Analytic solutions of relativistic dissipative spin hydrodynamics with radial expansion in Gubser flow

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We have derived the analytic solutions of dissipative relativistic spin hydrodynamics with Gubser expansion. Following the standard strategy of deriving the solutions in a Gubser flow, we take the Weyl rescaling and obtain the energy-momentum and angular momentum conservation equations in the $dS_3 \times \mathbb{R}$ space-time. We then derive the analytic solutions of spin density, spin potential and other thermodynamic in $dS_3 \times \mathbb{R}$ space-time and transform them back into Minkowski space-time $\mathbb{R}^{3,1}$. In the Minkowski space-time, the spin density and spin potential including the information of radial expansion decay as $\sim L^{-2}\tau^{-1}$ and $\sim L^{-2}\tau^{-1/3}$ in large L limit, with τ being proper time and L being the characteristic length of the system, respectively. Moreover, we observe the nonvanishing spin corrections to the energy density and other dissipative terms in the Belinfante form of dissipative spin hydrodynamics. Our results can also be used as test beds for future simulations of relativistic dissipative spin hydrodynamics.

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I. INTRODUCTION

A large amount of orbital angular momentum perpendicular to the reaction plane is generated in noncentral relativistic heavy-ion collisions at Relativistic Heavy Ion Collider (RHIC). Due to the spin-orbit coupling, particles produced in collisions are polarized along the direction of initial angular momentum. Such kinds of polarization can be measured through the weak decay of Λ and $\bar{\Lambda}$ hyperons [1–3]. The global polarization of Λ and $\bar{\Lambda}$ hyperons measured by STAR Collaboration [4–6] confirms the early theoretical predictions [1–3] and is described by phenomenological models well [7–21]. One can also see the recent reviews [22–24] and references therein. Very recently, the global spin polarization at low energy region attracts lots of attention and needs to be systematically studied [25–30].

Furthermore, the STAR experiments have also measured the local spin polarization of Λ and $\bar{\Lambda}$ hyperons along the beam and out-of-plane directions [31,32]. The experimental data shows that the local spin polarization along the beam direction as a function of the azimuthal angle is opposite to the theoretical predictions from both transport and hydrodynamic models [12,13,18,33–35]. Such difference can not be explained by feed-down effects [36–38]. It is referred as the "sign problem" in the local spin polarization. There are also many other approaches, e.g., by considering hadronic interactions [39,40], quantum kinetic approaches [41–53], simulation for the chiral kinetic theory with side-jump effects [54], and relativistic spin hydrodynamics [55–71]. Although some approaches [54,72] agree with the data qualitatively and certain progress has been made, this problem has not been solved completely till now and requires further investigation.

One possible problem related to the sign problem is that the spin degrees of freedom may not reach the global equilibrium so that the Cooper-Frye formula [9,11] at global equilibrium fails to reproduce local spin polarization. It suggests that we need to add the spin degree of freedom to the current phenomenological frameworks and consider the off-equilibrium effects [22–24,32,73–77].

Very recently, the shear-induced polarization (SIP) as one of the off-equilibrium effects [73–77] has been proposed and plays an important role to the spin polarization. The numerical results from relativistic hydrodynamics including SIP can give a correct sign in the comparison with the experimental data in the named the strange quark equilibrium scenarios [75], or in the isothermal equilibrium scenarios [77]. On the other hand, the total polarization in strange quark equilibrium scenarios is also found to be sensitive to the equation of state, freeze-out temperature, and other parameters [78]. Similar studies on the parameter dependence at the $\sqrt{s_{NN}} = 19.6$ GeV collisions are shown in Ref. [79], and one can also see Refs. [20,80–83] for other

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related discussions. It indicates that the off-equilibrium effects need to be systematically studied in the future. Moreover, the modified Cooper-Frye formula including off-equilibrium effects, such as the effects of collisions [84] and spin [74] has been discussed.

On the other hand, there are two general ways to add the spin degree of freedom to the current phenomenological frames. As a macroscopic effective theory, the relativistic spin hydrodynamic is one possible way to consider the spin effects in the heavy ion collisions. In ordinary relativistic hydrodynamics, spin is not encoded into the conservation equations of energy, momentum, and net charge or baryon number. To describe the influence of spin, there are several ways to construct the relativistic spin hydrodynamics, such as through effective Lagrangian approaches [85,86], entropy current analysis [58,64,65,67,68,70], quantum statistical operators approach [7,87–89], and quantum kinetic theory [43,55,56,59,69,84,90]. Meanwhile, the microscopic descriptions for spin dynamics are the quantum kinetic theories for massive fermions with collisions [41–43,46,48,49,51,52,91,92], which are a natural extension of chiral kinetic theory [93–107]. Also, see recent reviews [22,23,66,108,109] and the reference therein.

Moreover, in recent studies [63,74,110], a modified Cooper-Frye formula with spin potential at local equilibrium has been derived, in which the spin potential, just like the thermal vorticity and shear viscous tensor, contributes to spin polarization pseudovector.

Although there are intensive discussions on relativistic spin hydrodynamics, the codes for the numerical simulations have not been developed yet. It results in the lack of decaying behavior of spin density and spin potential. To see the influence of spin potential in the modified Cooper-Frye formula [74,110], we need to know the decaying behavior of these terms. Meanwhile, the numerical simulations also require some analytic solutions in special configurations as the test beds.

To estimate the decay behavior of spin potential and find the suitable test beds for the future numerical simulations, we search for the analytic solutions of relativistic spin hydrodynamics at some certain configurations. Based on the canonical form of relativistic spin hydrodynamics [58,64,70], we have already derived the analytic solutions in Bjorken expansion [71]. Our results show that the spin density and spin potential decay as τ^{-1} and $\tau^{-1/3}$, respectively, where τ is the proper time. We also find that only one component of spin density, S^{xy}, does not accelerate the Bjorken velocity. The transverse expansion of the medium and other components of spin density were not allowed in our previous study [71].

In this work, we search for the analytical solutions of relativistic dissipative spin hydrodynamics in Gubser expansion by following the similar strategy for the relativistic magnetohydrodynamics [111–116]. The Gubser flow [117–129], which can describe the radial expansion, is closer to the reality in heavy-ion collisions. As expected, our analytical solution in a Gubser flow will contain the information of transverse expansion.

We emphasize that we derive the analytic solutions of relativistic spin hydrodynamics with radial expansion in a Gubser flow and it is different with some other approaches, in which the Bjorken or Gubser expansion is treated as the expanding background [129–132].

The paper is organized as follows: In Sec. II, we review the basic idea of Gubser flow. In Sec. III, we introduce the canonical form of spin hydrodynamics, the conservation equations, and constitutive relations in both Minkowski space $\mathbb{R}^{3,1}$ and $dS_3 \times \mathbb{R}$ space-time. In Sec. IV, we simplify the differential equations in a Gubser flow and derive the analytic solutions. We also discuss the results in the Belinfante form of spin hydrodynamics in Sec. IV D. We conclude and summarize this work in Sec. V.

Throughout this work, we choose the metric $g_{\mu\nu}$ in Minkowski space-time with Cartesian coordinates as $\mathrm{diag}\{-,+,+,+\}$, the normalized fluid velocity u^μ satisfying $u_\mu u^\mu = -1$, and the transverse projection operator $\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$. In curved space-time, we often use the covariant derivative ∇_μ while ∂_μ denotes the ordinary derivative. In addition, the symbols $\partial_\perp^\mu \equiv \Delta^{\mu\nu}\partial_\nu$ and $\nabla_\perp^\mu \equiv \Delta^{\mu\nu}\nabla_\nu$ represent projection derivatives. For simplicity, we define the symmetric and antisymmetric parts of an arbitrary tensor $A^{\mu\nu}$ as $A^{(\mu\nu)} \equiv (A^{\mu\nu} + A^{\nu\mu})/2$ and $A^{[\mu\nu]} \equiv (A^{\mu\nu} - A^{\nu\mu})/2$. When considering the viscous tensor, we also introduce the symmetric and traceless part of $A^{\mu\nu}$, $A^{\langle\mu\nu\rangle} \equiv \frac{1}{2}[\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\nu\alpha}\Delta^{\mu\beta}]A_{\alpha\beta} - \frac{1}{3}(\Delta^{\alpha\beta}A_{\alpha\beta})\Delta^{\mu\nu}$. To avoid being misleading, a physical variable A with a hat, i.e., $\hat{A}$, denotes that it is defined in $dS_3 \times \mathbb{R}$ space-time.

