Three-point functions of a fermionic higher-spin current in 4D conformal field theory

Evgeny I. Buchbinder* and Benjamin J. Stone†
Department of Physics M013, The University of Western Australia, 35 Stirling Highway, Crawley Western Australia 6009, Australia

(Received 26 April 2022; accepted 26 May 2022; published 10 June 2022)

We investigate the properties of a four-dimensional conformal field theory possessing a fermionic higher-spin current $Q_{\alpha(2k)\dot{\alpha}}$. Using a computational approach, we examine the number of independent tensor structures contained in the three-point correlation functions of two fermionic higher-spin currents with the conserved vector current V_m and with the energy-momentum tensor T_{mn}. In particular, the $k = 1$ case corresponds to a "supersymmetrylike" current, that is, a fermionic conserved current with identical properties to the supersymmetry current which appears in $\mathcal{N} = 1$ superconformal field theories. However, we show that in general, the three-point correlation functions $\langle QQV \rangle$ and $\langle QQT \rangle$ are not consistent with $\mathcal{N} = 1$ supersymmetry.

DOI: 10.1103/PhysRevD.105.125004

I. INTRODUCTION

Correlation functions of conserved currents are among the most important observables in conformal field theory. It is a well-known fact that conformal symmetry determines the general form of two- and three-point correlation functions up to finitely many parameters; however, it remains an open problem to understand the structure of three-point functions of conserved currents for arbitrary spin. The systematic approach to study correlation functions of conserved currents was undertaken in [1,2] (see also Refs. [3–12] for earlier results), and was later extended to superconformal field theories in diverse dimensions [13–26].[1] The most important examples of conserved currents in conformal field theory are the energy-momentum tensor and vector currents; their three-point functions were studied in [1]. However, more general conformal field theories can possess higher-spin conserved currents. As was proven by Maldacena and Zhiboedov in [28], all correlation functions of higher-spin currents are equal to those of a free theory. This theorem was originally proven in three dimensions and was later generalized in [29–31] to four- and higher-dimensional cases. The general structure of the three-point functions of conserved higher-spin, bosonic, vector currents was found by Stanev [32] and Zhiboedov [33]; see also [34] for similar results in the embedding formalism [35–40] (and [41,42] for supersymmetric extensions). There are also some novel approaches to the construction of correlation functions of conserved currents which carry out the calculations in momentum space, using methods such as spinor-helicity variables [43–49].

The study of correlation functions in conformal field theory has mostly been devoted to bosonic operators with vector indices (except for supersymmetric settings); fermionic operators have practically not been studied.[2] Our interest in studying three-point functions of fermionic operators is twofold: first, any conformal field theory possessing fermionic operators naturally breaks the assumptions of the Maldacena-Zhiboedov theorem [28] discussed above. Indeed, the main assumption of the Maldacena-Zhiboedov theorem was that the conformal field theory under consideration possesses a unique conserved current of spin two, the energy-momentum tensor. However, in [28] it was also shown that if a conformal field theory possesses a conserved fermionic higher-spin current, then it has an additional conserved current of spin two. Hence, it is not clear whether correlation functions of fermionic higher-spin currents must coincide with those in a free theory. Second, fermionic operators are interesting due to their prevalence in supersymmetric field theories.

*evgeny.buchbinder@uwa.edu.au
†benjamin.stone@research.uwa.edu.au

[1]The approach of [1,2] performs the analysis in general dimensions and did not consider parity-violating structures relevant for three-dimensional conformal field theories. These structures were found later in [27].

[2]Recently, in [34], correlation functions involving fermionic operators were studied; however, these operators were not conserved currents.

In fact, there is a natural question: if a conformal field theory possesses a conserved fermionic current, is it necessarily supersymmetric?

The aim of this paper is to study correlation functions of the conserved fermionic higher-spin currents[3]

$$Q_{\alpha(2k)\dot{\alpha}}, \qquad \bar{Q}_{\alpha\dot{\alpha}(2k)}, \tag{1.1}$$

which obey the conservation equations

$$\partial^{\alpha\dot{\alpha}} Q_{\alpha\alpha(2k-1)\dot{\alpha}} = 0, \qquad \partial^{\alpha\dot{\alpha}} \bar{Q}_{\alpha\dot{\alpha}\dot{\alpha}(2k-1)} = 0. \tag{1.2}$$

The case $k = 1$ in (1.1) is quite interesting as it corresponds to currents of spin-$\frac{3}{2}$ which possess the same index structure and conservation properties as the supersymmetry currents. Indeed, one might expect that a conformal field theory possessing conserved spin-$\frac{3}{2}$ primary operators is supersymmetric. One way to explore this issue is to study the correlation functions involving such operators to see if they are consistent with supersymmetry. In particular, we must study the general form of the three-point functions involving combinations of the operators $Q_{\alpha(2)\dot{\alpha}}, \bar{Q}_{\alpha\dot{\alpha}(2)}$ [i.e., (1.1) for $k = 1$], the energy-momentum tensor T_{mn}, and the vector current V_m. Recall that in any superconformal field theory the supersymmetry current and the energy-momentum tensor are components of the supercurrent multiplet, $J_{\alpha\dot{\alpha}}(z)$, where $z = (x^m, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$ is a point in four-dimensional (4D) Minkowski superspace. This implies that in supersymmetric theories the three-point functions

$$\langle \bar{Q}_{\alpha\dot{\alpha}(2)}(x_1) Q_{\beta(2)\dot{\beta}}(x_2) T_{mn}(x_3) \rangle,$$
$$\langle Q_{\alpha(2)\dot{\alpha}}(x_1) Q_{\beta(2)\dot{\beta}}(x_2) T_{mn}(x_3) \rangle \tag{1.3}$$

must be contained in the three-point function of the supercurrent $\langle J_{\alpha\dot{\alpha}}(z_1) J_{\beta\dot{\beta}}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle$, which was shown in [14] to be fixed up to two independent tensor structures. Similarly, in supersymmetric theories the vector current V_m is a component of the flavor current multiplet, $L(z)$. Hence, the three-point functions

$$\langle \bar{Q}_{\alpha\dot{\alpha}(2)}(x_1) Q_{\beta(2)\dot{\beta}}(x_2) V_m(x_3) \rangle,$$
$$\langle Q_{\alpha(2)\dot{\alpha}}(x_1) Q_{\beta(2)\dot{\beta}}(x_2) V_m(x_3) \rangle \tag{1.4}$$

must be contained in the three-point function of the supercurrent and the flavor current $\langle J_{\alpha\dot{\alpha}}(z_1) J_{\beta\dot{\beta}}(z_2) L(z_3) \rangle$, which was shown to be fixed up to a single tensor structure [14].

In this paper, we study the general form of the three-point functions (1.3) and (1.4) and extend the results to the operators (1.1), using only the constraints of conformal symmetry; supersymmetry is not assumed. The analysis is highly nontrivial and requires significant use of computational methods. To streamline the calculations we develop a hybrid formalism which combines the approach of Osborn and Petkou [1] and the approach based on the contraction of tensor indices with auxiliary vectors/spinors. This method is widely used throughout the literature to construct correlation functions of more complicated tensor operators. Our particular approach, however, has some advantages as the correlation function is completely described in terms of a polynomial which is a function of a single conformally covariant three-point building block, X, and the auxiliary spinor variables $u, \bar{u}, v, \bar{v}, w, \bar{w}$. Hence, one does not have to work with the spacetime points explicitly when imposing conservation equations. To find all solutions for the polynomial, we construct a generating function which produces an exhaustive list of all possible linearly dependent structures for fixed (and in some cases, arbitrary) spins. The possible structures form a basis in which the polynomial may be decomposed, and are in correspondence with the solutions to a set of six linear inhomogeneous Diophantine equations, which can be solved computationally for any spin.

Using the methods outlined above, we find that the three-point functions (1.3) and (1.4), in general, are not consistent with supersymmetry as they are fixed up to more independent tensor structures than the three-point functions $\langle JJJ \rangle$ and $\langle JJL \rangle$. This means, based on the constraints of conformal symmetry alone, that the existence of spin-$\frac{3}{2}$ supersymmetry-like conserved currents in a conformal field theory does not necessarily imply that the theory is superconformal. We want to stress that our analysis is based only on symmetries and does not take into account other features of local field theory. We do not know how to realize a local nonsupersymmetric conformal field theory possessing conserved spin-$\frac{3}{2}$ currents, neither do we have a proof that it is impossible.

Our paper is organized as follows: in Sec. II, we discuss the general formalism to construct two- and three-point functions in conformal field theory. First, we review the constructions of Osborn and Petkou [1] and introduce our hybrid generating function formalism based on contractions of tensor operators with auxiliary spinors. We construct a generating function which, for a given choice of spins, generates all possible linearly dependent solutions for the correlation function. In Secs. III and IV, we find the most general form of the three-point functions (1.4). Our conclusions are that the three-point function $\langle \bar{Q}QV \rangle$ depends on three independent tensor structures (here and in all other cases the tensor structures are found explicitly) and the three-point function $\langle QQV \rangle$ depends on a single tensor structure. In Secs. V and VI, we find the most general form of the three-point functions (1.3). Our conclusions are that the three-point function $\langle \bar{Q}QT \rangle$ depends on four independent tensor structures and the three-point function $\langle QQT \rangle$ depends on a single tensor structure. Most of our analysis in Secs. III–VI was performed for an arbitrary k. However, due to computational limitations certain results were proven only for small values k. Nevertheless, we

[3]We use the standard notation $\Phi_{\alpha(m)\dot{\alpha}(n)} = \Phi_{(\alpha_1\cdots\alpha_m)(\dot{\alpha}_1\cdots\dot{\alpha}_n)}$.

believe that the results stated above hold for all values of k. Finally, in Sec. VII, we discuss whether our results are consistent with supersymmetry for $k = 1$, when Q possesses the same properties as the supersymmetry current. We show that, in general, the results obtained in Secs. III–VI are not consistent with supersymmetry. Our four-dimensional notation and conventions are summarized in the Appendix.

II. CONFORMAL BUILDING BLOCKS

In this section we will review the pertinent aspects of the group theoretic formalism used to compute correlation functions of primary operators in four-dimensional conformal field theories. For a more detailed review of the formalism as applied to correlation functions of bosonic primary fields, the reader may consult [1]. Our 4D conventions and notation are those of [50]; see the Appendix for a brief overview.

A. Two-point functions

Consider 4D Minkowski space $\mathbb{M}^{1,3}$, parametriZed by coordinates x^m, where $m = 0, 1, 2, 3$ are Lorentz indices. Given two points, x_1 and x_2, we can define the covariant two-point function

$$x_{12}^m = (x_1 - x_2)^m, \qquad x_{21}^m = -x_{12}^m. \tag{2.1}$$

Next, following Osborn and Petkou [1], we introduce the conformal inversion tensor, I_{mn}, which is defined as follows:

$$I_{mn}(x) = \eta_{mn} - 2\frac{x_m x_n}{x^2}, \qquad I_{ma}(x) I^{an}(x) = \delta_m^n. \tag{2.2}$$

This object played a pivotal role in the construction of correlation functions in [1], as the full conformal group may be generated by considering Poincaré transformations supplemented by inversions. However, in the context of this work, we require an analogous operator for the spinor representation. Hence, we convert the vector two-point functions (2.1) into spinor notation using the conventions outlined in the Appendix:

$$x_{12\alpha\dot{\alpha}} = (\sigma^m)_{\alpha\dot{\alpha}} x_{12m}, \qquad x_{12}^{\dot{\alpha}\alpha} = (\tilde{\sigma}^m)^{\dot{\alpha}\alpha} x_{12m},$$
$$x_{12}^2 = -\frac{1}{2} x_{12}^{\dot{\alpha}\alpha} x_{12\alpha\dot{\alpha}}. \tag{2.3}$$

In this form the two-point functions possess the following useful properties:

$$x_{12}^{\dot{\alpha}\alpha} x_{12\beta\dot{\alpha}} = -x_{12}^2 \delta_\beta^\alpha, \qquad x_{12}^{\dot{\alpha}\alpha} x_{12\alpha\dot{\beta}} = -x_{12}^2 \delta_{\dot{\beta}}^{\dot{\alpha}}. \tag{2.4}$$

Hence, we find

$$(x_{12}^{-1})^{\dot{\alpha}\alpha} = -\frac{x_{12}^{\dot{\alpha}\alpha}}{x_{12}^2}. \tag{2.5}$$

We also introduce the normalized two-point functions, denoted by $\hat{x}_{12}$,

$$\hat{x}_{12\alpha\dot{\alpha}} = \frac{x_{12\alpha\dot{\alpha}}}{(x_{12}^2)^{1/2}}, \qquad \hat{x}_{12}^{\dot{\alpha}\alpha} \hat{x}_{12\beta\dot{\alpha}} = -\delta_\alpha^\beta. \tag{2.6}$$

From here we can now construct an operator analogous to the conformal inversion tensor acting on the space of symmetric traceless tensors of arbitrary rank. Given a two-point function x, we define the operator

$$\mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(x) = \hat{x}_{(\alpha_1(\dot{\alpha}_1} \cdots \hat{x}_{\alpha_k)\dot{\alpha}_k)}, \tag{2.7}$$

along with its inverse

$$\bar{\mathcal{I}}^{\dot{\alpha}(k)\alpha(k)}(x) = \hat{x}^{(\dot{\alpha}_1(\alpha_1} \cdots \hat{x}^{\dot{\alpha}_k)\alpha_k)}. \tag{2.8}$$

The spinor indices may be raised and lowered using the standard conventions as follows:

$$\mathcal{I}^{\alpha(k)}{}_{\dot{\alpha}(k)}(x) = \varepsilon^{\alpha_1\gamma_1} \cdots \varepsilon^{\alpha_k\gamma_k} \mathcal{I}_{\gamma(k)\dot{\alpha}(k)}(x), \tag{2.9a}$$

$$\bar{\mathcal{I}}_{\dot{\alpha}(k)}{}^{\alpha(k)}(x) = \varepsilon_{\dot{\alpha}_1\dot{\gamma}_1} \cdots \varepsilon_{\dot{\alpha}_k\dot{\gamma}_k} \bar{\mathcal{I}}^{\dot{\gamma}(k)\alpha(k)}(x). \tag{2.9b}$$

Now due to the property

$$\mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(-x) = (-1)^k \mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(x), \tag{2.10}$$

we have the following useful relations:

$$\mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(x_{12}) \bar{\mathcal{I}}^{\dot{\alpha}(k)\beta(k)}(x_{21}) = \delta_{(\alpha_1}^{(\beta_1} \cdots \delta_{\alpha_k)}^{\beta_k)}, \tag{2.11a}$$

$$\bar{\mathcal{I}}^{\dot{\beta}(k)\alpha(k)}(x_{12}) \mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(x_{21}) = \delta_{(\dot{\alpha}_1}^{(\dot{\beta}_1} \cdots \delta_{\dot{\alpha}_k)}^{\dot{\beta}_k)}. \tag{2.11b}$$

The objects (2.7) and (2.8) prove to be essential in the construction of correlation functions of primary operators with arbitrary spin. Indeed, the vector representation of the inversion tensor may be recovered in terms of the spinor two-point functions as follows:

$$I_{mn}(x) = -\frac{1}{2}\mathrm{Tr}(\tilde{\sigma}_m \hat{x} \tilde{\sigma}_n \hat{x}). \tag{2.12}$$

Now let $\Phi_{\mathcal{A}}$ be a primary field with dimension Δ, where $\mathcal{A}$ denotes a collection of Lorentz spinor indices. The two-point correlation function of $\Phi_{\mathcal{A}}$ and its conjugate $\bar{\Phi}^{\bar{\mathcal{A}}}$ is fixed by conformal symmetry to the form

$$\langle \Phi_{\mathcal{A}}(x_1) \bar{\Phi}^{\bar{\mathcal{A}}}(x_2) \rangle = c \frac{\mathcal{I}_{\mathcal{A}}{}^{\bar{\mathcal{A}}}(x_{12})}{(x_{12}^2)^{\Delta}}, \tag{2.13}$$

where $\mathcal{I}$ is an appropriate representation of the inversion tensor and c is a constant complex parameter. The denominator of the two-point function is determined by the conformal dimension of $\Phi_{\mathcal{A}}$, which guarantees that the correlation function transforms with the appropriate weight under scale transformations. For example, in the case of the fermionic current field $Q_{\alpha(2k)\dot{\alpha}}$, the two-point function is uniquely fixed to the following form:

$$\langle Q_{\alpha(2k)\dot{\alpha}}(x_1)\bar{Q}^{\dot{\beta}(2k)\beta}(x_2)\rangle = c\,\frac{\mathcal{I}_{\alpha(2k)}{}^{\dot{\beta}(2k)}(x_{12})\bar{\mathcal{I}}_{\dot{\alpha}}{}^{\beta}(x_{12})}{(x_{12}^2)^{\Delta(Q)}}, \tag{2.14}$$

where in this case $\Delta(Q)$ is fixed by conservation of Q ($\bar{Q}$) at x_1 (x_2). It is not too difficult to show that $\Delta(Q) = k + \frac{5}{2}$.

