Neutrino oscillations at JUNO, the Born rule, and Sorkin's triple path interference

Patrick Huber,* Hisakazu Minakata,† Djordje Minic,‡ Rebekah Pestes,§ and Tatsu Takeuchi‖
Department of Physics, Center for Neutrino Physics, Virginia Tech, Blacksburg, Virginia 24061, USA

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We argue that neutrino oscillations at JUNO offer a unique opportunity to study Sorkin's triple path interference, which is predicted to be zero in canonical quantum mechanics by virtue of the Born rule. In particular, we compute the expected bounds on triple path interference at JUNO and demonstrate that they are comparable to those already available from electromagnetic probes. Furthermore, the neutrino probe of the Born rule is much more direct due to an intrinsic independence from any boundary conditions, whereas such dependence on boundary conditions is always present in the case of electromagnetic probes. Thus, neutrino oscillations present an ideal probe of this aspect of the foundations of quantum mechanics.

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I. INTRODUCTION

Obtaining a deeper understanding of quantum mechanics (QM) is homework leftover from the 20th century. The question is becoming more acute with the development of QM-based technologies already impacting our everyday lives (semiconductors, superconductors, etc.), as well as the promise of various quantum information technologies that may be realized in the not too distant future [1]. In this paper, we emphasize the relevance of neutrino physics to address various foundational questions in QM. In particular, we consider the potential of neutrino oscillations to probe the triple path interference of Sorkin [2] as a direct test of the Born rule and compare the expected bound to those currently available from electromagnetic experiments.

There are many features that distinguish QM from classical mechanics (CM). Though the statistical nature of QM as opposed to the deterministic nature of CM is often emphasized in textbooks, many other differences exist as well, for instance, in correlations [3–5] and in the presence or absence of interference. However, it has been noted that not only do QM correlations go beyond those of CM, but are themselves restricted [6,7] and are not as large as that allowed by logic and relativity [8]. QM interference is also restricted in that the Born rule only allows for pairwise interference between paths, but not for triple path interference or higher [2].

The fact that canonical QM itself is restricted, with no apparent physical or logical reason, points to the possible existence of consistent theories that go beyond QM boundaries, perhaps at the expense of some of our cherished principles that we currently hold to be fundamental. It is up to experiments to determine whether nature always stays within those QM boundaries or occasionally ventures outside, and under what conditions. Indeed, it has been argued that it would (or should) in the context of quantum gravity and cosmology [9,10], in the realm of quantum measurement [11], or in the domain of macroscopic quantum systems [12].

The celebrated Bell's inequality [3,4] and its generalization by Clauser *et al.*, the CHSH inequality [5], provide bounds that CM correlations cannot violate. That experimental correlations violate these bounds has been confirmed by various groups using photons [13], ions, and atoms, Josephson junctions, and NV centers in diamond (see Fig. 1 of [14]). However, Cirelson has shown that the CHSH correlator is bounded from above for QM as well [6,7], and while this bound is naturally larger than that of CM, it does not saturate the logically allowed maximum, despite the fact that such a saturation does not contradict relativistic causality as demonstrated by Popescu and Rohrlich [8]. Whether experimental correlations violate the QM Cirelson bound or not is being checked using photons [14,15].

The Bell/CHSH inequalities involve correlations between entangled pairs of spatially separated observables. In contrast, the Leggett-Garg inequalities (LGIs) [16] involve correlations between temporary separated measurements

*pahuber@vt.edu
†hisakazu.minakata@gmail.com
‡dminic@vt.edu
§rebhawk8@vt.edu
‖takeuchi@vt.edu

of a single observable and are expected to hold for "macroscopic" systems. Since QM satisfies neither of the two assumptions underlying the LGIs, namely, macroscopic realism and noninvasive measurability, QM correlations can be expected to violate them, though there is some subtlety in how those correlations should be defined in QM to reflect the setup of each experiment. Nevertheless, experimental checks that demonstrate the violation of some form of the LGIs have been performed using a variety of systems, including superconducting qubits, nuclear spins, and photons (see Table 1 of Refs. [17,18]), and more recently using the neutrino oscillation data from MINOS [19] and Daya-Bay [20].

