

Gravitational waves in metastable supersymmetry breaking

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If supersymmetry is broken in metastable vacua, it is not clear why we are now in there rather than supersymmetric vacua. Moreover, it is natural to expect that we were in supersymmetric vacua, which have higher symmetry than metastable vacua, in the early Universe. In this paper, we reexamine and improve the previous analysis on the cosmological evolution of the vacuum structure in the Intriligator, Seiberg, and Shih (ISS) model of metastable supersymmetry breaking by taking into account constraints on the reheating temperature, which is needed to avoid the overproduction of gravitinos. It turns out that the desired phase transition from a supersymmetric vacuum to a metastable vacuum is allowed only in the light gravitino mass region $m_{3/2} < 4.7$ eV. This is achieved by either rolling down potential or tunneling processes depending on the reheating temperature. We show that when the tunneling processes are realized, abundant gravitational waves could be produced from collisions of runaway bubbles. The resulting gravitational waves are detectable with the future gravitational wave interferometers like LISA and DECIGO.

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I. INTRODUCTION

Gravitational waves are a powerful tool to explore the early Universe. Indeed, gravitational waves from inflation [1–3] have been probed intensively by the temperature anisotropy and the B-mode polarization of the cosmic microwave background, and current null observations have put constraints on the energy scale of inflation [4,5]. Recently, pulsar timing arrays, NANOGrav, PPTA, EPTA, and IPTA reported detection of correlated signals among pulsars [6–9], which might be stochastic gravitational waves of primordial origin. In particular, it has been discussed that primordial gravitational waves from the (first order) QCD phase transition can account for the pulsar timing signals [10,11]. The electroweak phase transition, if it is first order, can also result in abundant gravitational waves [12–15], which could be detected by gravitational wave interferometers such as LIGO [16], Virgo [17], LISA [18], and DECIGO [19]. Therefore, the observation of gravitational waves enables us to explore high energy

physics in the early Universe and is a promising way to access physics that goes beyond current collider experiments $\sim \mathcal{O}(1)$ TeV.

Supersymmetry is often postulated to be a symmetry of the fundamental theory at high energy. However if it is true, then supersymmetry needs to be broken at some point in the cosmological history. In [20], it was pointed out that a first order phase transition along a pseudoflat direction associated with the spontaneous breaking of a $U(1)$ R-symmetry could give rise to detectable gravitational wave signals. The discussion applies to generic supersymmetry breaking models where the supersymmetry breaking vacua are global minima, since the existence and breaking of R-symmetry is usually required in this case [21]. An exception to the discussion is the metastable supersymmetry breaking model proposed by Intriligator, Seiberg, and Shih (ISS) [22], where the metastable supersymmetry breaking vacuum is local but not global minimum. The model has attracted substantial attention because it can avoid the difficulties associated with R-symmetry [21] and chirality [23]. Moreover the metastable vacuum can be made to be parametrically long-lived, making the metastable supersymmetry breaking a viable scenario.

Nevertheless, it is natural to expect that we were in a supersymmetric vacuum in the early Universe, which has higher symmetry, than a metastable vacuum. It is then necessary to understand how we ended up in the metastable vacuum now. To answer the question, [24–27] studied the

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phase structure of the theory by taking into account thermal corrections from particles in the ISS hidden sector to the effective potential and showed that the metastable vacuum could be chosen eventually in the cosmological evolution of the thermal Universe even if the Universe was in the supersymmetric vacuum initially. However, it should be stressed that an arbitrary high reheating temperature is allowed in the analysis of these early works. On the other hand, in gauge mediated supersymmetry breaking models, the reheating temperature actually cannot be too high in order to avoid the overproduction of gravitinos [28].

In this paper, we will first reexamine the previous analysis [24,25] in light of the gravitino problem. We will find that a phase transition from the supersymmetric vacuum to the metastable vacuum is not possible in the middle $0.4 \text{ keV} \lesssim m_{3/2} \lesssim 1 \text{ GeV}$ and heavy $600 \text{ GeV} \lesssim m_{3/2}$ allowed regions [29] of gravitino mass. As a result, in order for the Universe to be able to arrive at the metastable vacuum, a messenger model that is compatible with the light gravitino mass is needed. We will consider the light gravitino mass region $m_{3/2} < 4.7 \text{ eV}$ [29] and show that, as long as a compatible messenger sector is chosen, the desired phase transition is always possible by taking into account thermal corrections from the ISS hidden sector and the minimal supersymmetric standard model (MSSM) as a visible sector. The phase transition is achieved by either rolling down potential or tunneling processes depending on the reheating temperature. It will be argued that inclusion of effects from a messenger sector does not spoil the conclusion. Next, we will study gravitational wave production from the tunneling processes. It will turn out that from the tunneling, gravitational waves are produced mainly by collision of nucleated runaway bubbles. We will calculate the gravitational wave spectra and show that they are detectable with the future space borne gravitational wave interferometers such as LISA [18] and DECIGO [19]. This gives us a unique way to probe the metastable supersymmetry breaking.

The organization of the paper is as follows. In Sec. II, we review the ISS model of the dynamical supersymmetry breaking in metastable vacua. In Sec. III A, the cosmological gravitino problem is reviewed. To avoid their overproduction, we see that high reheating temperature is forbidden for certain ranges of gravitino mass. In Sec. III B, we investigate the thermal contributions from the ISS sector and the MSSM sector to the finite temperature effective potential. We show that phase transitions from the supersymmetric vacuum to the metastable vacuum are always possible in the light gravitino mass region. We argue that inclusion of contributions from a messenger sector does not spoil our conclusion. In Sec. IV, we study gravitational wave production from tunneling associated with the phase transition. It is shown that the produced gravitational waves are detectable with the future gravitational wave interferometers like LISA [18] and

DECIGO [19]. The final section is devoted to conclusion and further discussions.