B. Three-point functions

Given three distinct points in Minkowski space, x_i, with $i = 1, 2, 3$, we define conformally covariant three-point functions in terms of the two-point functions as in [1]

$$X_{ij} = \frac{x_{ik}}{x_{ik}^2} - \frac{x_{jk}}{x_{jk}^2}, \quad X_{ji} = -X_{ij}, \quad X_{ij}^2 = \frac{x_{ij}^2}{x_{ik}^2 x_{jk}^2}, \tag{2.15}$$

where (i, j, k) is a cyclic permutation of (1,2,3). For example, we have

$$X_{12}^m = \frac{x_{13}^m}{x_{13}^2} - \frac{x_{23}^m}{x_{23}^2}, \qquad X_{12}^2 = \frac{x_{12}^2}{x_{13}^2 x_{23}^2}. \tag{2.16}$$

There are several useful identities involving the two-point and three-point functions along with the conformal inversion tensor; for example, we have the useful algebraic relations

$$I_m{}^a(x_{13})I_{an}(x_{23}) = I_m{}^a(x_{12})I_{an}(X_{13}),$$
$$I_{mn}(x_{23})X_{12}^n = \frac{x_{12}^2}{x_{13}^2}X_{13m}, \tag{2.17a}$$

$$I_m{}^a(x_{23})I_{an}(x_{13}) = I_m{}^a(x_{21})I_{an}(X_{32}),$$
$$I_{mn}(x_{13})X_{12}^n = \frac{x_{12}^2}{x_{23}^2}X_{32m}, \tag{2.17b}$$

and the differential identities

$$\partial_{(1)m}X_{12n} = \frac{1}{x_{13}^2}I_{mn}(x_{13}),$$
$$\partial_{(2)m}X_{12n} = -\frac{1}{x_{23}^2}I_{mn}(x_{23}). \tag{2.18}$$

The three-point functions also may be represented in spinor notation as follows:

$$X_{ij,\alpha\dot{\alpha}} = (\sigma_m)_{\alpha\dot{\alpha}}X_{ij}^m, \quad X_{ij,\alpha\dot{\alpha}} = (x_{ik}^{-1})_{\alpha\dot{\gamma}}x_{ij}^{\dot{\gamma}\gamma}(x_{jk}^{-1})_{\gamma\dot{\alpha}}. \tag{2.19}$$

These objects satisfy properties similar to the two-point functions (2.4). Indeed, it is convenient to define the normalized three-point functions $\hat{X}_{ij}$ and the inverses (X_{ij}^{-1}),

$$\hat{X}_{ij,\alpha\dot{\alpha}} = \frac{X_{ij,\alpha\dot{\alpha}}}{(X_{ij}^2)^{1/2}}, \qquad (X_{ij}^{-1})^{\dot{\alpha}\alpha} = -\frac{X_{ij}^{\dot{\alpha}\alpha}}{X_{ij}^2}. \tag{2.20}$$

Now given an arbitrary three-point building block X, it is also useful to construct the following higher-spin operator:

$$\mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(X) = \hat{X}_{(\alpha_1(\dot{\alpha}_1}\cdots\hat{X}_{\alpha_k)\dot{\alpha}_k)}, \tag{2.21}$$

along with its inverse

$$\bar{\mathcal{I}}^{\dot{\alpha}(k)\alpha(k)}(X) = \hat{X}^{(\dot{\alpha}_1(\alpha_1}\ldots\hat{X}^{\dot{\alpha}_k)\alpha_k)}. \tag{2.22}$$

These operators have properties similar to the two-point higher-spin inversion operators (2.7) and (2.8). There are also some useful algebraic identities relating the two- and three-point functions at various points, such as

$$\mathcal{I}_{\alpha\dot{\alpha}}(X_{12}) = \mathcal{I}_{\alpha\dot{\gamma}}(x_{13})\bar{\mathcal{I}}^{\dot{\gamma}\gamma}(x_{12})\mathcal{I}_{\gamma\dot{\alpha}}(x_{23}),$$
$$\bar{\mathcal{I}}^{\dot{\alpha}\gamma}(x_{13})\mathcal{I}_{\gamma\dot{\gamma}}(X_{12})\bar{\mathcal{I}}^{\dot{\gamma}\alpha}(x_{13}) = \bar{\mathcal{I}}^{\dot{\alpha}\alpha}(X_{32}). \tag{2.23}$$

These identities (and cyclic permutations of them) are analogous to (2.17a) and (2.17b), and also admit higher-spin generalizations, for example,

$$\bar{\mathcal{I}}^{\dot{\alpha}(k)\gamma(k)}(x_{13})\mathcal{I}_{\gamma(k)\dot{\gamma}(k)}(X_{12})\bar{\mathcal{I}}^{\dot{\gamma}(k)\alpha(k)}(x_{13}) = \bar{\mathcal{I}}^{\dot{\alpha}(k)\alpha(k)}(X_{32}). \tag{2.24}$$

In addition, similar to (2.18), there are also the following useful identities:

$$\partial_{(1)\alpha\dot{\alpha}}X_{12}^{\dot{\sigma}\sigma} = -\frac{2}{x_{13}^2}\mathcal{I}_{\alpha}{}^{\dot{\sigma}}(x_{13})\bar{\mathcal{I}}_{\dot{\alpha}}{}^{\sigma}(x_{13}),$$
$$\partial_{(2)\alpha\dot{\alpha}}X_{12}^{\dot{\sigma}\sigma} = \frac{2}{x_{23}^2}\mathcal{I}_{\alpha}{}^{\dot{\sigma}}(x_{23})\bar{\mathcal{I}}_{\dot{\alpha}}{}^{\sigma}(x_{23}). \tag{2.25}$$

These identities allow us to account for the fact that correlation functions of primary fields obey differential constraints which can arise due to conservation equations. Indeed, given a tensor field $\mathcal{T}_{\mathcal{A}}(X)$, there are the following differential identities which arise as a consequence of (2.25):

$$\partial_{(1)\alpha\dot{\alpha}}\mathcal{T}_{\mathcal{A}}(X_{12}) = \frac{1}{x_{13}^2}\mathcal{I}_{\alpha}{}^{\dot{\sigma}}(x_{13})\bar{\mathcal{I}}_{\dot{\alpha}}{}^{\sigma}(x_{13})\frac{\partial}{\partial X_{12}^{\dot{\sigma}\sigma}}\mathcal{T}_{\mathcal{A}}(X_{12}), \tag{2.26a}$$

$$\partial_{(2)\alpha\dot{\alpha}}\mathcal{T}_{\mathcal{A}}(X_{12}) = -\frac{1}{x_{23}^2}\mathcal{I}_{\alpha}{}^{\dot{\sigma}}(x_{23})\bar{\mathcal{I}}_{\dot{\alpha}}{}^{\sigma}(x_{23})\frac{\partial}{\partial X_{12}^{\dot{\sigma}\sigma}}\mathcal{T}_{\mathcal{A}}(X_{12}). \tag{2.26b}$$

Now concerning three-point correlation functions, let Φ, Ψ, and Π be primary fields with scale dimensions Δ_1, Δ_2, and Δ_3, respectively. The three-point function may be constructed using the general ansatz

$$\langle\Phi_{\mathcal{A}_1}(x_1)\Psi_{\mathcal{A}_2}(x_2)\Pi_{\mathcal{A}_3}(x_3)\rangle = \frac{\mathcal{I}^{(1)}{}_{\mathcal{A}_1}{}^{\bar{\mathcal{A}}_1}(x_{13})\mathcal{I}^{(2)}{}_{\mathcal{A}_2}{}^{\bar{\mathcal{A}}_2}(x_{23})}{(x_{13}^2)^{\Delta_1}(x_{23}^2)^{\Delta_2}}\mathcal{H}_{\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2\mathcal{A}_3}(X_{12}), \tag{2.27}$$

where the tensor $\mathcal{H}_{\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2\mathcal{A}_3}$ encodes all information about the correlation function and is highly constrained by the conformal symmetry as follows:

(i) Under scale transformations of Minkowski space $x^m \mapsto x'^m = \lambda^{-2}x^m$, the three-point building blocks transform as $X^m \mapsto X'^m = \lambda^2 X^m$. As a consequence, the correlation function transforms as

$$\langle\Phi_{\mathcal{A}_1}(x_1')\Psi_{\mathcal{A}_2}(x_2')\Pi_{\mathcal{A}_3}(x_3')\rangle = (\lambda^2)^{\Delta_1+\Delta_2+\Delta_3}\langle\Phi_{\mathcal{A}_1}(x_1)\Psi_{\mathcal{A}_2}(x_2)\Pi_{\mathcal{A}_3}(x_3)\rangle, \tag{2.28}$$

which implies that $\mathcal{H}$ obeys the scaling property

$$\mathcal{H}_{\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2\mathcal{A}_3}(\lambda^2 X) = (\lambda^2)^{\Delta_3-\Delta_2-\Delta_1}\mathcal{H}_{\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2\mathcal{A}_3}(X), \quad \forall\,\lambda\in\mathbb{R}\backslash\{0\}. \tag{2.29}$$

This guarantees that the correlation function transforms correctly under scale transformations.

(ii) If any of the fields Φ, Ψ, and Π obey differential equations, such as conservation laws in the case of conserved current multiplets, then the tensor $\mathcal{H}$ is also constrained by differential equations. Such constraints may be derived with the aid of identities (2.26a) and (2.26b).

(iii) If any (or all) of the operators Φ, Ψ, and Π coincide, the correlation function possesses symmetries under permutations of spacetime points, e.g.,

$$\langle\Phi_{\mathcal{A}_1}(x_1)\Phi_{\mathcal{A}_2}(x_2)\Pi_{\mathcal{A}_3}(x_3)\rangle = (-1)^{\epsilon(\Phi)}\langle\Phi_{\mathcal{A}_2}(x_2)\Phi_{\mathcal{A}_1}(x_1)\Pi_{\mathcal{A}_3}(x_3)\rangle, \tag{2.30}$$

where $\epsilon(\Phi)$ is the Grassmann parity of Φ. As a consequence, the tensor $\mathcal{H}$ obeys constraints which will be referred to as "point-switch identities." Similar relations may also be derived for two fields which are related by complex conjugation.

The constraints above fix the functional form of $\mathcal{H}$ (and therefore the correlation function) up to finitely many independent parameters. Hence, using the general formula (2.31), the problem of computing three-point correlation functions is reduced to deriving the general structure of the tensor $\mathcal{H}$ subject to the above constraints.

C. Comments regarding differential constraints

An important aspect of this construction which requires further elaboration is that it is sensitive to the configuration of the fields in the correlation function. Indeed, depending on the exact way in which one constructs the general ansatz (2.31), it can be difficult to impose conservation equations on one of the three fields due to a lack of useful identities such as (2.26a) and (2.26b). To illustrate this more clearly, consider the following example: suppose we want to determine the solution for the correlation function $\langle\Phi_{\mathcal{A}_1}(x_1)\Psi_{\mathcal{A}_2}(x_2)\Pi_{\mathcal{A}_3}(x_3)\rangle$ with the ansatz

$$\langle\Phi_{\mathcal{A}_1}(x_1)\Psi_{\mathcal{A}_2}(x_2)\Pi_{\mathcal{A}_3}(x_3)\rangle = \frac{\mathcal{I}^{(1)}{}_{\mathcal{A}_1}{}^{\bar{\mathcal{A}}_1}(x_{13})\mathcal{I}^{(2)}{}_{\mathcal{A}_2}{}^{\bar{\mathcal{A}}_2}(x_{23})}{(x_{13}^2)^{\Delta_1}(x_{23}^2)^{\Delta_2}}\mathcal{H}_{\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2\mathcal{A}_3}(X_{12}). \tag{2.31}$$

All information about this correlation function is encoded in the tensor $\mathcal{H}$; however, this particular formulation of the problem prevents us from imposing conservation on the field Π in a straightforward way. To rectify this issue we reformulate the ansatz with Π at the front

$$\langle\Pi_{\mathcal{A}_3}(x_3)\Psi_{\mathcal{A}_2}(x_2)\Phi_{\mathcal{A}_1}(x_1)\rangle = \frac{\mathcal{I}^{(3)}{}_{\mathcal{A}_3}{}^{\bar{\mathcal{A}}_3}(x_{31})\mathcal{I}^{(2)}{}_{\mathcal{A}_2}{}^{\bar{\mathcal{A}}_2}(x_{21})}{(x_{31}^2)^{\Delta_3}(x_{21}^2)^{\Delta_2}}\tilde{\mathcal{H}}_{\bar{\mathcal{A}}_3\bar{\mathcal{A}}_2\mathcal{A}_1}(X_{32}). \tag{2.32}$$

In this case, all information about this correlation function is now encoded in the tensor $\tilde{\mathcal{H}}$, which is a completely different solution compared to $\mathcal{H}$. Conservation on Π can now be imposed by treating x_3 as the first point with the aid of identities analogous to (2.25), (2.26a), and (2.26b). What we now need is a simple equation relating the tensors $\mathcal{H}$ and $\tilde{\mathcal{H}}$, which correspond to different representations of the same correlation function. If we have equality between the two ansatz above, after some manipulations we obtain the following relation:

$$\begin{aligned}\tilde{\mathcal{H}}_{\bar{\mathcal{A}}_3\bar{\mathcal{A}}_2\mathcal{A}_1}(X_{32}) = (-1)^{\epsilon}(x_{13}^2)^{\Delta_3-\Delta_1}\left(\frac{x_{21}^2}{x_{23}^2}\right)^{\Delta_2}\mathcal{I}^{(1)}{}_{\mathcal{A}_1}{}^{\bar{\mathcal{A}}_1}(x_{13})\\ \times\bar{\mathcal{I}}^{(2)}{}_{\bar{\mathcal{A}}_2}{}^{\mathcal{A}_2'}(x_{12})\mathcal{I}^{(2)}{}_{\mathcal{A}_2'}{}^{\bar{\mathcal{A}}_2'}(x_{23})\\ \times\bar{\mathcal{I}}^{(3)}{}_{\bar{\mathcal{A}}_3}{}^{\mathcal{A}_3}(x_{13})\mathcal{H}_{\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2'\mathcal{A}_3}(X_{12}),\end{aligned} \tag{2.33}$$

where ϵ is either 0 or 1 depending on the Grassmann parity of the fields Φ, Ψ, and Π; since the overall sign is somewhat irrelevant for the purpose of this calculation we will absorb it into the overall sign of $\tilde{\mathcal{H}}$. In general, this equation is quite impractical to work with due to the presence of both two- and three-point functions; hence, further simplification is required. Let us now introduce some useful definitions; suppose $\mathcal{H}(X)$ (with indices suppressed) is composed out of a finite basis of linearly independent tensor structures $P_i(X)$, i.e., $\mathcal{H}(X) = \sum_i a_i P_i(X)$ where a_i are constant complex parameters. We define $\tilde{\mathcal{H}}(X) = \sum_i \bar{a}_i \bar{P}_i(X)$, the conjugate of $\mathcal{H}$, and also $\mathcal{H}^c(X) = \sum_i a_i \bar{P}_i(X)$, which we will call the complement of $\mathcal{H}$. As a consequence of (2.23), the following relation holds:

$$\begin{aligned}\mathcal{H}^c_{\mathcal{A}_1\mathcal{A}_2\bar{\mathcal{A}}_3}(X_{32}) &= (x_{13}^2 X_{32}^2)^{\Delta_3-\Delta_2-\Delta_1} \mathcal{I}^{(1)}{}_{\mathcal{A}_1}{}^{\bar{\mathcal{A}}_1}(x_{13}) \mathcal{I}^{(2)}{}_{\mathcal{A}_2}{}^{\bar{\mathcal{A}}_2}(x_{13}) \\ &\quad \times \bar{\mathcal{I}}^{(3)}{}_{\bar{\mathcal{A}}_3}{}^{\mathcal{A}_3}(x_{13}) \mathcal{H}_{\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2\mathcal{A}_3}(X_{12}). \qquad (2.34)\end{aligned}$$

This equation is an extension of (2.14) in [1] to the spinor representation, and it allows us to construct an equation relating different representations of the same correlation function. After inverting this identity and substituting it directly into (2.33), we apply identities such as (2.23) to obtain an equation relating $\mathcal{H}^c$ and $\tilde{\mathcal{H}}$,

$$\tilde{\mathcal{H}}_{\bar{\mathcal{A}}_3\bar{\mathcal{A}}_2\mathcal{A}_1}(X) = (X^2)^{\Delta_1-\Delta_3} \bar{\mathcal{I}}^{(2)}{}_{\bar{\mathcal{A}}_2}{}^{\mathcal{A}_2}(X) \mathcal{H}^c_{\mathcal{A}_1\mathcal{A}_2\bar{\mathcal{A}}_3}(X). \qquad (2.35)$$

It is important to note that this is now an equation in terms of a single variable, X, which vastly simplifies the calculations. Indeed, once $\tilde{\mathcal{H}}$ is obtained we can then impose conservation on Π as if it were located at the "first point." However, as we will see in the subsequent examples, this transformation is quite difficult to carry out for correlation functions of higher-spin primary operators due to the proliferation of tensor indices.

To summarize, in order to successfully impose all the relevant constraints on the fields in the correlator, we will adhere to the following three step approach:

1. Using ansatz (2.31), construct a solution for $\mathcal{H}$ that is consistent with the algebraic/tensorial symmetry properties of the fields Φ, Ψ, and Π.
2. Impose conservation equations on the first and second points using identities (2.25), (2.26a), and (2.26b) to constrain the functional form of the tensor $\mathcal{H}$.
3. Reformulate the correlation function using ansatz (2.32), which allows one to find an explicit relation for $\tilde{\mathcal{H}}$ in terms of $\mathcal{H}^c$. Conservation of Π may now be imposed as if it were located at the first point.