II. REVIEW ON GUBSER FLOW

In this section, following Refs. [117,118] we briefly review the main results in a Gubser flow. Besides the Bjorken boost invariance, Gubser flow can describe expansion with azimuthal symmetry in transverse plane [117,118].

Following [117,118], one can construct Gubser flow by imposing the "Gubser symmetry" $SO(3) \times SO(1,1) \times Z_2$, which strongly restricts the profile of fluid velocity. It is challenging to find the fluid velocity satisfying Gubser symmetry in Minkowski space-time $\mathbb{R}^{3,1}$. Fortunately, one can solve the hydrodynamic equations in the manifold $dS_3 \times \mathbb{R}$, in which dS_3 refers to the 3-dimensional de Sitter space-time, and transform the solution back to Minkowski space-time $\mathbb{R}^{3,1}$ through Weyl rescaling.

We rewrite the metric in Minkowski space-time $\mathbb{R}^{3,1}$ with coordinates (t, x, y, z) as

$$\begin{aligned} ds^2 &= -dt^2 + dx^2 + dy^2 + dz^2 \\ &= -d\tau^2 + dx_\perp^2 + x_\perp^2 d\varphi^2 + \tau^2 d\eta^2, \end{aligned} \tag{1}$$

where

$$x = x_\perp \cos\varphi, \qquad y = x_\perp \sin\varphi,$$
$$t = \tau\cosh\eta, \qquad z = \tau\sinh\eta. \tag{2}$$

The coordinates $\tau, x_\perp, \varphi$, and η denote longitudinal proper time, transverse plane radius, azimuthal angle, and rapidity, respectively. We then introduce a timelike hyperbola embedding into the manifold $\mathbb{R}^{3,1}$. The radius of this hyperbola is normalized to be 1, i.e.,

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 = 1, \tag{3}$$

where X_μ is the Cartesian coordinate in $\mathbb{R}^{3,1}$ and can be parametrized as:

$$X_0 = -\frac{L^2 - \tau^2 + x_\perp^2}{2\tau L}, \qquad X_1 = \frac{x_\perp \cos\varphi}{\tau},$$
$$X_3 = \frac{L^2 + \tau^2 - x_\perp^2}{2\tau L}, \qquad X_2 = \frac{x_\perp \sin\varphi}{\tau}. \tag{4}$$

Here, L is an adjustable parameter with dimension of length. The line element of dS_3 can be expressed as

$$ds_3^2 = \frac{1}{\tau^2}(-d\tau^2 + dx_\perp^2 + x_\perp^2 d\varphi^2). \tag{5}$$

Now, the metric of Minkowski space-time $\mathbb{R}^{3,1}$ can be transformed into the one of $dS_3 \times \mathbb{R}$ under Weyl rescaling with factor τ,

$$d\hat{s}^2 \equiv \frac{1}{\tau^2} ds^2 = \frac{1}{\tau^2}(-d\tau^2 + dx_\perp^2 + x_\perp^2 d\varphi^2) + d\eta^2, \tag{6}$$

where ds^2 is given by Eq. (1). Notice that Weyl rescaling is not a coordinate transformation [118]. Later, the metric (6) is rewritten by the Gubser coordinates $(\rho, \theta, \varphi, \eta)$,

$$d\hat{s}^2 = -d\rho^2 + \cosh^2\rho(d\theta^2 + \sin^2\theta d\varphi^2) + d\eta^2, \tag{7}$$

where

$$\sinh\rho = -\frac{L^2 - \tau^2 + x_\perp^2}{2L\tau},$$
$$\tan\theta = \frac{2Lx_\perp}{L^2 + \tau^2 - x_\perp^2}. \tag{8}$$

Calculations in the current work are performed mainly in these coordinates unless otherwise specified.

Now, we discuss the velocity profile in a Gubser flow. The Gubser symmetry requires that the normalized velocity must be

$$\hat{u}_\mu = (-1, 0, 0, 0) \tag{9}$$

in $dS_3 \times \mathbb{R}$ [117,118,133], which means the Gubser flow is static in the $(\rho, \theta, \varphi, \eta)$ coordinate system. Under the Weyl rescaling $u_\mu = \tau\hat{u}_\mu$ and coordinate transformation, one can derive the fluid velocity in the $(\tau, x_\perp, \varphi, \eta)$ coordinates in Minkowski space-time $\mathbb{R}^{3,1}$,

$$u_\mu = \left(-\frac{1}{\cosh\rho}\frac{L^2 + \tau^2 + x_\perp^2}{2L\tau}, \frac{1}{\cosh\rho}\frac{x_\perp}{L}, 0, 0\right), \tag{10}$$

where ρ is given by Eq. (8). Different with the standard Bjorken flow, velocity in a Gubser flow depends on the $x_\perp$.

Meanwhile, it can be proven that the fluid velocity (9) holds during the space-time evolution of the Gubser flows, i.e., the space-time covariant derivatives of fluid velocity (9) always vanish in a Gubser flow.

III. RELATIVISTIC DISSIPATIVE SPIN HYDRODYNAMICS

In this section, we extend the main equations in the relativistic dissipative spin hydrodynamics from Minkowski space-time $\mathbb{R}^{3,1}$ [58,64,65,68,70] to $dS_3 \times \mathbb{R}$ space-time. Note that, the energy momentum conservation and spin evolution equations are not invariant under Weyl rescaling [129,134].

A. Relativistic dissipative spin hydrodynamics in Minkowski space $\mathbb{R}^{3,1}$

The canonical form of energy momentum tensor $T^{\mu\nu}$ for dissipative spin hydrodynamics in Minkowski space-time reads [58,64,70],

$$T^{\mu\nu} = (e + p)u^\mu u^\nu + pg^{\mu\nu} + 2h^{(\mu}u^{\nu)} + \pi^{\mu\nu} + T^{[\mu\nu]}, \tag{11}$$

with energy density e, pressure p, heat flux h^μ, and viscosity tensor $\pi^{\mu\nu}$. The antisymmetric part $T^{[\mu\nu]}$ is further decomposed as

$$T^{[\mu\nu]} = 2q^{[\mu}u^{\nu]} + \phi^{\mu\nu}, \tag{12}$$

where $q^\mu \equiv u_\alpha \Delta^\mu_\beta T^{[\alpha\beta]}$ and $\phi^{\mu\nu} \equiv \Delta^\mu_\alpha \Delta^\nu_\beta T^{[\alpha\beta]}$. The q^μ and $\phi^{\mu\nu}$ play a role of the source to produce or absorb the spin.

The main equations for dissipative spin hydrodynamics are

$$\nabla_\mu T^{\mu\nu} = 0, \tag{13}$$

$$\nabla_\alpha \Sigma^{\alpha\mu\nu} = -2T^{[\mu\nu]}, \tag{14}$$

where $\Sigma^{\alpha\mu\nu}$ is the rank-three canonical spin component in the total angular momentum. Note that we have replaced the ordinary derivative ∂_μ with the covariant derivative ∇_μ for the general space-time. The Eq. (14) comes from the total angular momentum conservation and describes the spin evolution. Furthermore, one can decompose $\Sigma^{\alpha\mu\nu}$ as [58,64,70]

$$\Sigma^{\alpha\mu\nu} = u^\alpha S^{\mu\nu} + \Sigma_{(1)}^{\alpha\mu\nu}, \tag{15}$$

where $S^{\mu\nu} = -S^{\nu\mu}$ is named as the spin density and $\Sigma_{(1)}^{\alpha\mu\nu}$ is perpendicular to the fluid velocity $u_\alpha \Sigma_{(1)}^{\alpha\mu\nu} = 0$. The above decomposition for $\Sigma^{\alpha\mu\nu}$ is called the a non-anti-symmetric gauge which has been used in Refs. [55,58,64,65,67,68,85] and also in spin hydrodynamics for massless fermions [135]. One can construct a total antisymmetric tensor for the spin density, which has been used in Refs. [59,70].