II. METASTABLE VACUA IN SUPERSYMMETRIC GAUGE THEORY

For the sake of establishing the notations, let us briefly review the metastable supersymmetry breaking model proposed by Intriligator, Seiberg, and Shih (ISS) [22]. We consider a supersymmetric gauge theory as a hidden sector for supersymmetry breaking with chiral superfields Φ_{ij} , φ_c^i , and $\tilde{\varphi}^{ic}$ and a tree level superpotential

$$W_{\text{cl}} = h\text{Tr}\varphi\Phi\tilde{\varphi} - h\mu^2\text{Tr}\Phi, \quad (1)$$

where $i = 1 \dots N_f$ denotes flavors, $c = 1 \dots N$ denotes indexes of the $SU(N)$ gauge group, and h and μ are constants. The number of flavors is taken to satisfy the condition $N_f > 3N$ so that the theory is weakly coupled at energy scale much less than the Landau scale Λ_m . We mention that above the energy scale of the Landau pole, we have a UV free electric dual description in terms of a $SU(N_f - N)$ gauge theory thanks to the Seiberg's duality [30–32]. Supersymmetry is broken in the ISS model since the F -flatness condition cannot be satisfied due to the rank condition. The supersymmetry breaking vacuum has a nonvanishing energy density V_{meta} and is given by

$$\begin{aligned} \langle \Phi \rangle &= 0, & \langle \varphi_c^i \rangle &= \langle \tilde{\varphi}^{ic} \rangle = \mu \delta_{ci}, \\ V_{\text{meta}} &= (N_f - N)|h^2\mu^4|, \end{aligned} \quad (2)$$

where $c, i = 1, \dots, N$ in a suitable choice of bases. The vacuum is locally stable [22] due to the Coleman-Weinberg one-loop contributions to the potential [33]. However it is not globally stable because there exists a supersymmetric vacuum, which is generated dynamically.

To see this, let us consider the situation that Φ has a nonzero vacuum expectation value. Then from the superpotential (1), φ and $\tilde{\varphi}$ obtain a mass of $h\Phi$. Below the mass scale, one can integrate out φ and $\tilde{\varphi}$ so that we have a pure supersymmetric $SU(N)$ gauge theory effectively. The low energy effective theory is asymptotic free and we expect gaugino condensation. Actually, a superpotential is generated dynamically

$$W_{\text{dyn}} = N \left(\frac{h^{N_f} \det \Phi}{\Lambda_m^{N_f - 3N}} \right)^{1/N}, \quad (3)$$

and gives rise to the gaugino condensation [31,34]

$$\langle \text{Tr}\rho\rho \rangle = 32\pi^2 \left(\frac{h^{N_f} \det \Phi}{\Lambda_m^{N_f - 3N}} \right)^{\frac{1}{N}}, \quad (4)$$

where ρ denotes the gauginos. The gaugino condensate sets the mass scale for the massive $SU(N)$ degrees of freedom

such as the gauginos and the gauge fields of the ISS model and will play a role in our discussion later for the finite temperature effective potential. Moreover, the superpotential W_{dyn} leads to the emergence of a supersymmetry preserving stable vacuum at

$$\langle \Phi \rangle = \Phi_0 \delta_{ij} \equiv \frac{\mu}{h e^{(N_f - 3N)/(N_f - N)}} \delta_{ij}, \quad \langle \varphi \rangle = \langle \tilde{\varphi}^T \rangle = \mathbf{0}, \quad (5)$$

where $\epsilon := \frac{\mu}{\Lambda_m} \ll 1$. We require $\epsilon \ll 1$ to guarantee the perturbative treatment of the theory and the longevity of the metastable vacuum (2) [22]. Then the theory realizes supersymmetry breaking, assuming that we have already been in the metastable vacuum.

However it is actually more natural to expect that we were in the supersymmetric vacuum which has higher symmetry than the metastable vacuum in the early Universe. In [24–27], thermal correction from particles in the ISS sector to the structure of vacua was studied and it was shown that the metastable vacuum could be preferred in the thermal Universe with a sufficiently high temperature even if the Universe started out in the supersymmetric vacuum initially. Note that inclusion of thermal effects from a visible sector was also partly studied in [25]. However, an arbitrary high reheating temperature is allowed in the analysis of these early works and this may not be compatible with the cosmological gravitino problem [28]. In the next section, we will elaborate such cosmological evolution of the ISS model by including thermal corrections from particles in the ISS sector, a visible sector and partly a messenger sector in light of the cosmological gravitino problem.

III. COSMOLOGICAL PHASE TRANSITIONS IN THE ISS MODEL

In the early Universe, it is natural to expect that we were in the supersymmetric vacuum (5) where the scalar component of Φ has a nonzero expectation value. In [24–27] it was shown that if the ISS sector is in thermal equilibrium in the early Universe, then the thermally corrected effective potential $V_T(\Phi, \varphi)$ has a global minimum at the origin of the field space at sufficiently high temperature. Note that here we parametrize $\Phi_{ij} = \Phi \delta_{ij}$, $\varphi_i = \varphi \mathbf{1}_{N_f}$ so that the effective potential is determined by the two parameters. The field Φ can either tunnel or roll down on the potential, depending on the reheating temperature, from the supersymmetric vacuum $(\Phi_0, 0)$ to the origin $(0, 0)$ of the field space [24, 25]. As the temperature of the Universe decreases, a second order phase transition occurs from the origin to the supersymmetry breaking vacuum (2) rather than to the supersymmetric vacuum $(\Phi_0, 0)$ [26, 27].

Consider the potential $V_T(\Phi)$ ($\varphi = 0$) which describes the vacuum structure between the supersymmetric one (5) and the origin of the field space. From the superpotentials (1) and (3), we obtain the potential at zero temperature

$$V_0(\Phi) = |h^2 \mu^4| N_f \left(\left(\frac{\Phi}{\Phi_0} \right)^{\frac{N_f - N}{N}} - 1 \right)^2. \quad (6)$$

In the early Universe, there are thermal corrections to the above potential from particles in equilibrium with Φ . The thermal corrections to the potential (6) at temperature T is given by [35, 36]

$$\Delta V_T(\Phi) = \frac{T^4}{2\pi^2} \sum_i \pm n_i \int_0^\infty dp p^2 \ln(1 \mp e^{-\sqrt{p^2 + m_i^2(\Phi)}/T}), \quad (7)$$

where n_i denotes degrees of freedom of the i -th particle, $m_i(\Phi)$ represents tree level masses of particles, which may depend on the field expectation value of Φ , and the upper (lower) signs are for bosonic (fermionic) particles. Note that both of contributions from bosons and fermions are negative and those magnitudes are larger if their masses are lighter. Note also that the contributions from a Φ -independent mass is just a constant shift to the potential. We will neglect such contributions since they do not change the shape of the potential. We mention that although there is also the Coleman-Weinberg mechanism [33], which modifies the potential even at zero temperature, it is exactly zero around the supersymmetric vacuum and negligible around $\Phi = 0$ compared with the thermal correction.

