Cross section of the process $e^+e^- \to \Lambda\bar{\Lambda}$ in the vicinity of charmonium $\psi(3770)$ including three-gluon and D-meson loop contributions

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The total cross section of the process $e^+e^- \to \Lambda\bar{\Lambda}$ is calculated within the energy range close to the mass of the $\psi(3770)$ charmonium state. Two different contributions were considered: the D-meson loop and the three gluon charmonium annihilation one. Both of them contribute noticeably and in sum, fairly reproduce the data. Large relative phase for these contributions are generated with respect to the pure electromagnetic mechanism. As a by-product, the fit for the electromagnetic form factor of Λ hyperon is obtained for the large momentum transferred region.

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I. INTRODUCTION

The bound state of a pair of charmed quarks are the one of the most clear and simple system, which allows one to study the details of a confinement mechanism and to refine the conjectures of quantum chromodynamics (QCD). During recent decades, these states are under intensive experimental and theoretical study [1–4]. The special interest is focused on the electron-positron annihilation processes with the production of different mesonic and baryonic final states thus, giving us a clear way to produce and to study the charmonium in pure $J^{PC} = 1^{--}$ state. The binary final states (i.e., with two particles finally produced) give the possibility to further simplify the consideration of the processes with charmonium in the intermediate state [5].

Besides, it was shown a long time ago that these processes are an excellent way to measure the electromagnetic form factors of the particles produced [6,7]. There is an enormous set of the measurements of the electromagnetic form factors of the proton, for example, by the *BABAR* Collaboration [8,9] or by the BES III Collaboration [10]. But the other baryons are also under study: Λ^0 and Σ^0 baryons [11,12], $\Sigma^\pm$ baryons [13], Ξ^0, Ξ^-, and Ω^- baryons [14]. For a recent review of the situation on $\Lambda\bar{\Lambda}$ pair production, see a review [15]. A lot of interest has been paid to the near threshold behavior of these form factors [16,17], which demonstrates the nontrivial enhancement effect. Many theoretical ideas to explain this effect were proposed (see, for example, [18,19]).

In this paper, we want to consider the process of electron-positron annihilation into a pair of Λ hyperons, which was recently precisely measured in the vicinity of $\psi(3770)$ charmonium [20]. The charmonium $\psi(3770)$ is one of the intriguing states that was studied by many collaborations (for example, by KEDR-VEPP-4M [21,22], CLEO [23], and more recently, BES III [24,25]). In one paper [25], one can find the measurement of the cross section of the process $e^+e^- \to p\bar{p}$ with the specific care to the region near the mass of $\psi(3770)$, which demonstrates the dip instead of enhancing the Breit-Wigner peak. This is the manifestation of the large relative phase that is generated by the intermediate charmonium state with respect to the pure electromagnetic background. In papers [26,27], we showed that the source of this phase is mostly attributed to the three gluon mechanism of the charmonium decay. Here, we consider the similar process $e^+e^- \to \Lambda\bar{\Lambda}$, which also must have large relative phase coming from the same three gluon mechanism. Since all the calculations were already done in [27], here we just briefly recall the main formulas and put the main focus on the details that differ in the $\Lambda\bar{\Lambda}$ case.

The paper is organized in the following manner: in Sec. II, the total cross section of the process $e^+e^- \to \Lambda\bar{\Lambda}$ in the Born approximation is presented and the electromagnetic form factors of the Λ hyperon are discussed; Sec. III shows how the charmonium in the intermediate state contributes to the total cross section; two subsequent

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Secs. IV and V show the main formulas for the OZI-allowed mechanism with the D-meson loop and for the OZI-violating three gluons mechanism. Section VI gives some numerical estimations and comparison of our calculation with experimental data from BES III [20]; Sec. VII concludes our results and proposes some possible extension of this work in the future.

II. BORN APPROXIMATION

We consider the process of electron-positron annihilation into a pair of Λ baryons,

$$e^+(q_+) + e^-(q_-) \to \Lambda(p_1) + \bar{\Lambda}(p_2), \tag{1}$$

where quantities in parenthesis are the four momenta of the corresponding particles. The cross section for the process has the standard way,

$$d\sigma = \frac{1}{8s} \sum_{\text{spins}} |\mathcal{M}|^2 d\Phi_2, \tag{2}$$

where the summation of the amplitude square $|\mathcal{M}|^2$ runs over all possible initial and final particles spin states. We systematically neglect the mass of the electron m_e in this paper. The phase volume of final particles $d\Phi_2$ has the form,

$$d\Phi_2 = \frac{1}{(2\pi)^2} \delta(q_+ + q_- - p_1 - p_2) \frac{d\mathbf{p_1}}{2E_1} \frac{d\mathbf{p_2}}{2E_2} = \frac{|\mathbf{p}|}{2^4 \pi^2 \sqrt{s}} d\Omega_\Lambda = \frac{\beta}{2^4 \pi} d\cos\theta_\Lambda, \tag{3}$$