D. Generating function formalism

To study and impose constraints on correlation functions of primary fields with general spins it is often advantageous to use the formalism of generating functions to streamline the calculations. Suppose we must analyze the constraints on a general spin-tensor $\mathcal{H}_{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3}(X)$, where $\mathcal{A}_1 = \{\alpha(i_1), \dot{\alpha}(j_1)\}$, $\mathcal{A}_2 = \{\beta(i_2), \dot{\beta}(j_2)\}$, and $\mathcal{A}_3 = \{\gamma(i_3), \dot{\gamma}(j_3)\}$ represent sets of totally symmetric spinor indices associated with the fields at points x_1, x_2, and x_3, respectively. We introduce sets of commuting auxiliary spinors for each point: $U = \{u, \bar{u}\}$ at x_1, $V = \{v, \bar{v}\}$ at x_2, and $W = \{w, \bar{w}\}$ at x_3, where the spinors satisfy

$$\begin{aligned}&u^2 = \varepsilon_{\alpha\beta} u^\alpha u^\beta = 0, \qquad \bar{u}^2 = \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{u}^{\dot{\alpha}} \bar{u}^{\dot{\beta}} = 0, \\ &v^2 = \bar{v}^2 = 0, \qquad w^2 = \bar{w}^2 = 0. \qquad (2.36)\end{aligned}$$

Now if we define the objects

$$\mathbf{U}^{\mathcal{A}_1} \equiv \mathbf{U}^{\alpha(i_1)\dot{\alpha}(j_1)} = u^{\alpha_1} \cdots u^{\alpha_{i_1}} \bar{u}^{\dot{\alpha}_1} \cdots \bar{u}^{\dot{\alpha}_{j_1}}, \qquad (2.37a)$$

$$\mathbf{V}^{\mathcal{A}_2} \equiv \mathbf{V}^{\beta(i_2)\dot{\beta}(j_2)} = v^{\beta_1} \cdots v^{\beta_{i_2}} \bar{v}^{\dot{\beta}_1} \cdots \bar{v}^{\dot{\beta}_{j_2}}, \qquad (2.37b)$$

$$\mathbf{W}^{\mathcal{A}_3} \equiv \mathbf{W}^{\gamma(i_3)\dot{\gamma}(j_3)} = w^{\gamma_1} \cdots w^{\gamma_{i_3}} \bar{w}^{\dot{\gamma}_1} \cdots \bar{w}^{\dot{\gamma}_{j_3}}, \qquad (2.37c)$$

then the generating polynomial for $\mathcal{H}$ is constructed as follows:

$$\mathcal{H}(X; U, V, W) = \mathcal{H}_{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3}(X) \mathbf{U}^{\mathcal{A}_1} \mathbf{V}^{\mathcal{A}_2} \mathbf{W}^{\mathcal{A}_3}. \qquad (2.38)$$

There is in fact a one-to-one mapping between the space of symmetric traceless spin tensors and the polynomials constructed using the above method. The tensor $\mathcal{H}$ can then be extracted from the polynomial by acting on it with the following partial derivative operators:

$$\frac{\partial}{\partial \mathbf{U}^{\mathcal{A}_1}} \equiv \frac{\partial}{\partial \mathbf{U}^{\alpha(i_1)\dot{\alpha}(j_1)}} = \frac{1}{i_1! j_1!} \frac{\partial}{\partial u^{\alpha_1}} \cdots \frac{\partial}{\partial u^{\alpha_{i_1}}} \frac{\partial}{\partial \bar{u}^{\dot{\alpha}_1}} \cdots \frac{\partial}{\partial \bar{u}^{\dot{\alpha}_{j_1}}}, \qquad (2.39a)$$

$$\frac{\partial}{\partial \mathbf{V}^{\mathcal{A}_2}} \equiv \frac{\partial}{\partial \mathbf{V}^{\beta(i_2)\dot{\beta}(j_2)}} = \frac{1}{i_2! j_2!} \frac{\partial}{\partial v^{\beta_1}} \cdots \frac{\partial}{\partial v^{\beta_{i_2}}} \frac{\partial}{\partial \bar{v}^{\dot{\beta}_1}} \cdots \frac{\partial}{\partial \bar{v}^{\dot{\beta}_{j_2}}}, \qquad (2.39b)$$

$$\frac{\partial}{\partial \mathbf{W}^{\mathcal{A}_3}} \equiv \frac{\partial}{\partial \mathbf{W}^{\gamma(i_3)\dot{\gamma}(j_3)}} = \frac{1}{i_3! j_3!} \frac{\partial}{\partial w^{\gamma_1}} \cdots \frac{\partial}{\partial w^{\gamma_{i_3}}} \frac{\partial}{\partial \bar{w}^{\dot{\gamma}_1}} \cdots \frac{\partial}{\partial \bar{w}^{\dot{\gamma}_{j_3}}}. \qquad (2.39c)$$

The tensor $\mathcal{H}$ is then extracted from the polynomial as follows:

$$\mathcal{H}_{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3}(X) = \frac{\partial}{\partial \mathbf{U}^{\mathcal{A}_1}} \frac{\partial}{\partial \mathbf{V}^{\mathcal{A}_2}} \frac{\partial}{\partial \mathbf{W}^{\mathcal{A}_3}} \mathcal{H}(X; U, V, W). \qquad (2.40)$$

Let us point out that methods based on using auxiliary vectors/spinors to create a polynomial are widely used in the construction of correlation functions throughout the literature (see e.g., [19,27,32–34,38]). However, usually the entire correlator is contracted with auxiliary variables, and as a result one produces a polynomial depending on all three spacetime points and the auxiliary spinors. In our approach, however, we contract the auxiliary spinors with the tensor $\mathcal{H}_{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3}(X)$, which depends on only a single variable.

Our approach proves to be essential in the construction of correlation functions of higher-spin operators. It also proves to be more computationally tractable, as the polynomial $\mathcal{H}$, (2.38), is now constructed out of scalar combinations of X, and the auxiliary spinors U, V, and W with the appropriate homogeneity. Such a polynomial can be constructed out of the following scalar basis structures:

$$uv = u^\alpha v_\alpha, \qquad uw = u^\alpha w_\alpha, \qquad vw = v^\alpha w_\alpha, \tag{2.41a}$$

$$\bar{u}\,\bar{v} = \bar{u}^{\dot\alpha}\bar{v}_{\dot\alpha}, \qquad \bar{u}\,\bar{w} = \bar{u}^{\dot\alpha}\bar{w}_{\dot\alpha}, \qquad \bar{v}\,\bar{w} = \bar{v}^{\dot\alpha}\bar{w}_{\dot\alpha}, \tag{2.41b}$$

$$uX\bar{u} = u^\alpha \hat{X}_{\alpha\dot\alpha}\bar{u}^{\dot\alpha}, \qquad uX\bar{v} = u^\alpha \hat{X}_{\alpha\dot\alpha}\bar{v}^{\dot\alpha}, \qquad uX\bar{w} = u^\alpha \hat{X}_{\alpha\dot\alpha}\bar{w}^{\dot\alpha}, \tag{2.41c}$$

$$vX\bar{u} = v^\alpha \hat{X}_{\alpha\dot\alpha}\bar{u}^{\dot\alpha}, \qquad vX\bar{v} = v^\alpha \hat{X}_{\alpha\dot\alpha}\bar{v}^{\dot\alpha}, \qquad vX\bar{w} = v^\alpha \hat{X}_{\alpha\dot\alpha}\bar{w}^{\dot\alpha}, \tag{2.41d}$$

$$wX\bar{u} = w^\alpha \hat{X}_{\alpha\dot\alpha}\bar{u}^{\dot\alpha}, \qquad wX\bar{v} = w^\alpha \hat{X}_{\alpha\dot\alpha}\bar{v}^{\dot\alpha}, \qquad wX\bar{w} = w^\alpha \hat{X}_{\alpha\dot\alpha}\bar{w}^{\dot\alpha}, \tag{2.41e}$$

subject to cyclic permutations of linear dependence relations such as

$$(uX\bar{u})(\bar{v}\,\bar{w}) - (uX\bar{v})(\bar{u}\,\bar{w}) + (uX\bar{w})(\bar{u}\,\bar{v}) = 0. \tag{2.42}$$

There can be more general linear dependence relations for more complicated combinations of the basis structures (2.41); however, such relations can be obtained computationally.

In general, it is a nontrivial technical problem to come up with an exhaustive list of possible solutions for the polynomial $\mathcal{H}$ for a given set of spins. Hence, let us introduce a more convenient labeling scheme for the building blocks (2.41)

$$P_1 = uv, \qquad P_2 = uw, \qquad P_3 = vw, \tag{2.43a}$$

$$Q_1 = uX\bar{v}, \qquad Q_2 = uX\bar{w}, \qquad Q_3 = vX\bar{w}, \tag{2.43b}$$

$$Z_1 = uX\bar{u}, \qquad Z_2 = vX\bar{v}, \qquad Z_3 = wX\bar{w}. \tag{2.43c}$$

Now if we also define the objects

$$P(k_1, k_2, k_3) = P_1^{k_1} P_2^{k_2} P_3^{k_3}, \tag{2.44a}$$

$$Q(X, r_1, r_2, r_3) = Q_1^{r_1} Q_2^{r_2} Q_3^{r_3}, \tag{2.44b}$$

$$Z(X, s_1, s_2, s_3) = Z_1^{s_1} Z_2^{s_2} Z_3^{s_3}, \tag{2.44c}$$

then the generating function for the polynomial $\mathcal{H}(X; U, V, W)$ may be defined as follows:

$$\begin{aligned}\mathcal{F}(X; \Gamma, U, V, W) = X^{\Delta_3-\Delta_2-\Delta_1} &P(k_1, k_2, k_3)\bar{P}(\bar{k}_1, \bar{k}_2, \bar{k}_3) \\ &\times Q(X, r_1, r_2, r_3)\bar{Q}(X, \bar{r}_1, \bar{r}_2, \bar{r}_3) \\ &\times Z(X, s_1, s_2, s_3),\end{aligned} \tag{2.45}$$

where the non-negative integers, $\Gamma = \{k_i, \bar{k}_i, r_i, \bar{r}_i, s_i\}$, $i = 1, 2, 3$, are solutions to the following linear system:

$$k_1 + k_2 + s_1 + r_1 + r_2 = i_1, \qquad \bar{k}_1 + \bar{k}_2 + s_1 + \bar{r}_1 + \bar{r}_2 = j_1, \tag{2.46a}$$

$$k_1 + k_3 + s_2 + \bar{r}_1 + r_3 = i_2, \qquad \bar{k}_1 + \bar{k}_3 + s_2 + r_1 + \bar{r}_3 = j_2, \tag{2.46b}$$

$$k_2 + k_3 + s_3 + \bar{r}_2 + \bar{r}_3 = i_3, \qquad \bar{k}_2 + \bar{k}_3 + s_3 + r_2 + r_3 = j_3, \tag{2.46c}$$

and i_1, i_2, i_3, j_1, j_2, j_3 are fixed integers which specify the spin structure of the correlation function. These equations are obtained by comparing the homogeneity of the auxiliary spinors u, $\bar{u}$, etc., in the generating function (2.45), against the index structure of the tensor $\mathcal{H}$. Let us assume there exists a finite number of solutions Γ_I, $I = 1, \ldots, N$ to (2.46) for a given choice of i_1, i_2, i_3, j_1, j_2, j_3. Then the most general ansatz for the polynomial $\mathcal{H}$ in (2.38) is as follows:

$$\mathcal{H}(X; U, V, W) = \sum_{I=1}^{N} a_I \mathcal{F}(X; \Gamma_I, U, V, W), \tag{2.47}$$

where a_I are a set of complex constants. Hence, constructing the most general ansatz for the generating polynomial $\mathcal{H}$ is now equivalent to finding all non-negative integer solutions Γ_I of (2.46), where i_1, i_2, i_3 and j_1, j_2, j_3 are arbitrary non-negative integers. The solutions correspond to a linearly dependent basis of possible structures in which the polynomial $\mathcal{H}$ can be decomposed. Using computational methods, we can generate all possible solutions to (2.46) for fixed (and in some cases arbitrary) values of the spins.

In the remaining sections of this paper we will construct solutions for the three-point functions of the fermionic current field $Q_{\alpha(2k)\dot\alpha}$ with the vector current and the energy momentum tensor using the formalism outlined above. We use a combination of the method of systematic decomposition and the generating function approach to reduce the number of possible linearly dependent structures in each case. We present most of our results in terms of the scalar basis structures (2.41); however, the generating function (2.45) underpins most of the calculations.

III. CORRELATOR $\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3)\rangle$

In this section we will compute the correlation function $\langle \bar{Q}QV\rangle$, where V is a conserved vector field $V_{\gamma\dot{\gamma}}$ with scale dimension 3. The ansatz for this correlator consistent with the general results of Sec. II B is

$$\begin{aligned}\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3)\rangle &= \frac{1}{(x_{13}^2 x_{23}^2)^{k+\frac{5}{2}}} \mathcal{I}_{\alpha}{}^{\dot{\alpha}'}(x_{13}) \bar{\mathcal{I}}_{\dot{\alpha}(2k)}{}^{\alpha'(2k)}(x_{13}) \\ &\times \mathcal{I}_{\beta(2k)}{}^{\dot{\beta}'(2k)}(x_{23}) \bar{\mathcal{I}}_{\dot{\beta}}{}^{\beta'}(x_{23}) \\ &\times \mathcal{H}_{\alpha'(2k)\dot{\alpha}',\beta'\dot{\beta}'(2k),\gamma\dot{\gamma}}(X_{12}), \end{aligned} \tag{3.1}$$

where $\mathcal{H}$ is a homogeneous tensor field of degree $q = 3 - 2(k + \frac{5}{2}) = -2(k+1)$. It is constrained as follows:

(i) Under scale transformations of spacetime $x^m \mapsto x'^m = \lambda^{-2} x^m$ the three-point building blocks transform as $X^m \mapsto X'^m = \lambda^2 X^m$. As a consequence, the correlation function transforms as

$$\begin{aligned}\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1') Q_{\beta(2k)\dot{\beta}}(x_2') V_{\gamma\dot{\gamma}}(x_3')\rangle = (\lambda^2)^{2k+8} \langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3)\rangle, \end{aligned} \tag{3.2}$$

which implies that $\mathcal{H}$ obeys the scaling property

$$\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(\lambda^2 X) = (\lambda^2)^q \mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X), \quad \forall \lambda \in \mathbb{R}\backslash\{0\}. \tag{3.3}$$

This guarantees that the correlation function transforms correctly under scale transformations.

(ii) The conservation of the fields Q at x_1 and x_2 imply the following constraints on the correlation function:

$$\partial_{(1)}^{\dot{\alpha}\alpha} \langle \bar{Q}_{\alpha\dot{\alpha}\dot{\alpha}(2k-1)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3)\rangle = 0, \tag{3.4a}$$

$$\partial_{(2)}^{\dot{\beta}\beta} \langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k-1)\beta\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3)\rangle = 0. \tag{3.4b}$$

Using identities (2.26a) and (2.26b) we obtain the following differential constraints on the tensor $\mathcal{H}$:

$$\partial_X^{\dot{\alpha}\alpha} \mathcal{H}_{\alpha(2k-1)\alpha\dot{\alpha},\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X) = 0, \tag{3.5a}$$

$$\partial_X^{\dot{\beta}\beta} \mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}\dot{\beta}(2k-1),\gamma\dot{\gamma}}(X) = 0, \tag{3.5b}$$

where $\partial_X^{\dot{\alpha}\alpha} = (\tilde{\sigma}^a)^{\dot{\alpha}\alpha} \frac{\partial}{\partial X^a}$. There is also a third constraint equation arising from conservation of V at x_3,

$$\partial_{(3)}^{\dot{\gamma}\gamma} \langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3)\rangle = 0; \tag{3.6}$$

however, there are no identities analogous to (2.26a) and (2.26b) that allow the partial derivative operator acting on x_3 to pass through the prefactor of (4.1); hence, we use the procedure outlined in Sec. II C. First we construct an alternative ansatz with V at the front as follows:

$$\begin{aligned}\langle V_{\gamma\dot{\gamma}}(x_3) Q_{\beta(2k)\dot{\beta}}(x_2) \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1)\rangle &= \frac{1}{(x_{31}^2)^3 (x_{21}^2)^{k+\frac{5}{2}}} \mathcal{I}_{\gamma}{}^{\dot{\gamma}'}(x_{31}) \bar{\mathcal{I}}_{\dot{\gamma}}{}^{\gamma'}(x_{31}) \\ &\times \mathcal{I}_{\beta(2k)}{}^{\dot{\beta}'(2k)}(x_{21}) \bar{\mathcal{I}}_{\dot{\beta}}{}^{\beta'}(x_{21}) \\ &\times \tilde{\mathcal{H}}_{\gamma'\dot{\gamma}',\beta'\dot{\beta}'(2k),\alpha\dot{\alpha}(2k)}(X_{32}). \end{aligned} \tag{3.7}$$