We regard $S^{\mu\nu}$ as an independent variable and introduce the corresponding spin potential $\omega^{\mu\nu} = -\omega^{\nu\mu}$, which is conjugate to spin density $S^{\mu\nu}$. To add the effects of spin, the thermodynamic relations become

$$\begin{aligned} e + p &= Ts + \omega_{\mu\nu} S^{\mu\nu}, \\ dp &= s dT + S^{\mu\nu} d\omega_{\mu\nu}, \end{aligned} \tag{16}$$

where T and s denote the temperature and entropy density, respectively. For simplicity, here we set the particle number density and the charge or baryon chemical potential are always zero throughout the current work. To highlight the spin effect, we also neglect the heat flux h^μ in the energy-momentum tensor $T^{\mu\nu}$ in Eq. (11).

The power counting scheme can be assumed as $\omega_{\mu\nu} \sim \mathcal{O}(\partial^1)$, $S^{\mu\nu} \sim \mathcal{O}(\partial^0)$, and $\Sigma_{(1)}^{\alpha\mu\nu} \sim \mathcal{O}(\partial^1)$ [64]; or $\omega_{\mu\nu} \sim \mathcal{O}(\partial^1)$, $S^{\mu\nu} \sim \mathcal{O}(\partial^1)$, and $\Sigma_{(1)}^{\alpha\mu\nu} \sim \mathcal{O}(\partial^2)$ [58,70].

The spin evolution equation (14) becomes

$$\nabla_\alpha (u^\alpha S^{\mu\nu}) = -4 q^{[\mu} u^{\nu]} - 2\phi^{\mu\nu}. \tag{17}$$

According to the second law of thermodynamics [58,64] or the effective theories [70], constitutive equations are given by

$$\begin{aligned} \pi^{\mu\nu} &= -\eta_s \nabla^{\langle\mu} u^{\nu\rangle} - \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha, \\ q^\mu &= -\lambda \left(\frac{1}{T} \nabla_\perp^\mu T - u^\alpha \nabla_\alpha u^\mu - 4\omega^{\mu\nu} u_\nu \right), \\ \phi^{\mu\nu} &= -\gamma (\nabla_\perp^{[\mu} u^{\nu]} - 2\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta}), \end{aligned} \tag{18}$$

where η_s and ζ are the shear viscosity and bulk viscosity, respectively, and λ, γ are two new transport coefficients related to the spin. The four coefficients $\eta_s, \zeta, \lambda, \gamma$ in the constitutive relations are all non-negative.

B. Main equations in $dS_3 \times \mathbb{R}$ space-time

In this subsection, we transform the spin hydrodynamic equations in the previous subsection to the $dS_3 \times \mathbb{R}$ space-time.

The Weyl rescaling in Eq. (6) implies that $g_{\mu\nu} \to \hat{g}_{\mu\nu} = \frac{1}{\tau^2} g_{\mu\nu}$ and $g^{\mu\nu} \to \hat{g}^{\mu\nu} = \tau^2 g^{\mu\nu}$. The Christoffel symbols $\hat{\Gamma}^\lambda_{\mu\nu}$ in $dS_3 \times \mathbb{R}$ space-time are related to the one in the Minkowski space-time $\Gamma^\lambda_{\mu\nu}$ by the following relation [129,136,137]:

$$\Gamma^\lambda_{\mu\nu} = \hat{\Gamma}^\lambda_{\mu\nu} + \frac{1}{\tau} (\delta^\lambda_\nu \hat{\nabla}_\mu \tau + \delta^\lambda_\mu \hat{\nabla}_\nu \tau - \hat{g}_{\mu\nu} \hat{g}^{\lambda\alpha} \hat{\nabla}_\alpha \tau), \tag{19}$$

where $\hat{\nabla}_\mu$ is also defined in the $dS_3 \times \mathbb{R}$ space-time. Similarly, we introduce the energy-momentum tensor in the $dS_3 \times \mathbb{R}$ space-time,

$$\hat{T}^{\mu\nu} \equiv \tau^\alpha T^{\mu\nu}, \tag{20}$$

with α being a constant. The energy-momentum conservation (13) becomes [136]

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= \partial_\mu T^{\mu\nu} + \Gamma^\mu_{\mu\lambda} T^{\lambda\nu} + \Gamma^\nu_{\mu\lambda} T^{\mu\lambda} \\ &= \tau^{-\alpha} [\hat{\nabla}_\mu \hat{T}^{\mu\nu} - 2\tau^{-1} \hat{T}^{[\mu\nu]} \hat{\nabla}_\mu \tau] \\ &\quad + \tau^{-\alpha-1} [(6-\alpha) \hat{T}^{\lambda\nu} \hat{\nabla}_\lambda \tau - \hat{T}^\mu{}_\mu \hat{g}^{\nu\alpha} \hat{\nabla}_\lambda \tau]. \end{aligned} \tag{21}$$

Obviously, only when $\alpha = 6$ and $\hat{T}^\mu_\mu = \tau^\alpha T^\mu_\mu = 0$ is traceless, the last term proportional to $\tau^{-\alpha-1}$ vanishes. The traceless condition of $\hat{T}^{\mu\nu}$ is satisfied in a conformal fluid [138–144], in which the bulk viscosity is zero $\zeta = 0$ and the $e = 3p$.

For simplicity, following the common strategy in a Gubser flow [117,118], we choose

$$\alpha = 6 \tag{22}$$

and set $\zeta = 0$ from now on. The energy-momentum conservation (13) then reduces to

$$\hat{\nabla}_\mu \hat{T}^{\mu\nu} - 2\tau^{-1} \hat{T}^{[\mu\nu]} \hat{\nabla}_\mu \tau = 0. \tag{23}$$

In general, the transformation rule for a physical variable $\hat{A}$ in $dS_3 \times \mathbb{R}$ space-time to its corresponding quantity A in Minkowski space-time is [118,140,141,145]

$$\hat{A}^{\mu_1 \dots \mu_m}_{\nu_1 \dots \nu_n}(x) = \tau^{\Delta_A} A^{\mu_1 \dots \mu_m}_{\nu_1 \dots \nu_n}(x), \tag{24}$$

where $\Delta_A = [A] + m - n$, and $[A]$ is the mass dimension of A.

Next, we discuss the spin evolution equation (17). From Eq. (19), the spin evolution equation (17) is not covariant under Weyl rescaling. In $dS_3 \times \mathbb{R}$ space-time, Eq. (17) becomes

$$\begin{aligned} \hat{\nabla}_\alpha (\hat{u}^\alpha \hat{S}^{\mu\nu}) = {} & (\hat{u}_\alpha \hat{S}^{\alpha\nu} \hat{g}^{\mu\beta} + \hat{u}_\alpha \hat{S}^{\mu\alpha} \hat{g}^{\nu\beta} + \hat{u}^\mu \hat{S}^{\nu\beta} - \hat{u}^\nu \hat{S}^{\mu\beta}) \tau^{-1} \hat{\nabla}_\beta \tau \\ & - 4\hat{q}^{[\mu} \hat{u}^{\nu]} - 2\hat{\phi}^{\mu\nu}, \end{aligned} \tag{25}$$

where we have used

$$\nabla_\alpha(u^\alpha S^{\mu\nu}) = \tau^{-6}\hat{\nabla}_\alpha(\hat{u}^\alpha \hat{S}^{\mu\nu}) - \tau^{-7}(\hat{u}_\alpha \hat{S}^{\alpha\nu}\hat{g}^{\mu\beta} + \hat{u}_\alpha \hat{S}^{\mu\alpha}\hat{g}^{\nu\beta})\hat{\nabla}_\beta\tau - \tau^{-7}(\hat{u}^\mu \hat{S}^{\nu\beta} - \hat{u}^\nu \hat{S}^{\mu\beta})\hat{\nabla}_\beta\tau. \quad (26)$$

Equation (25) has several new terms proportional to $\tau^{-1}\hat{\nabla}_\beta\tau$.