where $d\Omega_\Lambda = d\phi_\Lambda d\cos\theta_\Lambda$ and ϕ_Λ and θ_Λ are the azimuthal and the polar angles of the final Λ-baryon momentum in the center-of-mass reference frame (center of mass system, c.m.s); i.e., θ_Λ is the angle between three momenta of the initial electron $\mathbf{q}_-$ and the final Λ-baryon $\mathbf{p_1}$ (see Fig. 1) and $|\mathbf{p}| \equiv |\mathbf{p_1}| = |\mathbf{p_2}| = \sqrt{s}\beta/2$ is the modulus of the three momenta of the final Λ or $\bar{\Lambda}$. Here, the quantity $\beta = \sqrt{1 - 4M_\Lambda^2/s}$ is the final Λ-baryon velocity, with M_Λ being the mass of Λ baryon.

In the Born approximation [see Fig. 2(a)], the amplitude $\mathcal{M} = \mathcal{M}_B$ in (2) describes the electron-positron pair annihilation into virtual photon, which then produces the

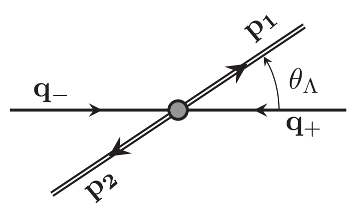

FIG. 1. The definition of the scattering angle θ_Λ from (3) in the center-of-mass reference frame.

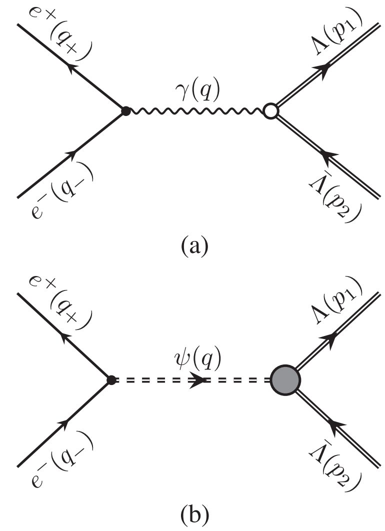

FIG. 2. Feynman diagrams of the process $e^+e^- \to \Lambda\bar{\Lambda}$ in Born approximation (a) and with the charmonium $\psi(3770)$ intermediate state (b).

$\Lambda\bar{\Lambda}$ pair. The amplitude $\mathcal{M}_B$ corresponding to this process has the form,

$$\mathcal{M}_B = \frac{1}{s} J_\mu^{e\bar{e}\to\gamma}(q) J^\mu_{\gamma\to\Lambda\bar{\Lambda}}(q), \tag{4}$$

where $s = q^2 = (q_+ + q_-)^2 = (p_1 + p_2)^2$ is the total invariant mass squared of the lepton pair (q is the momentum of the intermediate photon). The quantities $J_\mu^{e\bar{e}\to\gamma}$ and $J_\mu^{\gamma\to\Lambda\bar{\Lambda}}$ are electromagnetic currents,

$$J_\mu^{e\bar{e}\to\gamma}(q) = -e[\bar{v}(q_+)\gamma_\mu u(q_-)], \tag{5}$$

$$J_\mu^{\gamma\to\Lambda\bar{\Lambda}}(q) = e[\bar{u}(p_1)\Gamma_\mu(q) v(p_2)], \tag{6}$$

where e is the modulus of electron charge $e = \sqrt{4\pi\alpha}$ with α being the fine structure constant [28]. In general, the vertex of the photon with the Λ baryon has the form,

$$\Gamma_\mu(q) = F_1(q^2)\gamma_\mu - \frac{F_2(q^2)}{4M_\Lambda}(\gamma_\mu \hat{q} - \hat{q}\gamma_\mu), \tag{7}$$

where we use the notation $\hat{a} \equiv a_\mu \gamma^\mu$. Here, the functions $F_{1,2}(q^2)$ are the Λ-baryon electromagnetic form factors normalized as $F_1(0) = 0$ and $F_2(0) = \mu_\Lambda$, where μ_Λ is the Λ-baryon anomalous magnetic moment.

It was shown in [29] that the nontrivial structure of the baryon starts to manifest itself even at relatively low q^2, and thus, one must take into account these effects of the structure. Since experimental data at the moment do not allow one to extract the electric G_E and the magnetic G_M form factors of a baryons separately, we utilize the assumption that $|G_E| = |G_M|$, i.e., $F_2(q^2) = 0$. Then the total cross section in Born approximation has the form,

$$\sigma_B(s) = \frac{2\pi\alpha^2}{3s}\beta(3-\beta^2)|F_1(s)|^2. \quad (8)$$

The form factor F_1 is chosen to have pQCD inspired form [30,31], which takes into account the running of the QCD coupling constant α_s,

$$F_1(s) = \frac{C}{s^2 \log^2(s/\Lambda_{\mathrm{QCD}}^2)}, \quad (9)$$

where Λ_{QCD} is the QCD scale and the constant C should be fitted on the experimental data for baryon-antibaryon production in a wide energy range.

