


Decay properties of $N(1535)$ in the holographic QCD

Akihiro Iwanaka,^{*} Daisuke Fujii,[†] and Atsushi Hosaka^{‡,§}

Research Center for Nuclear Physics(RCNP), Osaka University, Ibaraki 567-0048, Japan

 (Received 20 May 2022; accepted 13 June 2022; published 30 June 2022)

We study one pion emission decay of the first excited state of the nucleon with negative parity $N(1535) \equiv N^*$ in the holographic model of QCD. The excited state is described as a vibrational mode along the extra z direction in the five-dimensional space-time of the model. We have obtained an analytic formula for the axial coupling of $g_A^{NN^*}$. The off diagonal axial coupling is obtained at the decaying pion momentum $|\mathbf{k}| = 448$ MeV as $g_A^{NN^*} \sim 0.32$ and hence, a partial decay width $\Gamma_{N^* \rightarrow \pi N} \sim 30$ MeV, which is smaller than but reasonably compared to the experimental data.

DOI: [10.1103/PhysRevD.105.114057](https://doi.org/10.1103/PhysRevD.105.114057)

I. INTRODUCTION

Baryons at low energies show various features due to the rich structure of QCD. Ground state properties, such as masses and magnetic moments, are now computed directly from QCD by the first principle method of the lattice QCD. Yet, effective model approaches are useful to describe dynamical processes, such as resonance formations or decays. A widely employed model is the quark model, where resonances emerge as single particle excitations of quarks [1–4].

When considering dynamical processes, it is important to include interactions with pions because many resonances are formed and decay through pions. The importance of the pion dynamics is also expected by spontaneous breaking of chiral symmetry [5,6]. One way to include pions in baryon dynamics is realized by the Skyrme model, where baryons emerge as solitons of the nonlinear sigma model [7,8]. The approach has been justified in the large N_c limit, where N_c is the number of colors, and various baryon resonances appear as collective excitations [9–11]. For instance, the $\Delta(1232)$ is a rotational excitation of the deformed hedgehog in the spin-isospin space, and the Roper resonance $N(1440)$ a monopole vibration of the radial motion [12]. Moreover, negative parity states may be described as composite states of a

ground state baryon and a negative parity meson such as $K\Sigma$ for $N(1535)$ and $\bar{K}N$ for $\Lambda(1450)$ [13–15].

A similar, but an alternative approach is the holographic QCD. A well-known model is the Sakai-Sugimoto model, which has been used to investigate various nonperturbative properties of QCD [16,17]. The model is based on flavor gauge theory in the five-dimensional space, the four-dimensional added space-time with one extra dimension (z direction). A solitonic picture for baryons is then realized by an instanton in that five-dimensional space [18]. As in the Skyrme model, baryon resonances are described by collective excitations of the hedgehog [Belavin-Polyakov-Schwartz-Tyupkin (BPST)] instanton. A somewhat differently looking feature of this approach is that negative parity states are described by the collective vibration along the z direction. Static properties have been investigated, with reasonable agreement with experimental data [18].

Now concerning dynamical properties, the one-pion decay of $N(1440)$ was investigated in Ref. [19]. A crucial point of this approach is that a finite value of the partial decay width is obtained in the long wavelength limit (of the pion) with good agreement with data. Moreover, a model independent relation was found for the axial coupling constants between the nucleon and Roper resonance. These features sharply contrast with nonrelativistic quark model calculations, where the decay is forbidden.¹

Now after having observed the good features in the decay of the $N(1440)$, in this paper, we investigate the decay of the negative parity baryon $N(1535)$, which is the first negative parity resonance, in the holographic QCD, the state is described as a collective vibration along the z direction. The method of the computation parallels the previous one [19]. What are new here are the computation

^{*}iwana@rcnp.osaka-u.ac.jp

[†]daisuke@rcnp.osaka-u.ac.jp

[‡]hosaka@rcnp.osaka-u.ac.jp

[§]Also at Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195 Japan.

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

¹Higher order terms in the relativistic expansion can make it finite [20].

of the zeroth (time) component of the axial current, and the evaluation of the matrix elements in the z valuable.

This paper is organized as follows. In Sec. II, we present the action used in this paper and obtain the classical instanton solution to the equations of motion. We then define the chiral currents and give their concrete expressions. In Sec. III, we compute the axial coupling and decay width of $N(1535)$ in comparison with experimental data. The final section is for discussions and summary.