In this subsection, we extend the energy-momentum and angular momentum conservation equations to the $dS_3\times\mathbb{R}$ space-time. Unfortunately, these two kinds of conservation equations are not conformal invariant, i.e., we find that there are extra terms $\sim\hat{\nabla}_\mu\tau$ in Eqs. (23) and (25) under Weyl rescaling.

Here, we emphasize that we do not consider the conformal fluid in the current study. In an ordinary fluid without spin, the antisymmetric part of energy-momentum tensor $T^{[\mu\nu]}$ is zero, and Eq. (23) reduces to the simplest expression $\hat{\nabla}_\mu \hat{T}^{\mu\nu} = 0$. However, as discussed in Sec. III A, the antisymmetric part of energy-momentum tensor $T^{[\mu\nu]}$ is nonvanishing in the spin hydrodynamics. We keep the general expression (23) for energy-momentum conservation here. Later, we will solve the conservation equation (23) in Sec. IV.

C. Constitutive equations in $dS_3\times\mathbb{R}$ space-time

In this subsection, we discuss the constitutive equations (11), (18) in $dS_3\times\mathbb{R}$ space-time. The decomposition of $\hat{T}^{\mu\nu}$ in $dS_3\times\mathbb{R}$ space-time is similar to Eq. (11), (12):

$$\hat{T}^{\mu\nu} = (\hat{e}+\hat{p})\hat{u}^\mu\hat{u}^\nu + \hat{p}\hat{g}^{\mu\nu} + 2\hat{h}^{(\mu}\hat{u}^{\nu)} + \hat{\pi}^{\mu\nu} + 2\hat{q}^{[\mu}\hat{u}^{\nu]} + \hat{\phi}^{\mu\nu}. \quad (27)$$

By using Eq. (24), we find that $\hat{u}^\mu = u^\mu/\tau$, $\hat{\pi}^{\mu\nu} = \tau^6\pi^{\mu\nu}$ and thermodynamic variables become

$$\hat{e} = \tau^4 e, \qquad \hat{T} = \tau T, \qquad \hat{s} = \tau^3 s. \quad (28)$$

Applying Eq. (19), the transformation of $\nabla_\mu u^\nu$ is given by [141]

$$\nabla_\mu u^\nu = \tau^{-1}\hat{\nabla}_\mu\hat{u}^\nu + \tau^{-2}\delta^\nu_\mu\hat{u}^\lambda\hat{\nabla}_\lambda\tau - \tau^{-2}\hat{u}_\mu\hat{g}^{\nu\alpha}\hat{\nabla}_\alpha\tau. \quad (29)$$

It is straightforward to show that the bulk viscosity term $\zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha$ does not transform homogeneously. Based on Eq. (29), we can get [140,141,145]

$$\nabla^{\langle\mu}u^{\nu\rangle} = \tau^{-3}\hat{\nabla}^{\langle\mu}\hat{u}^{\nu\rangle}, \qquad \nabla_\perp^{[\mu}u^{\nu]} = \tau^{-3}\hat{\nabla}_\perp^{[\mu}\hat{u}^{\nu]}, \quad (30)$$

which lead to a compact form for $\hat{\pi}^{\mu\nu}$ and $\hat{\phi}^{\mu\nu}$,

$$\hat{\pi}^{\mu\nu} = \tau^6\pi^{\mu\nu} = -\hat{\eta}_s\hat{\nabla}^{\langle\mu}\hat{u}^{\nu\rangle}$$
$$\hat{\phi}^{\mu\nu} = \tau^6\phi^{\mu\nu} = -\hat{\gamma}(\hat{\nabla}_\perp^{[\mu}\hat{u}^{\nu]} - 2\hat{\Delta}^{\mu\alpha}\hat{\Delta}^{\nu\beta}\hat{\omega}_{\alpha\beta}), \quad (31)$$

where

$$\hat{\gamma} = \tau^3\gamma, \qquad \hat{\eta}_s = \tau^3\eta_s. \quad (32)$$

Therefore, $\hat{\pi}^{\mu\nu}$ and $\hat{\phi}^{\mu\nu}$ have the same structure as $\pi^{\mu\nu}$ and $\phi^{\mu\nu}$. Note that we deliberately write $\hat{\eta}_s$ and $\hat{\gamma}$ as $(\hat{\eta}_s/\hat{s})\hat{s}$ and $(\hat{\gamma}/\hat{s})\hat{s}$, respectively. The $\hat{\eta}_s/\hat{s}$ and $\hat{\gamma}/\hat{s}$ are dimensionless scalars which do not get modified when passing from Minkowski space-time $\mathbb{R}^{3,1}$ to $dS_3\times\mathbb{R}$ space-time. We follow the standard strategy in Gubser flows and set

$$\frac{\hat{\eta}_s}{\hat{s}} = \frac{\eta}{s} = \text{constant}, \qquad \frac{\hat{\gamma}}{\hat{s}} = \frac{\gamma}{s} = \text{constant}. \quad (33)$$

Unfortunately, $\hat{q}^\mu$ becomes

$$\hat{q}^\mu = \tau^5 q^\mu = -\hat{\lambda}\left(\frac{1}{\hat{T}}\hat{\nabla}^\mu_\perp\hat{T} - \hat{u}^\alpha\hat{\nabla}_\alpha\hat{u}^\mu - 4\hat{\omega}^{\mu\nu}\hat{u}_\nu - 2\tau^{-1}\hat{\Delta}^{\mu\alpha}\hat{\nabla}_\alpha\tau\right), \quad (34)$$

where $\hat{\lambda} = \tau^3\lambda$. The last term in the bracket $-2\tau^{-1}\hat{\Delta}^{\mu\alpha}\hat{\nabla}_\alpha\tau$ is generated by the Weyl rescaling. For simplicity, we need to set $\lambda = \hat{\lambda} = 0$ in the current work, i.e., we set

$$q^\mu = \hat{q}^\mu = 0. \quad (35)$$

In fact, we have also checked that the nonzero $\hat{q}^\mu$ breaks the Gubser symmetry and will change the velocity $\hat{u}_\mu = (-1,0,0,0)$ due to the extra term $-2\tau^{-1}\hat{\Delta}^{\mu\alpha}\hat{\nabla}_\alpha\tau$.

In this section, we extend the conservation equations and constitutive equations for the relativistic spin hydrodynamics from Minkowski space-time $\mathbb{R}^{3,1}$ to the $dS_3\times\mathbb{R}$ space-time. We find that neither the energy momentum conservation equation (23) nor the spin evolution equation (25) is covariant after Weyl rescaling. We also get the constitutive equations in the $dS_3\times\mathbb{R}$ space-time shown in Eqs. (27) and (31). We further set the bulk viscosity $\zeta = 0$ and $\hat{q}^\mu = 0$ for simplicity.

IV. ANALYTIC SOLUTIONS IN GUBSER FLOW

In this section, we derive the analytic solutions of dissipative spin hydrodynamics in a Gubser flow in high temperature limit.

We adopt the strategy similar to our previous works [71,111–116]. First, we consider the thermodynamic relations and equations of state. We express the energy density $\hat{e}$ and entropy density $\hat{s}$ in the $dS_3\times\mathbb{R}$ space-time as polynomial functions of temperature $\hat{T}$ and spin chemical potential $\hat{\omega}^{\mu\nu}$ in Sec. IV A. Secondly, in Sec. IV B, we concentrate on the fluid acceleration equations and search for the special configurations for dissipative spin hydrodynamics, which do not modify the fluid velocity (9) in a Gubser flow. After that we succeed in finding self-consistent analytical solutions for $\hat{e}$ and $\hat{S}^{\mu\nu}$. Finally, in Sec. IV C, we convert the solutions obtained in $dS_3 \times \mathbb{R}$ space-time back to Minkowski space-time $\mathbb{R}^{3,1}$ and compare them with the our solutions in a Bjorken flow [71]. We also discuss the results for the spin hydrodynamics in the Belinfante form in Sec. IV D. Throughout this section, we use the Gubser coordinates $(\rho, \theta, \phi, \eta)$ in $dS_3 \times \mathbb{R}$ space-time if not specified.