Since the correlation function possesses the following property:

$$\begin{aligned}\langle V_{\gamma\dot{\gamma}}(x_3) Q_{\beta(2k)\dot{\beta}}(x_2) \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1)\rangle = -\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3)\rangle, \end{aligned} \tag{3.8}$$

we can now compute $\tilde{\mathcal{H}}$ in terms of $\mathcal{H}$. After some manipulations one finds the following relation:

$$\begin{aligned}\tilde{\mathcal{H}}_{\gamma\dot{\gamma},\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k)}(X_{32}) &= x_{13}^6 X_{12}^{2k+5} \mathcal{I}_{\gamma}{}^{\dot{\gamma}'}(x_{31}) \bar{\mathcal{I}}_{\dot{\gamma}}{}^{\gamma'}(x_{31}) \\ &\times \mathcal{I}_{\alpha}{}^{\dot{\alpha}'}(x_{13}) \bar{\mathcal{I}}_{\dot{\alpha}(2k)}{}^{\alpha'(2k)}(x_{13}) \\ &\times \mathcal{I}_{\beta\dot{\mu}}(x_{13}) \bar{\mathcal{I}}^{\dot{\mu}\beta'}(X_{12}) \mathcal{I}_{\mu(2k)\dot{\beta}(2k)}(x_{13}) \\ &\times \bar{\mathcal{I}}^{\dot{\beta}'(2k)\mu(2k)}(X_{12}) \mathcal{H}_{\alpha'(2k)\dot{\alpha}',\beta'\dot{\beta}'(2k)\gamma'\dot{\gamma}'}(X_{12}). \end{aligned} \tag{3.9}$$

This is quite impractical to work with due to the presence of both two-point functions and three-point functions; therefore we will make use of the following relation derived from (2.33):

$$\begin{aligned}\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X_{12}) &= (x_{13}^2 X_{32}^2)^q \mathcal{I}_{\alpha(2k)}{}^{\dot{\alpha}'(2k)}(x_{13}) \bar{\mathcal{I}}_{\dot{\alpha}}{}^{\alpha'}(x_{13}) \mathcal{I}_{\beta}{}^{\dot{\beta}'}(x_{13}) \\ &\times \bar{\mathcal{I}}_{\dot{\beta}(2k)}{}^{\beta'(2k)}(x_{13}) \mathcal{I}_{\gamma}{}^{\dot{\gamma}'}(x_{13}) \bar{\mathcal{I}}_{\dot{\gamma}}{}^{\gamma'}(x_{13}) \\ &\times \mathcal{H}^c_{\alpha'\dot{\alpha}'(2k),\beta'(2k)\dot{\beta}',\gamma'\dot{\gamma}'}(X_{32}). \end{aligned} \tag{3.10}$$

After substituting this relation directly into (3.9), and making use of (2.23), we obtain the following equation:

$$\begin{aligned}\tilde{\mathcal{H}}_{\gamma\dot{\gamma},\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k)}(X) = X^{2k-1} \mathcal{I}_{\beta}{}^{\dot{\beta}'}(X) \bar{\mathcal{I}}_{\dot{\beta}(2k)}{}^{\beta'(2k)}(X) \times \mathcal{H}^c_{\alpha\dot{\alpha}(2k),\beta'(2k)\dot{\beta}',\gamma\dot{\gamma}}(X). \end{aligned} \tag{3.11}$$

The equation relating $\tilde{\mathcal{H}}$ to $\mathcal{H}^c$ is now expressed in terms of a single variable, the building block vector X. Conservation on the third point is now equivalent to imposing the following constraint on the tensor $\tilde{\mathcal{H}}$:

$$\partial_X^{\dot{\gamma}\gamma}\tilde{\mathcal{H}}_{\gamma\dot{\gamma},\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k)}(X) = 0. \quad (3.12)$$

(iii) The correlation function is also constrained by the following reality condition:

$$\langle\bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1)Q_{\beta(2k)\dot{\beta}}(x_2)V_{\gamma\dot{\gamma}}(x_3)\rangle = \langle\bar{Q}_{\beta\dot{\beta}(2k)}(x_2)Q_{\alpha(2k)\dot{\alpha}}(x_1)V_{\gamma\dot{\gamma}}(x_3)\rangle^*, \quad (3.13)$$

which implies the following constraint on the tensor $\mathcal{H}$:

$$\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X) = -\bar{\mathcal{H}}_{\beta\dot{\beta}(2k),\alpha(2k)\dot{\alpha},\gamma\dot{\gamma}}(-X). \quad (3.14)$$

Hence, we have to solve for the tensor $\mathcal{H}$ subject to the above constraints. This is technically quite a challenging problem due to the complicated index structure of the tensor $\mathcal{H}$. Instead, we will streamline the calculations by constructing a generating function as outlined in Sec. II D. We introduce the commuting auxiliary spinors $u, \bar{u}, v, \bar{v}, w, \bar{w}$, which satisfy $u^2 = 0$, $\bar{u}^2 = 0$, etc., and define the generating function for $\mathcal{H}$ as follows:

$$\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = \mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X)\mathbf{U}^{\alpha(2k)\dot{\alpha}}\mathbf{V}^{\beta\dot{\beta}(2k)}\mathbf{W}^{\gamma\dot{\gamma}}. \quad (3.15)$$

The tensor $\mathcal{H}$ is then obtained from the generating polynomial by acting on it with partial derivatives

$$\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X) = \frac{\partial}{\partial\mathbf{U}^{\alpha(2k)\dot{\alpha}}}\frac{\partial}{\partial\mathbf{V}^{\beta\dot{\beta}(2k)}}\frac{\partial}{\partial\mathbf{W}^{\gamma\dot{\gamma}}}\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}). \quad (3.16)$$

Again, the generating function approach simplifies the various algebraic and differential constraints on the tensor $\mathcal{H}$. In particular, the differential constraints (3.5a) and (3.5b) become

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial u^{\sigma}}\frac{\partial}{\partial\bar{u}^{\dot{\sigma}}}\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = 0, \quad (3.17a)$$

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial v^{\sigma}}\frac{\partial}{\partial\bar{v}^{\dot{\sigma}}}\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = 0, \quad (3.17b)$$

while the homogeneity and reality condition (3.14) become

$$\mathcal{H}(\lambda^2 X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = (\lambda^2)^q\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}), \quad (3.18a)$$

$$\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = -\bar{\mathcal{H}}(-X; v, \bar{v}, u, \bar{u}, w, \bar{w}). \quad (3.18b)$$

Our task now is to construct the general solution for the polynomial $\mathcal{H}$ consistent with the above constraints.

The general expansion for the polynomial $\mathcal{H}$ is formed out of products of the basis objects introduced in (2.41). Let us start by decomposing the polynomial $\mathcal{H}$, and we have

$$\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = \frac{1}{X^{2k+2}}w^{\alpha}\bar{w}^{\dot{\alpha}}\mathcal{F}_{\alpha\dot{\alpha}}(X; u, \bar{u}, v, \bar{v}), \quad (3.19)$$

where have used the fact that $\mathcal{H}$ is homogeneous degree 1 in both w and $\bar{w}$. The vector object $\mathcal{F}$ is now homogeneous degree 0 in X, homogeneous degree 1 in $\bar{u}$, v, and homogeneous degree $2k$ in u, $\bar{v}$. It may be decomposed further by introducing the following basis vector structures:

$$\mathcal{Z}_{1,\alpha\dot{\alpha}} = \hat{X}_{\alpha\dot{\alpha}}, \quad \mathcal{Z}_{2,\alpha\dot{\alpha}} = u_{\alpha}\bar{u}_{\dot{\alpha}}, \quad \mathcal{Z}_{3,\alpha\dot{\alpha}} = u_{\alpha}\bar{v}_{\dot{\alpha}}, \quad (3.20a)$$

$$\mathcal{Z}_{4,\alpha\dot{\alpha}} = v_{\alpha}\bar{u}_{\dot{\alpha}}, \quad \mathcal{Z}_{5,\alpha\dot{\alpha}} = v_{\alpha}\bar{v}_{\dot{\alpha}}. \quad (3.20b)$$

We then have

$$\mathcal{F}_{\alpha\dot{\alpha}}(X; u, \bar{u}, v, \bar{v}) = \sum_{i=1}^{5}\mathcal{Z}_{i,\alpha\dot{\alpha}}\mathcal{F}_i(X; u, \bar{u}, v, \bar{v}), \quad (3.21)$$

where the $\mathcal{F}_i$ are polynomials that are homogeneous degree 0 in X, with the appropriate homogeneity in $u, \bar{u}, v, \bar{v}$. It is not too difficult to construct all possible polynomial structures for each $\mathcal{Z}_{i,\alpha\dot{\alpha}}$:

$\mathcal{Z}_1$ structures:

$$\mathcal{F}_1(X; u, \bar{u}, v, \bar{v}) = a_1(vX\bar{u})(uX\bar{v})^{2k} + a_2(uX\bar{u})(vX\bar{v})(uX\bar{v})^{2k-1} + a_3(uv)(\bar{u}\,\bar{v})(uX\bar{v})^{2k-1}, \quad (3.22a)$$

$\mathcal{Z}_2$ structures:

$$\mathcal{F}_2(X; u, \bar{u}, v, \bar{v}) = a_4(vX\bar{v})(uX\bar{v})^{2k-1}, \quad (3.22b)$$

$\mathcal{Z}_3$ structures:

$$\mathcal{F}_3(X; u, \bar{u}, v, \bar{v}) = a_5(vX\bar{u})(uX\bar{v})^{2k-1} + a_6(uX\bar{u})(vX\bar{v})(uX\bar{v})^{2k-2} + a_7(uv)(\bar{u}\,\bar{v})(uX\bar{v})^{2k-2}, \quad (3.22c)$$

$\mathcal{Z}_4$ structures:

$$\mathcal{F}_4(X; u, \bar{u}, v, \bar{v}) = a_8(uX\bar{v})^{2k}, \quad (3.22d)$$

$\mathcal{Z}_5$ structures:

$$\mathcal{F}_5(X; u, \bar{u}, v, \bar{v}) = a_9(uX\bar{u})(uX\bar{v})^{2k-1}. \quad (3.22e)$$

However, not all of these structures are linearly independent. In particular, it may be shown that $\mathcal{F}_{\alpha\dot{\alpha}}(X; u, \bar{u}, v, \bar{v}) = 0$ for the choice

$$a_2 = -a_1, \quad a_3 = a_1, \quad a_i = 0, \quad i = 4, \ldots, 9. \tag{3.23}$$

Therefore we can construct a linearly independent basis of polynomial structures by removing the a_1 structure, which leaves us with eight independent structures to consider. We now impose the differential constraints and point switch identities using *Mathematica*. After imposing (3.17a), we obtain the following k-dependent relations:

$$a_4 = \frac{(1-2k)a_2 + (1+2k)a_3}{1+2k}, \qquad a_7 = \frac{(-1+2k)(a_2 + (1+2k)a_3)}{2k(1+2k)}, \tag{3.24}$$

in addition to $a_5 = a_6 = 0$. Next we impose (3.17b), from which we obtain

$$a_9 = \frac{(1-2k)a_2 + (1+2k)a_3}{1+2k}. \tag{3.25}$$

Hence, the correlation function is determined up to three independent complex parameters, a_2, a_3, and a_8. We now must impose the reality condition (3.18b). Using *Mathematica*, we find that $a_2 = \mathrm{i}\tilde{a}_2$, $a_3 = \mathrm{i}\tilde{a}_3$, and $a_8 = \mathrm{i}\tilde{a}_8$, where $\tilde{a}_2$, $\tilde{a}_3$, and $\tilde{a}_8$ are three real constant parameters.

It remains to demonstrate that this correlation function is conserved at x_3 in accordance with conservation of the vector current. First, we compute the tensor $\tilde{\mathcal{H}}$ using (3.11). This may be written more compactly in the generating function formalism; to do this we introduce the following differential operators:

$$(vX\partial\bar{s}) = v^\alpha \hat{X}_\alpha{}^{\dot{\alpha}} \frac{\partial}{\partial \bar{s}^{\dot{\alpha}}}, \qquad (\partial s X \bar{v}) = \frac{\partial}{\partial s^\alpha} \hat{X}^\alpha{}_{\dot{\alpha}} \bar{v}^{\dot{\alpha}}. \tag{3.26}$$

The relation (3.11) is now equivalent to

$$\tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = \frac{1}{(2k)!} X^{2k-1} (vX\partial\bar{s})(\partial s X \bar{v})^{2k} \mathcal{H}^c(X; u, \bar{u}, s, \bar{s}, w, \bar{w}). \tag{3.27}$$

Conservation on the third point (3.12) is equivalent to imposing the following constraint on $\tilde{\mathcal{H}}$:

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}} \frac{\partial}{\partial w^\sigma} \frac{\partial}{\partial \bar{w}^{\dot{\sigma}}} \tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = 0. \tag{3.28}$$

It may be shown using *Mathematica* that this is satisfied up to $k = 4$. Beyond $k = 4$ the calculations for (3.11) seem to become very computationally intensive; however, we have no reason to expect that the result will change for higher values of k. Hence, we are reasonably confident that $\langle \bar{Q} Q V \rangle$ is fixed up to three independent real parameters.

IV. CORRELATOR $\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3) \rangle$

In this section we will compute the correlation function $\langle QQV \rangle$. The ansatz for this correlator consistent with the general results of Sec. II B is

$$\begin{aligned} &\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3) \rangle \\ &= \frac{1}{(x_{13}^2 x_{23}^2)^{k+\frac{5}{2}}} \mathcal{I}_{\alpha(2k)}{}^{\dot{\alpha}'(2k)}(x_{13}) \bar{\mathcal{I}}_{\dot{\alpha}}{}^{\alpha'}(x_{13}) \\ &\times \mathcal{I}_{\beta(2k)}{}^{\dot{\beta}'(2k)}(x_{23}) \bar{\mathcal{I}}_{\dot{\beta}}{}^{\beta'}(x_{23}) \\ &\times \mathcal{H}_{\alpha'\dot{\alpha}'(2k), \beta'\dot{\beta}'(2k), \gamma\dot{\gamma}}(X_{12}), \end{aligned} \tag{4.1}$$

where $\mathcal{H}$ is a homogeneous tensor field of degree $q = 3 - 2(k + \frac{5}{2}) = -2(k+1)$. It is constrained as follows:

(i) Under scale transformations of spacetime $x^m \mapsto x'^m = \lambda^{-2} x^m$ the three-point building blocks transform as $X^m \mapsto X'^m = \lambda^2 X^m$. As a consequence, the correlation function transforms as

$$\langle Q_{\alpha(2k)\dot{\alpha}}(x_1') Q_{\beta(2k)\dot{\beta}}(x_2') V_{\gamma\dot{\gamma}}(x_3') \rangle = (\lambda^2)^{2k+8} \langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3) \rangle, \tag{4.2}$$

which implies that $\mathcal{H}$ obeys the scaling property

$$\mathcal{H}_{\alpha\dot{\alpha}(2k), \beta\dot{\beta}(2k), \gamma\dot{\gamma}}(\lambda^2 X) = (\lambda^2)^q \mathcal{H}_{\alpha\dot{\alpha}(2k), \beta\dot{\beta}(2k), \gamma\dot{\gamma}}(X), \quad \forall \lambda \in \mathbb{R} \setminus \{0\}. \tag{4.3}$$

This guarantees that the correlation function transforms correctly under scale transformations.

(ii) The conservation of the fields Q at x_1 and x_2 imply the following constraints on the correlation function:

$$\partial_{(1)}^{\dot{\alpha}\alpha} \langle Q_{\alpha(2k-1)\alpha\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3) \rangle = 0, \tag{4.4a}$$

$$\partial_{(2)}^{\dot{\beta}\beta} \langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k-1)\beta\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3) \rangle = 0. \tag{4.4b}$$

Using identities (2.26a) and (2.26b), we obtain the following differential constraints on the tensor $\mathcal{H}$:

$$\partial_X^{\dot{\alpha}\alpha} \mathcal{H}_{\alpha\dot{\alpha}\dot{\alpha}(2k-1), \beta\dot{\beta}(2k), \gamma\dot{\gamma}}(X) = 0, \tag{4.5a}$$

$$\partial_X^{\dot{\beta}\beta} \mathcal{H}_{\alpha\dot{\alpha}(2k), \beta\dot{\beta}\dot{\beta}(2k-1), \gamma\dot{\gamma}}(X) = 0. \tag{4.5b}$$

There is also a third constraint equation arising from conservation of V at x_3:

$$\partial_{(3)}^{\dot{\gamma}\gamma} \langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) V_{\gamma\dot{\gamma}}(x_3) \rangle = 0. \tag{4.6}$$

Similar to the previous example, we use the procedure outlined in Sec. II C and find the following relation between $\mathcal{H}^c$ and $\tilde{\mathcal{H}}$:

$$\tilde{\mathcal{H}}_{\gamma\dot{\gamma},\beta\dot{\beta}(2k),\alpha(2k)\dot{\alpha}}(X) = X^{2k-1}\mathcal{I}_{\beta}{}^{\dot{\beta}'}(X)\bar{\mathcal{I}}_{\dot{\beta}(2k)}{}^{\beta'(2k)}(X)\mathcal{H}^c_{\alpha(2k)\dot{\alpha},\beta'(2k)\dot{\beta}',\gamma\dot{\gamma}}(X). \quad (4.7)$$

Conservation on the third point is now tantamount to imposing the constraint

$$\partial_X^{\dot{\gamma}\gamma}\tilde{\mathcal{H}}_{\gamma\dot{\gamma},\beta\dot{\beta}(2k),\alpha(2k)\dot{\alpha}}(X) = 0. \quad (4.8)$$

(iii) The correlation function possesses the following symmetry property under exchange of the fields at x_1 and x_2:

$$\langle Q_{\alpha(2k)\dot{\alpha}}(x_1)Q_{\beta(2k)\dot{\beta}}(x_2)V_{\gamma\dot{\gamma}}(x_3)\rangle = -\langle Q_{\beta(2k)\dot{\beta}}(x_2)Q_{\alpha(2k)\dot{\alpha}}(x_1)V_{\gamma\dot{\gamma}}(x_3)\rangle. \quad (4.9)$$

This implies the following constraint on the tensor $\mathcal{H}$:

$$\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X) = -\mathcal{H}_{\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k),\gamma\dot{\gamma}}(-X). \quad (4.10)$$

Hence, we have to solve for the tensor $\mathcal{H}$ subject to the above constraints. Analogous to the previous example in Sec. III, we streamline the calculations by constructing a generating function, which is defined as follows:

$$\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = \mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X)\mathbf{U}^{\alpha\dot{\alpha}(2k)}\mathbf{V}^{\beta\dot{\beta}(2k)}\mathbf{W}^{\gamma\dot{\gamma}}. \quad (4.11)$$

The tensor $\mathcal{H}$ is then extracted from the generating polynomial by acting on it with partial derivatives,

$$\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma\dot{\gamma}}(X) = \frac{\partial}{\partial\mathbf{U}^{\alpha\dot{\alpha}(2k)}}\frac{\partial}{\partial\mathbf{V}^{\beta\dot{\beta}(2k)}}\frac{\partial}{\partial\mathbf{W}^{\gamma\dot{\gamma}}}\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}). \quad (4.12)$$

As will be seen shortly, the generating function approach simplifies the various algebraic and differential constraints on the tensor $\mathcal{H}$. In particular, the differential constraints (4.5a) and (4.5b) become

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial u^{\sigma}}\frac{\partial}{\partial\bar{u}^{\dot{\sigma}}}\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = 0, \quad (4.13a)$$

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial v^{\sigma}}\frac{\partial}{\partial\bar{v}^{\dot{\sigma}}}\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = 0, \quad (4.13b)$$

while the homogeneity and point switch constraints become

$$\mathcal{H}(\lambda^2X;u,\bar{u},v,\bar{v},w,\bar{w}) = (\lambda^2)^q\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}), \quad (4.14a)$$

$$\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = -\mathcal{H}(-X;v,\bar{v},u,\bar{u},w,\bar{w}). \quad (4.14b)$$

Our task is now to construct the general solution for the polynomial $\mathcal{H}$ consistent with the above constraints.