In the next subsection, we will review the cosmological gravitino problem and find that there cannot be a phase transition from the supersymmetric vacuum $(\Phi_0, 0)$ to the origin $(0, 0)$ of the field space for middle and heavy mass regions of gravitino.

A. Constraints from the gravitino problem

In order to mediate the supersymmetry breaking of the ISS sector to a visible sector, one needs to include a messenger sector, which is charged under some gauge group of a visible sector. To be concrete, we will consider the MSSM as a visible sector here after. In the gauge mediated supersymmetry breaking of the ISS model, the gravitino would be the lightest supersymmetric particle among the relic particles. The mass of the gravitino $m_{3/2} = F/\sqrt{3}M_P$ is determined by the supersymmetry breaking scale F and the reduced Planck mass scale M_P [37]. In our present case, $F = V_{\text{meta}}^{1/2}$ and so

$$m_{3/2} = \frac{(N_f - N)^{1/2} |h\mu^2|}{\sqrt{3}M_P}. \quad (8)$$

Since its mass is suppressed by the Planck scale, the gravitino usually becomes the lightest supersymmetric particle and its presence could lead to serious cosmological problems. In order to avoid the overproduction of gravitinos, the reheating temperature T_R must be sufficiently low [28,29]. The observation of the cosmic microwave background lensing, the cosmic shear [38], the Lyman- α [39], and the light-element photodestruction [40] have excluded certain mass regions of the gravitino for any reheating temperature. Remaining mass regions are $m_{3/2} < 4.7$ eV, 0.4 keV $\lesssim m_{3/2} \lesssim 1$ GeV, and 600 GeV $\lesssim m_{3/2}$ [29]. For the middle region of 0.4 keV $\lesssim m_{3/2} \lesssim 1$ GeV and the heavy region of 600 GeV $\lesssim m_{3/2}$, higher reheating temperatures compared with the supersymmetry breaking scales are excluded to avoid the overproduction of gravitinos [41], and T_R is at most upper bounded as [29]

$$T_R < 0.1 \times V_{\text{meta}}^{1/4}. \quad (9)$$

On the other hand, we do not have any upper bound on the reheating temperature in the light gravitino mass region $m_{3/2} < 4.7$ eV.

The upper bound (9) on the reheating temperature has important implications on the thermal phase transitions. The thermal correction (7) to the potential is controlled by T^4 times a sum of integral factors. The bosonic and fermionic integrals vanish at $T = 0$ and reach their respective maximum values of order 1 at large T . Previously in [24,25], it was assumed that the reheating temperature can be as high as one wishes, and as a result the correction (7) is strong enough to drive the phase transition from the supersymmetric vacuum to the metastable vacuum. However, this assumption should be reexamined in view of the gravitino constraint (9) on the reheating temperature. In fact for temperature below the reheating temperature (9), the thermal correction is negligible compared to the zero temperature potential as long as we do not have unnaturally large number of species $\sum_i \pm n_i \gg 10^4$. Therefore, we conclude that in the middle and heavy region of gravitino mass, the supersymmetric vacuum will remain as the global minimum of the theory and the desired phase transition cannot occur. As a result, we will concentrate in the rest of the paper on the light gravitino mass region

$$m_{3/2} < 4.7 \text{ eV}, \quad (10)$$

where there is no upper bound on the reheating temperature.

Finally, let us comment that although there could be massless particles as Nambu-Goldstone bosons in the ISS sector, which we neglect in the thermal correction (7), they are harmless. They can be consistent with the upper limits on the abundance of radiation components [42] if they decoupled much before the epoch of the big bang nucleosynthesis (~ 0.1 MeV). This would be always possible by tuning parameters in the messenger sector.

B. Evolution of finite temperature effective potential

The thermal potential (7) is obtained by including the contributions of all particles whose masses depend on Φ in the ISS sector, the MSSM sector, and a messenger sector.

Let us first list the particles in the ISS sector whose masses have nontrivial dependence on Φ . The discussion basically follows the earlier works of [24,25], but is different in some details. The N_f flavors of φ and $\tilde{\varphi}$ are massless at $\Phi = 0$, but they obtain masses of

$$m_\varphi = h\Phi, \quad (11)$$

when Φ has a nonzero expectation value. In the $SU(N)$ gauge sector, the gauginos and the gauge fields get masses from the condensate (4) and we have

$$\begin{aligned} m_{\text{gauge}} &= \frac{(32\pi^2)^{1/3}}{2(N^2 - 1)} \left(\frac{h^{N_f} \det \Phi}{\Lambda_m^{N_f - 3N}} \right)^{1/3N} \\ &= \frac{(32\pi^2)^{1/3}}{2(N^2 - 1)} \epsilon^{(N_f - 3N)/3N} \left(\frac{m_\varphi}{\mu} \right)^{N_f/3N} \mu. \end{aligned} \quad (12)$$

Because the gauge confinement occurs when φ fields can be integrated out, we require the condition $m_{\text{gauge}} < m_\varphi$. Note that the condition is necessary for the existence of the supersymmetric vacuum (5). Substituting Eqs. (11) and (12) into Eq. (7), we get the thermal correction from the ISS sector to the finite temperature effective potential. Then the degrees of freedom are evaluated as $\sum_i \pm n_i = \pm 4NN_f$ for φ and $\tilde{\varphi}$, and $\sum_i \pm n_i = \pm 2(N^2 - 1)$ for the gauge sector, respectively.