A. Thermodynamic relations and equations of motion

According to Eqs. (16) and (24), we rewrite the thermodynamic relations in $dS_3 \times \mathbb{R}$ space-time:

$$\begin{aligned} \hat{e} + \hat{p} &= \hat{T}\,\hat{s} + \hat{\omega}_{\mu\nu}\hat{S}^{\mu\nu}, \\ d\hat{p} &= \hat{s}d\hat{T} + \hat{S}^{\mu\nu}d\hat{\omega}_{\mu\nu}. \end{aligned} \tag{36}$$

Again, for simplicity, we set the number density and chemical potential at zero. Following the standard Gubser flows [117,118], we can assume that the thermodynamic variables $\hat{e}, \hat{p}, \hat{T}, \hat{s}$ and transport coefficients $\hat{\gamma}, \hat{\eta}_s$ are only functions of de Sitter time ρ. It suggests a natural assignment that $\hat{\omega}_{\mu\nu}\hat{S}^{\mu\nu}$ depends on ρ only. We emphasize that due to the nontrivial metric $\hat{g}_{\mu\nu} = \mathrm{diag}\{-1, \cosh^2\rho, \cosh^2\rho \sin^2\theta, 1\}$ in the $dS_3 \times \mathbb{R}$ space-time, $\hat{S}^{\mu\nu}$ and $\hat{\omega}^{\mu\nu}$ may be the functions of both ρ and θ.

To close the system, we need the equations of state besides Eq. (36). In III B, we assume that

$$\hat{e} = c_s^{-2}\hat{p} = 3\hat{p}, \tag{37}$$

which is a reasonable approximation in the ultrarelativistic or high temperature limits. Here, c_s^2 is the speed of sound and usually one can choose $c_s^2 = 1/3$ for simplicity. We emphasize that EoS (37) does not imply the system is conformal invariant. In fact, there is no conformal symmetry in our system. More discussion will be shown in the next subsection. On the other hand, inspired by the relation between particle number density and chemical potential, we assume another equation of state in high temperature limit [71], i.e.,

$$\hat{S}^{\mu\nu} = a\hat{T}^2\hat{\omega}^{\mu\nu}, \tag{38}$$

with dimensionless constant a. Equations (37) and (38) are regarded as two given conditions in the subsequent discussion.

For convenience, we further define a new auxiliary variable $\bar{\omega}^2$:

$$\bar{\omega}^2 \equiv \frac{\omega^{\mu\nu}\omega_{\mu\nu}}{T^2} = \frac{\hat{\omega}^{\mu\nu}\hat{\omega}_{\mu\nu}}{\hat{T}^2}, \tag{39}$$

which is a dimensionless scalar and invariant under Weyl rescaling.

Utilizing these equations of state and transformation rule Eq. (24), we can rewrite the thermodynamic relations Eq. (36) as

$$\begin{aligned} \frac{4}{3}\hat{e} &= \hat{T}\,\hat{s} + a\hat{T}^4\bar{\omega}^2, \\ \frac{1}{3}d\hat{e} &= (\hat{s} + a\hat{T}^3\bar{\omega}^2)d\hat{T} + \frac{1}{2}a\hat{T}^4 d(\bar{\omega}^2). \end{aligned} \tag{40}$$

From Eq. (40), one can express $\hat{e} = \hat{e}(\hat{T}, \bar{\omega}^2)$ and $\hat{s} = \hat{s}(\hat{T}, \bar{\omega}^2)$ as,

$$\hat{e} = \hat{T}^4\left(c_0 + \frac{3}{2}a\bar{\omega}^2\right), \tag{41}$$

$$\hat{s} = \hat{T}^3\left(\frac{4}{3}c_0 + a\bar{\omega}^2\right), \tag{42}$$

where

$$c_0 \equiv \frac{\hat{e}_0}{\hat{T}_0^4} - \frac{3}{2}a\bar{\omega}_0^2 = \frac{3}{4}\frac{\hat{s}_0}{\hat{T}_0^3} - \frac{3}{4}a\bar{\omega}_0^2 \tag{43}$$

is a constant determined by initial values $\hat{e}_0 = \hat{e}(\rho_0)$, $\hat{s}_0 = \hat{s}(\rho_0)$, $\hat{T}_0 = \hat{T}(\rho_0)$, and $\bar{\omega}_0^2 = \bar{\omega}^2(\rho_0)$.

B. Simplify the differential equations

In this subsection, our task is to find special configuration to hold the fluid velocity in a Gubser flow and simplify main differential equations (23) and (25).

Contracting the projector $\hat{\Delta}_{\alpha\nu} = \hat{g}_{\alpha\nu} + \hat{u}_\alpha\hat{u}_\nu$ in the $dS_3 \times \mathbb{R}$ space-time with both sides of Eq. (23), yields the acceleration equation for the fluid velocity:

$$\hat{\Delta}_{\alpha\nu}[\hat{\nabla}_\mu\hat{T}^{\mu\nu} - 2\tau^{-1}\hat{T}^{[\mu\nu]}\hat{\nabla}_\mu\tau] = 0. \tag{44}$$

Plugging Eq. (27) into it, we get

$$\begin{aligned} \hat{u}^\mu\hat{\nabla}_\mu\hat{u}_\alpha = -\frac{1}{\hat{e}+\hat{p}}[&\hat{\Delta}^\mu_\alpha\hat{\nabla}_\mu\hat{p} + \hat{\Delta}_{\nu\alpha}\hat{\nabla}_\mu\hat{\pi}^{\mu\nu} + \hat{\Delta}_{\nu\alpha}\hat{\nabla}_\mu\hat{\phi}^{\mu\nu} \\ &- 2\tau^{-1}\hat{g}_{\alpha\nu}\hat{\phi}^{\mu\nu}\hat{\nabla}_\mu\tau]. \end{aligned} \tag{45}$$

To compute Eq. (45), we find that only six Christoffel symbols $\hat{\Gamma}^\lambda_{\mu\nu}$ in Gubser coordinates $(\rho, \theta, \varphi, \eta)$ are nonzero,

$$\begin{aligned} \hat{\Gamma}^\rho_{\theta\theta} &= \cosh\rho\sinh\rho, & \hat{\Gamma}^\rho_{\varphi\varphi} &= \cosh\rho\sinh\rho\sin^2\theta, \\ \hat{\Gamma}^\theta_{\rho\theta} &= \tanh\rho, & \hat{\Gamma}^\theta_{\varphi\varphi} &= -\sin\theta\cos\theta, \\ \hat{\Gamma}^\varphi_{\rho\varphi} &= \tanh\rho, & \hat{\Gamma}^\varphi_{\theta\varphi} &= \cot\theta. \end{aligned} \tag{46}$$

Then, it is straightforward to get the nonzero components of $\hat{\pi}^{\mu\nu}$ and $\hat{\phi}^{\mu\nu}$ from Eq. (31):

$$\begin{aligned} \hat{\pi}^{\theta\theta} &= -\frac{1}{3}\hat{\eta}_s \cosh^{-2}\rho \tanh\rho, \\ \hat{\pi}^{\varphi\varphi} &= -\frac{1}{3}\hat{\eta}_s \cosh^{-2}\rho \sin^{-2}\theta \tanh\rho, \\ \hat{\pi}^{\eta\eta} &= \frac{2}{3}\hat{\eta}_s \tanh\rho, \\ \hat{\phi}^{ij} &= 2\hat{\gamma}\hat{\omega}^{ij}, \qquad (i,j=\theta,\varphi,\eta). \end{aligned} \tag{47}$$