Neutrino experiments are, in many respects, ideal laboratories for foundational quantum research given the long coherence times that neutrino states have and the fundamental role of interference in neutrino oscillation phenomena. In addition to probing the LGIs [19,20], the violation of which is not surprising, the potential of neutrino oscillations to constrain models that generalize and go beyond canonical QM has also been explored. For instance, Refs. [21,22] investigate whether neutrino oscillations can constrain the continuous spontaneous localization model [23] (expected effect is too small to be observed), while Ref. [24] argues that atmospheric neutrino data can constrain Nambu QM extensions [25]. The possibility of utilizing Mössbauer neutrinos to probe the time-energy uncertainty relation has been discussed in Ref. [26].

In this paper, we explore the potential of the neutrino oscillation experiment JUNO [27] to look for the triple path interference of Sorkin [2]. Sorkin provides a classification scheme for theories that go beyond CM in terms of the existence of multipath interferences. Only double path interferences exist in QM due to the Born rule; though, in principle, triple and higher-order interferences are possible. Experimental bounds have been placed on the presence of triple path interference using photons [28] and liquid state NMR [29]. Both experiments report upper bounds on the ratio of triple path to double path interferences of order 10^{-2}. Improving the photon bound requires controlling the sensitivity of multislit interference to the change in the boundary condition due to the opening and closing of slits [30–34]. Neutrino oscillation, on the other hand, does not involve any slits and always has three mass eigenstates interfering with each other. However, at different baselines, neutrino energies, and neutrino energy resolutions, it effectively becomes a double path interference experiment due to the large separation in scale between δm^2_{21} and δm^2_{31}. Indeed, JUNO [27] is expected to be the first experiment in which the interference between the atmospheric and solar oscillation amplitudes is clearly visible [35].

In the following, we first review Sorkin's definition of multipath interference and the classification of theories based on their presence or absence [2]. We then review our previous work from Ref. [35], which looked at the potential of JUNO to detect deviations of the neutrino oscillation probabilities from their QM predictions. This is effectively the same problem as we are considering in this paper, albeit imposing a particular normalization for the possible triple path interference. The expected bounds on triple path interference at JUNO for several other normalization/parametrization choices with details of the analyses are presented next. The parametrizations are chosen under the caveat that the contribution of the triple path interference should be invisible to pre-JUNO experiments and includes the one that facilitates comparison of the bound with the photon/NMR results. We conclude with a discussion on the necessity of a concrete model that predicts triple path interference to derive a more physically meaningful bound.

II. THE BORN RULE AND SORKIN'S MULTIPATH INTERFERENCE

We follow the discussion of Sorkin in Ref. [2]. Let

$$P_n(A, B, C, \cdots) \tag{1}$$

denote the probability of a system to go from an initial state $|\alpha\rangle$ to a final state $|\beta\rangle$ when n pathways $A, B, C, \ldots$ connecting the two are available. Classically, we have

$$P_n(A, B, C, \cdots) = P_1(A) + P_1(B) + P_1(C) + \cdots \tag{2}$$

for any number of paths. Quantum mechanically, we have for two paths

$$\begin{aligned} P_2(A, B) &= |\psi_A + \psi_B|^2 \\ &= \underbrace{|\psi_A|^2}_{P_1(A)} + \underbrace{|\psi_B|^2}_{P_1(B)} + \underbrace{(\psi_A^*\psi_B + \psi_B^*\psi_A)}_{I_2(A,B)}. \end{aligned} \tag{3}$$

The extra term

$$I_2(A, B) = P_2(A, B) - P_1(A) - P_1(B) \tag{4}$$

is the "interference" of the two paths A and B. The nonvanishing of this double path interference, $I_2(A, B) \neq 0$, distinguishes QM from CM.

In QM the superposition principle allow us to superimpose an arbitrary number of "paths" on top of each other. Indeed, in the path integral approach we superimpose an infinite number of them [36]. However, the Born rule dictates that all the superimposed paths only interfere with each other in a pairwise manner. For instance, for three paths we have

$$\begin{aligned}P_3(A,B,C) &= |\psi_A+\psi_B+\psi_C|^2\\ &= \underbrace{|\psi_A|^2}_{P_1(A)}+\underbrace{|\psi_B|^2}_{P_1(B)}+\underbrace{|\psi_C|^2}_{P_1(C)}+\underbrace{(\psi_A^*\psi_B+\psi_B^*\psi_A)}_{I_2(A,B)}\\ &\quad+\underbrace{(\psi_B^*\psi_C+\psi_C^*\psi_B)}_{I_2(B,C)}+\underbrace{(\psi_C^*\psi_A+\psi_A^*\psi_C)}_{I_2(C,A)}\\ &= P_2(A,B)+P_2(B,C)+P_2(C,A)\\ &\quad-P_1(A)-P_1(B)-P_1(C). \end{aligned} \tag{5}$$