Next, we consider a messenger sector to mediate the supersymmetry breaking to the MSSM sector. It will turn out that the imposition of the light gravitino mass constraint (10) places constraints on the construction of a messenger sector. Let us illustrate this with the simplest model of the gauge mediation for the ISS model [43]. Consider messenger superfields f and \tilde{f} which are charged under a gauge group of a visible sector. The messenger fields interact with the ISS sector through a superpotential

$$W_{\text{mess}} = (M + \lambda \text{Tr} \Phi) f \tilde{f}, \quad (13)$$

where M, λ are constants and other possible higher order terms have been omitted. The soft supersymmetry breaking term is given by the ratio of the F -term to the scalar expectation value in the superpotential (13) at the metastable vacuum (2) [37]. In our case, it is

$$m_{\text{soft}} \sim \frac{g^2}{16\pi^2} \frac{\lambda V_{\text{meta}}^{1/2}}{M}, \quad (14)$$

where g represents generic standard model coupling constants. In order to avoid the tachyonic instability of the scalar components of the messenger fields in the metastable

vacuum, the messenger mass scale M is required to satisfy the condition [43]

$$M > \lambda^{1/2} V_{\text{meta}}^{1/4}. \quad (15)$$

Using Eqs. (8) and (14), the above inequality can be rewritten to give a lower bound on the gravitino mass:

$$m_{3/2} > \frac{(16\pi^2/g^2)^2 m_{\text{soft}}^2}{\sqrt{3}\lambda M_P}. \quad (16)$$

It should be mentioned that from the view point of the electric dual of the ISS model, λ naturally takes a small value because it is proportional to a ratio of two high energy scales like $\lambda \propto \Lambda_m/M_P$ [43]. At most, one would like to take $\lambda \lesssim 1$ in order to ensure perturbative treatment of Eq. (13). Since we have never observed superpartner particles in particle colliders such as the LHC, let us fix the soft mass to be $m_{\text{soft}} \gtrsim 3$ TeV, which is slightly above the current lower limits [44]. Taking also $g \lesssim 1$ in order to maintain the validity of perturbation theory, then the inequality (16) reads

$$m_{3/2} \gtrsim 50 \text{ eV}. \quad (17)$$

The condition excludes the light gravitino window, $m_{3/2} < 4.7$ eV, in the ISS model. On the other hand, as we have discussed in Sec. III A for the middle gravitino mass range $0.4 \text{ keV} \lesssim m_{3/2} \lesssim 1 \text{ GeV}$ and the heavy gravitino mass range $600 \text{ GeV} \lesssim m_{3/2}$, the thermal effects to the potential are too small to give rise to a phase transition from the supersymmetric vacuum to the origin $\Phi = 0$ due to the reheating temperature condition (9). Therefore, the ISS model with the simplest gauge mediation scenario [43] is not suitable for a metastable supersymmetry breaking scenario if the supersymmetric vacuum was chosen initially at the early Universe. In general, in order to allow for the desired supersymmetry breaking phase transition, a gauge mediation model, e.g., [45], which is compatible with the light mass gravitino region (10) is needed. This can always be arranged and the details are model dependent, see for example [46,47]. We remark that the mass of messenger fields usually depends on the expectation value of Φ , e.g., through a superpotential like that of Eq. (13) [37,46,47], such that it is lighter at $\Phi = 0$ than at the supersymmetric vacuum $\Phi = \Phi_0$. Thus, the thermal contribution of the messenger sector generally tends to assist the phase transition from the supersymmetric vacuum to $\Phi = 0$; namely, neglecting the contributions from the messenger sector will not spoil our following discussion on the existence of the desired phase transition. Therefore in this paper, in order to investigate the general features of the ISS model of metastable supersymmetry breaking, we will be entitled not to specify the messenger sector and focus only on the ISS and the MSSM sectors.

Finally, we consider the MSSM sector. We expect that the both of the standard model particles and the superpartners are massless in the supersymmetric vacuum. However the superpartners obtain masses when they are far away from the supersymmetric vacuum along the direction of Φ . One would then expect that they tend to stabilize the supersymmetric vacuum since their masses are lighter at the supersymmetric vacuum than at the origin $\Phi = 0$. From the superpotentials (1) and (3), we can deduce the F -term of Φ to be

$$F_\Phi|_{\varphi=\tilde{\varphi}=0} = N_f^{1/2} |h\mu^2| \left| 1 - \epsilon^{(N_f-3N)/N} \left(\frac{h\Phi}{\mu} \right)^{(N_f-N)/N} \right|. \quad (18)$$

As a result, the tree level mass of superpartner particles along the Φ direction is given by [37]

$$m_{\text{SP}} \sim \frac{g^2}{16\pi^2} \frac{\lambda F_\Phi|_{\varphi=\tilde{\varphi}=0}}{M} \sim \frac{m_{\text{soft}} F_\Phi|_{\varphi=\tilde{\varphi}=0}}{V_{\text{meta}}^{1/2}}, \quad (19)$$

where M is the messenger mass scale in Eqs. (13) and (14) has been used to get the second expression. From the fact that we have never observed superpartner particles in particle colliders such as the LHC, let us fix the soft mass to be $m_{\text{soft}} \sim 3$ TeV. For simplicity, we will assume that all the superpartner particles have the same mass of (19). Then the thermal corrections to the effective potential is obtained by summing over the 94 and 32 degrees of freedom for the bosonic and the fermionic particles other than the standard model particles in the MSSM sector. Note that the thermal effect from the standard model Higgs is negligible because its mass must be smaller than m_{soft} .