For a proton in our energy region, this fit was done in [25], giving $C = (62.6 \pm 4.1)\ \mathrm{GeV}^4$ (using $\Lambda_{\mathrm{QCD}} = 300$ MeV). This fit qualitatively agrees with the more recent result of paper [32], where this constant was fitted to be equal to $C = 72\ \mathrm{GeV}^4$ with $\Lambda_{\mathrm{QCD}} = 520$ MeV.

In our case of $\Lambda\bar{\Lambda}$ pair production, we fix this constant using the whole range of BES-III measurement [20] presented in Fig. 3. Fitting the Born cross section from (8) with respect to this data gives us the following parameter:

$$C = (43.1 \pm 1.4)\ \mathrm{GeV}^4, \quad (10)$$

which we use further for Λ-baryon electromagnetic form factor (9). We note that this expression for the form factor F_1 with the constant C from (10) works for a relatively large momentum transferred q^2. It does not pretend to work near the threshold since there are many delicate features playing important role there, such as Coulomb-like enhancement factor [33,34] or the manifestation of wavy nature of baryon stabilization after its emerging from the vacuum [29].

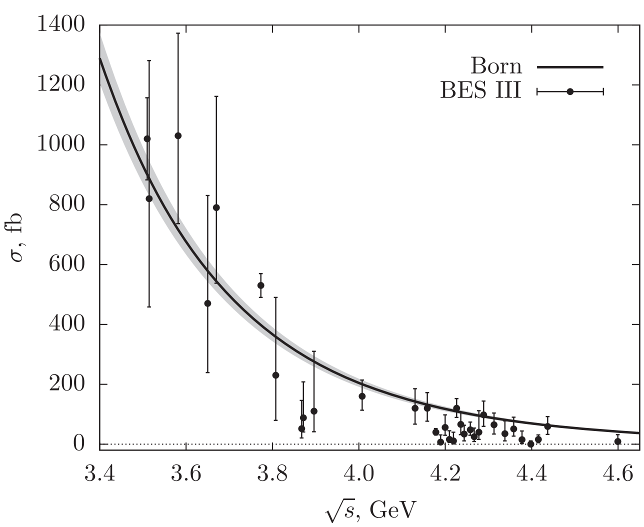

FIG. 3. The total cross section for the process $e^+e^- \to \Lambda\bar{\Lambda}$. The black line is the cross section in Born approximation (8). The curve errors origin from the form factor constant (10) fitting errors.

III. THE QUARKONIUM $\psi(3770)$ INTERMEDIATE STATE

The main task of our work is to describe the effect of the charmonium resonance $\psi(3770)$ excitation in the process (1). As one can see in Fig. 3, the experimental point for the cross section at $\sqrt{s} = M_\psi$ [where M_ψ is the $\psi(3770)$ mass] flies higher the Born cross section curve. In [20], there is a fit of this point with the use of some phenomenologically inspired formula [see Eq. (3) in [20]]. Further, we develop the model (based on our previous calculations [26,27]), which reveals the underlying mechanism for this point to be upstairs.

In the region of charmonium resonance $\psi(3770)$ excitation, one should take into account the additional contribution to the amplitude,

$$\mathcal{M} = \mathcal{M}_B + \mathcal{M}_\psi, \quad (11)$$

where $\mathcal{M}_\psi$ takes into account the mechanism with the charmonium $\psi(3770)$ in the intermediate state [see Fig. 2(b)], which is enhanced by the Breit-Wigner factor,

$$\mathcal{M}_\psi = \frac{g^{\mu\nu} - q^\mu q^\nu/M_\psi^2}{s - M_\psi^2 + iM_\psi\Gamma_\psi} J_\mu^{e\bar{e}\to\psi}(q) J_\nu^{\psi\to\Lambda\bar{\Lambda}}(q), \quad (12)$$

where Γ_ψ is the total decay width of the $\psi(3770)$ resonance and $J_\mu^{e\bar{e}\to\psi}$ and $J_\mu^{\psi\to\Lambda\bar{\Lambda}}$ are the currents that describe the transition of the lepton pair into the $\psi(3770)$ resonance and the transition of the $\psi(3770)$ resonance into a $\Lambda\bar{\Lambda}$ pair, correspondingly. Following [26], we assume that $J_\mu^{e\bar{e}\to\psi}$ has the same structure as $J_\mu^{e\bar{e}\to\gamma}$ from (5); i.e.,

$$J_\mu^{e\bar{e}\to\psi}(q) = g_e[\bar{v}(q_+)\gamma_\mu u(q_-)], \quad (13)$$

where the constant $g_e = F_1^{\psi\to e\bar{e}}(M_\psi^2)$ is the value of the form factor of the vertex $\psi \to e\bar{e}$ at the $\psi(3770)$ mass shell [here, we follow the same approximation as in the Born case and assume that $F_2^{\psi\to e\bar{e}}(M_\psi^2) = 0$]. We fix this constant g_e via the total decay width of $\psi \to e^+e^-$, which is known to be equal to $\Gamma_{\psi\to e^+e^-} = 261$ eV [28],

$$g_e = \sqrt{\frac{12\pi\Gamma_{\psi\to e^+e^-}}{M_\psi}} = 1.6 \times 10^{-3}. \quad (14)$$

We neglect a possible imaginary part of the vertex g_e since it was shown in [35] that it is small, less then 10% of the real part. We also note a mistake in our paper [27] where this constant is claimed to be related with the $\psi \to p\bar{p}$ decay.