With the assumption that $\hat{e}$ and $\hat{p}$ depend on ρ only, which leads to $\hat{\Delta}^\mu_\alpha \hat{\nabla}_\mu \hat{p}(\rho) = 0$, Eq. (45) then becomes:

$$\hat{u}^\mu \hat{\nabla}_\mu \hat{u}_\rho = 0, \tag{48}$$

$$\hat{u}^\mu \hat{\nabla}_\mu \hat{u}_\theta = 0, \tag{49}$$

$$\hat{u}^\mu \hat{\nabla}_\mu \hat{u}_\varphi = \frac{2\cosh^2\rho \sin^2\theta}{\hat{e}+\hat{p}}\left(\frac{\hat{\gamma}}{\hat{s}}\right)\hat{s}(-\partial_\theta \hat{\omega}^{\theta\varphi} - \cot\theta \hat{\omega}^{\theta\varphi} + 2\hat{\omega}^{\theta\varphi}\tau^{-1}\partial_\theta\tau), \tag{50}$$

$$\hat{u}^\mu \hat{\nabla}_\mu \hat{u}_\eta = \frac{2}{\hat{e}+\hat{p}}\left(\frac{\hat{\gamma}}{\hat{s}}\right)\hat{s}(-\partial_\theta \hat{\omega}^{\theta\eta} - \cot\theta \hat{\omega}^{\theta\eta} + 2\hat{\omega}^{\theta\eta}\tau^{-1}\partial_\theta\tau). \tag{51}$$

Obviously, when $\hat{\omega}^{\theta\varphi}, \hat{\omega}^{\theta\eta} = 0$, $\hat{u}^\mu \hat{\nabla}_\mu \hat{u}_\nu = 0$, i.e., the Gubser velocity (9) holds during the evolution if $\hat{\omega}^{\theta\varphi}, \hat{\omega}^{\theta\eta}$ always vanish. Later, we will check that $\hat{\omega}^{\theta\varphi}, \hat{\omega}^{\theta\eta}$ vanish under appropriate initial conditions and Gubser velocity (9).

Contacting $\hat{u}_\nu$ with both sides of Eq. (23) provides the conservation equation for energy:

$$\hat{u}_\nu[\hat{\nabla}_\mu \hat{T}^{\mu\nu} - 2\tau^{-1}\hat{T}^{[\mu\nu]}\hat{\nabla}_\mu\tau] = 0. \tag{52}$$

Using Eqs. (27), (37), and (47), the evolution of energy density (52) reads

$$\frac{d\hat{e}}{d\rho} + \frac{8}{3}\hat{e}\tanh\rho - \frac{2}{3}\left(\frac{\hat{\eta}_s}{\hat{s}}\right)\hat{s}\tanh^2\rho = 0. \tag{53}$$

Equation (53) is the same as the one in ordinary relativistic hydrodynamics without spin effect in a Gubser flow [117,118] (also see Refs. [121,123] for extensions).

Third, we compute the evolution of spin following Eq. (25). After a long and tedious calculation, we eventually obtain six independent equations for the evolution of spin from Eq. (25):

$$\partial_\rho \hat{S}^{\rho\varphi} + 3\tanh\rho \hat{S}^{\rho\varphi} + \hat{S}^{\theta\varphi}\tau^{-1}\partial_\theta\tau = 0, \tag{54}$$

$$\partial_\rho \hat{S}^{\rho\eta} + 2\tanh\rho \hat{S}^{\rho\eta} + \hat{S}^{\theta\eta}\tau^{-1}\partial_\theta\tau = 0, \tag{55}$$

$$\partial_\rho \hat{S}^{\theta\varphi} + 4\tanh\rho \hat{S}^{\theta\varphi} + \cosh^{-2}\rho \hat{S}^{\rho\varphi}\tau^{-1}\partial_\theta\tau + 4\left(\frac{\hat{\gamma}}{\hat{s}}\right)\hat{s}\hat{\omega}^{\theta\varphi} = 0, \tag{56}$$

$$\partial_\rho \hat{S}^{\theta\eta} + 3\tanh\rho \hat{S}^{\theta\eta} + \cosh^{-2}\rho \hat{S}^{\rho\eta}\tau^{-1}\partial_\theta\tau + 4\left(\frac{\hat{\gamma}}{\hat{s}}\right)\hat{s}\hat{\omega}^{\theta\eta} = 0, \tag{57}$$

and

$$\partial_\rho \hat{S}^{\varphi\eta} + 3\tanh\rho \hat{S}^{\varphi\eta} + 4\left(\frac{\hat{\gamma}}{\hat{s}}\right)\hat{s}\hat{\omega}^{\varphi\eta} = 0, \tag{58}$$

$$\partial_\rho \hat{S}^{\rho\theta} + 3\tanh\rho \hat{S}^{\rho\theta} = 0. \tag{59}$$

As mentioned in Eq. (51), the fixed Gubser velocity requires that $\hat{\omega}^{\theta\varphi}, \hat{\omega}^{\theta\eta} = 0$. This requirement leads to all $\hat{S}^{\theta\varphi}, \hat{S}^{\theta\eta}$ always being zero during the evolution through the EoS (38). Unfortunately, $\hat{S}^{\theta\varphi}, \hat{S}^{\theta\eta}$ are coupled to $\hat{S}^{\rho\varphi}, \hat{S}^{\rho\eta}$ through Eqs. (54)–(57). To satisfy the requirement from Eq. (51), we have to choose the trivial solutions of Eqs. (54)–(57), which is $\hat{S}^{\theta\varphi}(\rho,\theta) = \hat{S}^{\theta\eta}(\rho,\theta) = 0$ and $\hat{S}^{\rho\varphi}(\rho,\theta) = \hat{S}^{\rho\eta}(\rho,\theta) = 0$.

Remarkably, in the space-time $dS_3 \times \mathbb{R}$, there are extra terms proportional to $\hat{\nabla}_\alpha\tau$ in both energy-momentum conservation equation (23) and the evolution equations for spin (25). As mentioned in the previous subsection, these terms come from the Weyl rescaling and cannot be neglected in general. Fortunately, in the configuration for the Gubser flow, all of these terms vanish in Eqs. (53), (58), and (59). It is of great help for us to derive the analytic solutions in the relativistic spin hydrodynamics in a Gubser flow. At last, we only have three independent differential equations, i.e., conservation equation for energy (53) and evolution equations for spin (58), (59).

C. Analytic solutions in $dS_3 \times \mathbb{R}$ and $\mathbb{R}^{3,1}$ space-time

In this subsection, we solve the differential equations (53), (58), and (59) for the spin hydrodynamics in a Gubser flow. We then transform our solutions in $dS_3 \times \mathbb{R}$ space-time to the Minkowski space-time $\mathbb{R}^{3,1}$.

We consider the high temperature limit and the spin chemical potential is much smaller than temperature in the relativistic heavy ion collisions, i.e., $\omega^{\mu\nu} \ll T$, or

$$\bar{\omega}^2 = \frac{\omega^{\mu\nu}\omega_{\mu\nu}}{T^2} = \frac{\hat{\omega}^{\mu\nu}\hat{\omega}_{\mu\nu}}{\hat{T}^2} \ll 1. \tag{60}$$

We emphasize again that $\hat{\eta}_s/\hat{s}$ and $\hat{\gamma}/\hat{s}$ are small constants and we can assume $\hat{\eta}_s/\hat{s}, \hat{\gamma}/\hat{s} \ll 1$. Therefore, we can consider the $\bar{\omega}^2$ and $\hat{\eta}_s/\hat{s}, \hat{\gamma}/\hat{s}$ as small parameters and expand all the quantities in the power series of $\bar{\omega}^2$ and $\hat{\eta}_s/\hat{s}, \hat{\gamma}/\hat{s}$.

