Renormalizability of a Yang-Mills center-vortex ensemble

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Recently, a new procedure to quantize the $SU(N)$ Yang-Mills theory in the nonperturbative regime was proposed. The idea is to divide the configuration space $\{A_\mu\}$ into sectors labeled by different topological degrees of freedom and fix the gauge separately on each one of them. As Singer's theorem on gauge copies only refers to gauge fixing conditions that are global in $\{A_\mu\}$, this construction might avoid the Gribov problem. In this work, we present a proof of the renormalizability in the center-vortex sectors, thus establishing the calculability of the Yang-Mills center-vortex ensemble.

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I. INTRODUCTION

In 1978, I. M. Singer showed that any gauge-fixing condition in $SU(N)$ Yang-Mills (YM) theory that is global in configuration space $\{A_\mu\}$ will necessarily contain Gribov copies [1–3]. This is the fundamental reason behind the infrared problems faced when trying a quantization in the continuum. In the last many years, the main approach to circumvent this problem has been based on the restriction of the path integral to the first Gribov region, the ensuing Gribov-Zwanziger quantization procedure [4–6], as well as its refinement and improvement [7–10]. It is interesting to note that, in his work, Singer pointed to a different procedure based on a locally finite open covering $\{\vartheta_\alpha\}$ of the total space of gauge field configurations $\{A_\mu\}$, namely,

$$\{A_\mu\} = \cup_\alpha\, \vartheta_\alpha, \tag{1.1}$$

together with a subordinate partition of unity [11,12]

$$\sum_\alpha \rho_\alpha(A_\mu) = 1, \qquad \forall\ A_\mu \in \{A_\mu\}, \tag{1.2}$$

where the support of the function ρ_α is ϑ_α. Introducing this identity, the YM partition function can be rewritten as

$$Z_{\rm YM} = \sum_\alpha Z_{(\alpha)}, \quad Z_{(\alpha)} = \int_{\vartheta_\alpha} [DA]\rho_\alpha(A) e^{-S_{\rm YM}[A]}, \tag{1.3}$$

where $S_{\rm YM}$ is the YM action. Note that, in each term, the path-integral can be done on the support of $\rho_\alpha(A)$. Now, by choosing the components of the covering ϑ_α such that they admit local cross sections

$$f_\alpha(A) = 0, \tag{1.4}$$

without copies, the usual Faddeev-Popov procedure can be safely implemented on each term $Z_{(\alpha)}$

$$Z_{\rm YM} = \sum_\alpha \int_{\vartheta_\alpha} [DA]\rho_\alpha(A) e^{-S_{\rm YM}} \delta(f_\alpha(A)) {\rm Det} \frac{\delta f_\alpha(A^U)}{\delta U}\bigg|_{U=I}. \tag{1.5}$$

Over the years, this possibility was overlooked, certainly because of the difficulties to identify and characterize this type of covering and effectively implement the partition of unity. Along this line, if the covering were a partition of $\{A_\mu\}$,

$$\{A_\mu\} = \cup_\alpha\, \vartheta_\alpha, \qquad \vartheta_\alpha \cap \vartheta_\beta = \varnothing, \qquad \alpha \neq \beta, \tag{1.6}$$

then $\rho_\alpha(A)$ would be a characteristic function $\theta_\alpha(A)$, which is one if $A_\mu \in \vartheta_\alpha$ and is zero otherwise. This case was precisely implemented in Ref. [13]. The main idea is to introduce a map $S(A) \in SU(N)$ such that

$$S(A^U) = US(A), \tag{1.7}$$

where $U(x) \in SU(N)$ is regular and A^U stands for the gauge transformed field. This map is obtained by initially

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introducing an auxiliary tuple of adjoint scalar fields $\psi(A) = (\psi_1[A], \psi_2[A], \ldots)$ that minimize an auxiliary action $S_{\mathrm{H}}[A]$ with $SU(N) \to Z(N)$ spontaneous symmetry breaking (SSB),

$$\frac{\delta S_{\mathrm{H}}[A]}{\delta \zeta_I} = 0. \tag{1.8}$$

In the next step, a polar decomposition

$$(\psi_I, \psi_2, \ldots) = (S q_1 S^{-1},\, S q_2 S^{-1}, \ldots), \quad q_I \in \mathfrak{su}(N) \tag{1.9}$$

(I is the flavor index) is performed in terms of a "pure modulus" tuple $(q_1, q_2, \ldots)$ defined by the condition

$$f(q_1, q_2, \ldots) = 0, \qquad f \in \mathfrak{su}(N). \tag{1.10}$$

As the phases $S(A)$ are not always regular (even when A_μ is smooth), a nontrivial partition can be introduced by means of the equivalence relation between singular maps: $S' \sim S$, iff $S' = US$ for some regular $U \in SU(N)$. Then, A_μ is said to be in ϑ_{S_0} iff $S(A)$ is equivalent to some class representative $S_0 \in SU(N)$. The infinitely many labels S_0 correspond to different distributions of topological defects such as oriented and nonoriented center vortices (see Refs. [13–15]). Regarding the relation between center vortices and the infrared properties of YM theory, see Refs. [16–26]. Based on this construction, it is clear that the orbit of a given gauge field A_μ cannot intersect different sectors. This is because the orbits are generated by performing all possible *regular* gauge transformations. That is, as we move along the orbit of A_μ, the map changes according to Eq. (1.7). Therefore if $A_\mu \in \vartheta_{S_0}$, necessarily $A_\mu^U \in \vartheta_{S_0}$. Moreover, after fixing the gauge in a given sector ϑ_{S_0} by means of a condition

$$S(A) = S_0, \tag{1.11}$$

no Gribov copies are expected. In effect, because of the property (1.7), this condition together with $S(A^U) = S_0$ implies $U(x) = \mathbb{1}$. This type of argument was first given in Ref. [27] by relying on the N lowest eigenfunctions of the lattice covariant Laplacian defined in the fundamental representation. These objects were organized as the columns of an $N \times N$ matrix M, which transforms as $M \to UM$ under a gauge transformation U. Then, configurations were said to be gauge fixed if the $SU(N)$-phase obtained by means of the usual polar decomposition of M is the identity. A related gauge was based on the set of $N^2 - 1$ lowest eigenfunctions of the lattice covariant adjoint Laplacian [28]. As we will review in Sec. II, our procedure in the continuum is closer to the latter, with an important difference: in the lattice, the gauge fixing conditions can be defined globally as there is no concept of singular phase in a discrete spacetime. On the other hand, the possibility of singular phases in the continuum is precisely the reason that allows us to define physically inequivalent sectors and fix the gauge locally. To illustrate this situation, consider the Abelian Higgs model in $3+1$ dimensions. In this case, there is a vortex-free sector where the Higgs-phase can be fixed to be identically zero. On the other hand, as is well known, this theory admits static vortex nontrivial solutions where the phase changes by a multiple of 2π, when going around a loop encircling the vortex guiding-centers. Of course, because of topological reasons, the phase of the Higgs field cannot be fixed to be trivial in this case. Instead, a particular choice of multivalued phase can be defined. For example, when considering a single unit-charge vortex along the z-axis, the gauge can be fixed by requiring the Higgs phase to be the harmonic polar angle. Similarly, our procedure is a unitary gauge independently performed on each topological sector, but defined on auxiliary fields. The necessary conditions for this procedure to be well-defined will be briefly reviewed in Sec. II. These include an appropriate choice for the number of flavors, the pure modulus condition, and the SSB pattern of the auxiliary action. There, we shall also discuss its implementation at the quantum level, where the Yang-Mills partition function is written as a superposition of local contributions originated from ϑ_{S_0}. Of course, to make sense of the formal expressions, a fundamental requirement is renormalizability. In Ref. [29], relying on the algebraic method, we showed that this property is valid in the vortex-free sector ($S_0 = I$), where the possible counterterms are restricted by the Ward identities of the gauge-fixed action. In this work, we present a proof of renormalizability for the center-vortex sectors. In Sec. III, we give some preliminary definitions and review the gauge-fixed action in the vortex-free sector from a BRST perspective. In Sec. IV, we extend this procedure to a general sector labeled by center vortices, and we introduce the required boundary conditions in a way that maximizes the symmetries of the full action. In Sec. V, we list these symmetries and use them to establish the renormalizability of a general center-vortex sector ϑ_{S_0}. Finally, in Sec. VI we present our conclusions.