Only pairwise interferences between the pairs (A,B), (B,C), and (C,A) appear. Therefore, it makes sense to define any deviation from this relation as the triple path interference,

$$\begin{aligned}&I_3(A,B,C)\\ &\quad= P_3(A,B,C)-P_2(A,B)-P_2(B,C)-P_2(C,A)\\ &\qquad+P_1(A)+P_1(B)+P_1(C). \end{aligned} \tag{6}$$

For both CM and QM, this triple path interference is zero for any triplet of paths.

In a similar fashion, the n-path interference for $n \geq 4$ can be defined as

$$\begin{aligned}&I_n(A_1,A_2,\ldots,A_n)\\ &\quad= P_n(A_1,A_2,\ldots,A_n)-\sum P_{n-1}(A_i,A_j,\cdots)\\ &\qquad+\sum P_{n-2}(A_i,\cdots)-\cdots-(-1)^n\sum P_1(A_i), \end{aligned} \tag{7}$$

which are always zero for both CM and QM. Therefore, CM and QM can be characterized by

$$\begin{aligned}&\text{CM: } I_n = 0 \quad \text{for } n\geq 2,\\ &\text{QM: } I_2 \neq 0, \qquad I_n = 0 \quad \text{for } n \geq 3. \end{aligned} \tag{8}$$

Experimental confirmation of $I_3 = 0$ would be a confirmation of the Born rule. In Refs. [28,29], bounds were placed on the parameter

$$\kappa = \frac{\varepsilon}{\delta}, \tag{9}$$

where

$$\begin{aligned}\varepsilon &= I_3(A,B,C),\\ \delta &= |I_2(A,B)|+|I_2(B,C)|+|I_2(C,A)|. \end{aligned} \tag{10}$$

Reference [28] reports $\kappa = 0.0064 \pm 0.0119$ for a multislit experiment with a single photon source, while Ref. [29] reports $\kappa = 0.007 \pm 0.003$ based on a liquid state NMR experiment. Thus, the 1σ deviation of κ from zero allowed by these experiments is $|\kappa| < 0.01 \sim 0.02$.

III. HUBER *ET AL.*

Here, we review the analysis of Ref. [35]. According to canonical QM, the neutrino oscillation amplitude for $\nu_\beta \to \nu_\alpha$ at distance x from the source is given by the superposition of the contributions of the three mass eigenstates,

$$\begin{aligned}S_{\alpha\beta}^{(123)} &= U_{\alpha1}U_{\beta1}^* + U_{\alpha2}U_{\beta2}^*e^{-i\Delta_{21}x} + U_{\alpha3}U_{\beta3}^*e^{-i\Delta_{31}x}\\ &= \underbrace{(U_{\alpha1}U_{\beta1}^*+U_{\alpha2}U_{\beta2}^*+U_{\alpha3}U_{\beta3}^*)}_{\delta_{\alpha\beta}}\\ &\quad+\underbrace{U_{\alpha2}U_{\beta2}^*(e^{-i\Delta_{21}x}-1)}_{S_{\alpha\beta}^{\mathrm{sol}}}+\underbrace{U_{\alpha3}U_{\beta3}^*(e^{-i\Delta_{31}x}-1)}_{S_{\alpha\beta}^{\mathrm{atm}}}\\ &= \delta_{\alpha\beta}+S_{\alpha\beta}^{\mathrm{sol}}+S_{\alpha\beta}^{\mathrm{atm}}, \end{aligned} \tag{11}$$

where

$$\Delta_{ij} = \frac{\delta m_{ij}^2}{2E}. \tag{12}$$

The Born rule gives the QM oscillation probability for $\nu_\beta \to \nu_\alpha$ as

$$\begin{aligned}P_{\mathrm{QM}}(\nu_\beta\to\nu_\alpha) &= |S_{\alpha\beta}^{(123)}|^2\\ &= \underbrace{\delta_{\alpha\beta}+|S_{\alpha\beta}^{\mathrm{sol}}|^2+|S_{\alpha\beta}^{\mathrm{atm}}|^2+2\delta_{\alpha\beta}\Re(S_{\alpha\beta}^{\mathrm{sol}}+S_{\alpha\beta}^{\mathrm{atm}})}_{P_{\beta\alpha}^{\text{non-int-fer}}}\\ &\quad+\underbrace{(S_{\alpha\beta}^{\mathrm{sol}*}S_{\alpha\beta}^{\mathrm{atm}}+S_{\alpha\beta}^{\mathrm{atm}*}S_{\alpha\beta}^{\mathrm{atm}})}_{P_{\beta\alpha}^{\text{int-fer}}}\\ &= P_{\beta\alpha}^{\text{non-int-fer}}+P_{\beta\alpha}^{\text{int-fer}}. \end{aligned} \tag{13}$$