The general expansion for the polynomial $\mathcal{H}$ is then formed out of products of the basis objects above. Let us start by decomposing the polynomial $\mathcal{H}$, we have

$$\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = \frac{1}{X^{2k+2}}w^{\alpha}\bar{w}^{\dot{\alpha}}\mathcal{F}_{\alpha\dot{\alpha}}(X;u,\bar{u},v,\bar{v}), \quad (4.15)$$

where we have used the fact that $\mathcal{H}$ is homogeneous degree 1 in both w and $\bar{w}$. The vector object $\mathcal{F}$ is now homogeneous degree 0 in X, degree 1 in u and v, and degree $2k$ in $\bar{u}$ and $\bar{v}$. It may be decomposed further using the structures defined in (3.20):

$$\mathcal{F}_{\alpha\dot{\alpha}}(X;u,\bar{u},v,\bar{v}) = \sum_{i=1}^{5}\mathcal{Z}_{i,\alpha\dot{\alpha}}\mathcal{F}_i(X;u,\bar{u},v,\bar{v}), \quad (4.16)$$

where the $\mathcal{F}_i$ are polynomials that are homogeneous degree 0 in X, with the appropriate homogeneity in $u, \bar{u}, v, \bar{v}$. It is not too difficult to construct all possible polynomial structures for each $\mathcal{Z}_{i,\alpha\dot{\alpha}}$, and we find

$\mathcal{Z}_1$ structures:

$$\mathcal{F}_1(X;u,\bar{u},v,\bar{v}) = a_1(uv)(\bar{u}\bar{v})^{2k} + a_2(uX\bar{u})(vX\bar{v})(\bar{u}\bar{v})^{2k-1} + a_3(uX\bar{v})(vX\bar{u})(\bar{u}\bar{v})^{2k-1}, \quad (4.17a)$$

$\mathcal{Z}_2$ structures:

$$\mathcal{F}_2(X;u,\bar{u},v,\bar{v}) = a_4(vX\bar{v})(\bar{u}\bar{v})^{2k-1}, \quad (4.17b)$$

$\mathcal{Z}_3$ structures:

$$\mathcal{F}_3(X;u,\bar{u},v,\bar{v}) = a_5(vX\bar{u})(\bar{u}\bar{v})^{2k-1}, \quad (4.17c)$$

$\mathcal{Z}_4$ structures:

$$\mathcal{F}_4(X;u,\bar{u},v,\bar{v}) = a_6(uX\bar{v})(\bar{u}\bar{v})^{2k-1}, \quad (4.17d)$$

$\mathcal{Z}_5$ structures:

$$\mathcal{F}_5(X;u,\bar{u},v,\bar{v}) = a_7(uX\bar{u})(\bar{u}\bar{v})^{2k-1}. \quad (4.17e)$$

However, not all of these structures are linearly independent. In particular, it may be shown that $\mathcal{F}_{\alpha\dot{\alpha}}(X;u,\bar{u},v,\bar{v}) = 0$ for the choice

$$a_3 = -a_2, \qquad a_4 = -a_1 + a_2, \qquad a_5 = a_1 - a_2,$$
$$a_6 = a_1 - a_2, \qquad a_7 = -a_1 + a_2. \tag{4.18}$$

Therefore we can construct a linearly independent basis of polynomial structures by removing the a_1 and a_2 structures; hence, there are only five independent structures remaining. We now impose the differential constraints and point switch identities using *Mathematica*. After imposing (4.13a), we obtain the following k-dependent relations between the coefficients:

$$a_4 = -\frac{1}{2k}a_3, \qquad a_5 = \frac{3(1+2k)}{2k(3+2k)}a_3,$$
$$a_7 = -\frac{(1+2k)a_3 + (3+2k)a_6}{3+8k+4k^2}. \tag{4.19}$$

At this stage only two independent coefficients remain. Next we impose (4.13b), from which we obtain

$$a_4 = -\frac{1}{2k}a_3, \qquad a_5 = \frac{3(1+2k)}{2k(3+2k)}a_3,$$
$$a_6 = \frac{3(1+2k)}{2k(3+2k)}a_3, \qquad a_7 = -\frac{1}{2k}a_3. \tag{4.20}$$

Hence, the correlation function is determined up to a single complex parameter, $a_3 = a$. It may then be shown using *Mathematica* that the point switch identity (4.14b) is satisfied for this choice of coefficients.

It remains to demonstrate that this correlation function is conserved at x_3 in accordance with conservation of the vector current. First, we need to compute $\tilde{\mathcal{H}}$ using (4.7), which is also more convenient to work with in the generating function formalism. It may then be shown that (4.7) is equivalent to

$$\tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = \frac{1}{(2k)!}X^{2k-1}(vX\partial\bar{s})(\partial sX\bar{v})^{2k}$$
$$\times \mathcal{H}^c(X; u, \bar{u}, s, \bar{s}, w, \bar{w}). \tag{4.21}$$

Therefore, given the solution for the tensor $\mathcal{H}$, we compute $\mathcal{H}^c$ by conjugating the polynomial structures and then compute $\tilde{\mathcal{H}}$ using (4.7). The differential constraint (4.8) may also be written in the generating function formalism

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial w^{\sigma}}\frac{\partial}{\partial \bar{w}^{\dot{\sigma}}}\tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = 0. \tag{4.22}$$

It may be shown using *Mathematica* that this constraint is automatically satisfied for the coefficient relations (4.20), up to $k = 4$. Again, beyond this point, the calculations seem to be quite computationally intensive. However it is reasonable to expect that the same results will hold for all k. Hence, after imposing all the constraints, the correlation function $\langle QQV \rangle$ is determined up to a single complex parameter.

V. CORRELATOR $\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3) \rangle$

In this section we will compute the correlation function $\langle \bar{Q}QT \rangle$, where T is the energy momentum tensor $T_{\gamma(2)\dot{\gamma}(2)}$ with scale dimension 4. The ansatz for this correlator consistent with the general results of Sec. II B is

$$\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3) \rangle$$
$$= \frac{1}{(x_{13}^2 x_{23}^2)^{k+\frac{5}{2}}} \mathcal{I}_{\alpha}{}^{\dot{\alpha}'}(x_{13}) \bar{\mathcal{I}}_{\dot{\alpha}(2k)}{}^{\alpha'(2k)}(x_{13})$$
$$\times \mathcal{I}_{\beta(2k)}{}^{\dot{\beta}'(2k)}(x_{23}) \bar{\mathcal{I}}_{\dot{\beta}}{}^{\beta'}(x_{23})$$
$$\times \mathcal{H}_{\alpha'(2k)\dot{\alpha}',\beta'\dot{\beta}'(2k),\gamma(2)\dot{\gamma}(2)}(X_{12}), \tag{5.1}$$

where $\mathcal{H}$ is a homogeneous tensor field of degree $q = 4 - 2(k + \frac{5}{2}) = -2k - 1$. It is constrained as follows:

(i) Under scale transformations of spacetime $x^m \mapsto x'^m = \lambda^{-2}x^m$ the three-point building blocks transform as $X^m \mapsto X'^m = \lambda^2 X^m$. As a consequence, the correlation function transforms as

$$\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x'_1) Q_{\beta(2k)\dot{\beta}}(x'_2) T_{\gamma(2)\dot{\gamma}(2)}(x'_3) \rangle$$
$$= (\lambda^2)^{2k+9} \langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3) \rangle, \tag{5.2}$$

which implies that $\mathcal{H}$ obeys the scaling property

$$\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(\lambda^2 X)$$
$$= (\lambda^2)^q \mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X), \quad \forall\, \lambda \in \mathbb{R}\backslash\{0\}. \tag{5.3}$$

This guarantees that the correlation function transforms correctly under scale transformations.

(ii) The conservation of the fields Q at x_1 and x_2 imply the following constraints on the correlation function:

$$\partial_{(1)}^{\dot{\alpha}\alpha} \langle \bar{Q}_{\alpha\dot{\alpha}\dot{\alpha}(2k-1)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3) \rangle = 0, \tag{5.4a}$$

$$\partial_{(2)}^{\dot{\beta}\beta} \langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k-1)\beta\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3) \rangle = 0. \tag{5.4b}$$

Using identities (2.26a) and (2.26b) we obtain the following differential constraints on the tensor $\mathcal{H}$:

$$\partial_X^{\dot{\alpha}\alpha} \mathcal{H}_{\alpha(2k-1)\alpha\dot{\alpha},\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X) = 0, \tag{5.5a}$$

$$\partial_X^{\dot{\beta}\beta} \mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}\dot{\beta}(2k-1),\gamma(2)\dot{\gamma}(2)}(X) = 0. \tag{5.5b}$$

There is also a third constraint equation arising from conservation of V at x_3,

$$\partial^{\dot{\gamma}\gamma}_{(3)}\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle = 0. \quad (5.6)$$

Using the same procedure as the previous examples, we construct an alternative ansatz for the correlation function as follows:

$$\begin{aligned}\langle T_{\gamma(2)\dot{\gamma}(2)}(x_3) Q_{\beta(2k)\dot{\beta}}(x_2)\bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1)\rangle &= \frac{1}{(x_{31}^2)^3 (x_{21}^2)^{\frac{7}{2}}}\mathcal{I}_{\gamma(2)}{}^{\dot{\gamma}'(2)}(x_{31})\bar{\mathcal{I}}_{\dot{\gamma}(2)}{}^{\gamma'(2)}(x_{31})\\ &\times \mathcal{I}_{\beta(2k)}{}^{\dot{\beta}'(2k)}(x_{21})\bar{\mathcal{I}}_{\dot{\beta}}{}^{\beta'}(x_{21})\\ &\times \tilde{\mathcal{H}}_{\gamma'(2)\dot{\gamma}'(2),\beta'\dot{\beta}'(2k),\alpha\dot{\alpha}(2k)}(X_{32}). \end{aligned} \quad (5.7)$$

Now due to the property

$$\begin{aligned}\langle T_{\gamma(2)\dot{\gamma}(2)}(x_3) Q_{\beta(2k)\dot{\beta}}(x_2)\bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1)\rangle &= -\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle, \end{aligned} \quad (5.8)$$

we have a way to compute $\tilde{\mathcal{H}}$ in terms of $\mathcal{H}$. After some manipulations we find

$$\begin{aligned}\tilde{\mathcal{H}}_{\gamma(2)\dot{\gamma}(2),\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k)}(X_{32}) &= x_{13}^8 X_{12}^{2k+5}\mathcal{I}_{\gamma(2)}{}^{\dot{\gamma}'(2)}(x_{31})\bar{\mathcal{I}}_{\dot{\gamma}(2)}{}^{\gamma'(2)}(x_{31})\\ &\times \mathcal{I}_{\alpha}{}^{\dot{\alpha}'}(x_{13})\bar{\mathcal{I}}_{\dot{\alpha}(2k)}{}^{\alpha'(2k)}(x_{13})\mathcal{I}_{\beta\dot{\mu}}(x_{13})\bar{\mathcal{I}}^{\dot{\mu}\beta'}(X_{12})\\ &\times \mathcal{I}_{\mu(2k)\dot{\beta}(2k)}(x_{13})\bar{\mathcal{I}}^{\dot{\beta}'(2k)\mu(2k)}(X_{12})\\ &\times \mathcal{H}_{\alpha'(2k)\dot{\alpha}',\beta'\dot{\beta}'(2k),\gamma'(2)\dot{\gamma}'(2)}(X_{12}). \end{aligned} \quad (5.9)$$

We now make use of the following identity derived from (2.34):

$$\begin{aligned}\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X_{12}) &= (x_{13}^2 X_{32}^2)^q \mathcal{I}_{\alpha(2k)}{}^{\dot{\alpha}'(2k)}(x_{13})\bar{\mathcal{I}}_{\dot{\alpha}}{}^{\alpha'}(x_{13})\\ &\times \mathcal{I}_{\beta}{}^{\dot{\beta}'}(x_{13})\bar{\mathcal{I}}_{\dot{\beta}(2k)}{}^{\beta'(2k)}(x_{13})\\ &\times \mathcal{I}_{\gamma(2)}{}^{\dot{\gamma}'(2)}(x_{13})\bar{\mathcal{I}}_{\dot{\gamma}(2)}{}^{\dot{\gamma}'(2)}(x_{13})\\ &\times \mathcal{H}^c_{\alpha'\dot{\alpha}'(2k),\beta'(2k)\dot{\beta}',\gamma'(2)\dot{\gamma}'(2)}(X_{32}). \end{aligned} \quad (5.10)$$

After substituting this equation into (5.9), we obtain the relation

$$\begin{aligned}\tilde{\mathcal{H}}_{\gamma(2)\dot{\gamma}(2),\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k)}(X) &= X^{2k-3}\mathcal{I}_{\beta}{}^{\dot{\beta}'}(X)\bar{\mathcal{I}}_{\dot{\beta}(2k)}{}^{\beta'(2k)}(X)\\ &\times \mathcal{H}^c_{\alpha\dot{\alpha}(2k),\beta'(2k)\dot{\beta}',\gamma(2)\dot{\gamma}(2)}(X). \end{aligned} \quad (5.11)$$

Conservation at x_3 is now equivalent to imposing the following constraint on the tensor $\tilde{\mathcal{H}}$:

$$\partial_X^{\dot{\sigma}\sigma}\tilde{\mathcal{H}}_{\sigma\dot{\sigma}\gamma\dot{\gamma},\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k)}(X) = 0. \quad (5.12)$$

(iii) The correlation function is also constrained by the reality condition

$$\begin{aligned}&\langle \bar{Q}_{\alpha\dot{\alpha}(2k)}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle\\ &\quad = \langle \bar{Q}_{\beta\dot{\beta}(2k)}(x_2) Q_{\alpha(2k)\dot{\alpha}}(x_1) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle^*. \end{aligned} \quad (5.13)$$

This implies the following constraint on the tensor $\mathcal{H}$:

$$\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X) = -\bar{\mathcal{H}}_{\beta\dot{\beta}(2k),\alpha(2k)\dot{\alpha},\gamma(2)\dot{\gamma}(2)}(-X). \quad (5.14)$$

Hence, we have to solve for the tensor $\mathcal{H}$ subject to the above constraints. Analogous to the previous examples we streamline the calculations by constructing a generating function, which is defined as follows:

$$\begin{aligned}&\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w})\\ &\quad = \mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X)\mathbf{U}^{\alpha(2k)\dot{\alpha}}\mathbf{V}^{\beta\dot{\beta}(2k)}\mathbf{W}^{\gamma(2)\dot{\gamma}(2)}. \end{aligned} \quad (5.15)$$

The tensor $\mathcal{H}$ is then obtained from the generating polynomial by acting on it with partial derivatives

$$\begin{aligned}&\mathcal{H}_{\alpha(2k)\dot{\alpha},\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X)\\ &\quad = \frac{\partial}{\partial \mathbf{U}^{\alpha(2k)\dot{\alpha}}}\frac{\partial}{\partial \mathbf{V}^{\beta\dot{\beta}(2k)}}\frac{\partial}{\partial \mathbf{W}^{\gamma(2)\dot{\gamma}(2)}}\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}). \end{aligned} \quad (5.16)$$