II. DETAILING THE PROCEDURE

In order for the maps $A \to \psi(A) \to S(A)$ given by Eqs. (1.8) and (1.9) to be well defined, natural conditions leading to a unique solution must be specified. Besides the regularity of the fields, we consider the asymptotic behavior $\psi_I(A) \to \bar{\psi}_I(A) \in \mathcal{M}$, where $\mathcal{M}$ is the vacua manifold of S_{H}, together with $D(A)\psi_I \to 0$. In addition, the solution must satisfy

$$(\psi_1(A^U), \psi_2(A^U), \ldots) = (\psi_1(A), \psi_2(A), \ldots) \Rightarrow U \in Z(N), \tag{2.1}$$

otherwise, any attempt at gauge fixing will surely have Gribov copies. In regions where the gauge field is close to $(i/g)\bar{S}\partial_\mu \bar{S}^{-1}$, $\bar{S} \in SU(N)$, the solution is expected to be close to an element of $\mathcal{M}$ whose phase accompanies $\bar{S}$. Then, it is clear that the elements of $\mathcal{M}$ must satisfy Eq. (2.1), which corresponds to choosing an $SU(N) \to Z(N)$ SSB pattern for $S_{\rm H}$. For this choice, showing that for every A the regular solution $(\psi_1(A), \psi_2(A)\ldots)$ satisfies Eq. (2.1) is a hard mathematical problem. Nonetheless, it is clear that this pattern favors this condition, as it plays a similar role to the orthonormality conditions in the lattice for the lowest eigenfunctions of the covariant adjoint Laplacian. For example, in Ref. [29], we considered an auxiliary action based on N^2-1 adjoint scalar fields ζ_I, where the elements of $\mathcal{M}$ are given by $\psi_I = vST_IS^{-1}$, $S \in SU(N)$. These elements satisfy $(\psi_I, \psi_J) = \delta_{IJ}$, thus forming orthonormal bases. The action reads

$$S_{\rm H} = \int d^4x (D_\mu^{ab}\zeta_I^b D_\mu^{a'b'}\zeta_I^{b'} + V_{\rm H}),$$
$$V_{\rm H} = \frac{\mu^2}{2}\zeta_I^a\zeta_I^a + \frac{\kappa}{3}f^{abc}f_{IJK}\zeta_I^a\zeta_J^b\zeta_K^c + \frac{\lambda}{4}\gamma_{IJKL}^{abcd}\zeta_I^a\zeta_J^b\zeta_K^c\zeta_L^d, \tag{2.2}$$

where γ is a combination of antisymmetric structure constants and deltas compatible with color and flavor symmetry. This generalizes the model introduced in Ref. [30]. Furthermore, in Ref. [15], Eq. (2.1) was shown to be valid for some examples of A, which include center-vortex configurations. As for the polar decomposition of the tuple of auxiliary fields in Eq. (1.9), the proposal was to define the pure modulus tuple $(q_1, q_2, \ldots)$ as the one that minimizes the average distance to the Lie basis vT_I,

$$\sum_{I=1}^{N^2-1}(q_I - vT_I)^2. \tag{2.3}$$

This leads to the condition

$$f(q_1, q_2, \ldots) = \sum_{I=1}^{N^2-1}[q_I, vT_I] = 0. \tag{2.4}$$

The uniqueness of this decomposition in sectors labeled by center vortices was also studied in Ref. [15]. In each sector ϑ_{S_0}, the gauge is then fixed by means of the condition (1.11), or, equivalently,

$$f_{S_0}(A) = f(S_0^{-1}\psi_1 S_0, S_0^{-1}\psi_2 S_0, \ldots) = 0. \tag{2.5}$$

In order to avoid the presence of multivalued fields, the Lie algebra components of q_I that rotate under S_0 must satisfy regularity conditions at the center-vortex guiding centers.

Let us now discuss the implementation of this procedure at the quantum level. The tuple $\psi(A)$ is introduced by means of an identity

$$\mathbb{1} = \int D\zeta \delta(\zeta - \psi(A)),$$
$$Z_{\rm YM} = \int [DA][D\zeta]\delta(\zeta - \psi(A))e^{-S_{\rm YM}(A)}. \tag{2.6}$$

The information that $\psi(A)$ is a solution to the auxiliary equations of motion is given through the representation

$$\delta(\zeta - \psi(A)) = \det\left(\frac{\delta^2 S_{\rm H}}{\delta\zeta_I\delta\zeta_J}\right)\delta\left(\frac{\delta S_{\rm H}}{\delta\zeta_I}\right). \tag{2.7}$$

Next, by definition (see Sec. I), the contribution to $Z_{\rm YM}$ originated from the sector ϑ_{S_0} is given by restricting the ζ path-integral to auxiliary fields of the form

$$\zeta = (Sq_1S^{-1}, Sq_2S^{-1}, \ldots), \quad S = US_0, \quad f(q_1, q_2, \ldots) = 0 \tag{2.8}$$

[cf. Eq. (2.4)], i.e.,

$$Z_{\rm YM}^{S_0} = \int [DA][Dq]_{\rm r.c.}[DU]\delta(f(q))J(q) \times \delta(SqS^{-1} - \psi(A))e^{-S_{\rm YM}(A)}. \tag{2.9}$$

Here, J is the Jacobian that arises as a consequence of switching from the integral over ζ to the integral over its modulus q and phase S [13]. The gauge can then be fixed upon redefining $A^U \to A$, $\zeta^U \to \zeta$. The presence of S_0 occurs as it cannot be eliminated by a regular gauge transformation U. The classification of all possible defects in this non-Abelian context is a difficult problem which is out of the scope of the present work. Instead, we will focus on some examples. In sectors labeled by center vortices, in order to ensure regularity, the components of q_I that rotate under S_0 must vanish at the guiding centers. For example, in a sector labeled by an elementary center vortex with guiding center along some closed world surface Ω, the label is given by $S_0 = e^{i\chi\beta\cdot T}$, χ being an angle that changes by 2π when going around a loop that links Ω, and β being proportional to a fundamental weight. Here, we use the definition $\beta\cdot T \equiv \beta_q T_q$, where β_q are the components of the $(N-1)$-tuple β, and T_q are the Cartan subalgebra generators.[1] Because of the relation $e^{i2\pi\beta\cdot T} = e^{-i2\pi/N} \in Z(N)$, the rotated Lie basis $S_0 T_I S_0^{-1} = \mathrm{Ad}(S_0)_{JI}T_J$ describes a topologically nontrivial loop in $\mathrm{Ad}(SU(N))$, as we travel around a path in real space that