The amplitude $\mathcal{M}_\psi$ from (12) interferes with the Born one $\mathcal{M}_B$, giving the standard interference contribution to the cross section,

$$d\sigma_{\rm int} = \frac{1}{8s}\sum_{\rm spins} 2\,{\rm Re}[\mathcal{M}_B^+ \mathcal{M}_\psi] d\Phi_2, \tag{15}$$

which leads to the following form of the interference contribution to the total cross section:

$$\sigma_{\rm int}(s) = {\rm Re}\left(\frac{S_i(s)}{s - M_\psi^2 + iM_\psi\Gamma_\psi}\right), \tag{16}$$

where $S_i(s)$ contains all the dynamics of the transformation of charmonium into $\Lambda\bar{\Lambda}$ pair and has the following explicit form:

$$S_i(s) = \frac{e g_e \beta}{48\pi s}\int_{-1}^{1} d\cos\theta_\Lambda \sum_{s'} (J^\alpha_{\gamma\to\Lambda\bar{\Lambda}})^* J_\alpha^{\psi\to\Lambda\bar{\Lambda}}. \tag{17}$$

Here, we used the following trick of invariant phase space integration:

$$\sum_{s'}\int d\Phi_2 (J^\mu_{\gamma\to\Lambda\bar{\Lambda}})^* J^\nu_{\psi\to\Lambda\bar{\Lambda}} = \frac{1}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\sum_{s'}\int d\Phi_2 (J^\alpha_{\gamma\to\Lambda\bar{\Lambda}})^* J_\alpha^{\psi\to\Lambda\bar{\Lambda}}, \tag{18}$$

where summation $\sum_{s'}$ goes over all spin states of final particles. This trick significantly simplifies the treatment of Lorenz indexes, but it is applied only for the total phase space integration.

The subscript index i in (17) denotes the type of mechanism of this transformation. Since the mass of $\psi(3770)$ is higher than the threshold of the D-meson pair production, it is natural to expect that the D-meson loop will be the main mechanism in this reaction (see Fig. 4), and we consider it below in Sec. IV. However, we need also to consider the OZI-violating three gluon mechanism (see Fig. 5), which we briefly recall in Sec. V.

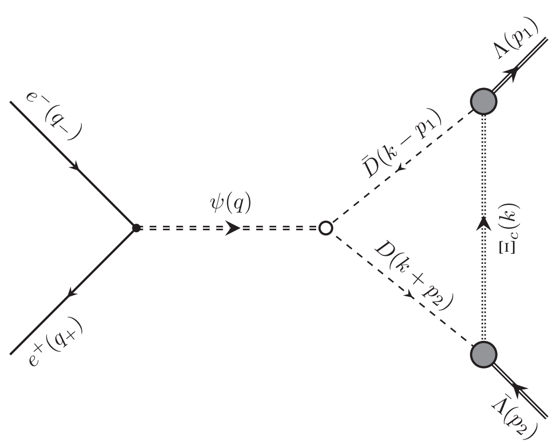

FIG. 4. D-meson loop mechanism.

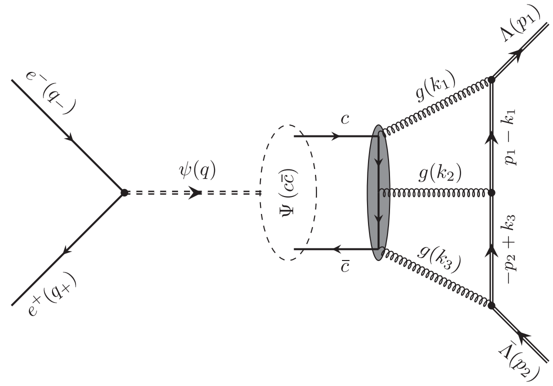

FIG. 5. Three gluon mechanism.

Having the interference contribution (16) with the total relative phase between Born amplitude $\mathcal{M}_B$ and the charmonium contribution one $\mathcal{M}_\psi$, we can restore the total cross section using the procedure described in [27] [see Eqs. (15) and (16) there].