II. AXIAL CURRENT

A. Classical solutions and collective quantization

We briefly review how baryons are formed in the Sakai-Sugimoto model. We calculate the classical instanton solution and use collective coordinate method to obtain the wave function of baryons. In this model, baryons appear as quantized five-dimensional instanton.

The Sakai-Sugimoto model is an effective model of hadrons based on gauge/string duality. It is described by the $U(N_f)$ Yang-Mills-Chern-Simons theory in curved five-dimensional space-time, where N_f is the number of flavors. The action is given by the Yang-Mills part S_{YM} and Chern-Simons part S_{CS} as follows:

$$S = S_{\text{YM}} + S_{\text{CS}}, \quad (1)$$

$$S_{\text{YM}} = -\kappa \int d^4x dz \text{tr} \left[\frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 + k(z) \mathcal{F}_{\mu z}^2 \right], \quad (2)$$

$$S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_{M^4 \times \mathbb{R}} \omega_5(\mathcal{A}). \quad (3)$$

Here, $\mu, \nu = 0, 1, 2, 3$ are four-dimensional Lorentz indices, and z is the coordinate of the fifth dimension. The warp factors $k(z)$ and $h(z)$ are defined as $k(z) = 1 + z^2$ and $h(z) = (1 + z^2)^{-1/3}$, respectively. The number of colors N_c is related to κ as

$$\kappa = \frac{\lambda N_c}{216\pi^3},$$

where λ is the 't Hooft coupling. $\mathcal{A} = \mathcal{A}_\alpha dx^\alpha = \mathcal{A}_\mu dx^\mu + \mathcal{A}_z dz$ ($\alpha = 0, 1, 2, 3, z$) is the five-dimensional $U(N_f)$ gauge field, and $\mathcal{F} = \frac{1}{2} \mathcal{F}_{\alpha\beta} dx^\alpha \wedge dx^\beta = d\mathcal{A} + i\mathcal{A} \wedge \mathcal{A}$ is the field strength. In this paper, we consider the case of $N_f = 2$. We decompose $U(2)$ gauge field as

$$\mathcal{A} = A^a \frac{\tau^a}{2} + \hat{A} \frac{I_2}{2},$$

where \hat{A} is the $U(1)$ part, A the $SU(2)$ part, τ^a ($a = 1, 2, 3$) the Pauli matrices and I_2 the 2×2 unit matrix. The Chern-Simons five-form is

$$\omega_5(\mathcal{A}) = \text{tr} \left(\mathcal{A} \mathcal{F}^2 - \frac{i}{2} \mathcal{A}^3 \mathcal{F} - \frac{1}{10} \mathcal{A}^5 \right).$$

In this paper, we mainly work with the unit of the Kaluza-Klein mass $M_{\text{KK}} = 1$.

Because of the warp factors $k(z)$ and $h(z)$, it is difficult to solve the equation of motion in general. However, the size is small in the large λ limit, since the instanton size is proportional to $\lambda^{-1/2}$. Then, we can consider the solution localized at the origin $z \sim 0$. In this case, we can set $k(z) = h(z) = 1$, and the solution is given by the BPST instanton solution,

$$\begin{aligned} A_M^{\text{cl}}(\mathbf{x}, z) &= -if(\xi)g\partial_M g^{-1}, \\ A_0^{\text{cl}} &= 0, \\ \hat{A}_M^{\text{cl}} &= 0, \\ \hat{A}_0^{\text{cl}} &= \frac{1}{8\pi^2 a} \frac{1}{\xi^2} \left[1 - \frac{\rho^4}{(\xi^2 + \rho^2)^2} \right], \end{aligned} \quad (4)$$

with

$$\begin{aligned} f(\xi) &= \frac{\xi^2}{\xi^2 + \rho^2}, & g(x) &= \frac{(z - Z) - i(\mathbf{x} - \mathbf{X}) \cdot \boldsymbol{\tau}}{\xi}, \\ \xi &= \sqrt{(z - Z)^2 + |\mathbf{x} - \mathbf{X}|^2}, \end{aligned}$$

where $X^M = (X^1, X^2, X^3, Z) = (\mathbf{X}, Z)$ is the positions of the instanton in four-dimensional space and ρ is a parameter of the instanton size.