[1]The conventions for the Lie algebra generators are described in Appendix.

links Ω. This means that the basis necessarily becomes ill-defined when the path size is shrunk to zero. Indeed, the transformed Lie components read

$$\begin{aligned} S_0 T_\alpha S_0^{-1} &= \cos(\alpha\cdot\beta\chi)T_\alpha - \sin(\alpha\cdot\beta\chi)T_{\bar\alpha}, \\ S_0 T_{\bar\alpha} S_0^{-1} &= \cos(\alpha\cdot\beta\chi)T_\alpha + \sin(\alpha\cdot\beta\chi)T_{\bar\alpha}. \end{aligned} \tag{2.10}$$

The $(N-1)$-tuples α are the roots of $\mathfrak{su}(N)$, i.e., they are formed by eigenvalues of the adjoint action of the Cartan generators on $\mathfrak{su}(N)$ (see Appendix). Then, the elements T_α, $T_{\bar\alpha}$, with $\alpha\cdot\beta \neq 0$, are ill-defined at the vortex guiding centers. For the corresponding field components, we must require the regularity conditions

$$\zeta_I^\alpha = \zeta_I^{\bar\alpha} = 0, \qquad x \in \Omega, \tag{2.11}$$

which can be imposed with the introduction of appropriate Lagrange multipliers defined on Ω. This will be discussed carefully in Sec. IV.

Summarizing, after introducing an identity in the integrand of $Z_{\rm YM}$, and then restricting the auxiliary fields, the gauge fields get restricted to be associated with solutions $\psi(A)$ of the form in Eq. (2.8). When S_0 is inequivalent to S_0', the gauge fields that contribute to $Z_{\rm YM}^{S_0}$ and $Z_{\rm YM}^{S_0'}$ are expected to be physically inequivalent. Consider, for example, a pair of sectors with the same weight β, but phases χ, χ' multivalued at different guiding centers Ω and Ω'. As we move on a gauge orbit, which is done with regular transformations, the guiding centers cannot be changed, so that a pair of gauge fields on a given orbit cannot be in different sectors. Note also that by increasing the vacuum parameter v, the solutions outside Ω corresponding to $A \in \vartheta_{S_0}$ are expected to be close to $vS_0T_IS_0^{-1}$. Then, the implemented SSB pattern tends to rule out the possibility of multiple counting, and the sum over all possible labels is expected to yield the full YM partition function,

$$Z_{\rm YM} = \sum_{S_0} Z_{\rm YM}^{S_0}. \tag{2.12}$$

III. PRELIMINARY DEFINITIONS AND THE VORTEX-FREE SECTOR

To construct the complete action, in a BRST formal manner [31], we start by defining the usual YM action in 4-dimensional Euclidean spacetime[2]

$$S_{\rm YM} = \frac{1}{4}\int_x F_{\mu\nu}^a F_{\mu\nu}^a, \tag{3.1}$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$ is the field strength for the $SU(N)$ gauge-field A_μ^a, while g is the coupling constant. Lower case greek and latin indices take the values $\{0,1,2,3\}$ and $\{1,\ldots,N^2-1\}$, respectively.

Another field naturally appearing in YM theory is the Faddeev-Popov ghost field c^a. This field appears in the gauge-fixing procedure and, together with A_μ, can be interpreted geometrically [32–34]. For now, we can define the nilpotent BRST operator s and the corresponding BRST transformations of the fields

$$\begin{aligned} sA_\mu^a &= \frac{i}{g} D_\mu^{ab} c^c, \\ sc^a &= -\frac{i}{g} f^{abc} c^b c^c, \end{aligned} \tag{3.2}$$

with the covariant derivative defined as $D_\mu^{ab} = \delta^{ab}\partial_\mu - g f^{abc} A_\mu^c$.

To fix the gauge in the vortex-free sector, we follow the procedure developed in [13,29]. The first step is to introduce a set of auxiliary fields ζ_I^a, b_I^a, c_I^a, $\bar c_I^a$, where the flavor index takes values in $\{1,2,\ldots,N^2-1\}$. In order to keep the physical degrees of freedom of pure YM theory unaltered, these fields are introduced as BRST doublets [31],

$$\begin{aligned} s\zeta_I^a &= i f^{abc}\zeta_I^b c^c + c_I^a, \\ sc_I^a &= -i f^{abc} c_I^b c^c, \\ s\bar c_I^a &= -i f^{abc} \bar c_I^b c^c - b_I^a, \\ sb_I^a &= i f^{abc} b_I^b c^c. \end{aligned} \tag{3.3}$$

Moreover, a set of BRST doublet parameters $\{\mu, U, \kappa, \mathcal{K}, \lambda, \Lambda\}$ are introduced

$$\begin{aligned} s\mu^2 &= U^2, \\ sU^2 &= 0, \\ s\kappa &= \mathcal{K}, \\ s\mathcal{K} &= 0, \\ s\lambda &= \Lambda, \\ s\Lambda &= 0. \end{aligned} \tag{3.4}$$

The parameters μ, κ, and λ are required to implement the $SU(N) \to Z(N)$ spontaneous symmetry breaking in the auxiliary action, thus producing the correlation between A_μ and the phases $S(A)$ containing defects. Their respective doublet partners $U^2, \mathcal{K}, \Lambda$ are required to guarantee that the observables remain independent from the gauge-fixing parameters [31,35]. The auxiliary action is introduced as a BRST-exact term in the form

[2]In this paper, we employ a condensed notation for integrals as $\int_x \equiv \int d^4x$.

$$S_{\rm aux} = -s\int_x (D_\mu^{ab}\bar{c}_I^b D_\mu^{ac}\zeta_I^c + \mu^2\bar{c}_I^a\zeta_I^a + \kappa f^{abc}f_{IJK}\bar{c}_I^a\zeta_J^b\zeta_K^c + \lambda\gamma_{IJKL}^{abcd}b_I^a\zeta_J^b\zeta_K^c\zeta_L^d)$$
$$= \int_x (D_\mu^{ab}b_I^b D_\mu^{ac}\zeta_I^c + D_\mu^{ab}\bar{c}_I^b D_\mu^{ac}c_I^c + \mu^2(\bar{c}_I^a c_I^a + b_I^a\zeta_I^b) + \kappa f_{IJK}f^{abc}(b_I^a\zeta_J^b\zeta_K^c - 2\bar{c}_I^a\zeta_K^b c_J^c)$$
$$+ \lambda\gamma_{IJKL}^{abcd}(b_I^a\zeta_J^b\zeta_K^c\zeta_L^d + 3\bar{c}_I^a c_J^b\zeta_K^c\zeta_L^d) - U^2\bar{c}_I^a\zeta_I^a - \Lambda f^{abc}f^{cde}\bar{c}_I^a\zeta_J^b\zeta_I^d\zeta_J^e - \mathcal{K}f^{IJK}f^{abc}\bar{c}_I^a\zeta_J^b\zeta_K^c). \qquad (3.5)$$