IV. D-MESON LOOP MECHANISM

Here, we follow exactly to our previous calculations [26,27] with only one systematic modification, which is needed to describe the $\Lambda\bar{\Lambda}$ pair final state instead of the proton-antiproton one. The D-meson loop mechanism (presented in Fig. 4) contributes to the interference of a charmonium state with the Born amplitude [see (17)] as

$$S_D(s) = \alpha_D(s) Z_D(s), \tag{19}$$

where

$$\alpha_D(s) = \frac{\alpha g_e}{2^4 3\pi^2}\beta F_1(s),$$
$$Z_D(s) = \frac{1}{s}\int\frac{dk}{i\pi^2} \times \frac{SpD(s,k^2)}{(k^2 - M_\Xi^2)((k-p_1)^2 - M_D^2)((k+p_2)^2 - M_D^2)} \times G_{\psi D\bar{D}}(s,(k+p_2)^2,(k-p_1)^2) \times G_{\Lambda D\Xi}(k^2,(k-p_1)^2) G_{\Lambda D\Xi}(k^2,(k+p_2)^2), \tag{20}$$

and $SpD(s,k^2)$ is the trace of γ-matrices over the baryon line,

$$SpD(s,k^2) = {\rm Sp}[(\hat{p}_1 + M_\Lambda)\gamma_5(\hat{k} + M_\Xi)\gamma_5(\hat{p}_2 - M_\Lambda)(\hat{k} - M_\Lambda)] = 2((k^2)^2 + k^2(s - 2(M_D^2 + M_\Lambda M_\Xi)) - sM_\Lambda M_\Xi + c_D), \tag{21}$$

$$c_D = M_D^4 + 2M_\Lambda M_\Xi M_D^2 + 2M_\Xi M_\Lambda^3 - M_\Lambda^4. \tag{22}$$

The quantities $G_{\psi D\overline{D}}$ and $G_{D\Xi\Lambda}$ in (20) are the form factors for the vertexes $\psi \to D\bar{D}$ and $D \to \Xi\Lambda$.

The details of the calculation of quantity $Z_D(s)$ can be found in [27], but here we just recall that technically we calculate imaginary part of this quantity and then restore the real part of it by using the dispersion relation with one subtraction at $q^2 = 0$. First, we need to mention that the subtraction constant here also vanishes since the Λ hyperon (which is the *uds* quarks state) do not have open charm, and thus, the vertex $\psi \to \Lambda\bar{\Lambda}$ at $q^2 = 0$ is zero.

Next, cutting the diagram by D-meson propagators, we get the vertex $\psi \to D\bar{D}$ with the only dependence over charmonium virtuality $q^2 = s$ since D-meson legs are on-mass-shell,

$$G_{\psi D\bar{D}}(s, M_D^2, M_D^2) = g_{\psi D\bar{D}} \frac{M_\psi^2}{s} \frac{\log(M_\psi^2/\Lambda_D^2)}{\log(s/\Lambda_D^2)}, \qquad (23)$$

where the scale Λ_D we fix on the characteristic value of the reaction $\Lambda_D = 2M_D$ and the constant $g_{\psi D\bar{D}}$ is fixed by the $\psi \to D\bar{D}$ decay width,

$$g_{\psi D\bar{D}} \equiv G_{\psi D\bar{D}}(M_\psi^2, M_D^2, M_D^2) = 4\sqrt{\frac{3\pi\Gamma_{\psi\to D\bar{D}}}{M_\psi \beta_D^3}} = 18.4, \qquad (24)$$

where $\beta_D = \sqrt{1 - 4M_D^2/M_\psi^2}$ is the D-meson velocity in this decay.

Next, we consider the function $G_{\Lambda D\Xi}(k^2, p^2)$ from (20). Again, the only dependence left in the imaginary part of Z_D is the off-mass-shellness of Ξ baryon in the t-channel since $k^2 < 0$. In [26,27], we used the following form of ΛDP-vertex based on the results of [36,37]:

$$G_{\Lambda DP}(k^2, M_D^2) = \frac{f_D g_{DN\Lambda}}{m_u + m_c}, \qquad k^2 < 0, \qquad (25)$$

where $f_D \approx 180$ MeV and

$$\frac{g_{DN\Lambda}}{\sqrt{4\pi}} = 1.9 \pm 0.6. \qquad (26)$$

For quark masses, the following values are used: $m_u \approx 280$ MeV and $m_c = 1.27$ GeV [28]. The $SU(4)$ symmetry leads us to the same result for $G_{\Lambda D\Xi}$,

$$G_{\Lambda D\Xi}(k^2, M_D^2) = \frac{f_D g_{\Lambda D\Xi}}{m_u + m_c}, \qquad k^2 < 0, \qquad (27)$$

where

$$g_{\Lambda D\Xi} \approx g_{DN\Lambda} = 6.7 \pm 2.1. \qquad (28)$$

V. THREE GLUON MECHANISM

The three gluon mechanism first was considered in [26], and in [27], it was refined and some misprints and minor mistakes were corrected. So here we just present the final formulas for its contribution to the interference of a charmonium state with a Born amplitude [see (17)]:

$$S_{3g}(s) = \alpha_{3g}(s) Z_{3g}(s), \qquad (29)$$

where

$$\alpha_{3g}(s) = \frac{\alpha\alpha_s^3}{2^3 3} g_e g_{col} \phi\beta F_1(s) G_\psi(s), \qquad (30)$$