Let us quantize the classical instanton solution by the collective coordinate method. In order to do that, we introduce time-dependent parameters $\mathbf{X}(t), Z(t), \rho(t)$, and $SU(2)$ orientation $V(t, x, a(t))$ with $V(z \rightarrow \pm\infty) \rightarrow \mathbf{a}(t)$, where $\mathbf{a}(t) = a_4(t) + ia_a(t)\tau^a$ is a variable representing rotations in spin or isospin space. In this case, the rotating $SU(2)$ gauge field is conveniently expressed by the following gauge transformation [18,21]:

$$A_M(t, x) = V A_M^{\text{cl}}(x; X(t), \rho(t)) V^{-1} - iV \partial_M V^{-1}, \quad (5)$$

where A_M^{cl} is the classical solution and V satisfies

$$\begin{aligned} -iV^{-1} \dot{V} &= -\dot{X}^M(t) A_M^{\text{cl}} + \chi^a f(\xi) g \frac{\tau^a}{2} g^{-1}, \\ \chi^a &= -i \text{tr}(\tau^a \mathbf{a}^{-1} \dot{\mathbf{a}}). \end{aligned}$$

We insert this gauge field in (1) and integrate with respect to (x^μ, z) . As a result, the instanton is quantized by the collective coordinate method. We obtain the collective Hamiltonian,

$$H = -\frac{1}{2M_0}(\partial_X^2 + \partial_Z^2) - \frac{1}{4M_0}\partial_{y_I}^2 + U(\rho, Z),$$

$$U(\rho, Z) = M_0 + \frac{M_0}{6}\rho^2 + \frac{N_c^2}{5M_0\rho^2} + \frac{M_0}{3}Z^2, \quad (6)$$

where $M_0 = 8\pi^2\kappa$ is the instanton mass and $y_I = \rho a_I$ ($I = 0, 1, 2, 3$).

The eigenstates of this Hamiltonian are characterized by quantum numbers $(l, I_3, s_3, n_\rho, n_z)$ and momentum \mathbf{p} : $l = 1, 3, 5, \dots$ are related to spin J and isospin I as $l/2 = J = I$. I_3 and s_3 are third components of the isospin and spin, respectively, and n_ρ and n_z the quantum numbers representing the oscillations in the radial and z directions, respectively. The ground state with $n_\rho = n_z = 0$ is the nucleon. In addition, the first excited states of $n_\rho = 1$ or of $n_z = 1$ corresponds to the $N(1440)$ or $N(1535)$, respectively. The wave functions of the ground states and the $N(1535)$ resonant state of the spin-upward proton are as follows [18,22]:

$$\psi_N \propto e^{i\mathbf{p}\cdot\mathbf{X}} R_N(\rho) \psi_Z(Z) (a_1 + ia_2),$$

$$\psi_{N^*} \propto e^{i\mathbf{p}\cdot\mathbf{X}} R_N(\rho) \psi'_Z(Z) (a_1 + ia_2), \quad (7)$$

where $N^* = N(1535)$ and

$$R_N(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}} e^{-\frac{M_0\rho^2}{\sqrt{6}}},$$

$$\psi_Z(Z) = e^{-\frac{M_0 Z^2}{\sqrt{6}}},$$

$$\psi'_Z(Z) = Z\psi_Z(Z).$$

B. The asymptotic solution of the instanton

In order to calculate the coupling constant of the interaction, we need to calculate the current. To define the current in the holographic model, we need to investigate the physics of the boundary based on the Gubser-Klebanov-Polyakov-Witten (GKP-W) method [23,24].

We have already seen the BPST instanton solution which is localized at $z \sim 0$ in the large λ limit. However, as we will see later, the current is defined at $z \rightarrow \pm\infty$, and therefore, this solution is not appropriate. We need to investigate properly the behavior of the soliton solution in a large z region. It is given in Ref. [22] as follows:

$$\hat{A}_0 = -\frac{1}{2a\lambda} G(\mathbf{x}, z; \mathbf{X}, Z),$$

$$\hat{A}_i = \frac{1}{2a\lambda} \left[\dot{X}^i + \frac{\rho^2}{2} \left(\frac{\chi^a}{2} \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z} \right) + \frac{\dot{\rho}}{\rho} \frac{\partial}{\partial X^i} \right) \right]$$

$$\times G(\mathbf{x}, z; \mathbf{X}, Z),$$

$$\hat{A}_z = \frac{1}{2a\lambda} \left[\dot{Z} + \frac{\rho^2}{2} \left(\frac{\chi^a}{2} \frac{\partial}{\partial X^a} + \frac{\dot{\rho}}{\rho} \frac{\partial}{\partial Z} \right) \right] H(\mathbf{x}, z; \mathbf{X}, Z),$$