In leading order of $\bar{\varpi}^2$, the Eq. (53) becomes

$$\frac{d\hat{T}}{d\rho}+\frac{2}{3}\hat{T}\tanh\rho-\frac{2}{9}\left(\frac{\hat{\eta}_s}{\hat{s}}\right)\tanh^2\rho+\mathcal{O}(\bar{\varpi}^2)=0, \quad (61)$$

$$\partial_\rho\hat{S}^{\varphi\eta}+3\tanh\rho\hat{S}^{\varphi\eta}+\frac{4}{a\hat{T}^2}\left(\frac{\hat{\gamma}}{\hat{s}}\right)\hat{s}\hat{S}^{\varphi\eta}=0, \quad (62)$$

$$\partial_\rho\hat{S}^{\rho\theta}+3\tanh\rho\hat{S}^{\rho\theta}=0, \quad (63)$$

whose solution is given by

$$\hat{T}=\hat{T}_0\left(\frac{\cosh\rho_0}{\cosh\rho}\right)^{\frac{2}{3}}\left(1+\frac{\hat{\eta}_s}{\hat{s}}B(\rho)\right)+\mathcal{O}(\bar{\varpi}^2), \quad (64)$$

with initial value $\hat{T}_0\equiv\hat{T}(\rho_0)$. Here, the auxiliary function $B(\rho)$ is

$$B(\rho)\equiv\frac{2}{27}\frac{1}{\hat{T}_0}\cosh^{-\frac{2}{3}}\rho_0\left[\sinh^3\rho F\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};-\sinh^2\rho\right)-\sinh^3\rho_0 F\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};-\sinh^2\rho_0\right)\right], \quad (65)$$

where $F(a,b;c;z)$ is the hypergeometric function. Substituting Eq. (64) to Eq. (41), we obtain the expression for the energy density $\hat{e}$:

$$\hat{e}=\hat{e}_0\left(\frac{\cosh\rho_0}{\cosh\rho}\right)^{\frac{8}{3}}\left(1+4\frac{\hat{\eta}_s}{\hat{s}}B(\rho)\right)+\mathcal{O}\left(\bar{\varpi}^2,\left(\frac{\hat{\eta}_s}{\hat{s}}\right)^2\right). \quad (66)$$

With the EoS (38), the solutions of evolution equations for spin (58), (59) are

$$\hat{S}^{\rho\theta}=c_1\cosh^{-3}\rho, \quad (67)$$

$$\hat{S}^{\varphi\eta}=f(\theta)\cosh^{-3}\rho A(\rho)+\mathcal{O}(\bar{\varpi}^2), \quad (68)$$

where

$$c_1=\hat{S}^{\rho\theta}(\rho_0,\theta_0)\cosh^3\rho_0 \quad (69)$$

is a constant determined by the initial condition and

$$A(\rho)\equiv\exp\left[-\frac{4}{a}\int_{\rho_0}^{\rho}d\rho'\left(\frac{\hat{\gamma}}{\hat{s}}\right)\frac{\hat{s}(\rho')}{\hat{T}(\rho')^2}\right]. \quad (70)$$

The $f(\theta)$ in Eq. (68) is a function of θ. As explained in Sec. IV A, although the Gubser flow requires the scalars $\hat{\omega}_{\mu\nu}\hat{S}^{\mu\nu}$, and both $\hat{\omega}_{\mu\nu}\hat{\omega}^{\mu\nu}$ and $\hat{S}_{\mu\nu}\hat{S}^{\mu\nu}$ depend on ρ only, EoS (38) implies that $\hat{\omega}^{\mu\nu}$ or $\hat{S}^{\mu\nu}$ could also depend on θ due to the metric $\hat{g}_{\mu\nu}=\mathrm{diag}(-1,\cosh^2\rho,\cosh^2\rho\sin^2\theta,1)$. Using Eq. (67), we find that

$$\hat{S}^{\mu\nu}\hat{S}_{\mu\nu}=-2\cosh^{-4}\rho[c_1^2-2f^2(\theta)\sin^2\theta A(\rho)^2]+\mathcal{O}(\bar{\varpi}^2). \quad (71)$$

Unless

$$f(\theta)=c_2\sin^{-1}\theta, \quad (72)$$

with a constant

$$c_2=\hat{S}_0^{\varphi\eta}A^{-1}(\rho_0)\cosh^3\rho_0\sin\theta_0, \quad (73)$$

determined by initial value of $\hat{S}_0^{\varphi\eta}=\hat{S}^{\varphi\eta}(\rho_0,\theta_0)$, the $\hat{S}^{\mu\nu}\hat{S}_{\mu\nu}$ would not be independent on θ. Finally, the expression for the spin density $\hat{S}^{\varphi\eta}$ becomes

$$\hat{S}^{\varphi\eta}=c_2\cosh^{-3}\rho\sin^{-1}\theta A(\rho)+\mathcal{O}(\bar{\varpi}^2). \quad (74)$$

If the dimensionless quantity $\hat{\gamma}/\hat{s}$ can be regarded as a small constant, i.e., $\hat{\gamma}/\hat{s}\ll 1$, we can obtain

$$A(\rho)=1+\left(\frac{\hat{\gamma}}{\hat{s}}\right)\frac{6}{a}\frac{\hat{s}_0}{\hat{T}_0^2}\cosh^{\frac{2}{3}}\rho_0\left[\mathrm{Sech}^{\frac{2}{3}}\rho_0F\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\mathrm{Sech}^2\rho_0\right)-\mathrm{Sech}^{\frac{2}{3}}\rho F\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\mathrm{Sech}^2\rho\right)\right]+\mathcal{O}\left(\frac{\hat{\omega}^{\alpha\beta}\hat{\omega}_{\alpha\beta}}{\hat{T}^2},\left(\frac{\hat{\eta}_s}{\hat{s}}\right)^2,\left(\frac{\hat{\gamma}}{\hat{s}}\right)^2,\frac{\hat{\gamma}\hat{\eta}_s}{\hat{s}^2}\right). \quad (75)$$

Next, we transform the analytic solutions (64), (66), (67), (74) in the $dS_3\times\mathbb{R}$ space-time to the Minkowski space-time $\mathbb{R}^{3,1}$. From Eq. (28), the energy density in Minkowski space-time $\mathbb{R}^{3,1}$ is

$$e=\frac{\hat{e}_0}{\tau_0^4}\left(\frac{\tau_0}{\tau}\right)^{\frac{4}{3}}\left[\frac{G(L,\tau_0,x_{\perp 0})}{G(L,\tau,x_\perp)}\right]^{\frac{4}{3}}\times\left(1+4\frac{\eta_s}{s}B(\rho)\right)+\mathcal{O}\left(\bar{\varpi}^2,\left(\frac{\eta_s}{s}\right)^2\right), \quad (76)$$

where we introduce the

$$G(L,\tau,x_\perp)\equiv 4L^2\tau^2+(L^2-\tau^2+x_\perp^2)^2, \quad (77)$$

with an adjustable parameter L defined in Eq. (4), and the τ_0 and $x_{\perp 0}$ stands for the initial proper time and the transverse position $x_\perp$. Here, we have used the identity

$$\hat{\eta}_s/\hat{s}=\eta_s/s, \quad (78)$$

since $\hat{\eta}_s/\hat{s}$ is a scalar under the Weyl rescaling. Using the same method, it is straightforward to get the expression for temperature T from Eq. (64). The T as a function of $\tau,x_\perp$ is similar to the $e(\tau,x_\perp)$.

Next, we take the Weyl rescaling and the coordinate transformation to the spin density. By using Eqs. (8) and (24), the nonzero spin density $S^{\mu\nu}$ in the $(\tau, x_\perp, \varphi, \eta)$ coordinate system of Minkowski space-time $\mathbb{R}^{3,1}$ is given by

$$S^{\tau x_\perp} = c_1 \frac{4L^2}{\tau} G(L,\tau,x_\perp)^{-1},$$
$$S^{\varphi\eta} = c_2 \frac{4L^2}{x_\perp \tau^2} G(L,\tau,x_\perp)^{-1} A(\rho). \tag{79}$$

We find that the exponential factor $A(\rho)$ in Eq. (70) is always less than 1. It means that dissipative effects $\propto \hat{\gamma}$ accelerate decaying. Furthermore, we express the spin density in Cartesian coordinates (t,x,y,z) by coordinate transformation Eq. (2), i.e.,

$$S^{0x} = \frac{4L^2}{\tau} C_+ G(L,\tau,x_\perp)^{-1}, \tag{80}$$

$$S^{0y} = \frac{4L^2}{\tau} C_- G(L,\tau,x_\perp)^{-1}, \tag{81}$$

$$S^{xz} = \frac{4L^2}{\tau} D_+ G(L,\tau,x_\perp)^{-1}, \tag{82}$$

$$S^{yz} = \frac{4L^2}{\tau} D_- G(L,\tau,x_\perp)^{-1}, \tag{83}$$

where we introduce that

$$C_\pm(t,x,y,z) = c_1 \cosh\eta \cos\varphi \pm c_2 \sinh\eta \sin\varphi A(\rho), \tag{84}$$

$$D_\pm(t,x,y,z) = -c_1 \sinh\eta \cos\varphi \pm c_2 \cosh\eta \sin\varphi A(\rho), \tag{85}$$

and η, φ, ρ are the functions of (t,x,y,z) given by Eqs. (2) and (8). The other two components, S^{0z} and S^{xy} vanish.