$$Z_{3g}(s) = \frac{4}{\pi^5 s} \int \frac{dk_1}{k_1^2}\frac{dk_2}{k_2^2}\frac{dk_3}{k_3^2} \times \frac{Sp3g\,\delta(q - k_1 - k_2 - k_3)}{((p_1 - k_1)^2 - M_\Lambda^2)((p_2 - k_3)^2 - M_\Lambda^2)}, \qquad (31)$$

where the quantity $g_{\mathrm{col}} = (1/4)\langle\Lambda|d^{ijk} T^i T^j T^k|\Lambda\rangle = 15/2$ is the color factor and $Sp3g$ is the product of traces over the Λ-hyperon and the c-quark lines,

$$Sp3g = \mathrm{Sp}[\hat{Q}_{\alpha\beta\gamma}(\hat{p}_c + m_c)\gamma^\mu(\hat{p}_{\bar{c}} - m_c)] \times \mathrm{Sp}[(\hat{p}_1 + M_\Lambda)\gamma^\alpha(\hat{p}_1 - \hat{k}_1 + M_\Lambda)\gamma^\beta \times (-\hat{p}_2 + \hat{k}_3 + M_\Lambda)\gamma^\gamma(\hat{p}_2 - M_\Lambda)\gamma_\mu],$$

with

$$\hat{Q}_{\alpha\beta\gamma} = \frac{\gamma_\gamma(-\hat{p}_{\bar{c}} + \hat{k}_3 + m_c)\gamma_\beta(\hat{p}_c - \hat{k}_1 + m_c)\gamma_\alpha}{((p_{\bar{c}} - k_3)^2 - m_c^2)((p_c - k_1)^2 - m_c^2)} + [\text{gluon permutations}], \qquad (32)$$

where the permutations over gluon vertices are performed in the gray block in Fig. 5.

The quantity ϕ in (30) is related to the charmonium wave function $\psi(\mathbf{r})$ as

$$\phi = \frac{|\psi(\mathbf{r} = \mathbf{0})|}{M_\psi^{3/2}} = \frac{\alpha_s^{3/2}}{3\sqrt{3\pi}}, \qquad (33)$$

where α_s is the QCD coupling constant. We should note that this three gluon mechanism is very sensitive to this quantity since it depends on its value in a rather high degree [see Eqs. (30) and (33)]. We use the value $\alpha_s(M_c) = 0.28$, which is expected by the QCD evolution of α_s from the b-quark scale to the c-quark scale, i.e., to $s \sim M_c^2$. We note that this value differs from the one for the J/ψ charmonium, which tends to feel much smaller value of parameter $\alpha_s(M_c) = 0.19$ [38].

The factor $G_\psi(s)$ in (30) is the form factor that describes the mechanism of transition of three gluons (with a total angular momentum equal to 1) into the final $\Lambda\bar{\Lambda}$ pair. Following [27], we suggest that this mechanism has much in common with the proton-antiproton pair production from the photon,

$$|G_\psi(s)| = \frac{C_\psi}{s^2 \log^2(s/\Lambda^2)}. \tag{34}$$

For the constant C_ψ, here we use the same value as it was obtained in the case of proton-antiproton production [27] since gluons to not feel the flavor of the quarks in the final baryons,

$$C_\psi = (45 \pm 9)\ \mathrm{GeV}^4. \tag{35}$$

VI. NUMERICAL RESULTS

The main building blocks for the cross section are the quantities $Z_D(s)$ from (20) and $Z_{3g}(s)$ from (31), which give the corresponding (D-meson loop and three gluon) contributions. In Fig. 6, we present the dependence of $Z_D(s)$ as a function of total energy $\sqrt{s}$ in the range starting from the threshold of the reaction $\sqrt{s} = 2M_\Lambda$ up to 4.5 GeV. One can see that the shape and the numerical values of the real and the imaginary parts of this quantity remain the same as it were in the case of proton-antiproton final state [see Fig. 7(a) in [27]]. As for the Fig. 7 presenting real and imaginary parts of the quantity $Z_{3g}(s)$, it shows the similar general behavior of the curves as it were in the case of proton-antiproton final state [see Fig. 7(b) in [27]] but the numerical difference is much more noticeable. Nevertheless, the characteristic large negative values of this quantity still remains thus, giving a large relative phase with respect to the Born contribution in the amplitude.

In Fig. 8, one can see the contributions from pure D-meson loop, while in Fig. 9, the pure three gluons contribution is present. Both of these contributions are compared with the data of BES III Collaboration [20] in the vicinity of $\psi(3770)$ resonance. For D-meson loop contributions, Fig. 8, the error bands are provided by the errors of the parameter $g_{\Lambda D\Xi}$ from (28). One can see that the central value of the curve goes lower then the experimental point, while the upper error band touches it. For the three gluons contribution, Fig. 9, the error bands represent the uncertainty of the parameter C_ψ from (35). We do not include to this error bands the possible uncertainties due to the strong dependence of this mechanism of parameter α_s [see text after Eq. (33)]. Here, we see a good agreement of this pure three gluon mechanism with the data point at $\sqrt{s} = M_\psi$.