We are now ready to calculate the thermal corrections (7) from the ISS and the MSSM sectors. The finite temperature effective potential is given by

$$V_T(\Phi) = V_0(\Phi) + \Delta V_T(\Phi), \quad (20)$$

where

$$\begin{aligned} \Delta V_T(\Phi) = & \frac{T^4}{2\pi^2} \left[\pm 4NN_f \int_0^\infty dp p^2 \ln \left(1 \mp e^{-\sqrt{p^2+m_\varphi^2(\Phi)}/T} \right) \right. \\ & \pm 2(N^2-1) \int_0^\infty dp p^2 \ln \left(1 \mp e^{-\sqrt{p^2+m_{\text{gauge}}^2(\Phi)}/T} \right) \\ & + 96 \int_0^\infty dp p^2 \ln \left(1 - e^{-\sqrt{p^2+m_{\text{SP}}^2(\Phi)}/T} \right) \\ & \left. - 32 \int_0^\infty dp p^2 \ln \left(1 + e^{-\sqrt{p^2+m_{\text{SP}}^2(\Phi)}/T} \right) \right]. \quad (21) \end{aligned}$$

We are interested in the temperature dependence of the finite temperature effective potential in the light gravitino mass region. In Fig. 1, we show the temperature

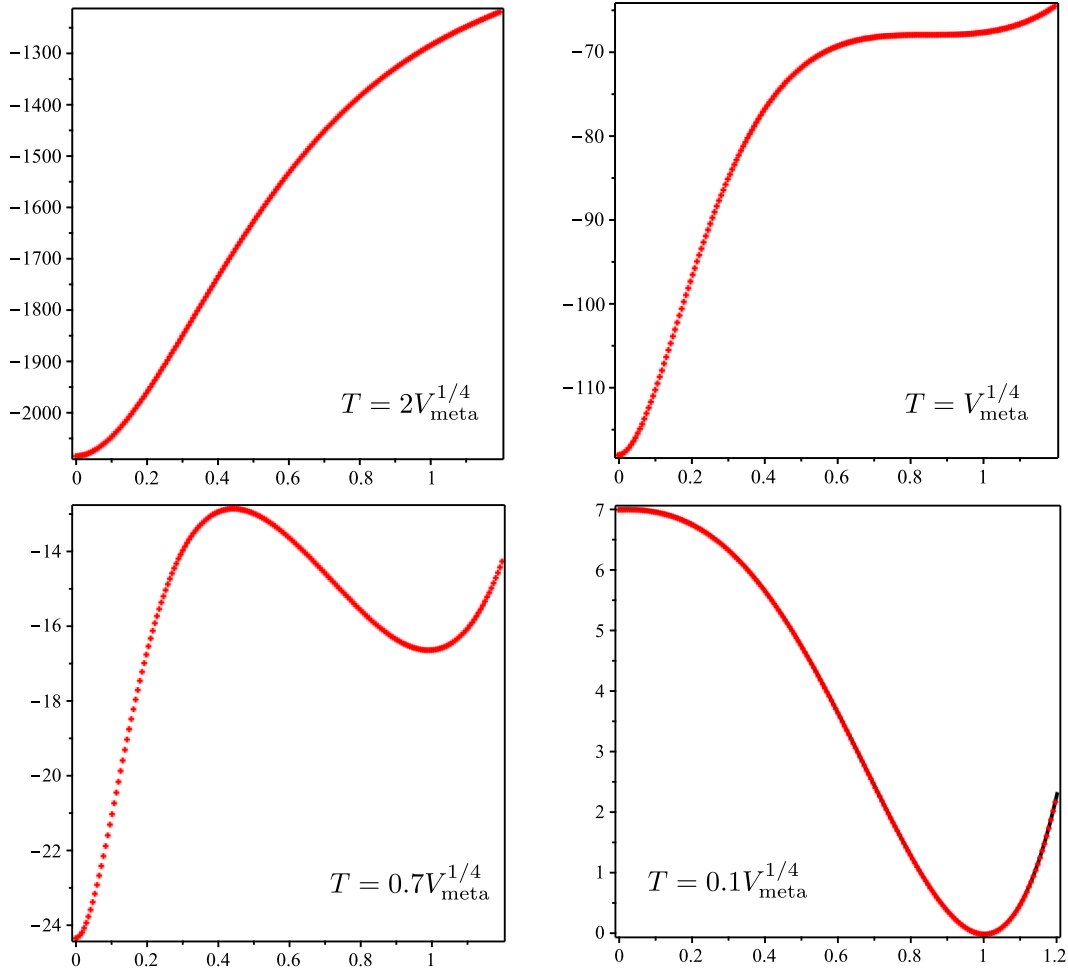


FIG. 1. Temperature dependence of the effective potential $V_T(\Phi)$ is shown for the gravitino mass of $m_{3/2} = 1$ eV. The horizontal and the longitudinal axes are normalized as Φ/Φ_0 and $V_T(\Phi)/|h^2\mu^4|$, respectively. Each parameter is set as $h = 1$, $\Lambda_m = 10^{10}$ GeV, $N = 2$, $N_f = 7$. When $T > V_{\text{meta}}^{1/4}$, the origin is only a minimum. Around $T = V_{\text{meta}}^{1/4}$, an inflection point comes along, namely a second minimum around the supersymmetric vacuum appears. When the temperature is right below the supersymmetry breaking scale $V_{\text{meta}}^{1/4}$, we have two local minima. In the figure of $T = 0.1V_{\text{meta}}^{1/4}$, the zero temperature potential $V_0(\Phi)$ is also depicted by a black line for reference. It shows that the thermal correction is already negligible at $T = 0.1V_{\text{meta}}^{1/4}$.

dependence of the potential for the gravitino mass $m_{3/2} = 1$ eV. The corresponding supersymmetry breaking scale is $V_{\text{meta}}^{1/4} = 6.5 \times 10^4$ GeV and the parameters of the ISS model are taken as $h = 1$, $\Lambda_m = 10^{10}$ GeV, $N = 2$, $N_f = 7$. From Fig. 1, we can see that the origin is only a minimum of the potential when the temperature is high compared with the supersymmetry breaking scale. During this epoch, the scalar field can roll down to the origin. Around $T = V_{\text{meta}}^{1/4}$, an inflection point appears around the supersymmetric vacuum. We will call this temperature T_* . Below T_* , there is a critical temperature T_{crit} at which the energy of the two vacua degenerates. For $T_{\text{crit}} < T < T_*$, tunneling process can occur from around the supersymmetric vacuum to the origin. For $T < T_{\text{crit}}$, the supersymmetric vacuum becomes the global minimum and we can no longer have any more phase transitions from the

supersymmetric vacuum to the origin. Instead, one may have tunneling from the origin to the supersymmetric vacuum. However, it is known that the tunneling would not occur because the phase transition from the origin to the metastable vacuum (2) is more efficient [26,27]. If the reheating temperature T_R is lower than T_{crit} , the Universe will stay in the supersymmetry vacuum and will never come to the metastable vacuum. A phenomenologically interesting case is when

$$T_{\text{crit}} < T_R < T_*, \tag{22}$$

where many bubbles can be nucleated from the tunneling process and abundant gravitational waves could be produced. We will study this possibility in the next section.