Again, the generating function approach simplifies the various algebraic and differential constraints on the tensor $\mathcal{H}$. In particular, the differential constraints (5.5a) and (5.5b) become

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial u^{\sigma}}\frac{\partial}{\partial \bar{u}^{\dot{\sigma}}}\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = 0, \quad (5.17a)$$

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial v^{\sigma}}\frac{\partial}{\partial \bar{v}^{\dot{\sigma}}}\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = 0, \quad (5.17b)$$

while the homogeneity and point switch constraints become

$$\mathcal{H}(\lambda^2 X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = (\lambda^2)^q \mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}), \quad (5.18a)$$

$$\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = -\bar{\mathcal{H}}(-X; v, \bar{v}, u, \bar{u}, w, \bar{w}). \quad (5.18b)$$

Let us now construct the general solution for the polynomial $\mathcal{H}$ consistent with the above constraints. We start by decomposing the polynomial $\mathcal{H}$ as follows:

$$\mathcal{H}(X; u, \bar{u}, v, \bar{v}, w, \bar{w}) = \frac{1}{X^{2k+1}} \bar{u}^{\dot{\alpha}} v^{\alpha} \mathcal{F}_{\alpha\dot{\alpha}}(X; u, \bar{v}, w, \bar{w}), \tag{5.19}$$

where have used the fact that $\mathcal{H}$ is homogeneous degree 1 in both $\bar{u}$ and v. The vector object $\mathcal{F}$ is now homogeneous degree 0 in X, homogeneous degree 2 in w and $\bar{w}$, and homogeneous degree $2k$ in u and $\bar{v}$. It may be decomposed further by defining the following basis vectors:

$$\mathcal{Z}_{1,\alpha\dot{\alpha}} = \hat{X}_{\alpha\dot{\alpha}}, \quad \mathcal{Z}_{2,\alpha\dot{\alpha}} = u_{\alpha}\bar{v}_{\dot{\alpha}}, \quad \mathcal{Z}_{3,\alpha\dot{\alpha}} = u_{\alpha}\bar{w}_{\dot{\alpha}}, \tag{5.20a}$$

$$\mathcal{Z}_{4,\alpha\dot{\alpha}} = w_{\alpha}\bar{v}_{\dot{\alpha}}, \quad \mathcal{Z}_{5,\alpha\dot{\alpha}} = w_{\alpha}\bar{w}_{\dot{\alpha}}. \tag{5.20b}$$

We then have

$$\mathcal{F}_{\alpha\dot{\alpha}}(X; u, \bar{v}, w, \bar{w}) = \sum_{i=1}^{5} \mathcal{Z}_{i,\alpha\dot{\alpha}} \mathcal{F}_i(X; u, \bar{v}, w, \bar{w}), \tag{5.21}$$

where the $\mathcal{F}_i$ are polynomials that are homogeneous degree 0 in X, with the appropriate homogeneity in $u, \bar{v}, w, \bar{w}$. The complete list of possible polynomial structures for each $\mathcal{Z}_{i,\alpha\dot{\alpha}}$ is

$$\begin{aligned}
&\mathcal{Z}_1 \text{ structures}: \\
&\mathcal{F}_1(X; u, \bar{v}, w, \bar{w}) = a_1(uw)^2(\bar{v}\,\bar{w})(uX\bar{v})^{2k-2} + a_2(wX\bar{w})^2(uX\bar{v})^{2k} \\
&\qquad + a_3(wX\bar{w})(uw)(\bar{v}\,\bar{w})(uX\bar{v})^{2k-1} + a_4(wX\bar{v})(uX\bar{w})(uw)(\bar{v}\,\bar{w})(uX\bar{v})^{2k-2} \\
&\qquad + a_5(wX\bar{v})^2(uX\bar{w})(\bar{v}\,\bar{w})(uX\bar{v})^{2k-2} + a_6(uX\bar{w})(wX\bar{w})(wX\bar{v})(uX\bar{v})^{2k-1},
\end{aligned} \tag{5.22a}$$

$$\begin{aligned}
&\mathcal{Z}_2 \text{ structures}: \\
&\mathcal{F}_2(X; u, \bar{v}, w, \bar{w}) = a_7(wX\bar{w})(uw)(\bar{v}\,\bar{w})(uX\bar{v})^{2k-2} + a_8(wX\bar{w})^2(uX\bar{v})^{2k-1} + a_9(uX\bar{w})(wX\bar{v})(wX\bar{w})(uX\bar{v})^{2k-2},
\end{aligned} \tag{5.22b}$$

$$\begin{aligned}
&\mathcal{Z}_3 \text{ structures}: \\
&\mathcal{F}_3(X; u, \bar{v}, w, \bar{w}) = a_{10}(wX\bar{v})(uw)(\bar{v}\,\bar{w})(uX\bar{v})^{2k-2} + a_{11}(wX\bar{w})(wX\bar{v})(uX\bar{v})^{2k-1} + a_{12}(wX\bar{v})^2(uX\bar{w})(uX\bar{v})^{2k-2},
\end{aligned} \tag{5.22c}$$

$$\begin{aligned}
&\mathcal{Z}_4 \text{ structures}: \\
&\mathcal{F}_4(X; u, \bar{v}, w, \bar{w}) = a_{13}(uX\bar{w})(uw)(\bar{v}\,\bar{w})(uX\bar{v})^{2k-2} + a_{14}(uX\bar{w})^2(wX\bar{v})(uX\bar{v})^{2k-1} + a_{15}(wX\bar{w})(uX\bar{w})(uX\bar{v})^{2k-1},
\end{aligned} \tag{5.22d}$$

$$\begin{aligned}
&\mathcal{Z}_5 \text{ structures}: \\
&\mathcal{F}_5(X; u, \bar{v}, w, \bar{w}) = a_{16}(wX\bar{v})(uX\bar{w})(uX\bar{v})^{2k-1} + a_{17}(wX\bar{w})(uX\bar{v})^{2k} + a_{18}(uw)(\bar{v}\,\bar{w})(uX\bar{v})^{2k-1}.
\end{aligned} \tag{5.22e}$$

There are also the additional "higher spin" structures, which appear only for $k > 1$:

$$\begin{aligned}
&\mathcal{Z}_2 \text{ structures}: \\
&\tilde{\mathcal{F}}_2(X; u, \bar{v}, w, \bar{w}) = a_{19}(uw)^2(\bar{v}\,\bar{w})^2(uX\bar{v})^{2k-3} + a_{20}(wX\bar{v})^2(uX\bar{w})^2(uX\bar{v})^{2k-3} + a_{21}(uX\bar{w})(wX\bar{v})(uw)(\bar{v}\,\bar{w})(uX\bar{v})^{2k-3}.
\end{aligned} \tag{5.23a}$$

Hence, we will need to treat the cases $k = 1$ and $k > 1$ separately. First, we will consider $k = 1$, which corresponds to a field with the same properties as the supersymmetry current, $Q_{\alpha(2),\dot{\alpha}}$.

A. Analysis for $k = 1$

In this subsection we will determine the constraints on the coefficients for general k. First, we must determine any linear dependence relations between the various polynomial structures. Using *Mathematica* it may be shown that $\mathcal{F}_{\alpha\dot{\alpha}}(X; u, \bar{v}, w, \bar{w}) = 0$ for the following relations between the coefficients:

$$a_3 = a_1 + a_2 - a_{10} + a_{11}, \tag{5.24a}$$

$$a_5 = -a_1 - a_4 + a_{10} + a_{12}, \tag{5.24b}$$

$$a_6 = a_1 - a_2 + a_4 - a_{10} - a_{12}, \tag{5.24c}$$

$$a_8 = a_7 + a_{10} - a_{11}, \tag{5.24d}$$

$$a_9 = -a_7 - a_{10} - a_{12}, \tag{5.24e}$$

$$a_{14} = a_{10} + a_{12} - a_{13}, \tag{5.24f}$$

$$a_{15} = -a_{10} + a_{11} + a_{13}, \tag{5.24g}$$

$$a_{17} = -a_{11} - a_{12} - a_{16}, \tag{5.24h}$$

$$a_{18} = -a_{10} - a_{12} - a_{16}. \tag{5.24i}$$

Therefore a linearly independent basis may be obtained by neglecting the structures corresponding to the coefficients $a_1, a_2, a_4, a_7, a_{10}, a_{11}, a_{12}, a_{13}, a_{16}$. There are only nine structures remaining, corresponding to the coefficients $a_3, a_5, a_6, a_8, a_9, a_{14}, a_{15}, a_{17}, a_{18}$, respectively. Now that we have identified any possible linear dependence between the polynomial structures, we impose the differential constraints and point-switch identities using *Mathematica*. After imposing the conservation equations (5.17a) and (5.17b), we obtain the following relations between the coefficients:

$$\begin{aligned} a_6 &= \frac{1}{3}(a_3 - 2a_5 + 4a_{18}), \\ a_8 &= \frac{1}{3}(-2a_3 - 2a_5 + a_{18}), \\ a_9 &= \frac{1}{3}(4a_3 + 4a_5 + 7a_{18}). \end{aligned} \tag{5.25}$$

Hence, the differential constraints are sufficient to fix the correlation function up to four independent complex parameters, a_3, a_5, a_{17}, and a_{18}. The next constraint to impose is the reality condition (5.18b), from which we determine $a_3 = \mathrm{i}\tilde{a}_3$, $a_5 = \mathrm{i}\tilde{a}_5$, $a_{17} = \mathrm{i}\tilde{a}_{17}$, and $a_{18} = \mathrm{i}\tilde{a}_{18}$, where $\tilde{a}_3$, $\tilde{a}_5$, $\tilde{a}_{17}$, and $\tilde{a}_{18}$ are four constant real parameters.

Finally, we must check that the correlation function satisfies the differential constraint (5.12) in accordance with conservation of the energy-momentum tensor. We begin by computing $\tilde{\mathcal{H}}$ using (5.11); in the generating function formalism this may be written as

$$\tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = \frac{1}{(2k)!} X^{2k-3}(vX\partial\bar{s})(\partial sX\bar{v})^{2k}\mathcal{H}^c(X; u, \bar{u}, s, \bar{s}, w, \bar{w}). \tag{5.26}$$

Conservation of the energy-momentum tensor at x_3 (5.12) is now equivalent to imposing the following differential constraint on the tensor $\tilde{\mathcal{H}}$:

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}} \frac{\partial}{\partial w^{\sigma}} \frac{\partial}{\partial \bar{w}^{\dot{\sigma}}} \tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = 0. \tag{5.27}$$

At this point we set $k = 1$ and proceed with the analysis. Using *Mathematica* it may be shown that this constraint is automatically satisfied for the coefficient constraints above; hence, the correlation function $\langle \bar{Q}QT \rangle$ is determined up to four independent real parameters.

B. Analysis for general k

Now let us carry out the analysis for general k; we must determine any linear dependence relations between the various polynomial structures. Indeed, we find that introducing the higher-spin contributions (5.23a) results in the following supplementary linear dependence relation for $k > 1$, i.e., $\mathcal{F}_{\alpha\dot{\alpha}}(X, u, \bar{v}, w, \bar{w}) = 0$ for the coefficient relations

$$a_8 = -a_{19}, \quad a_9 = a_{19} - a_{20}, \quad a_{21} = -a_{19} - a_{20}. \tag{5.28}$$

Therefore the complete list of independent structures corresponds to the coefficients $a_3, a_5, a_6, a_8, a_9, a_{14}, a_{15}, a_{17}, a_{18}, a_{21}$. We now impose the differential constraints and point switch identities using *Mathematica*. After imposing the differential constraints arising from requiring conservation on the first and second points, that is (5.17a) and (5.17b), we obtain the k-dependent relations

$$a_6 = \frac{a_3(-1 + k + 4k^2 - 4k^3) - 2a_5(1 - k - 12k^2 + 12k^3) + a_{21}(-2k + 8k^3)}{1 + 3k - 16k^2 + 12k^3}, \tag{5.29a}$$

$$a_8 = \frac{6(1 - k)(2a_3k(-1 + 2k) + a_5(1 + 2k)) + a_{21}(-1 + 4k^2)}{2 + 6k - 32k^2 + 24k^3}, \tag{5.29b}$$

$$a_9 = \frac{5a_5(-1 + k) + a_{21}(1 - 4k)}{2 - 2k}, \tag{5.29c}$$

$$a_{18} = \frac{2(1 - k)(4a_3k(-1 + 2k) + a_5(1 + 12k^2))}{2 + 6k - 32k^2 + 24k^3}. \tag{5.29d}$$

The remaining free coefficients are a_3, a_5, a_{17}, and a_{21}; the relations are also defined only for $k > 1$. Next we must impose the reality condition (5.18b), from which we obtain $a_3 = \mathrm{i}\tilde{a}_3$, $a_5 = \mathrm{i}\tilde{a}_5$, $a_{17} = \mathrm{i}\tilde{a}_{17}$, and $a_{21} = \mathrm{i}\tilde{a}_{21}$ where $\tilde{a}_3, \tilde{a}_5, \tilde{a}_{17}$, and $\tilde{a}_{21}$ are four real constants. Hence, we find that the correlation function is determined up to four independent real parameters.

Finally, we must impose the differential constraint on x_3 which arises due to conservation of the energy-momentum tensor, that is, Eq. (5.27). Indeed, we have shown using *Mathematica* that (5.27) is satisfied up to $k = 4$, and for higher values of k the computations of $\tilde{\mathcal{H}}$ seem to be beyond our computer power. However, we believe that the results will hold for higher values of k, so we can be reasonably confident that the correlation function is determined up to four independent real parameters for general k.

VI. CORRELATOR $\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle$

In this section we will compute the correlation function $\langle QQT\rangle$. The ansatz for this correlator consistent with the general results of Sec. II B is

$$\begin{aligned}&\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle\\&\quad=\frac{1}{(x_{13}^2 x_{23}^2)^{k+\frac{5}{2}}}\mathcal{I}_{\alpha(2k)}{}^{\dot{\alpha}'(2k)}(x_{13})\bar{\mathcal{I}}_{\dot{\alpha}}{}^{\alpha'}(x_{13})\\&\qquad\times\mathcal{I}_{\beta(2k)}{}^{\dot{\beta}'(2k)}(x_{23})\bar{\mathcal{I}}_{\dot{\beta}}{}^{\beta'}(x_{23})\\&\qquad\times\mathcal{H}_{\alpha'\dot{\alpha}'(2k),\beta'\dot{\beta}'(2k),\gamma(2)\dot{\gamma}(2)}(X_{12}),\end{aligned}\tag{6.1}$$

where $\mathcal{H}$ is a homogeneous tensor field of degree $q = 4 - 2(k + \frac{5}{2}) = -2k - 1$. It is constrained as follows:

(i) Under scale transformations of spacetime $x^m \mapsto x'^m = \lambda^{-2} x^m$ the three-point building blocks transform as $X^m \mapsto X'^m = \lambda^2 X^m$. As a consequence, the correlation function transforms as

$$\begin{aligned}&\langle Q_{\alpha(2k)\dot{\alpha}}(x_1') Q_{\beta(2k)\dot{\beta}}(x_2') T_{\gamma(2)\dot{\gamma}(2)}(x_3')\rangle\\&\quad=(\lambda^2)^{2k+9}\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle,\end{aligned}\tag{6.2}$$

which implies that $\mathcal{H}$ obeys the scaling property

$$\begin{aligned}&\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(\lambda^2 X)\\&\quad=(\lambda^2)^q\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X),\quad\forall\,\lambda\in\mathbb{R}\backslash\{0\}.\end{aligned}\tag{6.3}$$

This guarantees that the correlation function transforms correctly under conformal transformations.