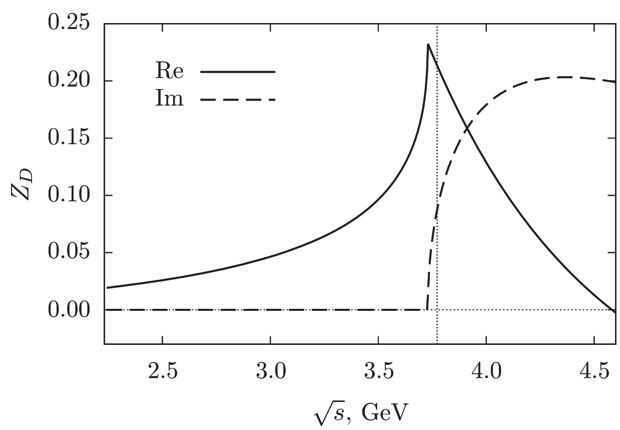

FIG. 6. The quantity $Z_D(s)$ from (20) as a function of $\sqrt{s}$ starting from the threshold $\sqrt{s} = 2M_\Lambda$. The vertical dashed line shows the position of $\psi(3770)$.

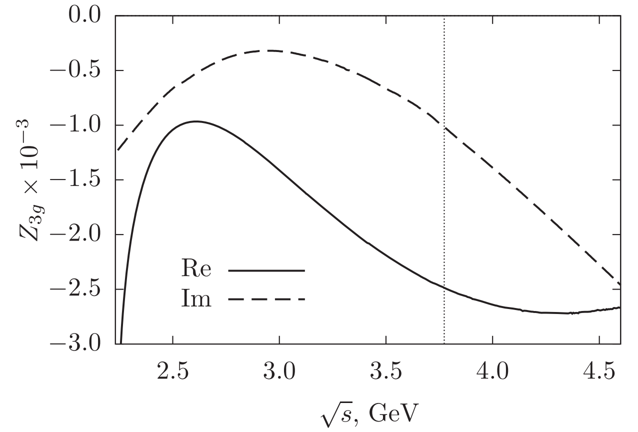

FIG. 7. The quantity $Z_{3g}(s)$ from (31) as a function of $\sqrt{s}$ starting from the threshold $\sqrt{s} = 2M_\Lambda$. The vertical dashed line shows the position of $\psi(3770)$.

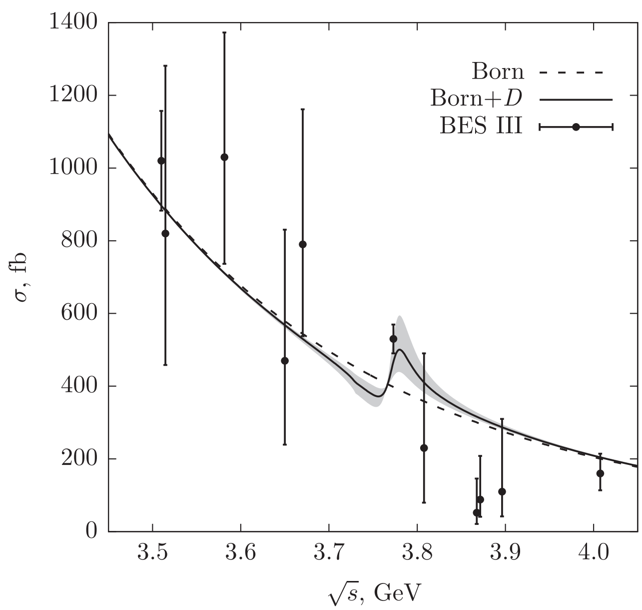

FIG. 8. The D-meson loop contributing to the total cross section with respect to the BES III data [20].

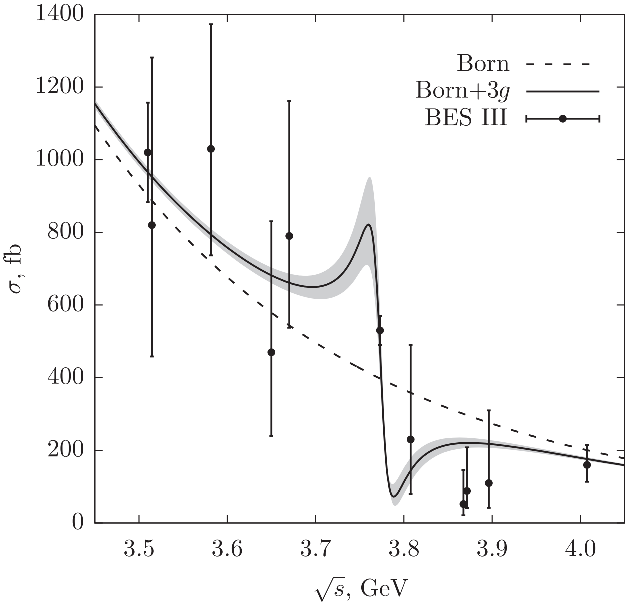

FIG. 9. The three gluon contributing to the total cross section with respect to the BES III data [20].