This is the QM prediction. To check this relation, Ref. [35] introduces the parameter q as

$$P_{\exp}(\nu_\beta\to\nu_\alpha) = P_{\beta\alpha}^{\text{non-int-fer}} + qP_{\beta\alpha}^{\text{int-fer}} \tag{14}$$

and discusses the bounds that can be placed on q by the JUNO experiment [27]; that is, the expected $P_{\exp}(\nu_\beta\to\nu_\alpha)$ at JUNO assuming canonical QM is simulated using GLoBES [37,38] to which the right-hand side of Eq. (14) is fit to calculate the expected bound on q. Since the $P_{\beta\alpha}^{\text{non-int-fer}}$ term includes interference effects between the 1–2 and 1–3 mass eigenstates, the parameter q checks for 2–3 interference. However, any deviation of $P_{\exp}(\nu_\beta\to\nu_\alpha)$ from the QM prediction can also be interpreted as due to Sorkin's triple path interference. Indeed, Eq. (14) effectively parametrizes the size of triple path interference as

$$\begin{aligned}\varepsilon &= P_{\exp}(\nu_\beta\to\nu_\alpha)-P_{\mathrm{QM}}(\nu_\beta\to\nu_\alpha)\\ &= (q-1)P_{\beta\alpha}^{\text{int-fer}}. \end{aligned} \tag{15}$$

Note that this parametrization of the triple path interference renders it invisible to all pre-JUNO experiments. Thus, the bound on $q-1$ can be reinterpreted as a bound on ε normalized to $P_{\beta\alpha}^{\text{int-fer}}$. Using Wilks's theorem [39], we find that the analysis of Ref. [35] imposes a 1σ allowed range on $q-1$ given by

$$-0.17 < (q-1) < 0.12. \tag{16}$$

Note that, in the absence of an actual theory with triple path interference that predicts the energy and baseline dependence of ε, we must make a somewhat arbitrary choice as in Eq. (15). By fitting the data with constant $\kappa = \varepsilon/\delta$, Refs. [28,29] are assuming that $\varepsilon \propto \delta$, cf. Eq. (10), which is another arbitrary choice. However, δ does have the advantage over $P_{\beta\alpha}^{\text{int-fer}}$ in that all three paths are treated equally. Because of this, and also for the ease of comparison with the photon and NMR results, we redo the analysis of [35] with this normalization.

IV. POSSIBLE JUNO BOUND ON κ

The parameter κ is introduced as

$$\begin{aligned}\varepsilon &= P_{\text{exp}}(\nu_\beta \to \nu_\alpha) - P_{\text{QM}}(\nu_\beta \to \nu_\alpha)\\ &= \kappa(|I_{\alpha\beta}(1,2)| + |I_{\alpha\beta}(1,3)| + |I_{\alpha\beta}(2,3)|),\end{aligned} \tag{17}$$

where

$$I_{\alpha\beta}(i,j) = 2\Re(U_{\alpha i}U_{\beta i}^* U_{\alpha j}^* U_{\beta j} e^{-i\Delta_{ij}x}) \tag{18}$$

is the interference between the ith and jth mass eigenstates. Strictly speaking, one should perform a global fit to all neutrino oscillation experiments to bound κ. However, $|I_{\alpha\beta}(2,3)|$ was invisible to all pre-JUNO experiments and would have averaged to a small constant, while the other terms would have been absorbed into the uncertainties in the mixing angles. This justifies our JUNO-only analysis.

We use GLoBES [37,38] to simulate the JUNO experiment as detailed in Ref. [35]. The simulation is set up with two detectors: a JUNO-like far detector, with a fiducial mass of 20 kt and an energy resolution of $3\%/\sqrt{E}$ at a distance of 53 km from a nuclear reactor source with a total power of 36 GWth, and a TAO-like [40] near detector, with a fiducial mass of 1 ton and an energy resolution of $1.7\%/\sqrt{E}$ at a distance of 30 m from a 4.6 GWth nuclear reactor core; we assume a total data taking time of six years.