$$A_0 = 4\pi^2 \rho^2 ia \dot{a}^{-1} G(\mathbf{x}, z; \mathbf{X}, Z)$$

$$+ 2\pi^2 \rho^2 a \tau^a a^{-1} \left(\dot{X}^i \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z} \right) + \dot{Z} \frac{\partial}{\partial X^a} \right)$$

$$\times G(\mathbf{x}, z; \mathbf{X}, Z),$$

$$A_i = -2\pi^2 \rho^2 a \tau^a a^{-1} \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z} \right) G(\mathbf{x}, z; \mathbf{X}, Z),$$

$$A_z = -2\pi^2 \rho^2 a \tau^a a^{-1} \frac{\partial}{\partial X^a} H(\mathbf{x}, z; \mathbf{X}, Z). \quad (8)$$

Here, $i = 1, 2, 3$, ϵ^{abc} is the completely antisymmetric tensor, and δ^{ab} the Kronecker delta. Green's functions $G(\mathbf{x}, z; \mathbf{X}, Z), H(\mathbf{x}, z; \mathbf{X}, Z)$ are

$$G(\mathbf{x}, z; \mathbf{X}, Z) = \kappa \sum_{n=1}^{\infty} \psi_n(z) \psi_n(Z) Y_n(|\mathbf{x} - \mathbf{X}|),$$

$$H(\mathbf{x}, z; \mathbf{X}, Z) = \kappa \sum_{n=0}^{\infty} \phi_n(z) \phi_n(Z) Y_n(|\mathbf{x} - \mathbf{X}|), \quad (9)$$

with

$$\phi_0(z) = \frac{1}{\sqrt{\kappa\pi}} \frac{1}{k(z)},$$

$$\phi_n(z) = \frac{1}{\sqrt{\lambda_n}} \partial_z \psi_n(z),$$

$$Y_n(r) = -\frac{1}{4\pi} \frac{e^{-\sqrt{\lambda_n} r}}{r}, \quad r = |\mathbf{x}|.$$

Here, $\{\psi_n(z)\}$ ($n = 1, 2, \dots$) is a complete set of functions of z consisting of the eigenfunctions of the eigenequation,

$$-h(z)^{-1} \partial_z (k(z) \partial_z \psi_n) = \lambda_n \psi_n, \quad (10)$$

with the normalization condition,

$$\kappa \int dz h(z) \psi_n \psi_m = \delta_{mn}.$$

We note that $\psi_{2n-1}(Z)$ and $\psi_{2n}(Z)$ are even and odd function of Z , respectively.

C. Currents

In Ref. [22], the left and right currents $\mathcal{J}_L^\mu, \mathcal{J}_R^\mu$ are defined by coupling with external fields $\mathcal{A}_{L\mu}, \mathcal{A}_{R\mu}$, respectively, at $z = \pm\infty$,

$$S|_{\mathcal{O}(\mathcal{A}_L, \mathcal{A}_R)} = -2 \int d^4x \text{tr}(\mathcal{A}_{L\mu} \mathcal{J}_L^\mu + \mathcal{A}_{R\mu} \mathcal{J}_R^\mu). \quad (11)$$

We introduce the external fields in the GKP-W method,

$$\begin{aligned}
 \mathcal{A}_\alpha(x^\mu, z) &= \mathcal{A}_\alpha^{\text{cl}}(x^\mu, z) + \delta\mathcal{A}_\alpha(x^\mu, z), \\
 \delta\mathcal{A}_\mu(x^\nu, z \rightarrow +\infty) &= \mathcal{A}_{L\mu}(x^\nu), \\
 \delta\mathcal{A}_\mu(x^\nu, z \rightarrow -\infty) &= \mathcal{A}_{R\mu}(x^\nu),
 \end{aligned} \tag{12}$$

where \mathcal{A}^{cl} is the classical solution and $\delta\mathcal{A}_\alpha$ an infinitesimal deviation. Substituting these fields in the action and taking only linear terms, we obtain

$$S|_{\mathcal{O}(\mathcal{A}_L, \mathcal{A}_R)} = \kappa \int d^4x 2 [\text{tr}(\delta\mathcal{A}^\mu k(z) \mathcal{F}_{\mu z}^{\text{cl}})]_{z=-\infty}^{z=+\infty}, \tag{13}$$

and so

$$\begin{aligned}
 \mathcal{J}_{L\mu} &= -\kappa(k(z) \mathcal{F}_{\mu z}^{\text{cl}})|_{z=+\infty}, \\
 \mathcal{J}_{R\mu} &= +\kappa(k(z) \mathcal{F}_{\mu z}^{\text{cl}})|_{z=-\infty}.
 \end{aligned} \tag{14}$$