In Fig. 10, we present the total cross section including both of these mechanisms in comparison with the BES III data [20]. We see a rather fair agreement of our calculation with the data: the point at $\sqrt{s} = M_\psi$ is rather close to the curve and the left and right shoulders of the curve catch the tendency of the data. Here, we must remind that we do not do any extra fit of the parameters. All the parameters of our model are fixed by the calculation for the case of the proton-antiproton final state [27].

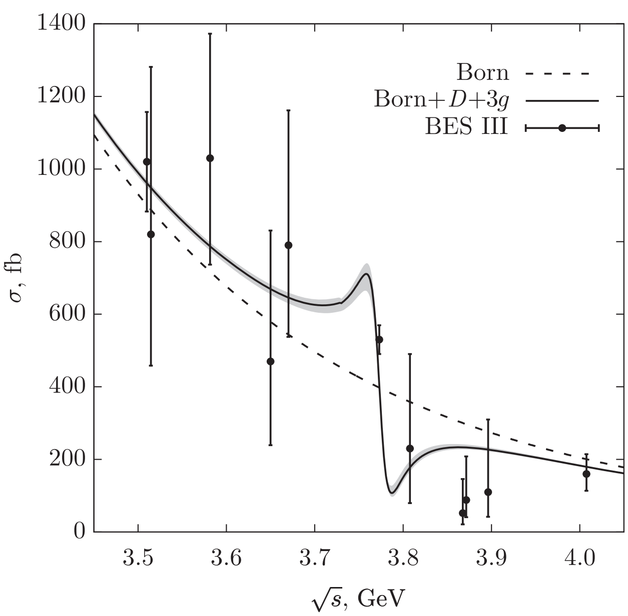

FIG. 10. The total cross section including two mechanisms in comparison with the BES III data [20].

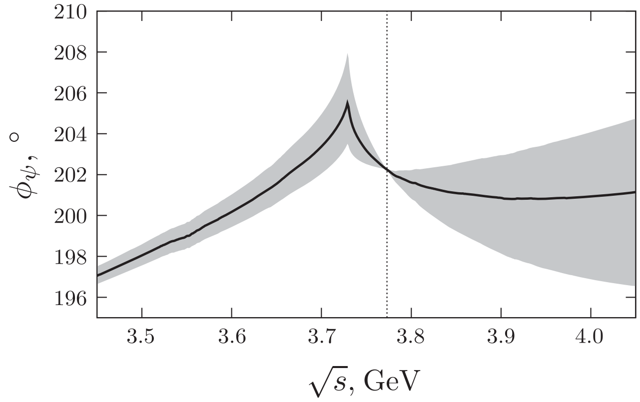

FIG. 11. The relative phase of total GeV charmonium $\psi(3770)$ contribution as a function of $\sqrt{s}$.

In Fig. 11, one can see the total relative phase ϕ_ψ of the charmonium contribution $\mathcal{M}_\psi$ to the amplitude with respect to the Born contribution $\mathcal{M}_B$ without Breit-Wigner factor; i.e.,

$$S_D(s) + S_{3g}(s) = |S(s)|e^{i\phi_\psi}, \tag{36}$$

where $S_D(s)$ was defined in (19) and $S_{3g}(s)$ is from (29). The error bands on this plot are due to both the $g_{\Lambda D\Xi}$ and C_ψ parameters uncertainties. As one can see at the point of the $\psi(3770)$ charmonium, the relative phase is rather large,

$$\phi_\psi \approx 202°. \tag{37}$$

It seems that this feature is common for the charmonium decay into the two baryons final state. We showed this in the proton-antiproton final state for the charmonium $\psi(3770)$ in papers [26,27] and for the charmonium $\chi_{c2}(1P)(3556)$ in paper [35].

VII. CONCLUSION

We considered the process of the electron-positron annihilation into a $\Lambda\bar{\Lambda}$ pair in the vicinity of charmonium $\psi(3770)$ resonance. Besides the Born mechanism, which is the pure QED, there are two contributions related with the intermediate charmonium $\psi(3770)$ state. One of them is the D-meson loop, and the other is the three gluon mechanism.

It was shown that both mechanisms contribute noticeably and give much of the final result. The total sum of them gives a rather good agreement with the experimental point at $\sqrt{s} = M_\psi$. It is also important that our curve reproduces the tendency of the experimental points at the left and at the right shoulders with respect to the central point. It is worth to notice again that we do not use any fitting procedure in this calculation. The parameters were fixed for the proton-antiproton production channel in the paper [27].

It is extremely desirable to make a precise scan over the energy region around the charmonium $\psi(3770)$ resonance with the small steps. This could support the conclusion that in the charmonium decay, the phase of $\psi \to p\bar{p}$ and $\psi \to \Lambda\bar{\Lambda}$ vertexes are large ($\phi_\psi \sim 200°$) and can be precisely measured in this channels. We showed this large phase generation in a set of papers [26,27,35].

In the future, we plan to consider another binary final states production process induced by the charmonium annihilation. Polarized effects can also be considered since the formalism is ready, and the data are present [39].

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