For the purpose of producing the simulated data $P_{\text{exp}}(\nu_\beta \to \nu_\alpha)$, we assume canonical QM with normal ordering to be the true mass ordering and the relevant oscillation parameters to be $\Delta m_{21}^2 = 7.54 \times 10^{-5}\ \text{eV}^2$, $\Delta m_{31}^2 = 2.43 \times 10^{-3}\ \text{eV}^2$, $\theta_{12} = 33.6°$, and $\theta_{13} = 8.9°$

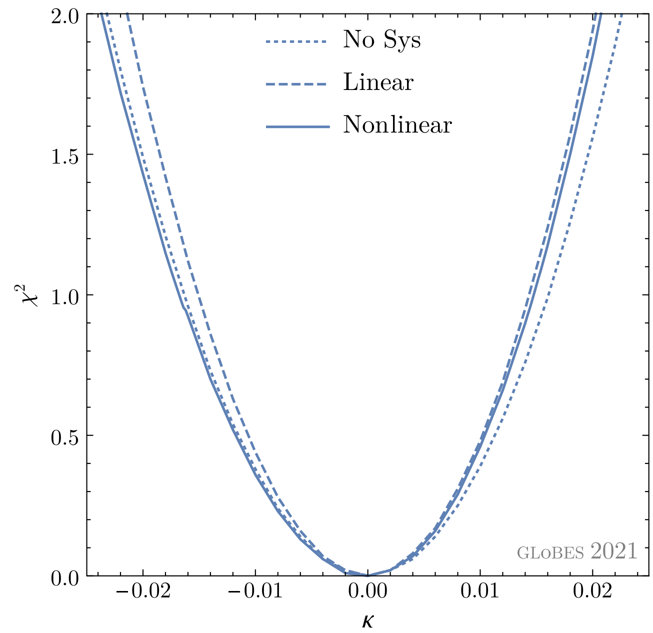

FIG. 1. χ^2 curves for κ as defined in Eq. (17). The (i) solid, (ii) dashed, and (iii) dotted curves correspond to simulations in which the uncertainties are treated differently as detailed in the main text.

[41]. The theoretical QM rates $P_{\text{QM}}(\nu_\beta \to \nu_\alpha)$ are calculated with the same inputs and the difference between $P_{\text{exp}}(\nu_\beta \to \nu_\alpha)$ and $P_{\text{QM}}(\nu_\beta \to \nu_\alpha)$ is fit with Eq. (17). The results of our simulation are shown in Fig. 1 for the following three analyses:

(i) Solid line: For each detector, we use a model for nonlinear effects in the reconstruction of the positron energy as described in Ref. [42], which includes terms up to the cubic in the positron energy. To account for the uncertainties in the reactor antineutrino flux prediction, we conservatively introduce a nuisance parameter to each of our 100 energy bins with the spectrum computed before applying the energy resolution function. This is equivalent to the assumption of *no* prior knowledge of fluxes, as in Ref. [42].

(ii) Dashed line: The same analysis is repeated except assuming that the energy calibration error for each detector is linear.

(iii) Dotted line: Simulation without a near detector while assuming perfect knowledge of detector and source systematics.

An interesting feature of Fig. 1 is that analysis (iii) with no systematic uncertainties provides a weaker constraint for κ than analyses (i) and (ii) with systematic uncertainties and a near detector. This demonstrates that the presence of a near detector not only constrains the neutrino flux, but also provides extra constraints on possible deviations of the neutrino oscillation probabilities from canonical QM.

Using Wilks's theorem, the 1σ allowed range of κ for analysis (i) is found to be

$$-0.017 < \kappa < 0.015. \tag{19}$$

V. CONSTRAINING OTHER FORMS OF THE INTERFERENCE TERM

In addition to Eq. (17), which we will refer to as case (1), we consider two other forms for the triple path interference ε: (2) constant ε, i.e., ε independent of L/E, and (3) ε proportional $1 - P_{\mathrm{QM}}(\nu_\beta \to \nu_\alpha)$, that is,

$$\begin{aligned}\varepsilon &= P_{\mathrm{exp}}(\nu_\beta \to \nu_\alpha) - P_{\mathrm{QM}}(\nu_\beta \to \nu_\alpha)\\ &= k(1 - P_{\mathrm{QM}}(\nu_\beta \to \nu_\alpha)),\end{aligned} \tag{20}$$

where k is a constant. In case (2), we are considering the possibility that triple interference is hidden in the uncertainty of the overall count rate, while in case (3) we are assuming

$$P_{\mathrm{exp}}(\nu_\beta \to \nu_\alpha) = k + (1-k)P_{\mathrm{QM}}(\nu_\beta \to \nu_\alpha). \tag{21}$$