The axial current is defined by

$$\mathcal{J}_A^\mu = \mathcal{J}_L^\mu - \mathcal{J}_R^\mu = -\kappa[\psi_0(z)k(z)\mathcal{F}_{\mu z}^{\text{cl}}]_{z=-\infty}^{z=+\infty}, \tag{15}$$

with $\psi_0(z) = (2/\pi) \arctan z$.

We define

$$\mathcal{J}_A^\mu = J_A^{a\mu} \frac{\tau^a}{2} + \hat{J}_A^\mu \frac{I_2}{2},$$

and use the asymptotic solutions. Thus, we obtain

$$\begin{aligned}
 J_A^0(r; \mathbf{X}, Z, \rho, \mathbf{a}) &= 2\pi^2 \kappa [\partial_0(\rho^2 \mathbf{a} \tau^a \mathbf{a}^{-1}) \partial_a H^A - 2\rho^2 i \mathbf{a} \dot{\mathbf{a}}^{-1} G^A \\
 &\quad - \rho^2 \mathbf{a} \tau^a \mathbf{a}^{-1} \dot{X}^i ((\partial_a \partial_i - \delta^{ia} \partial_j^2) H^A - \epsilon^{iaj} \partial_j G^A)], \\
 J_A^i(r; \mathbf{X}, Z, \rho, \mathbf{a}) &= -2\pi^2 \kappa \rho^2 \mathbf{a} \tau^a \mathbf{a}^{-1} ((\partial_i \partial_a - \delta^{ia} \partial_j^2) H^A - \epsilon^{iaj} \partial_j G^A),
 \end{aligned} \tag{16}$$

where

$$\begin{aligned}
 G^A(Z, r) &\equiv [\psi_0(z)k(z)\partial_z G]_{z=-\infty}^{z=+\infty} \\
 &= -\sum_{n=1}^{\infty} g_{a^n} \psi_{2n}(Z) Y_{2n}(r), \\
 H^A(Z, r) &\equiv [\psi_0(z)k(z)H]_{z=-\infty}^{z=+\infty} \\
 &= -\frac{1}{2\pi^2} \frac{1}{k(Z)} \frac{1}{r} - \sum_{n=1}^{\infty} \frac{g_{a^n}}{\lambda_{2n}} \partial_Z \psi_{2n}(Z) Y_{2n}(r), \\
 g_{a^n} &= \lambda_{2n} \kappa \int dz h(z) \psi_{2n} \psi_0,
 \end{aligned}$$

and $r = |\mathbf{x} - \mathbf{X}|$. Note that G^A and H^A are even and odd function of Z , respectively. In momentum space, we have the axial current as follows:

$$\tilde{J}_A^\mu(\mathbf{k}) = \int d^3x e^{-ik \cdot x} J_A^\mu(r), \tag{17}$$

$$\begin{aligned}
 \tilde{J}_A^{a0}(\mathbf{k}) &= 2\pi^2 \kappa \left\{ i \frac{k_a}{\mathbf{k}^2} \text{tr}[\tau^a \partial_0(\rho^2 \mathbf{a} \tau^b \mathbf{a}^{-1})] \right. \\
 &\quad \left. + \frac{P_i}{M_0} \rho^2 \text{tr}[\tau^a \mathbf{a} \tau^b \mathbf{a}^{-1}] \left(\delta^{ib} - \frac{k_i k_b}{\mathbf{k}^2} \right) \right\} \\
 &\quad \times e^{-ikX} \sum_{n=1}^{\infty} \frac{g_{a^n} \partial_Z \psi_{2n}}{\mathbf{k}^2 + \lambda_{2n}} \\
 &\quad + \left(I_a - 2\pi^2 \kappa \rho^2 \frac{P_i}{M_0} \epsilon^{iaj} k_j \right) e^{-ikX} \sum_{n=1}^{\infty} \frac{g_{a^n} \psi_{2n}}{\mathbf{k}^2 + \lambda_{2n}},
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 \tilde{J}_A^{ai}(\mathbf{k}) &= 2\pi^2 \kappa \rho^2 \text{tr}(\tau^a \mathbf{a} \tau^b \mathbf{a}^{-1}) \\
 &\quad \times e^{-ik \cdot X} \left[\left(\delta_{bi} - \frac{k_b k_i}{\mathbf{k}^2} \right) \sum_{n=1}^{\infty} \frac{g_{a^n} \partial_Z \psi_{2n}}{\mathbf{k}^2 + \lambda_{2n}} \right. \\
 &\quad \left. - i \epsilon^{ibj} k_j \sum_{n=1}^{\infty} \frac{g_{a^n} \psi_{2n}}{\mathbf{k}^2 + \lambda_{2n}} \right],
 \end{aligned} \tag{19}$$