Here, γ is a general color-flavor tensor, which is invariant under the adjoint global symmetry group $\mathrm{Ad}(SU(N))$. For later use, when implementing the different symmetries, we shall consider tensors formed by antisymmetric structure constants and Kronecker deltas such that $s\gamma_{IJKL}^{abcd}b_I^a\zeta_J^b\zeta_K^c\zeta_L^d$ is c-independent, i.e.,

$$\gamma_{IJKL}^{mbcd}f^{ame} + \gamma_{IJKL}^{amcd}f^{bme} + \gamma_{IJKL}^{abmd}f^{cme} + \gamma_{IJKL}^{abcm}f^{dme} = 0. \qquad (3.6)$$

The gauge fixing *per se* is performed in an indirect way by imposing some condition on the auxiliary fields (see for instance [13,29]). With this purpose, the usual BRST doublet $\{\bar{c}^a, b^a\}$ is introduced [31]

$$s\bar{c}^a = -b^a,$$
$$sb^a = 0, \qquad (3.7)$$

with $\bar{c}^a$ being the Faddeev-Popov antighost field and b^a the Lautrup-Nakanishi field. In the vortex-free sector, the representative S_0 can be taken as the identity, so the gauge fixing condition reads

$$f^{abc}\eta_I^b\zeta_I^c = 0, \qquad \eta_I = vT_I. \qquad (3.8)$$

The parameter v has mass dimension. Indeed, the field η_I can be thought of as a reference element in the classical vacua manifold $\mathcal{M}$ of the auxiliary action [13,15,29]. The gauge fixing is essentially a condition setting the local frame $\zeta_I^a T^a \in \mathfrak{su}(N)$ to lie as close as possible to the global frame vT_I. Such a condition is realized by the gauge-fixing action

$$S_{\rm gf} = -s\int_x if^{abc}\bar{c}^a\eta_I^b\zeta_I^c,$$
$$= \int_x [if^{abc}(b^a\eta_I^b\zeta_I^c + \bar{c}^a\eta_I^b c_I^c) + f^{ecd}f^{eba}\bar{c}^a\eta_I^b\zeta_I^c c^d]. \qquad (3.9)$$

The full gauge-fixed action in the vortex-free sector then reads

$$S_{\rm vf} = S_{\rm YM} + S_{\rm aux} + S_{\rm gf}. \qquad (3.10)$$

Note that, as the terms $S_{\rm aux}$ and $S_{\rm gf}$ are BRST exact, the theory continues to be pure YM, in spite of the SSB properties of the auxiliary sector. Another important feature of action (3.10) is a global flavor symmetry, which implies an extra conserved charge (besides the ghost number), the $\mathcal{Q}$-charge. Such symmetry and others play a crucial role in the proof of renormalizability of the vortex-free sector [29]. For completeness and further use, we display in Tables I and II the quantum numbers of fields and parameters so far introduced.

IV. CENTER-VORTEX SECTORS

Let us consider a sector labeled by n elementary center vortices located at arbitrary closed surfaces Ω_i. When they carry the same fundamental weight β, the associated phase can be written as $S_0 = e^{i\chi\beta\cdot T}$, where χ is multivalued when going around $\Omega = \Omega_1 \cup \ldots \cup \Omega_n$. In this case, the gauge-fixed configurations of auxiliary fields will be of the form $\zeta_I = S_0 q_I S_0^{-1}$, with $[q_I, T_I] = 0$ [cf. Eq. (2.9)]. Then, to assure regularity, the components of ζ_I that rotate under S_0 must vanish at Ω. These are given by the fields ζ_I^α and $\zeta_I^{\bar{\alpha}}$ along the off-diagonal directions T_α, $T_{\bar{\alpha}}$, with $\alpha\cdot\beta \neq 0$. A well-known manner to implement this type of boundary condition is to introduce a δ–functional in the partition function, and exponentiate it using auxiliary fields that only exist in Ω [36],

TABLE I. Quantum numbers of the fields.

Fields	A_μ	ζ_I	c_I	$\bar{c}_I$	b_I	η_I	$\bar{c}$	c	b	ξ_I	λ_I
Mass dimension	1	1	1	1	1	1	2	0	2	1	1
Ghost number	0	0	1	−1	0	0	−1	1	0	−1	0
$\mathcal{Q}$-charge	0	1	1	−1	−1	−1	0	0	0	−1	−1
Nature	B	B	F	F	B	B	F	F	B	F	B

TABLE II. Quantum numbers of the parameters.

Parameters	U^2	$\mathcal{K}$	Λ	μ^2	κ	λ
Mass dimension	2	1	0	2	1	0
Ghost number	1	1	1	0	0	0
$\mathcal{Q}$-charge	0	−1	−2	0	−1	−2
Nature	F	F	F	B	B	B

$$\prod_\gamma \delta_\Omega(\zeta_I^\gamma)\delta_\Omega(\zeta_I^{\bar\gamma}) = \int [D\lambda] e^{i\sum_\gamma \int d\sigma_1 d\sigma_2 \sqrt{g(\sigma_1,\sigma_2)}(\lambda_I^\gamma(\sigma_1,\sigma_2)\zeta_I^\gamma(x(\sigma_1,\sigma_2))+\lambda_I^{\bar\gamma}(\sigma_1,\sigma_2)\zeta_I^{\bar\gamma}(x(\sigma_1,\sigma_2)))}, \tag{4.1}$$

where $x(\sigma_1,\sigma_2)$ is a parametrization of Ω, λ_I^γ and $\lambda_I^{\bar\gamma}$ are auxiliary fields, g is the determinant of the worldsheet metric, and we are denoting by γ the roots that satisfy $\gamma\cdot\beta\neq 0$. By introducing a source localized on Ω, this expression can also be written in terms of a field λ_I defined on the whole spacetime

$$\prod_\gamma \delta_\Omega(\zeta_I^\gamma)\delta_\Omega(\zeta_I^{\bar\gamma}) = \int [D\lambda] e^{i\int dx\, J_\Omega(x)\sum_\gamma(\lambda_I^\gamma(x)\zeta_I^\gamma(x)+\lambda_I^{\bar\gamma}(x)\zeta_I^{\bar\gamma}(x))}, \tag{4.2}$$

$$J_\Omega(x) = \int d\sigma_1 d\sigma_2 \sqrt{g(\sigma_1,\sigma_2)}\delta(x-x(\sigma_1,\sigma_2)). \tag{4.3}$$

This procedure was proposed in Ref. [29]. However, these terms break the color-flavor symmetry, which would allow too many new counter-terms in the renormalizability analysis. For instance, the single term $\bar c^a c^a$ would generate $(N^2-1)^2$ independent contributions. To circumvent this problem we can invoke the Symanzik method [37] to promote J_Ω to a set of generic Schwinger sources $J^a(x)$, so the color-flavor symmetry can be restored. At the end, we choose J^a so as to recover the initial theory. One possibility to perform the trick is to consider the replacement

$$\prod_\gamma \delta_\Omega(\zeta_I^\gamma)\delta_\Omega(\zeta_I^{\bar\gamma}) \to \int [D\lambda] e^{-\int dx\, f^{abc}J^a\lambda_I^b\zeta_I^c}. \tag{4.4}$$