IV. GRAVITATIONAL WAVES FROM BUBBLE NUCLEATION

As shown in the previous section, there could be tunneling process from the false vacuum around $\Phi = \Phi_0$ to the true vacuum at $\Phi = 0$. Indeed it happens if the reheating temperature is in the region, $T_{\text{crit}} < T_R < T_*$, and then bubbles are nucleated.¹ In this section, we calculate gravitational waves produced by the dynamics of bubbles [48–52]. For simplicity, we will calculate gravitational wave production at $T_R = T_*$ at which the gravitational wave production is expected to be the most efficient.

The transition rate per unit volume in thermal Universe is given by [53]

$$\Gamma \simeq e^{-S_3/T}, \quad (23)$$

where S_3 is a three dimensional bounce action for $O(3)$ symmetric configurations, which gives the lowest action [53,54] and dominates the transition. The classical equation for a bounce solution is given by

$$\Phi''(r) + \frac{2}{r}\Phi'(r) = \frac{1}{N_f} \partial_\Phi V_T(\Phi), \quad (24)$$

where r is the spatial radial distance. Since it is difficult to obtain an analytic form of the bounce solution for general potentials, we approximately parametrize our potential as [24,55]:

$$V_T(\Phi) = N_f K (\Phi - \eta) \theta(\eta - \Phi), \quad (25)$$

where K and η are positive constants, θ represents the step function and N_f has been put for later convenience. For the potential (25), one can find an analytic bounce solution of the Eq. (24) with the bounce boundary condition $\Phi(\infty) = \Phi_f$ and $\frac{d\Phi(r)}{dr}|_{r=0} = 0$. Here Φ_f denotes the location of the supersymmetric vacuum after the finite temperature thermal effects are included. The bounce solution is given by

$$\Phi(r) = \begin{cases} \frac{K}{6}(r^2 - r_m^2) + \eta & \text{for } r \leq r_m, \\ \Phi_f - \frac{Kr_m^3}{3r} & \text{for } r \geq r_m, \end{cases} \quad (26)$$

where $r_m = \sqrt{3(\Phi_f - \eta)/K}$. This bounce solution is parametrized by the position of the false vacuum Φ_f , the turning point η of the potential, and K which characterizes the tilt of the potential. We note that for a wide range of parameters, which is compatible with the light gravitino mass region and of interest to us for the observation of gravitational waves, the position of the false vacuum can be

¹We assume an instantaneous reheating in this paper so that the reheating completes in shorter duration compared with other physical scales like the Hubble constant.

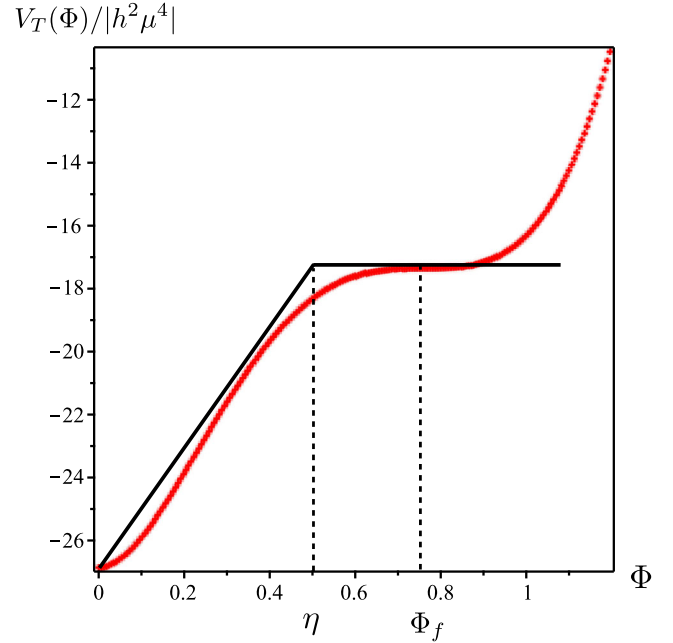


FIG. 2. The red dot line represents the finite temperature effective potential for a parameter set of $m_{3/2} = 1$ eV, $h = 1$, $\Lambda_m = 10^{10}$ GeV, $N = 5$, $N_f = 16$. The horizontal axis is normalized by Φ_0 . The black line represents the flat potential approximation (25) with $\eta = 2\Phi_f/3$.

approximated by $\Phi_f = 3\eta/2$ (see Fig. 2) to a very good approximation. Then the bounce action is given by

$$\frac{S_3}{T} = \frac{\sqrt{6}\pi N_f \eta^{5/2}}{5\sqrt{KT}}. \quad (27)$$

As a consequence of the phase transition from the false vacuum to the true vacuum, bubbles are nucleated and gravitational waves can be produced [48–52]. The dynamics of the bubbles and the produced gravitational waves are characterized by two parameters. The first one is the vacuum energy density released in the phase transition to the energy density of the radiation bath [56],

$$\alpha = \frac{V_f - V_T(\Phi = 0)}{g_* \pi^2 T_*^4 / 30}, \quad (28)$$

where g_* is the effective degrees of freedom of particles at the temperature T_* . The second parameter is

$$\beta = -H_* T_* \left(\frac{d S_3}{dT} \right)_{T=T_*}, \quad (29)$$

which approximately represents the duration of the phase transition [52]. Here $H_* = \sqrt{\frac{\pi^2 g_* T_*^4}{90 M_p^2}}$ is the Hubble constant at the time of phase transition. We note the sign of the function inside the parenthesis is negative in the situations

we consider, so that β has been defined to be positive. There are two kinds of scenarios for the dynamics of nucleated bubbles in the radiation dominated era [56]: the runaway bubbles and the no runaway bubbles. In the former scenario, the bubble wall accelerates continuously and approaches the speed of light. In the no runaway scenario, the speed of the bubble walls converges to a value smaller than the speed of light. A useful parameter which can be used to discriminate between the two scenarios is [57]