where

$$\begin{aligned}
 P_i &= M_0 \dot{X}_i, \\
 I_a &= \frac{i}{2} \left(y_4 \frac{\partial}{\partial y_a} - y_a \frac{\partial}{\partial y_4} - \epsilon_{abc} y_b \frac{\partial}{\partial y_c} \right).
 \end{aligned}$$

III. DECAY PROPERTIES OF $N(1535)$

A. Axial coupling

Let us calculate the axial coupling of the $N(1535) \rightarrow N\pi$ decay process using the current we have calculated. It is given by

$$g_A^{NN^*}(\tau^a)_{I_3 I_3'} = 2 \int d^3x \langle N, I_3' | J_A^{a0} | N^*, I_3 \rangle e^{ik \cdot x}. \tag{20}$$

As we can see from (19), the spatial components of the currents $\tilde{J}_A^{ai}(\mathbf{k})$ do not contribute to the axial coupling because of the parity of the wave functions $\psi(z)$, $\psi'(z)$ in (7) with respect to Z and completely antisymmetric tensor. This fact is consistent with the following property. In the nonrelativistic limit, the axial current between N and N^* has the structure,

$$J_A^\mu = \bar{\psi}_N \gamma^\mu \psi_{N^*}, \quad \psi_B = \begin{pmatrix} u_B \\ 0 \end{pmatrix}, \quad B = N, N^*.$$

Note that γ_5 does not appear because of the difference in the parities of the initial and final states. Then, we see that spatial component vanishes while the time component contributes.

The axial coupling constant at an arbitrary value of \mathbf{k} is calculated by (20) to find the result, with

$$g_A^{NN^*}(\mathbf{k}^2) = \int_{-\infty}^{\infty} dZ \psi_Z(Z) \psi'_Z(Z) \sum_{n=1}^{\infty} \frac{g_{a^n} \Psi_{2n}(Z)}{\mathbf{k}^2 + \lambda_{2n}}, \quad (21)$$

where $\psi_Z(Z)$ and $\psi'_Z(Z)$ are normalized wave functions.

Before discussing the actual decay at a finite \mathbf{k} , we would like to briefly show the case at $\mathbf{k} = 0$, where some analytic calculations are possible. Using

$$\sum_{n=1}^{\infty} \frac{g_{a^n} \Psi_{2n}(Z)}{\lambda_{2n}} = \frac{2}{\pi} \arctan Z = \psi_0(Z), \quad (22)$$

we get

$$g_A^{NN^*} = \int_{-\infty}^{\infty} dZ \psi_Z(Z) \psi'_Z(Z) \psi_0(Z).$$

We obtain analytically with the result,

$$g_A^{NN^*} = \sqrt{\frac{2}{\pi}} e^{\frac{2}{\sqrt{6}} M_0} \operatorname{erfc}\left(\sqrt{\frac{2}{\sqrt{6}}} M_0\right), \quad (23)$$

where $\operatorname{erfc}(x)$ is the complementary error function.

For numerical estimate, we choose the parameters,

$$\begin{aligned} \kappa &= 0.00745, \\ M_{KK} &= 940 \text{ MeV}, \end{aligned}$$

which are related to

$$\begin{aligned} M_0 &= 8\pi^2 \kappa = 0.588, \\ \kappa &= \frac{\pi}{4} \frac{f_\pi^2}{M_{KK}^2}, \end{aligned}$$

and so $f_\pi = 92.4$ MeV. M_{KK} is determined to reproduce the mass of $\rho(776 \text{ MeV})$ [16]. Using the parameters, we obtain the value,

$$g_A^{NN^*} = 0.42. \quad (24)$$

B. Decay width

Let us calculate the decay width of the $N(1535) \rightarrow N\pi$ decay proses from the axial coupling we calculated, and compare it with the experimental value.