Expression (4.2) is then recovered by setting the source J^a to its physical values, namely,

$$J^\alpha|_{\rm phys} = J^{\bar\alpha}|_{\rm phys} = 0,$$
$$J^q|_{\rm phys} = i\beta_q \int d\sigma_1 d\sigma_2 \sqrt{g(\sigma_1,\sigma_2)}\delta(x-x(\sigma_1,\sigma_2)). \tag{4.5}$$

In this case, we have

$$f^{abc}J^a\lambda_I^b\zeta_I^c = J^q f^{qbc}\lambda_I^b\zeta_I^c, \tag{4.6}$$

and taking into account that the only structure constants which contribute are $f^{q\alpha\bar\alpha} = \alpha|_q$, Eq. (4.6) becomes

$$\sum_{\alpha>0} J\cdot\alpha(\lambda_I^\alpha\zeta_I^{\bar\alpha} - \lambda_I^{\bar\alpha}\zeta_I^\alpha)$$
$$= \sum_{i=1}^n \sum_{\alpha>0} i\beta\cdot\alpha \int d\sigma_1 d\sigma_2 \sqrt{g(\sigma_1,\sigma_2)}\delta(x-x(\sigma_1,\sigma_2)) \times (\lambda_I^\alpha\zeta_I^{\bar\alpha} - \lambda_I^{\bar\alpha}\zeta_I^\alpha). \tag{4.7}$$

Since the scalar product $\alpha\cdot\beta$ is either $1, -1$, or 0, the desired expression is recovered. However, we still need to worry about the BRST invariance of these boundary conditions, which imply $s\zeta_I^\gamma = s\zeta_I^{\bar\gamma} = 0$ on the vortex surface [38]. For this purpose, it is convenient to work with BRST doublets and write the J^a-term as a BRST-exact quantity. Thus, we introduce the auxiliary field ξ_I such that the pair $\{\lambda_I, \xi_I\}$ forms a BRST doublet

$$s\xi_I^a = \lambda_I^a,$$
$$s\lambda_I^a = 0. \tag{4.8}$$

This ensures that these fields cannot be part of the physical spectrum of the theory [31]. The source J^a is assumed to be BRST invariant. Hence,

$$S_J = s\int_x f^{abc}J^a\xi_I^b\zeta_I^c$$
$$= \int_x f^{abc}J^a[\lambda_I^b\zeta_I^c - \xi_I^b(if^{cde}\zeta_I^c c^e + c_I^c)]. \tag{4.9}$$

Up to this point, the full action in the vortex sector reads

$$S = S_{\rm vf} + S_J. \tag{4.10}$$

Note that the discussion can be trivially extended to a sector labeled by vortices carrying a distribution of weights $\beta^1,\ldots,\beta^n$, i.e., $S_0 = e^{i\chi^1\beta^1\cdot T}\ldots e^{i\chi^n\beta^n\cdot T}$, where χ^i is multivalued when going around Ω_i, and each β^i takes values among the N different fundamental weights. In this case, the physical values of the source would be

$$J^\alpha|_{\rm phys} = J^{\bar\alpha}|_{\rm phys} = 0,$$
$$J^q|_{\rm phys} = i\sum_{i=1}^n \beta_q^i \int d\sigma_1^i d\sigma_2^i \sqrt{g(\sigma_1^i,\sigma_2^i)}\delta(x-x(\sigma_1^i,\sigma_2^i)), \tag{4.11}$$

where $x(\sigma_1^i,\sigma_2^i)$ is a parametrization of Ω_i.

V. ALGEBRAIC ANALYSIS OF RENORMALIZABILITY

In this section we analyze the center vortex sectors by employing the algebraic renormalization technique [31]. We will prove it at first order. Nevertheless, since this technique is recursive, the proof is valid to all orders in perturbation theory.

A. Ward identities

As discussed in [29], the action (3.10) for the vortex-free sector displays a rich set of Ward identities. It turns out that the same set of Ward identities can be accommodated for the action (4.10) of the center vortex sectors. For this aim, we have to include an additional external source term

$$
\begin{aligned}
S_{\rm ext} &= s\int_x (K^a_\mu A^a_\mu + \bar{C}^a c^a + \bar{L}^a_I c^a_I + Q^a_I \zeta^a_I + B^a_I b^a_I + \bar{L}^a_I \bar{c}^a_I + m^{ab}_{IJ}\zeta^a_I \xi^b_J + M^{ab}_I \bar{c}^a \zeta^b_I),\\
&= \int_x \Big\{ \frac{i}{g} K^a_\mu (D^{ab}_\mu c^b) - \frac{1}{2} i\bar{C}^a f^{abc} c^b c^c - i f^{abc} \bar{L}^a_I c^b_I c^c + Q^a_I (i f^{abc}\zeta^b_I c^c + c^a_I) + i f^{abc} B^a_I b^b_I c^c \\
&\quad - L^a_I (i f^{abc} \bar{c}^b_I c^c + b^a_I) + N^{ab}_I \bar{c}^a \zeta^b_I - M^{ab}_I b^a \zeta^b_I - M^{ab}_I \bar{c}^a (i f^{bmn}\zeta^m_I c^n + c^b_I) \\
&\quad - n^{ab}_{IJ}\zeta^a_I \xi^b_J + m^{ab}_{IJ}[\lambda^a_I \zeta^b_J - \xi^a_I (i f^{bcd}\zeta^c_I c^d + c^b_I)] \Big\},
\end{aligned} \tag{5.1}
$$

with K, $\bar{C}$, $\bar{L}$, Q, B, and L being BRST invariant, while

$$
\begin{aligned}
sM^{ab}_I &= N^{ab}_I,\\
sN^{ab}_I &= 0,\\
sm^{ab}_{IJ} &= -n^{ab}_{IJ},\\
sn^{ab}_{IJ} &= 0.
\end{aligned} \tag{5.2}
$$

The large amount of sources introduced is necessary to control nonlinear symmetries as well as to ensure important symmetries to establish the renormalization of the theory. The quantum numbers of all sources are displayed in Table III. The final full action in the vortex sectors is then

$$
\Sigma = S + S_{\rm ext}. \tag{5.3}
$$

This action enjoys the following set of Ward identities:

(i) The Slavnov-Taylor identity,

$$
\begin{aligned}
S(\Sigma) = \int_x \Big(& \frac{\delta\Sigma}{\delta K^a_\mu}\frac{\delta\Sigma}{A^a_\mu} + \frac{\delta\Sigma}{\delta\bar{L}^a_I}\frac{\delta\Sigma}{\delta c^a_I} + \frac{\delta\Sigma}{\delta L^a_I}\frac{\delta\Sigma}{\delta\bar{c}^a_I} + \frac{\delta\Sigma}{\delta B^a_I}\frac{\delta\Sigma}{\delta b_I} \\
& + \frac{\delta\Sigma}{\delta Q^a_I}\frac{\delta\Sigma}{\delta\zeta^a_I} + -b^a \frac{\delta\Sigma}{\delta\bar{c}^a} + \frac{\delta\Sigma}{\delta\bar{C}^a}\frac{\delta\Sigma}{\delta c^a} + N^{ab}_I \frac{\delta\Sigma}{\delta M^{ab}_I} \\
& + \lambda^a_I \frac{\delta\Sigma}{\delta\xi^a_I} - n^{ab}_{IJ}\frac{\delta\Sigma}{\delta m^{ab}_{IJ}} \Big) \\
& + U^2 \frac{\delta\Sigma}{\delta\mu^2} + \mathcal{K}\frac{\delta\Sigma}{\delta\kappa} + \Lambda\frac{\delta\Sigma}{\delta\lambda} = 0.
\end{aligned} \tag{5.4}
$$

TABLE III. Quantum numbers of the sources.