$$\alpha_\infty := \frac{30}{24\pi} \frac{\sum_i c_i \Delta m_i^2}{g_* T_*^2}, \quad (30)$$

where the sum is taken for particles which is lighter in the false vacuum than in the true vacuum, c_i stands for degrees of freedom of the particle (a factor of 1/2 is additionally multiplied for fermions), and Δm_i is the difference in mass of the particle between the two vacua. If $\alpha > \alpha_\infty$, the energy of the phase transition can convert to the acceleration of bubble walls and the runaway bubbles scenario is realized. If $\alpha < \alpha_\infty$, all the phase transition energy is deposited into the plasma and the no runaway bubbles scenario is realized. In our case, the superpartner particles in the MSSM sector contribute to α_∞ . We will only consider the case in which $m_{SP} \sim \text{TeV} \ll T_*$. This implies that $\alpha \gg \alpha_\infty$ and so we have the runaway bubbles scenario. In this case, bubble collisions [50,51,58–61] would be the dominant source of gravitational waves compared with the contributions from the sound wave [62–65] and the magnetohydrodynamic turbulence [66–70] in the plasma. For bubble collisions, the gravitational wave spectrum in terms of the energy density is given by [56,61]

$$h_0^2 \Omega_f(f) = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{1/3} \times \left(\frac{0.11 v_w^3}{0.42 + v_w^2} \right) \left(\frac{3.8 (f/f_p)^{2.8}}{1 + 2.8 (f/f_p)^{3.8}} \right), \quad (31)$$

where $\kappa = 1 - \alpha_\infty/\alpha$ and we will assume the bubble wall velocity v_w is the speed of light from now on. f_p is the peak frequency of the gravitational wave spectrum at present,

$$f_p = 16.5 \times 10^{-6} \text{ Hz} \left(\frac{0.62}{1.8 - 0.1 v_w + v_w^2} \right) \left(\frac{\beta}{H_*} \right) \times \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6}. \quad (32)$$

We calculated several gravitational wave spectra from bubble collisions for the gravitino mass $m_{3/2} = 0.01 \text{ eV}$ ($V_{\text{meta}}^{1/4} = 6.5 \times 10^3 \text{ GeV}$) and 1 eV ($V_{\text{meta}}^{1/4} = 6.5 \times 10^4 \text{ GeV}$) with the fitting formula (31) and (32). Then α and β were numerically evaluated from potentials at $T = T_*$. The results are depicted in Figs. 3 and 4. In Fig. 3, the energy density of gravitational waves is depicted for $h = 1$, $\Lambda_m = 10^{10} \text{ GeV}$. The gravitino masses are set $m_{3/2} = 0.01 \text{ eV}$ in the left side figure and $m_{3/2} = 1 \text{ eV}$ in the right side figure. The red line and the black line correspond to the parameter sets of $N = 2$, $N_f = 7$ and $N = 5$, $N_f = 16$, respectively. One can see that both of the produced gravitational waves are within the sensitivity of DECIGO [71], whose sensitivity is represented by the blue line. In Fig. 4, the energy density of gravitational waves is depicted for $h = 3$, $\Lambda_m = 10^{10} \text{ GeV}$. The gravitino masses are set $m_{3/2} = 0.01 \text{ eV}$ in the left side figure and

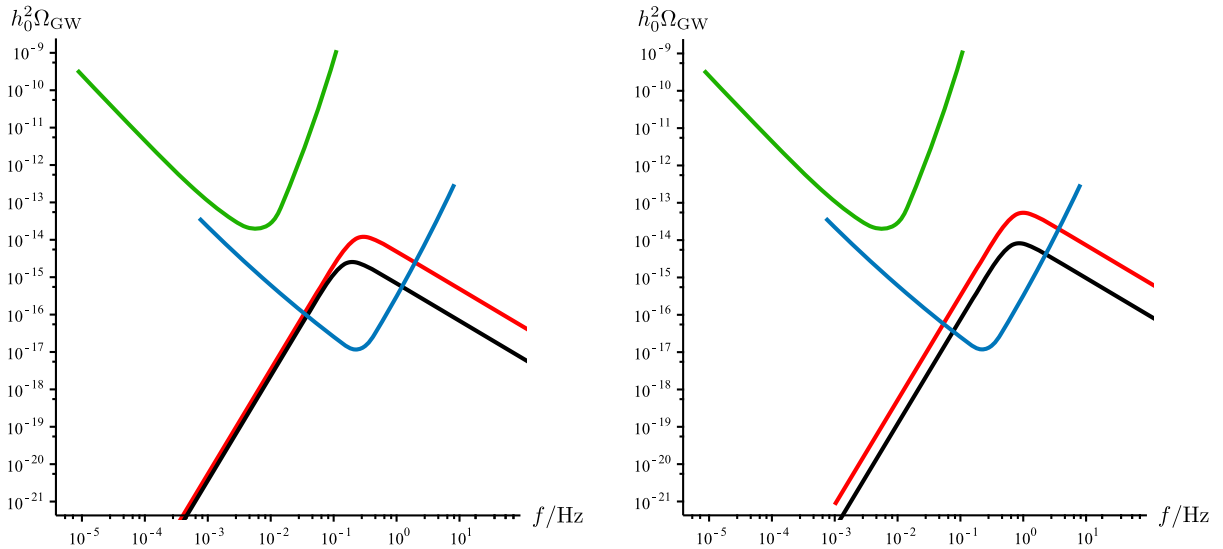


FIG. 3. Gravitational wave spectra from bubble collisions for $h = 1$, $\Lambda_m = 10^{10} \text{ GeV}$ are depicted. The gravitino masses are set $m_{3/2} = 0.01 \text{ eV}$ in the left side figure and $m_{3/2} = 1 \text{ eV}$ in the right side figure. In each figure, the red and black lines represent the gravitational wave spectra for $N = 2$, $N_f = 7$ and $N = 5$, $N_f = 16$, respectively. The green line is the sensitivity curve of LISA of the C1 configuration [56]. The blue line is the sensitivity curve of DECIGO for two clusters [71].