The decay width is given by

$$\begin{aligned} \Gamma_{N^* \rightarrow N\pi} &= \frac{1}{2m_{N^*}} \int \frac{d^3 p_N}{(2\pi)^3 2E_N} \frac{d^3 k}{(2\pi)^3 2E_\pi} \\ &\times (2\pi)^4 \delta^4(p_N + k - p_{N^*}) |t_{fi}|^2, \quad (25) \end{aligned}$$

$$\begin{aligned} t_{fi} &= \langle N(-k); \pi(k) | \mathcal{L} | N^*(0) \rangle \\ &= i \frac{g_A^{NN^*}}{2f_\pi} E_\pi \sqrt{E_N + m_N} \sqrt{2m_{N^*}} \delta_{s_3 s'_3}, \\ \mathcal{L} &= i \frac{g_A^{NN^*}}{2f_\pi} \bar{\psi}_N \gamma^0 \partial_0 \pi^a \tau^a \psi_{N^*}, \end{aligned}$$

where m_{N^*} is the mass of $N(1535)$, m_N the nucleon mass, E_N the energy of nucleon, $E_\pi = \sqrt{\mathbf{k}^2 + m_\pi^2}$, and m_π the pion mass. Hence, we obtain

$$\Gamma_{N^* \rightarrow N\pi} = \frac{g_A^{NN^*2} |\mathbf{k}| E_\pi^2 (E_N + m_N)}{16\pi f_\pi^2 m_{N^*}}, \quad (26)$$

where \mathbf{k} is the momentum of the emitted pion,

$$|\mathbf{k}| = \left[\frac{m_{N^*}^4 + m_N^4 + m_\pi^4}{4m_{N^*}^2} - \frac{2(m_{N^*}^2 m_N^2 + m_N^2 m_\pi^2 + m_\pi^2 m_{N^*}^2)}{4m_{N^*}^2} \right]^{1/2}.$$

Here, to obtain πNN^* coupling constant, we have used the Goldberger-Treiman relation,

$$g^{\pi NN^*} = \frac{m_{N^*} - m_N}{2f_\pi} g_A^{NN^*}. \quad (27)$$

From PDG, $m_{N^*} = 1510$ MeV, $m_N = 940$ MeV, $m_\pi = 140$ MeV, and $|\mathbf{k}| = 448$ MeV, where the mass of $N(1535)$ is the pole position. In addition, we need to compute $g_A(\mathbf{k}^2)$ at $|\mathbf{k}| = 448$ MeV. We have numerically computed the sum over n in (21) and verified a good convergence for the maximum value $n \sim 15$. The result is shown in Fig. 1 in the form of the form factor, $F(k^2) = g_A(\mathbf{k}^2)/g_A(0)$, as a function of $k^2 = |\mathbf{k}|^2$ in units of M_{KK}^2 . The actually emitted pion momentum $|\mathbf{k}| = 448$ MeV corresponds to $k^2 \sim 0.25$, where $F(k^2) \sim 0.75$. Thus, the value of the axial coupling

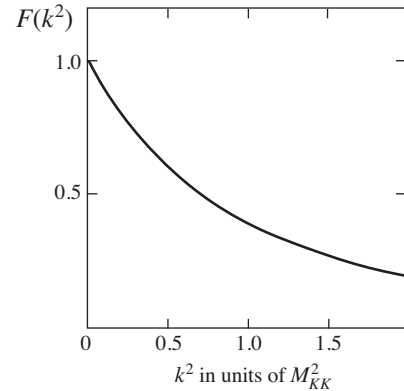


FIG. 1. The emitted pion momentum \mathbf{k} dependance of the form factor $F_A(\mathbf{k}^2)$.

$g_A^{NN^*}$ at this pion momentum is about $g_A^{NN^*}(|\mathbf{k}| \sim 450 \text{ MeV}) \sim 0.42 \times 0.75 \sim 0.32$ and the resulting decay width is $\Gamma_{N^* \rightarrow N\pi}(|\mathbf{k}| \sim 450 \text{ MeV}) \sim 30 \text{ MeV}$. This value is slightly smaller than the experimental value, which however is acceptable.