For each case, we perform the same three analyses as in the previous section and plot the results in Figs. 2 and 3, respectively. For case (2), the 1σ allowed range for ε from analysis (i) is

$$-0.065 < \varepsilon < 0.042, \tag{22}$$

while for case (3), the bound on k from analysis (i) is

$$-0.040 < k < 0.069. \tag{23}$$

In order to compare the constraints for cases (1)–(3), we define

$$\begin{aligned}&q_1 \equiv \kappa\langle\delta(E)\rangle, \qquad q_2 \equiv \varepsilon, \quad \text{and}\\ &q_3 \equiv k\langle 1 - P_{\mathrm{QM}}(\nu_\alpha \to \nu_\beta)\rangle,\end{aligned} \tag{24}$$

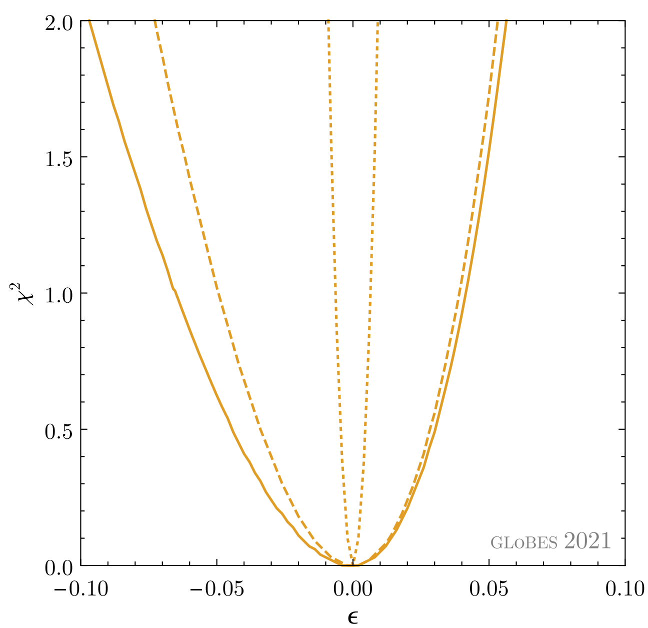

FIG. 2. χ^2 curves for ε as a constant. The (i) solid, (ii) dashed, and (iii) dotted curves correspond to simulations in which the uncertainties are treated differently, as detailed in the main text.

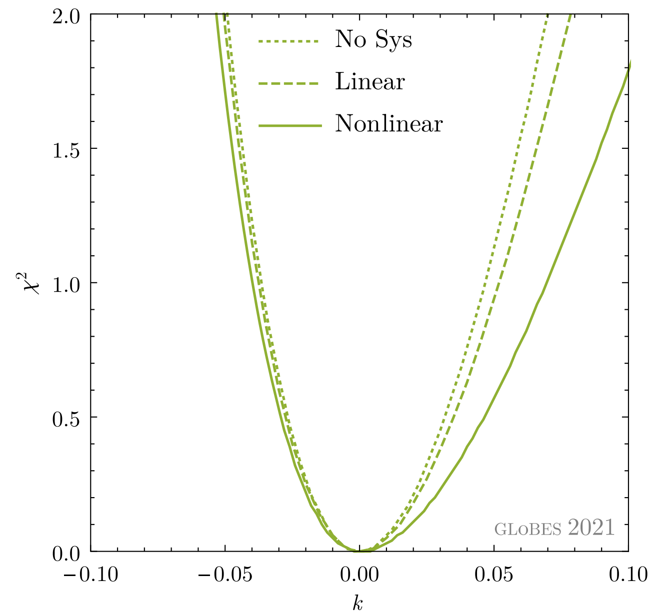

FIG. 3. χ^2 curves for k as defined in Eq. (20). The (i) solid, (ii) dashed, and (iii) dotted curves correspond to simulations in which the uncertainties are treated differently, as detailed in the main text.

