys. **B298**, 726 (1988).

[40] P. Ellicott and D. Toms, On the new effective action in quantum field theory, Nucl. Phys. **B312**, 700 (1989).

[41] L. F. Abbott, Introduction to the background field method, Acta Phys. Pol. B **13**, 33 (1982), https://inspirehep.net/literature/166273.

[42] B. S. DeWitt, *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965).

[43] J. L. Synge, *Relativity: The General Theory* (North-Holland, Amsterdam, 1960).

[44] G. Kunstatter, Vilkovisky's unique effective action: An introduction and explicit calculation, in *Super Field Theories, Proceedings of NATO Advanced Research Workshop on Superfield Theories*, edited by H. C. Lee, V. Elias, G. Kunstatter, R. B. Mann, and K. S. Viswanathan, NATO ASI Series B Vol. 160 (Plenum, New York, 1987).

[45] A. Rebhan, The Vilkovisky-DeWitt effective action and its application to Yang-Mills theories, Nucl. Phys. **B288**, 832 (1987).

[46] G. A. Vilkovisky, Effective action in quantum gravity, Classical Quantum Gravity **9**, 895 (1992).