By using the form factor, we can also estimate the distribution (size) of the axial density, the axial radius. It is defined by

$$\frac{\partial F_A(k^2)}{\partial k^2} = -\frac{1}{6} \langle r^2 \rangle_A, \quad (28)$$

which is estimated to be $\langle r^2 \rangle^{1/2} \sim 0.5 \text{ fm}$. This corresponds to the axial mass around $1 \text{ GeV} \sim \sqrt{6} \langle r^2 \rangle_A^{-1/2}$. This value is close to the mass of the axial vector meson which is expected to be $m_{a_1} = \sqrt{2} m_\rho \sim 1080 \text{ MeV}$ [25].

IV. SUMMARY AND DISCUSSIONS

In this paper, the one-pion emission decay process $N(1535) \rightarrow \pi N$ is investigated using the Sakai-Sugimoto model, which is a model of holographic QCD. In this model, baryons are formed as instantons in five-dimensional spacetime. We defined the axial current according to the

formulation of [22] and calculated the value of the axial coupling for the $N(1535) \rightarrow \pi N$ process. Using the obtained axial coupling, we calculated the decay width of the $N(1535) \rightarrow \pi N$ process and obtained a value of 30 MeV. This value is in good agreement with the experimental value at the level of accuracy that is to be expected for models of this type.

In general, when the axial coupling constants g_A of various baryons are calculated using a soliton picture-based model, the obtained values are known to be about 30% smaller [26]. This tendency is also observed in the present and the recent studies for resonance properties. In Ref. [19], the decay of the Roper resonance $N(1440) \rightarrow \pi N$ was estimated to be 64 MeV which is some smaller than the corresponding experimental partial decay width, 90–140 MeV. Nevertheless, it is amusing that the holographic approach explains qualitatively well dynamical properties of baryon resonances as well as static properties.

ACKNOWLEDGMENTS

This work was supported in part by Grants-in-Aid for Scientific Research [Grant No. 21H04478(A)] and the one on Innovative Areas (Grant No. 18H05407).

-
- [1] N. Isgur and G. Karl, *Phys. Rev. D* **18**, 4187 (1978).
 - [2] N. Isgur and G. Karl, *Phys. Rev. D* **19**, 2653 (1979).
 - [3] N. Isgur and G. Karl, *Phys. Rev. D* **20**, 1191 (1979).
 - [4] A. J. Hey and R. L. Kelly, *Phys. Rep.* **96**, 71 (1983).
 - [5] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **124**, 246 (1961).
 - [6] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961).
 - [7] T. H. R. Skyrme, *Nucl. Phys.* **31**, 556 (1962).
 - [8] I. Zahed and G. Brown, *Phys. Rep.* **142**, 1 (1986).
 - [9] E. Witten, *Nucl. Phys.* **B160**, 57 (1979).
 - [10] E. Witten, *Nucl. Phys.* **B223**, 422 (1983).
 - [11] E. Witten, *Nucl. Phys.* **B223**, 433 (1983).
 - [12] U. B. Kaulfuss and U. G. Meissner, *Phys. Lett.* **154B**, 193 (1985).
 - [13] C. G. Callan and I. R. Klebanov, *Nucl. Phys.* **B262**, 365 (1985).
 - [14] C. G. Callan, K. Hornbostel, and I. R. Klebanov, *Phys. Lett. B* **202**, 269 (1988).
 - [15] T. Ezo and A. Hosaka, *Phys. Rev. D* **102**, 014046 (2020).
 - [16] T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **113**, 843 (2005).
 - [17] T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **114**, 1083 (2005).
 - [18] H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, *Prog. Theor. Phys.* **117**, 1157 (2007).
 - [19] D. Fujii and A. Hosaka, *Phys. Rev. D* **104**, 014022 (2021).
 - [20] A. J. Arifi, D. Suenaga, and A. Hosaka, *Phys. Rev. D* **103**, 094003 (2021).
 - [21] H. Hata, M. Murata, and S. Yamato, *Phys. Rev. D* **78**, 086006 (2008).
 - [22] K. Hashimoto, T. Sakai, and S. Sugimoto, *Prog. Theor. Phys.* **120**, 1093 (2008).
 - [23] S. Gubser, I. Klebanov, and A. Polyakov, *Phys. Lett. B* **428**, 105 (1998).
 - [24] E. Witten, *Adv. Theor. Math. Phys.* **2**, 235 (1998).
 - [25] S. Weinberg, *Phys. Rev.* **177**, 2604 (1969).
 - [26] G. S. Adkins, C. R. Nappi, and E. Witten, *Nucl. Phys.* **B228**, 552 (1983).