Sources	K_μ	$\bar{C}$	$\bar{L}_I$	L_I	B_I	Q_I	M_I	N_I	m_{IJ}	n_{IJ}	J
Mass dimension	3	4	3	3	3	3	1	1	2	2	2
Ghost number	−1	−2	−2	0	−1	−1	0	1	0	1	0
$\mathcal{Q}$-charge	0	0	−1	1	1	−1	−1	−1	0	0	0
Nature	F	B	B	B	F	F	B	F	B	F	B

(ii) The gauge-fixing equation,

$$
\frac{\delta\Sigma}{\delta b^a} = i f^{abc}\eta^b_I \zeta^c_I - M^{ab}_I \zeta^b_I, \tag{5.5}
$$

(iii) The antighost equation,

$$
\bar{\mathcal{G}}^a\Sigma = \left(\frac{\delta}{\delta\bar{c}^a} + M^{ab}_I \frac{\delta}{\delta Q^b_I} - i f^{abc}\eta^b_I \frac{\delta}{\delta Q^c_I}\right)\Sigma = N^{ab}_I \zeta^b_I. \tag{5.6}
$$

(iv) The ghost number equation,

$$
\begin{aligned}
\mathcal{N}_{\rm gh}\Sigma = \int d^4x \Big(& c^a_I \frac{\delta}{\delta c^a_I} - \bar{c}^a_I \frac{\delta}{\delta\bar{c}^a_I} + c^a \frac{\delta}{\delta c^a} - \bar{c}^a \frac{\delta}{\delta\bar{c}^a} \\
& + U^2 \frac{\delta}{\delta U^2} + \mathcal{K}\frac{\delta}{\delta\mathcal{K}} + \Lambda\frac{\delta}{\delta\Lambda} \\
& - K^a \frac{\delta}{\delta K^a} - 2\bar{C}^a \frac{\delta}{\delta\bar{C}^a} - 2\bar{L}^a_I \frac{\delta}{\delta\bar{L}^a_I} - Q^a_I \frac{\delta}{\delta Q^a_I} \\
& - B^a_I \frac{\delta}{\delta B^a_I} + N^{ab}_I \frac{\delta}{\delta N^{ab}_I} \\
& - \xi^a_I \frac{\delta}{\delta\xi^a_I} + m^{ab}_{IJ}\frac{\delta}{\delta m^{ab}_{IJ}} \Big)\Sigma^{S_0} = 0.
\end{aligned} \tag{5.7}
$$

(v) The global flavor symmetry,

$$
\begin{aligned}
\mathcal{Q}\Sigma = \Big(& \zeta^a_I \frac{\delta}{\delta\zeta^a_I} - b^a_I \frac{\delta}{\delta b^a_I} - \bar{c}^a_I \frac{\delta}{\delta\bar{c}^a_I} + c^a_I \frac{\delta}{\delta c^a_I} - u^a_I \frac{\delta}{\delta u^a_I} \\
& - Q^a_I \frac{\delta}{\delta Q^a_I} + B^a_I \frac{\delta}{\delta B^a_I} + L^a_I \frac{\delta}{\delta L^a_I} \\
& - \bar{L}^a_I \frac{\delta}{\delta\bar{L}^a_I} - \kappa\frac{\delta}{\delta\kappa} - 2\lambda\frac{\delta}{\delta\lambda} - \mathcal{K}\frac{\delta}{\delta\mathcal{K}} - 2\Lambda\frac{\delta}{\delta\Lambda} \\
& - N^{ab}_I \frac{\delta}{\delta N^{ab}_I} - M^{ab}_I \frac{\delta}{\delta M^{ab}_I} \\
& - \xi^a_I \frac{\delta}{\delta\xi^a_I} - \lambda^a_I \frac{\delta}{\delta\lambda^a_I} \Big)\Sigma = 0.
\end{aligned} \tag{5.8}
$$

(vi) The linearly broken rigid symmetry,

$$\mathcal{R}\Sigma=\left(\bar{c}_I^a\frac{\delta}{\delta b_I^a}+\zeta_I^a\frac{\delta}{\delta c_I^a}-if^{abc}\eta_I^a\frac{\delta}{\delta N_I^{bc}}-B_I^a\frac{\delta}{\delta\bar{L}_I^a}+\bar{L}_I^a\frac{\delta}{\delta Q_I^a}-\kappa\frac{\delta}{\delta\mathcal{K}}-2\lambda\frac{\delta}{\delta\Lambda}-M_I^{ab}\frac{\delta}{\delta N_I^{ab}}-\xi_I^a\frac{\delta}{\delta\lambda_I^a}\right)\Sigma$$
$$=\bar{L}_I^a c_I^a+L_I^a\bar{c}_I^a-\zeta_I^a Q_I^a. \tag{5.9}$$

(vii) The ghost equation,

$$\mathcal{G}^a\Sigma=\left(\frac{\delta}{\delta c^a}+(f^{abc}f^{cnm}\eta_I^n+if^{abn}M_I^{mn})\frac{\delta}{\delta N_I^{mb}}+i(f^{blc}f^{cma}J^l+im_{IJ}^{db}f^{dma})\frac{\delta}{\delta n_{IJ}^{mb}}\right)\Sigma$$
$$=if^{abc}(\bar{C}^b c^c+Q_I^b\zeta_I^c+\bar{L}_I^b c_I^c+L_I^b\bar{c}_I^c+B_I^b b_I^c)+\frac{i}{g}D_\mu^{ab}K_\mu^b. \tag{5.10}$$

(viii) The J equation,[3]

$$\mathcal{J}^a\Sigma=\frac{\delta\Sigma}{\delta J^a}-f^{abc}\delta_{IJ}\frac{\delta\Sigma}{\delta m_{IJ}^{bc}}=0. \tag{5.11}$$

(ix) Global symmetry in the boundary-conditions sector,

$$\mathcal{F}\Sigma=\lambda_I^a\frac{\delta\Sigma}{\delta\lambda_I^a}+\xi_I^a\frac{\delta\Sigma}{\delta\xi_I^a}-J^a\frac{\delta\Sigma}{\delta J^a}-n_{IJ}^{ab}\frac{\delta\Sigma}{\delta n_{IJ}^{ab}}-m_{IJ}^{ab}\frac{\delta\Sigma}{\delta m_{IJ}^{ab}}=0. \tag{5.12}$$

(x) The linearly broken λ equation,

$$\Lambda_I^a\Sigma=\frac{\delta\Sigma}{\delta\lambda_I^a}=f^{abc}\zeta_I^b J^c. \tag{5.13}$$