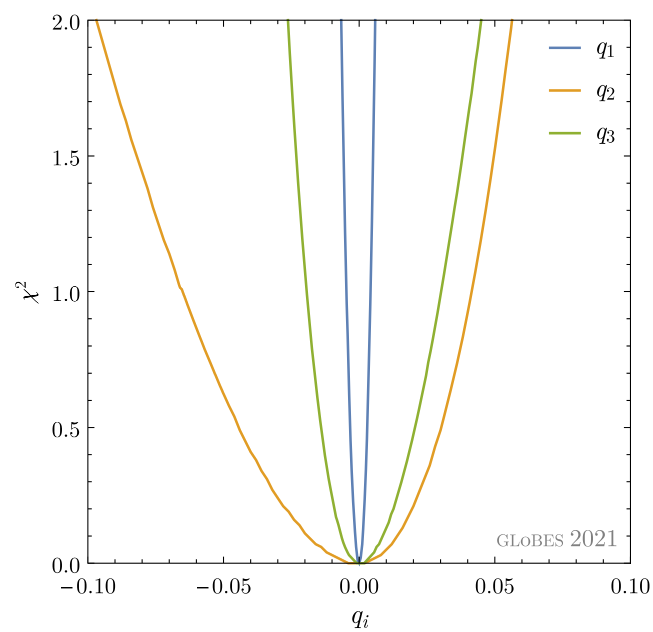

FIG. 4. χ^2 curves for q_i as defined in Eq. (24) with all the systematic uncertainties listed in Ref. [35].

where $\langle f(E) \rangle$ is the average of $f(E)$ over the interval $1.8 \leq E \leq 8.0$ MeV using the oscillation parameters that minimized the χ^2 at each point. The results for analysis (i) including all systematic uncertainties are shown in Fig. 4.

VI. DISCUSSION

In this paper, we point out the relevance of neutrino physics for addressing foundational questions in QM. In particular, we have examined the potential of the JUNO experiment to probe for the triple path interference among the three neutrino mass eigenstates and thereby test the Born rule. The potential JUNO 1σ bound of $-0.017 < \kappa < 0.015$ is competitive with those available from electromagnetic probes [28,29]. Moreover, the prospects of electromagnetic probes to improve their bounds is limited due to the sensitivity of the interference pattern on the change in boundary condition caused by the opening and closing of slits [30–34], whereas neutrino oscillations are independent of such considerations.

One drawback of our analysis is that we currently lack a theory that can model departures from the Born rule and predict how triple path interferences would depend on experimental parameters. This is reflected in the arbitrary choices we must make to normalize the triple path interference ε in our fits. We are also assuming that triple path interference is introduced while the existing double path interferences remain unmodified, which may not be the case for a complete theory. Without such a theory, we also cannot disentangle or distinguish triple path interference from other effects, such as the presence of nonstandard interactions of the neutrino [43–45], small matter effects [46,47], superlight sterile neutrinos [48], quasi-Dirac neutrino oscillations [49], and *CPT* violation [50]. However, since we expect the triple path interferences in a consistent extension of QM to maintain unitarity, it may be distinguishable from neutrino decay [51,52] or the effect of randomly fluctuating matter [53], which both involve nonunitary time evolution, or Lorentz violation [50], which could lead to sidereal modulation in the oscillation spectrum.

A promising candidate theory that could potentially incorporate triple path interferences within its framework is Nambu QM [25]. This theory generalizes the time evolution of QM states in a way that is noncanonical yet unitary. In essence, Nambu QM generalizes the space in which the "phase" (in the sense of the phase of a complex number) of a state is allowed to evolve, leading to noncanonical time evolution as well as noncanonical double path interference. Indeed, we have recently discussed how the vanilla version of the theory can be constrained using atmospheric neutrinos by looking at interference effects [24]. Given the larger freedom that the phase is allowed in Nambu QM, we envision generalizations (most probably a nonassociative one) in which the triple path interference and the departure from the Born rule could be precisely modeled.

We close this discussion by recalling that neutrinos can also probe the Leggett-Garg inequalities [19,20], spontaneous collapse models of quantum measurement [21,22], nonstandard time evolution of quantum states [24], and, if Mössbauer neutrinos can be realized, the time-energy uncertainty relation [26]. Further studies will indubitably lead to other ways to utilize neutrinos for foundational QM studies.

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