(ii) The conservation of the fields Q at x_1 and x_2 imply the following constraints on the correlation function:

$$\partial_{(1)}^{\dot{\alpha}\alpha}\langle Q_{\alpha(2k-1)\alpha\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle = 0,\tag{6.4a}$$

$$\partial_{(2)}^{\dot{\beta}\beta}\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k-1)\beta\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle = 0.\tag{6.4b}$$

Using identities (2.26a) and (2.26b), we obtain the following differential constraints on $\mathcal{H}$:

$$\partial_X^{\dot{\alpha}\alpha}\mathcal{H}_{\alpha\dot{\alpha}\,\dot{\alpha}(2k-1),\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X) = 0,\tag{6.5a}$$

$$\partial_X^{\dot{\beta}\beta}\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}\,\dot{\beta}(2k-1),\gamma(2)\dot{\gamma}(2)}(X) = 0.\tag{6.5b}$$

There is also a third constraint equation arising from conservation of V at x_3,

$$\partial_{(3)}^{\dot{\gamma}\gamma}\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle = 0.\tag{6.6}$$

Similar to the previous example, we use the procedure outlined in Sec. II C and find the following relation between $\mathcal{H}^c$ and $\tilde{\mathcal{H}}$:

$$\begin{aligned}&\tilde{\mathcal{H}}_{\gamma(2)\dot{\gamma}(2),\beta\dot{\beta}(2k),\alpha(2k)\dot{\alpha}}(X)\\&\quad=X^{2k-3}\mathcal{I}_{\beta}{}^{\dot{\beta}'}(X)\bar{\mathcal{I}}_{\dot{\beta}(2k)}{}^{\beta'(2k)}(X)\\&\qquad\times\mathcal{H}^c_{\alpha(2k)\dot{\alpha},\beta'(2k)\dot{\beta}',\gamma(2)\dot{\gamma}(2)}(X).\end{aligned}\tag{6.7}$$

Conservation on the third point is now equivalent to the following constraint on $\tilde{\mathcal{H}}$:

$$\partial_X^{\dot{\sigma}\sigma}\tilde{\mathcal{H}}_{\sigma\dot{\sigma}\gamma\dot{\gamma},\beta\dot{\beta}(2k),\alpha(2k)\dot{\alpha}}(X) = 0.\tag{6.8}$$

(iii) The correlation function possesses the following symmetry property under exchange of the fields at x_1 and x_2:

$$\begin{aligned}&\langle Q_{\alpha(2k)\dot{\alpha}}(x_1) Q_{\beta(2k)\dot{\beta}}(x_2) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle\\&\quad=-\langle Q_{\beta(2k)\dot{\beta}}(x_2) Q_{\alpha(2k)\dot{\alpha}}(x_1) T_{\gamma(2)\dot{\gamma}(2)}(x_3)\rangle.\end{aligned}\tag{6.9}$$

This implies the following constraint on the tensor $\mathcal{H}$:

$$\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X) = -\mathcal{H}_{\beta\dot{\beta}(2k),\alpha\dot{\alpha}(2k),\gamma(2)\dot{\gamma}(2)}(-X).\tag{6.10}$$

Hence, we have to solve for the tensor $\mathcal{H}$ subject to the above constraints. Let us now streamline the calculations by constructing the generating function:

$$\begin{aligned}&\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w})\\&\quad=\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X)\mathbf{U}^{\alpha\dot{\alpha}(2k)}\mathbf{V}^{\beta\dot{\beta}(2k)}\mathbf{W}^{\gamma(2)\dot{\gamma}(2)}.\end{aligned}\tag{6.11}$$

The tensor $\mathcal{H}$ is then obtained from the generating polynomial by acting on it with partial derivatives as follows:

$$\mathcal{H}_{\alpha\dot{\alpha}(2k),\beta\dot{\beta}(2k),\gamma(2)\dot{\gamma}(2)}(X) = \frac{\partial}{\partial \mathbf{U}^{\alpha\dot{\alpha}(2k)}}\frac{\partial}{\partial \mathbf{V}^{\beta\dot{\beta}(2k)}}\frac{\partial}{\partial \mathbf{W}^{\gamma(2)\dot{\gamma}(2)}}\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}). \tag{6.12}$$

Let us now convert our constraints on the tensor $\mathcal{H}$ to constraints on the generating function. In particular, the differential constraints (6.5a) and (6.5b) become

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial u^{\sigma}}\frac{\partial}{\partial \bar{u}^{\dot{\sigma}}}\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = 0, \tag{6.13a}$$

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}}\frac{\partial}{\partial v^{\sigma}}\frac{\partial}{\partial \bar{v}^{\dot{\sigma}}}\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = 0, \tag{6.13b}$$

while the homogeneity and point-switch constraints become

$$\mathcal{H}(\lambda^2 X;u,\bar{u},v,\bar{v},w,\bar{w}) = (\lambda^2)^q\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}), \tag{6.14a}$$

$$\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = -\mathcal{H}(-X;v,\bar{v},u,\bar{u},w,\bar{w}). \tag{6.14b}$$

Our task is now to construct the general solution for the polynomial $\mathcal{H}$ consistent with the above constraints. The general expansion for $\mathcal{H}$ is formed out of products of the basis objects (2.41). Let us start by decomposing the polynomial $\mathcal{H}$, and we have

$$\mathcal{H}(X;u,\bar{u},v,\bar{v},w,\bar{w}) = \frac{1}{X^{2k+1}}u^{\alpha}v^{\beta}\mathcal{F}_{\alpha\beta}(X;\bar{u},\bar{v},w,\bar{w}), \tag{6.15}$$

where we have used the fact that $\mathcal{H}$ is homogeneous degree 1 in both u and v. The tensor $\mathcal{F}$ is now homogeneous degree 0 in X, homogeneous degree 2 in w and $\bar{w}$, and homogeneous degree $2k$ in $\bar{u}$ and $\bar{v}$. It may be decomposed into symmetric and antisymmetric parts as follows:

$$\mathcal{F}_{\alpha\beta}(X;\bar{u},\bar{v},w,\bar{w}) = \varepsilon_{\alpha\beta}A(X;\bar{u},\bar{v},w,\bar{w}) + B_{(\alpha\beta)}(X;\bar{u},\bar{v},w,\bar{w}). \tag{6.16}$$

It is straightforward to identify the possible structures in the expansion for A. We find

$$\begin{aligned} A(X;\bar{u},\bar{v},w,\bar{w}) = {}& a_1(wX\bar{w})^2(\bar{u}\,\bar{v})^{2k} + a_2(wX\bar{u})^2(\bar{v}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-2} + a_3(wX\bar{w})(wX\bar{u})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-1} \\ &+ a_4(wX\bar{w})(wX\bar{v})(\bar{u}\,\bar{w})(\bar{u}\,\bar{v})^{2k-1} + a_5(wX\bar{v})^2(\bar{u}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-2} \\ &+ a_6(wX\bar{u})(wX\bar{v})(\bar{u}\,\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}. \end{aligned} \tag{6.17}$$

However, identifying all possible structures for the tensor B is more challenging. To this end we introduce a basis of spinor structures, $Y_{i,\alpha}$:

$$Y_{1,\alpha} = w_{\alpha}, \qquad Y_{2,\alpha} = \hat{X}_{\alpha\dot{\alpha}}\bar{u}^{\dot{\alpha}}, \qquad Y_{3,\alpha} = \hat{X}_{\alpha\dot{\alpha}}\bar{v}^{\dot{\alpha}}, \qquad Y_{4,\alpha} = \hat{X}_{\alpha\dot{\alpha}}\bar{w}^{\dot{\alpha}}. \tag{6.18}$$

From these basis spinors, we construct a set of symmetric objects, $\mathcal{Y}_{ij,\alpha\beta}$, defined as follows:

$$\mathcal{Y}_{ij,\alpha\beta} = \frac{1}{2}(Y_{i,\alpha}Y_{j,\beta} + Y_{i,\beta}Y_{j,\alpha}). \tag{6.19}$$

These objects are symmetric in α, β; hence, they form a basis in which the tensor B may be decomposed. However, since these objects are also symmetric in i, j, only ten of them are unique; therefore we form the list $\mathcal{Z}_{i,\alpha\beta}$ out of the unique structures. We then have the decomposition

$$B_{(\alpha\beta)}(X;\bar{u},\bar{v},w,\bar{w}) = \sum_{i=1}^{10}\mathcal{Z}_{i,\alpha\beta}B_i(X;\bar{u},\bar{v},w,\bar{w}), \tag{6.20}$$

where the polynomials B_i are homogeneous degree 0 in X, with the appropriate homogeneity in $\bar{u},\bar{v},w,\bar{w}$. We now construct all possible polynomial structures for each $\mathcal{Z}_{i,\alpha\beta}$:

$$\mathcal{Z}_1 \coloneqq \mathcal{Y}_{11}\ \text{structures:}$$
$$B_1(X;\bar{u},\bar{v},w,\bar{w}) = b_1(\bar{u}\,\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-1}, \tag{6.21a}$$

$$\mathcal{Z}_2 \coloneqq \mathcal{Y}_{12}\ \text{structures:}$$
$$B_2(X;\bar{u},\bar{v},w,\bar{w}) = b_2(wX\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-1} + b_3(wX\bar{u})(\bar{v}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-2} + b_4(wX\bar{v})(\bar{u}\,\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}, \tag{6.21b}$$

$\mathcal{Z}_3 := \mathcal{Y}_{13}$ structures:

$$B_3(X;\bar{u},\bar{v},w,\bar{w}) = b_5(wX\bar{w})(\bar{u}\,\bar{w})(\bar{u}\,\bar{v})^{2k-1} + b_6(wX\bar{v})(\bar{u}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-2} + b_7(wX\bar{u})(\bar{u}\,\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}, \tag{6.21c}$$

$\mathcal{Z}_4 := \mathcal{Y}_{14}$ structures:

$$B_4(X;\bar{u},\bar{v},w,\bar{w}) = b_8(wX\bar{w})(\bar{u}\,\bar{v})^{2k} + b_9(wX\bar{u})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-1} + b_{10}(wX\bar{v})(\bar{u}\,\bar{w})(\bar{u}\,\bar{v})^{2k-1}, \tag{6.21d}$$

$\mathcal{Z}_5 := \mathcal{Y}_{22}$ structures:

$$B_5(X;\bar{u},\bar{v},w,\bar{w}) = b_{11}(wX\bar{w})(wX\bar{v})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}, \tag{6.21e}$$

$\mathcal{Z}_6 := \mathcal{Y}_{23}$ structures:

$$B_6(X;\bar{u},\bar{v},w,\bar{w}) = b_{12}(wX\bar{w})^2(\bar{u}\,\bar{v})^{2k-1} + b_{13}(wX\bar{w})(wX\bar{u})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2} + b_{14}(wX\bar{w})(wX\bar{v})(\bar{u}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}, \tag{6.21f}$$

$\mathcal{Z}_7 := \mathcal{Y}_{24}$ structures:

$$B_7(X;\bar{u},\bar{v},w,\bar{w}) = b_{15}(wX\bar{w})(wX\bar{v})(\bar{u}\,\bar{v})^{2k-1} + b_{16}(wX\bar{u})(wX\bar{v})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2} + b_{17}(wX\bar{v})^2(\bar{u}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}, \tag{6.21g}$$

$\mathcal{Z}_8 := \mathcal{Y}_{33}$ structures:

$$B_8(X;\bar{u},\bar{v},w,\bar{w}) = b_{18}(wX\bar{w})(wX\bar{u})(\bar{u}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}, \tag{6.21h}$$

$\mathcal{Z}_9 := \mathcal{Y}_{34}$ structures:

$$B_9(X;\bar{u},\bar{v},w,\bar{w}) = b_{19}(wX\bar{w})(wX\bar{u})(\bar{u}\,\bar{v})^{2k-1} + b_{20}(wX\bar{u})^2(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2} + b_{21}(wX\bar{u})(wX\bar{v})(\bar{u}\,\bar{w})(\bar{u}\,\bar{v})^{2k-2}, \tag{6.21i}$$

$\mathcal{Z}_{10} := \mathcal{Y}_{44}$ structures:

$$B_{10}(X;\bar{u},\bar{v},w,\bar{w}) = b_{22}(wX\bar{u})(wX\bar{v})(\bar{u}\,\bar{v})^{2k-1}. \tag{6.21j}$$

There are also additional structures that are defined only for $k > 1$. Such structures will be denoted by $\tilde{B}$.

$\mathcal{Z}_5$ structures:

$$\tilde{B}_5(X;\bar{u},\bar{v},w,\bar{w}) = b_{23}(wX\bar{v})^2(\bar{u}\,\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-3} + b_{24}(wX\bar{u})(wX\bar{v})(\bar{v}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-3}, \tag{6.22a}$$

$\mathcal{Z}_6$ structures:

$$\tilde{B}_6(X;\bar{u},\bar{v},w,\bar{w}) = b_{25}(wX\bar{u})^2(\bar{v}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-3} + b_{26}(wX\bar{v})^2(\bar{u}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-3} + b_{27}(wX\bar{u})(wX\bar{v})(\bar{u}\,\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-3}, \tag{6.22b}$$

$\mathcal{Z}_8$ structures:

$$\tilde{B}_8(X;\bar{u},\bar{v},w,\bar{w}) = b_{28}(wX\bar{u})^2(\bar{u}\,\bar{w})(\bar{v}\,\bar{w})(\bar{u}\,\bar{v})^{2k-3} + b_{29}(wX\bar{u})(wX\bar{v})(\bar{u}\,\bar{w})^2(\bar{u}\,\bar{v})^{2k-3}. \tag{6.22c}$$

Therefore we must analyze the $k = 1$ and $k > 1$ cases separately.

A. Analysis for $k = 1$

First, we must determine any linear dependence relations between the various polynomial structures. In this case, since there are many structures, the linear dependence relations are rather complicated. For the A structures, we find $A(X;\bar{u},\bar{v},w,\bar{w}) = 0$ for the choice of coefficients

$$a_4 = -a_1 + a_2, \qquad a_5 = -a_1 - a_3, \qquad a_6 = 2a_1 - a_2 + a_3. \tag{6.23}$$

Hence, the structures corresponding to a_1, a_2, and a_3 may be neglected, and we are left with only the structures with coefficients a_4, a_5, a_6. Next we find linear dependence among the B structures, and we find $B_{(\alpha\beta)}(X;\bar{u},\bar{v},w,\bar{w}) = 0$ for the choices

$$b_6 = b_1 - b_2 - b_4 - b_5 + b_{11} + b_{12} + b_{14}, \tag{6.24a}$$

$$b_7 = -b_1 + b_2 - b_3 + b_5 - b_{12} + b_{13} + b_{18}, \tag{6.24b}$$

$$b_8 = -b_1 + b_2 + b_4 - b_{10} - b_{11} + b_{15} + b_{17}, \tag{6.24c}$$

$$b_9 = -b_1 + b_3 + b_4 - b_{10} - b_{11} + b_{12} - b_{13} + b_{15} + b_{17} + b_{19} - b_{20}, \tag{6.24d}$$

$$b_{21} = -b_{11} - b_{13} - b_{14} - b_{16} - b_{17} - b_{18} - b_{20}, \tag{6.24e}$$

$$b_{22} = b_{11} - b_{12} + b_{13} - b_{15} + b_{16} - b_{19} + b_{20}. \tag{6.24f}$$

Therefore a linearly independent basis may be constructed out of the structures corresponding to the coefficients b_6, b_7, b_8, b_9, b_{21}, and b_{22}. Overall there are nine independent structures to consider. We now impose the differential constraints and point switch identities using *Mathematica*. After imposing (6.5b) and (6.13b) we obtain the following relations between the coefficients:

$$b_6 = \frac{1}{90}(-126a_4 - 114a_5 - 115a_6), \tag{6.25a}$$

$$b_7 = \frac{1}{90}(114a_4 + 126a_5 + 115a_6), \tag{6.25b}$$

$$b_8 = \frac{1}{45}(42a_4 - 12a_5 + 5a_6), \tag{6.25c}$$

$$b_9 = \frac{1}{90}(-54a_4 - 66a_5 - 95a_6 - 4b_{21} - 44b_{22}), \tag{6.25d}$$

$$b_{21} = \frac{2}{3}(a_4 - a_5), \tag{6.25e}$$

$$b_{22} = \frac{1}{6}(-2a_4 + 2a_5 + 5a_6). \tag{6.25f}$$

Hence, the differential constraints fix the correlation function up to three parameters. Next we must impose the point switch identity (6.14b), from which we obtain $a_5 = a_4$, and hence, we are left with the free parameters a_4 and a_6.

We must now impose (6.8) in accordance with the conservation of the energy momentum tensor. First, we compute $\tilde{\mathcal{H}}$ using (6.7), which in the generating function formalism may be written as

$$\tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = \frac{1}{(2k)!} X^{2k-3} (vX\partial\bar{s})(\partial s X \bar{v})^{2k} \mathcal{H}^c(X; u, \bar{u}, s, \bar{s}, w, \bar{w}), \tag{6.26}$$

while the differential constraint (6.8) is equivalent to

$$\frac{\partial}{\partial X_{\sigma\dot{\sigma}}} \frac{\partial}{\partial w^{\sigma}} \frac{\partial}{\partial \bar{w}^{\dot{\sigma}}} \tilde{\mathcal{H}}(X; w, \bar{w}, v, \bar{v}, u, \bar{u}) = 0. \tag{6.27}$$

At this point we can freely set $k = 1$ and check whether our solution is consistent with conservation at x_3. Using *Mathematica*, it may be shown that (6.27) is satisfied provided that $a_6 = -\frac{12}{5} a_4$; hence, the correlation function $\langle QQT \rangle$ is determined up to a single complex parameter.