B. The most general counterterm

With the full action (5.3) at hand, we are now able to construct the most general counterterm Σ_C compatible with all Ward identities in Sec. VA. Hence, we write the perturbative expansion of the quantum action Γ at first order,

$$\Gamma^{(1)}=\Sigma+\epsilon\Sigma_C, \tag{5.14}$$

and impose on it all the Ward identities respected by the classical action Σ. A straightforward calculation leads to the following constraints for the counterterm,

$$\begin{aligned}\mathcal{B}_\Sigma\Sigma_C&=0,\\ \frac{\delta\Sigma_C}{\delta b^a}&=0,\\ \bar{\mathcal{G}}^a\Sigma_C&=0,\\ \mathcal{N}_{gh}\Sigma_C&=0,\\ \mathcal{Q}\Sigma_C&=0,\\ \mathcal{R}\Sigma_C&=0,\\ \mathcal{G}^a\Sigma_C&=0,\\ \mathcal{J}^a\Sigma_C&=0,\\ \mathcal{F}\Sigma_C&=0,\\ \Lambda_I^a\Sigma_C&=0.\end{aligned} \tag{5.15}$$

Here,

$$\mathcal{B}_\Sigma=\int_x\left(\frac{\delta\Sigma}{\delta K_\mu^a}\frac{\delta}{A_\mu^a}+\frac{\delta\Sigma}{\delta A_\mu^a}\frac{\delta}{\delta K_\mu^a}+\frac{\delta\Sigma}{\delta L_I^a}\frac{\delta}{\delta\bar{c}_I^a}+\frac{\delta\Sigma}{\delta\bar{c}_I^a}\frac{\delta}{\delta L_I^a}+\frac{\delta\Sigma}{\delta\bar{L}_I^a}\frac{\delta}{\delta c_I^a}+\frac{\delta\Sigma}{\delta c_I^a}\frac{\delta}{\delta\bar{L}_I^a}+\frac{\delta\Sigma}{\delta B_I^a}\frac{\delta}{\delta b_I}+\frac{\delta\Sigma}{\delta b_I^a}\frac{\delta}{\delta B_I^a}+\frac{\delta\Sigma}{\delta Q_I^a}\frac{\delta}{\delta\zeta_I^a}+\frac{\delta\Sigma}{\delta\zeta_I^a}\frac{\delta}{\delta Q_I^a}+\frac{\delta\Sigma}{\delta\bar{C}^a}\frac{\delta}{\delta c^a}+\frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta\bar{C}^a}+N_I^{ab}\frac{\delta}{\delta M_I^{ab}}-b^a\frac{\delta}{\delta\bar{c}^a}-n_{IJ}^{ab}\frac{\delta}{\delta m_{IJ}^{ab}}+\lambda_I^a\frac{\delta}{\delta\xi_I^a}\right)+U^2\frac{\delta}{\delta\mu^2}+\mathcal{K}\frac{\delta}{\delta\kappa}+\Lambda\frac{\delta}{\delta\lambda} \tag{5.16}$$

is the linearized Slavnov-Taylor operator, which turns out to be nilpotent. Thence, the first equation in (5.15) defines a cohomology problem for $\mathcal{B}_\Sigma$. The solution reads [31]

$$\Sigma_C=\Delta_0+\mathcal{B}_\Sigma\Delta^{-1}, \tag{5.17}$$

where Δ_0 is the nontrivial part of the cohomology and $\mathcal{B}_\Sigma\Delta^{-1}$ is the trivial one. The nontrivial part is an integrated functional, polynomial in the fields, sources and their derivatives, with dimension 4, and vanishing ghost number. The quantity Δ^{-1} is also an integrated functional, polynomial in the fields, sources and their derivatives, with dimension 4, but with ghost number -1. Due to the rich set of constraints (5.15), it is a straightforward exercise to show that the nontrivial cohomology is the usual one in YM theory, namely

$$\Delta_0=a_0S_{\mathrm{YM}}, \tag{5.18}$$

with a_0 being an independent renormalization constant. For the trivial sector of the cohomology, we can write

$$\Delta^{-1}=\bar{\Delta}^{-1}(\varphi)+D^{-1}(\varphi,\phi), \tag{5.19}$$

[3] Notice that, due to this Ward identity, the variables (J,m_{IJ}) can enter the counterterm only through the combination $\delta_{IJ}f^{abc}J^a-m_{IJ}^{bc}$.

where $\phi \equiv \{J, \lambda_I, \xi_I, m_{IJ}, n_{IJ}\}$ and φ stands for all the other fields, sources and parameters. This decomposition is a direct consequence of the Ward identity (5.12) together with the quantum numbers of the sources involved. Then, it follows that $\bar{\Delta}^{-1}(\varphi)$ is identical to the full Δ^{-1} of the vortex-free sector obtained in Ref. [29]. Remarkably, after applying all remaining constraints in (5.15), one finds

$$\bar{\Delta}^{-1} = \int_x [a_1(\bar{c}_I^a \partial^2 \zeta_I^a + g f^{abc}\partial_\mu A_\mu^a \bar{c}_I^b \zeta_I^c + g^2 f^{acm} f^{dbm} A_\mu^a A_\mu^b \bar{c}_I^c \zeta_I^d) + a_2 f^{IJK} f_{abc} \kappa \bar{c}_I^a \zeta_J^b \zeta_K^c + a_{3,IJKL}^{abcd} \lambda \bar{c}_I^a \zeta_J^b \zeta_K^c \zeta_L^d + a_4 \mu^2 \bar{c}_I^a \zeta_I^a] \tag{5.20}$$

$$D^{-1} = 0, \tag{5.21}$$

with a_i being independent renormalization parameters. The tensor $a_{3,IJKL}^{abcd}$ has the same structure of γ_{IJKL}^{abcd}. Therefore,

$$\Sigma_C = \Sigma_C(\varphi), \tag{5.22}$$

where $\Sigma_C(\varphi)$, given by

$$\begin{aligned}\Sigma_C(\varphi) = \int_x \Big[&\frac{a_0}{2}(\partial_\mu A_\nu^a)^2 - \frac{a_0}{2}\partial_\nu A_\mu^a \partial_\mu A_\nu^a + \frac{a_0}{2} g f^{abc} A_\mu^a A_\nu^b \partial_\mu A_\nu^c + \frac{a_0}{4} g^2 f^{abc} f^{cde} A_\mu^a A_\nu^b A_\mu^d A_\nu^e \\ &+ a_1(\partial_\mu b_I^a \partial_\mu \zeta_I^a + g f^{abc}\partial_\mu b_I^a A_\mu^b \zeta_I^c + g f^{abc} b_I^a \partial_\mu \zeta_I^b A_\mu^c + g^2 f^{abe} f^{cde} A_\mu^a b_I^b A_\mu^c \zeta_I^d \\ &+ \partial_\mu \bar{c}_I^a \partial_\mu c_I^a + g f^{abc}\partial_\mu \bar{c}_I^a A_\mu^b c_I^c + g f^{abc} \bar{c}_I^a \partial_\mu c_I^b A_\mu^c + g^2 f^{abe} f^{cde} A_\mu^a \bar{c}_I^b A_\mu^c c_I^d) \\ &+ a_2 f^{IJK} f^{abc}(\mathcal{K}\bar{c}_I^a \zeta_J^b \zeta_K^c - \kappa b_I^a \zeta_J^b \zeta_K^c - 2\kappa \bar{c}_I^a c_J^b \zeta_K^c) + a_{3,IJKL}^{abcd}(\Lambda \bar{c}_I^a \zeta_J^b \zeta_K^c \zeta_L^d - \lambda b_I^a \zeta_J^b \zeta_K^c \zeta_L^d - 3\lambda \bar{c}_I^a c_J^b \zeta_K^c \zeta_L^d) \\ &+ a_4(U^2 \bar{c}_I^a \zeta_I^a - \mu^2 b_I^a \zeta_I^a - \mu^2 \bar{c}_I^a c_I^a)\Big],\end{aligned} \tag{5.23}$$

is the vortex-free counterterm found in [29].