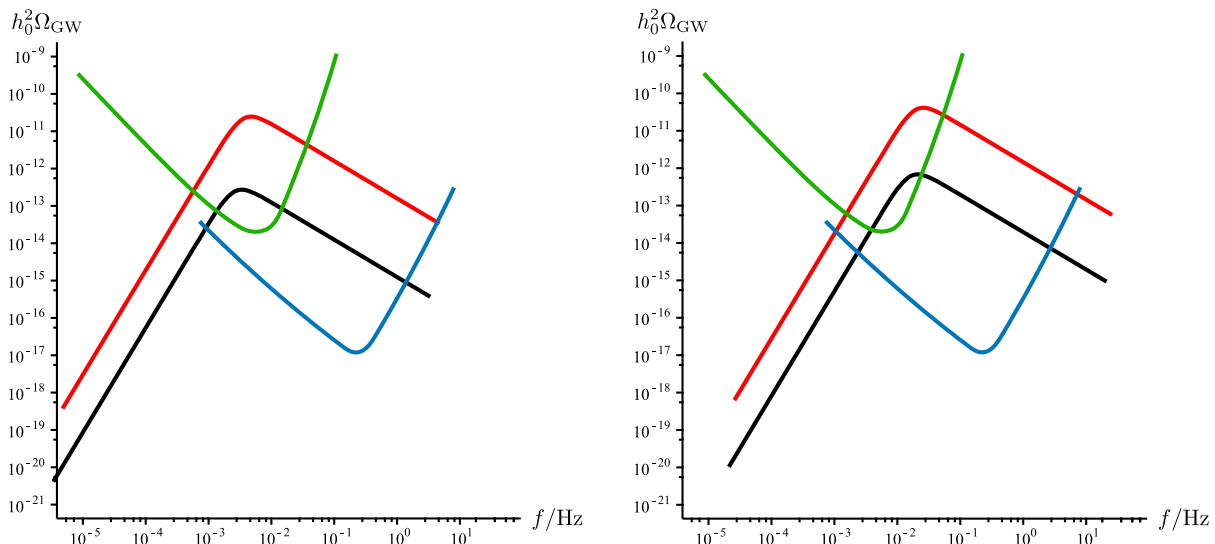


FIG. 4. Gravitational wave spectra from bubble collisions for $h = 3$, $\Lambda_m = 10^{10}$ GeV are depicted. The gravitino masses are set $m_{3/2} = 0.01$ eV in the left side figure and $m_{3/2} = 1$ eV in the right side figure. In each figure, the red and black lines represent the gravitational wave spectra for $N = 2$, $N_f = 7$ and $N = 5$, $N_f = 16$, respectively. The green line is the sensitivity curve of LISA of the C1 configuration [56]. The blue line is the sensitivity curve of DECIGO for two clusters [71].

$m_{3/2} = 1$ eV in the right side figure. Again, the red line and the black line correspond to the parameter sets of $N = 2$, $N_f = 7$ and $N = 5$, $N_f = 16$, respectively. The spectra are within the sensitivity of DECIGO [71] (blue line) and LISA [56] (green line). From Figs. 3 and 4, one can see that the produced gravitational waves for the stated parameter sets are detectable with the future gravitational wave interferometers. One finds that amplitude of gravitational waves becomes larger for bigger h . We also see that because lighter gravitino masses correspond to lower supersymmetry breaking energy scales, the peak frequency of the spectra tend to be lower for lighter gravitino masses. Potentially, we could explore the parameters of the ISS model of the metastable supersymmetry breaking scenario with gravitational wave observations. We finally mention that since adding contributions from a messenger sector to the thermal correction (7) assists the phase transitions as explained in the Sec. III B, more gravitational wave production is expected if a messenger sector is considered explicitly.

V. CONCLUSION

In this paper, we first elaborated the cosmological evolution of the vacuum structure of the ISS model in light of the cosmological gravitino problem. The incorporation of the gravitino constraints is crucial to our analysis. We showed that in the middle gravitino mass region 0.4 keV $\lesssim m_{3/2} \lesssim 1$ GeV and the heavy gravitino mass region 600 GeV $\lesssim m_{3/2}$, there are insufficient thermal effects to bring forth the desired phase transitions from the supersymmetric vacuum to the metastable vacuum since the reheating temperature is stringently bounded. Thus only in the light gravitino mass region, $m_{3/2} < 4.7$ eV, we have

enough thermal effects to allow for the phase transition. This is achieved by either rolling down potential or tunneling processes depending on the reheating temperature.

We also calculated gravitational wave spectra associated with the tunneling from the supersymmetry vacuum to the metastable vacuum. Abundant gravitational waves could be produced by collisions of runaway bubbles and they are detectable with the future gravitational wave interferometers like LISA and DECIGO. This gives us a unique way to probe the metastable supersymmetry breaking. Note that although the messenger sector was not included in the calculation in order to allow a model independent treatment, our conclusion is robust because the thermal effects from the messenger sector assists the phase transition and more gravitational wave production is expected in general when the thermal contributions of the messenger sector are included.

It should be mentioned that we did not consider the mass spectrum of the two Higgs doublets and superpartner particles seriously, since it goes beyond the purpose of this paper. In order to do so, we need to specify a specific model of the messenger sector for the gauge mediation. Fortunately, it seems that there has already been a gauge mediation model which is compatible with our scenario [45]. It would be interesting to study the correlation between the mass spectrum of superpartner particles and gravitational wave spectra in detail [20]. We note that the employment of a visible sector other than the MSSM is straightforward and would not change our conclusion as long as the mass of superpartner particles is not so heavy compared with the metastable supersymmetry breaking scale. We also mention that although we calculated the gravitational wave spectra by assuming the reheating temperature to be $T_R = T_*$, such a situation may be

necessarily realized if inflation is embedded in the ISS model [72] because T_* is usually close to the supersymmetry breaking scale as shown in Fig. 1. It is also interesting to extend our discussion to the ordinal O’Raifeartaigh type models of metastable supersymmetry breaking [73,74]. We leave these issues for future work.

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