B. Analysis for general k

Now let us complete the analysis for $k > 1$. Again we must find a linearly independent basis of polynomial structures. If we supplement the set of basis structures corresponding to b_6, b_7, b_8, b_9, b_{21}, and b_{22} with the $\tilde{B}$ structures defined in (6.22a), (6.22b), and (6.22c), then it may be shown that $B_{(\alpha\beta)}(X; \bar{u}, \bar{v}, w, \bar{w}) = 0$ for the choices

$$b_6 = b_{23} + b_{26}, \tag{6.28a}$$

$$b_7 = -b_{21} - b_{22} + b_{23} + b_{24} + b_{27}, \tag{6.28b}$$

$$b_8 = -b_{23}, \tag{6.28c}$$

$$b_9 = -b_{22} - b_{23} - b_{24}, \tag{6.28d}$$

$$b_{25} = -b_{22} - b_{24}, \tag{6.28e}$$

$$b_{28} = b_{21} + 2b_{22} - b_{23} - b_{27}, \tag{6.28f}$$

$$b_{29} = -b_{21} - b_{22} - b_{26}. \tag{6.28g}$$

Hence, there are ten independent structures to consider, corresponding to the coefficients $a_4, a_5, a_6, b_6, b_7, b_8, b_9, b_{25}, b_{28}$, and b_{29}. We now impose the differential constraints and point-switch identities; after imposing (6.5b), (6.13b), and (6.14b) we obtain $a_5 = a_4$, supplemented by the following k-dependent relations between the b coefficients:

$$b_6 = \frac{2a_4(24 + 45k + k^2 + 14k^3 - 4k^4 - 8k^5) + a_6(24 + 39k + 26k^2 - 12k^3 - 8k^4)}{(-5 + 2k)(1 + 2k)(-2 + k + 5k^2 + 2k^3)}, \tag{6.29a}$$

$$b_7 = -\frac{2(a_4(30 + 58k - 8k^2 - 8k^3) + a_6(15 + 23k + 5k^2 - 6k^3 + 12k^4 + 8k^5))}{(-5 + 2k)(1 + 2k)(-2 + k + 5k^2 + 2k^3)}, \tag{6.29b}$$

$$b_8 = \frac{-2a_4(19+25k-30k^2-4k^3+8k^4)+a_6(-19-17k+2k^2+20k^3+8k^4)}{(-5+2k)(1+2k)(-2+k+5k^2+2k^3)}, \tag{6.29c}$$

$$b_9 = \frac{4a_4(-7-7k+20k^2+12k^3)+2a_6(-7+k+8k^2+4k^3)}{(-5+2k)(1+2k)(-2+k+5k^2+2k^3)}, \tag{6.29d}$$

$$b_{25} = \frac{k(3+2k)(2a_4(-3+k+2k^2)-a_6(1+2k))}{10-9k-23k^2+4k^4}, \tag{6.29e}$$

$$b_{28} = \frac{2(3+2k)(a_4(2+2k-4k^2)+a_6(1-k+k^2+2k^3))}{10-9k-23k^2+4k^4}, \tag{6.29f}$$

$$b_{29} = \frac{k(3+2k)(2a_4(-3+k+2k^2)-a_6(1+2k))}{10-9k-23k^2+4k^4}. \tag{6.29g}$$

Hence, after imposing conservation on the first and second points, we find there are two free complex coefficients remaining. The last constraint to impose is conservation on x_3, that is, Eq. (6.27). We cannot obtain a relation for arbitrary k, as from a computational standpoint one must fix k in order to compute $\tilde{\mathcal{H}}$ as in (6.26). However, we find that the correlation function is fixed up to a single parameter up to $k = 4$, after which the computations become incredibly long and beyond our computer power. For $k = 2$ we find $a_6 = -\frac{20}{7}a_4$; for $k = 3$, $a_6 = -\frac{28}{9}a_4$; and for $k = 4$, $a_6 = -\frac{36}{11}a_4$. We anticipate that similar results will hold for general k as well.

VII. DISCUSSION ON SUPERSYMMETRY

In this section we will concentrate on the case $k = 1$, which corresponds to a supersym-metrylike current $Q_{\alpha\beta\dot{\alpha}} = (\sigma^m)_{\alpha\dot{\alpha}}Q_{m,\beta}$ of dimension $\frac{5}{2}$ satisfying the conservation equation

$$\partial^{\alpha\dot{\alpha}}Q_{\alpha\beta\dot{\alpha}} = 0. \tag{7.1}$$

However, our analysis in the previous sections did not assume supersymmetry. The question that naturally arises is whether the supersymmetrylike current actually is the supersymmetry current. That is, whether a conformal field theory possessing a conserved fermionic current of spin-$\frac{3}{2}$ is superconformal.

In any supersymmetric field theory the supersymmetry current is a component of the supercurrent $J_{\alpha\dot{\alpha}}(z)$, which also contains the energy-momentum tensor. As was explained in the Introduction, this implies that the three-point functions $\langle QQT\rangle$ and $\langle\bar{Q}QT\rangle$ must be contained in the three-point function of the supercurrent $\langle JJJ\rangle$. It is known that the general form of $\langle JJJ\rangle$ is fixed by superconformal symmetry up to two independent structures [14]. Hence, this implies that in any superconformal field theory, $\langle QQT\rangle$ and $\langle\bar{Q}QT\rangle$ must also be fixed up to at most two independent structures. Moreover, the three-point function $\langle QQT\rangle$ must actually vanish. Indeed, in a supersymmetric theory Q carries an R-symmetry charge and, hence, the entire correlator $\langle QQT\rangle$ carries an R-symmetry charge. However, by performing a simple change of variables in the path integral it then follows that $\langle QQT\rangle = 0$. In addition, our analysis in Sec. VI showed that, in general, conformal symmetry fixes $\langle QQT\rangle$ up one overall parameter, which is inconsistent with supersymmetry. We also found in Sec. V that the three-point function $\langle\bar{Q}QT\rangle$ is fixed up to four rather than two independent parameters, which, in general, is also inconsistent with the general form of $\langle JJJ\rangle$.

Similarly, we can examine the three-point functions $\langle\bar{Q}QV\rangle$ and $\langle QQV\rangle$ studied in Secs. III and IV, respectively. In supersymmetric theories, the vector current V_m belongs to the flavor current multiplet $L(z)$. Hence, the correlation functions $\langle\bar{Q}QV\rangle$ and $\langle\bar{Q}QV\rangle$ are contained in the three-point function $\langle JJL\rangle$. It is known [14] that $\langle JJL\rangle$ is fixed by superconformal symmetry up to an overall real coefficient. Hence, $\langle\bar{Q}QV\rangle$ must also be fixed up to an overall coefficient. As for $\langle QQV\rangle$, it must vanish just as $\langle QQT\rangle$. However, our analysis in Secs. III and IV showed that $\langle\bar{Q}QV\rangle$ is fixed up to three independent coefficients and $\langle QQV\rangle$ is fixed up to one overall coefficient. Both of these results are, in general, inconsistent with the general form of $\langle JJL\rangle$.

ACKNOWLEDGMENTS

The authors thank Sergei Kuzenko and Jessica Hutomo for their comments on the main results, and Daniel Hutchings, Nowar Koning, Michael Ponds, Emmanouil Raptakis, and Kai Turner for valuable discussions. The work of E. I. B. is supported in part by the Australian Research Council, Project No. DP200101944. The work of B. S. is supported by the *Bruce and Betty Green Postgraduate Research Scholarship* under the Australian Government Research Training Program.

APPENDIX: 4D CONVENTIONS AND NOTATION

Our conventions closely follow that of [50]. For the Minkowski metric η_{mn} we use the "mostly plus" convention: $\eta_{mn} = \mathrm{diag}(-1,1,1,1)$. Spinor indices on spin tensors are raised and lowered with the $\mathrm{SL}(2,\mathbb{C})$ invariant spinor metrics

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon_{\alpha\gamma}\varepsilon^{\gamma\beta} = \delta_\alpha{}^\beta, \tag{A1}$$

$$\varepsilon_{\dot\alpha\dot\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varepsilon^{\dot\alpha\dot\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon_{\dot\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\beta} = \delta_{\dot\alpha}{}^{\dot\beta}. \tag{A2}$$

Given the spinor fields ϕ_α and $\bar\phi_{\dot\alpha}$, the spinor indices $\alpha = 1, 2$ and $\dot\alpha = \bar 1, \bar 2$ are raised and lowered according to the following rules:

$$\begin{aligned} \phi_\alpha &= \varepsilon_{\alpha\beta}\phi^\beta, & \phi^\alpha &= \varepsilon^{\alpha\beta}\phi_\beta, \\ \bar\phi_{\dot\alpha} &= \varepsilon_{\dot\alpha\dot\beta}\bar\phi^{\dot\beta}, & \bar\phi^{\dot\alpha} &= \varepsilon^{\dot\alpha\dot\beta}\bar\phi_{\dot\beta}. \end{aligned} \tag{A3}$$

It is also useful to introduce the complex 2×2 σ-matrices, defined as follows:

$$\begin{aligned} \sigma_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_2 &= \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, & \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \tag{A4}$$

The σ-matrices span the Lie group $\mathrm{SL}(2,\mathbb{C})$, the universal covering group of the Lorentz group $\mathrm{SO}(3,1)$. Now let $\sigma_m = (\sigma_0, \vec\sigma)$; we denote the components of σ_m as $(\sigma_m)_{\alpha\dot\alpha}$ and define

$$(\tilde\sigma_m)^{\dot\alpha\alpha} \equiv \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}(\sigma_m)_{\beta\dot\beta}. \tag{A5}$$

It can be shown that the σ-matrices possess the following useful properties:

$$(\sigma_m\tilde\sigma_n + \sigma_n\tilde\sigma_m)_\alpha{}^\beta = -2\eta_{mn}\delta_\alpha^\beta, \tag{A6}$$

$$(\tilde\sigma_m\sigma_n + \tilde\sigma_n\sigma_m)^{\dot\alpha}{}_{\dot\beta} = -2\eta_{mn}\delta_{\dot\beta}^{\dot\alpha}, \tag{A7}$$

$$\mathrm{Tr}(\sigma_m\tilde\sigma_n) = -2\eta_{mn}, \tag{A8}$$

$$(\sigma^m)_{\alpha\dot\alpha}(\tilde\sigma_m)^{\dot\beta\beta} = -2\delta_\alpha^{\dot\alpha}\delta_{\dot\alpha}^{\dot\beta}. \tag{A9}$$

The σ-matrices are then used to convert spacetime indices into spinor ones and vice versa according to the following rules:

$$X_{\alpha\dot\alpha} = (\sigma^m)_{\alpha\dot\alpha}X_m, \qquad X_m = -\frac{1}{2}(\tilde\sigma_m)^{\dot\alpha\alpha}X_{\alpha\dot\alpha}. \tag{A10}$$

[1] H. Osborn and A. C. Petkou, Implications of conformal invariance in field theories for general dimensions, Ann. Phys. (N.Y.) **231**, 311 (1994).
[2] J. Erdmenger and H. Osborn, Conserved currents and the energy momentum tensor in conformally invariant theories for general dimensions, Nucl. Phys. **B483**, 431 (1997).
[3] A. M. Polyakov, Conformal symmetry of critical fluctuations, JETP Lett. **12**, 381 (1970).
[4] E. J. Schreier, Conformal symmetry and three-point functions, Phys. Rev. D **3**, 980 (1971).
[5] A. A. Migdal, On hadronic interactions at small distances, Phys. Lett. **37B**, 98 (1971).
[6] A. A. Migdal, Conformal invariance and bootstrap, Phys. Lett. **37B**, 386 (1971).
[7] S. Ferrara, A. F. Grillo, and R. Gatto, Manifestly conformal-covariant expansion on the light cone, Phys. Rev. D **5**, 3102 (1972).
[8] S. Ferrara, A. F. Grillo, and R. Gatto, Tensor representations of conformal algebra and conformally covariant operator product expansion, Ann. Phys. (N.Y.) **76**, 161 (1973).
[9] K. Koller, The significance of conformal inversion in quantum field theory, Commun. Math. Phys. **40**, 15 (1975).
[10] G. Mack, Convergence of operator product expansions on the vacuum in conformal invariant quantum field theory, Commun. Math. Phys. **53**, 155 (1977).
[11] Y. S. Stanev, Stress-energy tensor and U(1) current operator product expansions in conformal QFT, Bulgarian Journal of Physics **15**, 93 (1988).
[12] E. S. Fradkin and M. Y. Palchik, Recent developments in conformal invariant quantum field theory, Phys. Rep. **44**, 249 (1978).
[13] J. H. Park, $N = 1$ superconformal symmetry in four-dimensions, Int. J. Mod. Phys. A **13**, 1743 (1998).
[14] H. Osborn, $N = 1$ superconformal symmetry in four-dimensional quantum field theory, Ann. Phys. (N.Y.) **272**, 243 (1999).
[15] J. H. Park, Superconformal symmetry and correlation functions, Nucl. Phys. **B559**, 455 (1999).
[16] J.-H. Park, Superconformal symmetry in six-dimensions and its reduction to four-dimensions, Nucl. Phys. **B539**, 599 (1999).

[17] J.-H. Park, Superconformal symmetry in three-dimensions, J. Math. Phys. (N.Y.) **41**, 7129 (2000).

[18] S. M. Kuzenko and S. Theisen, Correlation functions of conserved currents in $N = 2$ superconformal theory, Classical Quantum Gravity **17**, 665 (2000).

[19] A. A. Nizami, T. Sharma, and V. Umesh, Superspace formulation and correlation functions of 3d superconformal field theories, J. High Energy Phys. 07 (2014) 022.

[20] E. I. Buchbinder, S. M. Kuzenko, and I. B. Samsonov, Superconformal field theory in three dimensions: Correlation functions of conserved currents, J. High Energy Phys. 06 (2015) 138.

[21] E. I. Buchbinder, S. M. Kuzenko, and I. B. Samsonov, Implications of $\mathcal{N} = 4$ superconformal symmetry in three spacetime dimensions, J. High Energy Phys. 08 (2015) 125.

[22] S. M. Kuzenko and I. B. Samsonov, Implications of $\mathcal{N} = 5$, 6 superconformal symmetry in three spacetime dimensions, J. High Energy Phys. 08 (2016) 084.

[23] E. I. Buchbinder and B. J. Stone, Mixed three-point functions of conserved currents in three-dimensional superconformal field theory, Phys. Rev. D **103**, 086023 (2021).

[24] E. I. Buchbinder, J. Hutomo, and S. M. Kuzenko, Correlation functions of spinor current multiplets in $\mathcal{N} = 1$ superconformal theory, J. High Energy Phys. 07 (2021) 165.

[25] E. I. Buchbinder, J. Hutomo, and S. M. Kuzenko, Three-point functions of higher-spin spinor current multiplets in $\mathcal{N} = 1$ superconformal theory, J. High Energy Phys. 10 (2021) 058.

[26] E. I. Buchbinder and B. J. Stone, Three-point functions of a superspin-2 current multiplet in 3D, $N = 1$ superconformal theory, Phys. Rev. D **104**, 106004 (2021).

[27] S. Giombi, S. Prakash, and X. Yin, A note on CFT correlators in three dimensions, J. High Energy Phys. 07 (2013) 105.

[28] J. Maldacena and A. Zhiboedov, Constraining conformal field theories with a higher spin symmetry, J. Phys. A **46**, 214011 (2013).

[29] Y. S. Stanev, Constraining conformal field theory with higher spin symmetry in four dimensions, Nucl. Phys. **876**, 651(R) (2013).

[30] V. Alba and K. Diab, Constraining conformal field theories with a higher spin symmetry in $d = 4$, arXiv:1307.8092.

[31] V. Alba and K. Diab, Constraining conformal field theories with a higher spin symmetry in $d > 3$ dimensions, J. High Energy Phys. 03 (2016) 044.

[32] Y. S. Stanev, Correlation functions of conserved currents in four dimensional conformal field theory, Nucl. Phys. **B865**, 200 (2012).

[33] A. Zhiboedov, A note on three-point functions of conserved currents, arXiv:1206.6370.

[34] E. Elkhidir, D. Karateev, and M. Serone, General three-point functions in 4D CFT, J. High Energy Phys. 01 (2015) 133.

[35] S. Weinberg, Six-dimensional methods for four-dimensional conformal field theories, Phys. Rev. D **82**, 045031 (2010).

[36] S. Weinberg, Six-dimensional methods for four-dimensional conformal field theories II: Irreducible fields, Phys. Rev. D **86**, 085013 (2012).

[37] M. S. Costa, J. Penedones, D. Poland, and S. Rychkov, Spinning conformal blocks, J. High Energy Phys. 11 (2011) 154.

[38] M. S. Costa, J. Penedones, D. Poland, and S. Rychkov, Spinning conformal correlators, J. High Energy Phys. 11 (2011) 071.

[39] M. S. Costa and T. Hansen, Conformal correlators of mixed-symmetry tensors, J. High Energy Phys. 02 (2015) 151.

[40] J.-F. Fortin, W.-J. Ma, V. Prilepina, and W. Skiba, Conformal conserved currents in embedding space, J. High Energy Phys. 01 (2022) 185.

[41] W. D. Goldberger, W. Skiba, and M. Son, Superembedding methods for 4d $N = 1$ SCFTs, Phys. Rev. D **86**, 025019 (2012).

[42] W. D. Goldberger, Z. U. Khandker, D. Li, and W. Skiba, Superembedding methods for current superfields, Phys. Rev. D **88**, 125010 (2013).

[43] S. Jain, R. R. John, and V. Malvimat, Constraining momentum space correlators using slightly broken higher spin symmetry, J. High Energy Phys. 04 (2021) 231.

[44] S. Jain, R. R. John, and V. Malvimat, Momentum space spinning correlators and higher spin equations in three dimensions, J. High Energy Phys. 11 (2020) 049.

[45] S. Jain and R. R. John, Relation between parity-even and parity-odd CFT correlation functions in three dimensions, J. High Energy Phys. 12 (**2021**) 067.

[46] S. Jain, R. R. John, A. Mehta, A. A. Nizami, and A. Suresh, Higher spin 3-point functions in 3d CFT using spinor-helicity variables, J. High Energy Phys. 09 (2021) 041.

[47] S. Jain, R. R. John, A. Mehta, A. A. Nizami, and A. Suresh, Momentum space parity-odd CFT 3-point functions, J. High Energy Phys. 08 (2021) 089.

[48] H. Isono, T. Noumi, and T. Takeuchi, Momentum space conformal three-point functions of conserved currents and a general spinning operator, J. High Energy Phys. 05 (2019) 057.

[49] T. Bautista and H. Godazgar, Lorentzian CFT 3-point functions in momentum space, J. High Energy Phys. 01 (2020) 142.

[50] I. Buchbinder and S. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity: Or a Walk Through Superspace* (IOP, Bristol, UK, 1998).