C. Quantum stability

To prove stability, one has to show that the counterterm (5.23) can be absorbed in the original action (5.3) by means of a multiplicative redefinition of the fields, sources and parameters, i.e.,

$$\Sigma(\Phi, \mathcal{S}, P) + \epsilon \Sigma_C(\Phi, \mathcal{S}, P) = \Sigma(\Phi_0, \mathcal{S}_0, P_0), \tag{5.24}$$

where Φ stands for the fields, $\mathcal{S}$ collects the sources, and P contains the parameters. The bare fields are defined by the multiplicative renormalization

$$\begin{aligned}\Phi_0 &= \left(1 + \frac{\epsilon}{2} z_\Phi\right)\Phi, \\ \mathcal{S}_0 &= (1 + \epsilon z_{\mathcal{S}})\mathcal{S}, \\ P_0 &= (1 + \epsilon z_P)P.\end{aligned} \tag{5.25}$$

As proven in [29], the $\Sigma(\varphi)$ part is stable, and the factors z_φ are the same as those of the vortex-free sector. Specifically,

$$\begin{aligned} z_A &= a_0, & z_g &= -\frac{a_0}{2}, \\ z_{c_I} &= 0, & z_{\bar{c}_I} &= 2a_1, \\ z_{\zeta_I} &= 0, & z_{b_I} &= 2a_1, \\ z_\kappa &= -a_1 - a_2, & z_{\mathcal{K}} &= -a_1 - a_2, \\ z_\lambda &= -a_1 - a_4, & z_\Lambda &= -a_1 - a_4, \\ z_{\mu^2} &= -a_1 - a_3, & z_{U^2} &= -a_1 - a_3, \\ z_c &= 0, & z_{\bar{c}} &= 0, \\ z_{\bar{C}} &= 0, & z_b &= 0, \\ z_L &= -a_1, & z_{\bar{L}} &= 0, \\ z_K &= -\frac{a_0}{2}, & z_B &= -a_1, \\ z_Q &= 0, & z_M &= 0, \\ z_N &= 0. \end{aligned} \tag{5.26}$$

Moreover,

$$z_n = z_m = z_J = -\frac{z_{\lambda_I}}{2} = -\frac{z_{\xi_I}}{2}. \tag{5.27}$$

As there is no counterterm containing J, and c_I and ζ_I do not renormalize, it is safe to set $z_n = z_m = z_J = z_{\lambda_I} = z_{\xi_I} = 0$. Therefore, since the algebraic technique is recursive, the renormalizability of the model at all orders in perturbation theory is proven. The number of independent renormalizations is given by the number of independent renormalization parameters a_i, namely, five.

VI. CONCLUSIONS

The search for a well defined quantization procedure for the Yang-Mills theory in the nonperturbative regime has attracted a lot of activity for many years. Many proposals have been analyzed, always considering gauge fixing procedures which are global in configuration space. Global conditions lead to the Gribov problem, which has been tackled by restricting the configuration space to be path-integrated. A different way out was raised at the end of Ref. [1], where a superposition of infinitely many local gauge-fixings was proposed. Recently, a particular realization of this general scenario was implemented by means of a partition of the configuration space into sectors labeled by topological degrees of freedom. The conditions for this realization to be well defined were discussed in Ref. [15], and briefly reviewed in Sec. II of the present work. In this work, we showed for the first time that this path is in principle calculable. Namely, we established the all-orders perturbative renormalizability of the procedure in sectors labeled by oriented center vortices. Remarkably, as the counterterms are the same as those of the vortex-free sector, no new parameters had to be introduced. In a future work, it would be important to explicitly calculate an approximation to the partial contributions defined in Eq. (2.9). At large distances, they are expected to contain terms proportional to the area and to the square of the extrinsic curvature of Ω, the closed worldsurface where the center-vortex guiding centers are located. This points to the idea that Singer's no go theorem is the fundamental reason behind a first-principles YM center-vortex ensemble. Furthermore, this could establish a connection with phenomenological ensembles of center vortices, which are known to successfully reproduce the properties of the confining string (see the reviews [39,40], and references therein).

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APPENDIX: LIE ALGEBRA CONVENTIONS

In this work, following Ref. [30] and references therein, we use a basis for $\mathfrak{su}(N)$, the Lie algebra of SU(N), which relies on the Cartan decomposition. The first $N-1$ elements are given by the generators T_q, $q = 1, \ldots, N-1$ of the Cartan subalgebra, also known as the maximal torus of $\mathfrak{su}(N)$, since all of its elements commute with each other:

$$[T_q, T_p] = 0. \tag{A1}$$

Then, we define the eigenvectors E_α of the adjoint action of T_q:

$$[T_q, E_\alpha] = \alpha_q E_\alpha. \tag{A2}$$

The eigenvalues α_q, $q = 1, \ldots, N-1$ are known as the roots of $\mathfrak{su}(N)$. It is possible to define a notion of ordering of these objects, where a root is said to be positive if and only if its last nonvanishing component is positive. The number of negative and positive roots is the same. This follows from the fact that if α is a root, then $-\alpha$ is also a root, with $E_{-\alpha} = E_\alpha^\dagger$. This may be obtained by taking the Hermitian conjugate of Eq. (A2). Then, the remaining $N(N-1)$ Hermitian generators are defined as

$$T_\alpha = \frac{E_\alpha + E_\alpha^\dagger}{\sqrt{2}} \tag{A3}$$

$$T_{\bar{\alpha}} = \frac{E_\alpha - E_\alpha^\dagger}{i\sqrt{2}}. \tag{A4}$$

We denote the elements of the basis $(T_q, T_\alpha, T_{\bar{\alpha}})$ collectively by T^a, always with a latin index different than p, q, which we use only for the Cartan generators. The commutation relations of this basis which are relevant for the purposes of this work are

$$[T_q, T_p] = 0, \tag{A5}$$

$$[T_q, T_\alpha] = i\alpha_q T_{\bar{\alpha}}, \tag{A6}$$

$$[T_q, T_{\bar{\alpha}}] = -i\alpha_q T_\alpha. \tag{A7}$$

These relations, together with the fact that the commutators between root generators never involve Cartan generators, imply that f^{qbc} is nonvanishing only when $b = \alpha$ and $c = \bar{\alpha}$, or $b = \bar{\alpha}$ and $c = \alpha$. Finally, we remark that this basis is orthonormal with respect to the Killing metric

$$(A, B) = \mathrm{Tr}(\mathrm{Ad}(A)\mathrm{Ad}(B)), \tag{A8}$$

where Ad() stands for the adjoint representation of the Lie Algebra.

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