Let us comment on our results here. In large L limit, i.e., $x_\perp, \tau \ll L$, we have $G(L,\tau,x_\perp) \sim L^4$ and the energy density and temperature become,

$$e \propto \tau^{-4/3}, \qquad T \propto \tau^{-1/3}. \tag{86}$$

The spin density in Eqs. (80)–(83) reduces to

$$S^{0x}, \qquad S^{0y}, \qquad S^{xz}, \qquad S^{yz} \propto L^{-2}\tau^{-1}. \tag{87}$$

From EoS (38), the spin chemical potential $\omega^{\mu\nu}$ decays like

$$\omega^{0x}, \qquad \omega^{0y}, \qquad \omega^{xz}, \qquad \omega^{yz} \propto L^{-2}\tau^{-1/3}. \tag{88}$$

The decay behavior of e, T, in large L limit is the same as those in the spin hydrodynamics in a Bjorken expansion [71]. Meanwhile, due to the dissipative effects, the spin density and spin potential can decay more rapidly. We find that the exponential factor $A(\rho)$ in Eq. (70) is always less than 1. It means that dissipative effects $\propto \hat{\gamma}$ accelerate decaying.

Notably, in a Bjorken flow, we only get the nonzero solutions for the spin component S^{xy} [71]. Here, there are four nonvanishing spin density components found in the current work due to the radial expansion in a Gubser flow.

Before we end this section, let us discuss the terms in the modified Cooper-Frye formula. As discussed in recent works for shear induced polarization [73–77] (also see Refs. [74,84,110] for other terms related to spin density), the thermal vortical $\Omega^{\mu\nu}$ and thermal shear tensor $\xi^{\mu\nu}$

$$\Omega^{\mu\nu} \equiv \frac{1}{2}\nabla^\nu(u^\mu/T) - \frac{1}{2}\nabla^\mu(u^\nu/T),$$
$$\xi^{\mu\nu} \equiv \frac{1}{2}\nabla^\mu(u^\nu/T) + \frac{1}{2}\nabla^\nu(u^\mu/T), \tag{89}$$

and spin potential $\omega^{\mu\nu} = S^{\mu\nu}/(aT^2)$ appear in modified Cooper-Frye formula.

From Eqs. (10), (24), (64), we find the nonzero components of $\Omega^{\mu\nu}$ are $\Omega^{\tau x_\perp}$ and $\Omega^{x_\perp\tau}$. Note that, in $dS_3 \times \mathbb{R}$ space-time, the nonzero thermal vortical tensors $\hat{\Omega}^{\tau x_\perp}$ and $\hat{\Omega}^{x_\perp\tau}$ come from the space-time derivatives of the temperature and the extra terms proportional to $\hat{\nabla}_\mu \tau$ shown in Eq. (29) from Weyl rescaling. In the $\mathbb{R}^{3,1}$ space-time, the $\nabla_\mu u_\nu$ with u_μ given by Eq. (10) is obviously nonzero and can contribute to $\Omega^{\tau x_\perp}$ and $\Omega^{x_\perp\tau}$.

We now analyze the evolution behavior of thermal vortical and shear tensors.

In large L but small η/s, γ/s limits, by using Eqs. (10), (38), (64), (79), we notice that

$$\cosh\rho \propto L\tau^{-1}, \qquad u^\tau \propto \tau^0, \qquad u^{x_\perp} \propto L^{-2}\tau x_\perp, \tag{90}$$

and obtain the evolution behavior of nonzero components of $\Omega^{\mu\nu}$ and $\xi^{\mu\nu}$ listed in Table I. We find that the only nonzero component of thermal vortical tensor, $\Omega^{\tau x_\perp}$, is much smaller than the maximum component of thermal

TABLE I. Evolution behavior of nonzero components for thermal vortical tensor $\Omega^{\mu\nu}$ and shear tensor $\xi^{\mu\nu}$ and spin potential $\omega^{\mu\nu}$ in $\mathbb{R}^{3,1}$ space-time in large L but small η/s, γ/s limits.

Nonzero components	$\Omega^{\tau x_\perp}$	$\xi^{\tau\tau}$	$\xi^{\tau x_\perp}$	$\xi^{x_\perp x_\perp}$	$\xi^{\varphi\varphi}$	$\xi^{\eta\eta}$	$\omega^{0x}, \omega^{0y}, \omega^{xz}, \omega^{yz}$
Evolution behavior in large L and small η/s, γ/s limits	$L^{-2}\tau^{1/3}$	$L^0\tau^{-2/3}$	$L^{-2}\tau^{1/3}$	$L^{-2}\tau^{4/3}$	$L^{-2}\tau^{4/3}$	$L^0\tau^{-8/3}$	$L^{-2}\tau^{-1/3}$

shear tensor, $\xi^{\tau\tau}$, but it has the same order of magnitude as the spin potential in Eq. (88).

From Eq. (18), in the global equilibrium, one of the most important conclusions for spin hydrodynamics is that the $\nabla_\perp^{[\mu}u^{\nu]} - 2\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta} = T[\Delta^{\mu\alpha}\Delta^{\nu\beta}\Omega_{\alpha\beta} - T^{-1}\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta}]$ should be zero [58,64,88]. Here, we compare the evolution behavior of the following quantities:

$$\begin{aligned} \Delta^{\mu\alpha}\Delta^{\nu\beta}\Omega_{\alpha\beta} &\propto L^{-2}\tau^{1/3}, \\ \Delta^{\mu\alpha}\Delta^{\nu\beta}\xi_{\alpha\beta} &\propto L^{0}\tau^{-2/3}, \\ T^{-1}\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta} &\propto L^{-2}\tau^{0}. \end{aligned} \tag{91}$$

We conclude that in large L but small η/s, γ/s limits the thermal shear tensor is more important than than spin potential and thermal vortical tensor.

We also discuss the evolution behavior at finite L case. We follow the in-viscid case of Gubser flow in Ref. [117,118] and take $\hat{e}_0 = 880$ at $\rho_0 = 0$ with $\eta_s = 0$, $c_0 = 11$ and $\hat{T}_0 = (\hat{e}_0/c_0)^{1/4}$, and $L = 4.3$ fm. To see the power law behavior, we plot $d[\log(A/A_0)]/d[\log(\tau/\tau_0)]$ as a function of τ. Here, we choose $A = \Omega^{\tau x_\perp}, \xi^{\tau x_\perp}, \omega^{\tau x_\perp}$, and A_0 stands for the value of these quantities at initial proper time τ_0. In Fig. 1, we choose $x_\perp = 0.5$ fm for simplicity. We have checked numerically that the power law behavior of these quantities for other fixed $x_\perp$ ($\lesssim 4$ fm) is almost the same. The maximum proper time is chosen as 4 fm/c. After 4 fm/c, the temperature will be less than the typical freeze out one [117,118].

In Fig. 1(a), we find that within 4 fm/c the $\Omega^{\tau x_\perp}$ and $\omega^{\tau x_\perp}$ always increases or decreases, respectively. While the thermal shear tensor $\xi^{\tau x_\perp}$ increases as $\propto \tau^{1/3}$ at early time but decreases rapidly as $\propto \tau^{-1}$ after 2.3 fm/c. Surprisingly, the thermal shear tensor decays much faster than the spin potential in this model. As a comparison, when $L = 50$ fm in Fig. 1(b), we observe a consistent result as expected in Table I.

We conclude that in finite L case, the evolution behavior of the quantities mentioned above depends on the parameters L. Therefore, we can not naively drop any one of them in the modified Cooper-Frye formula. To clarify it further, studies based on spin hydrodynamics are needed in the future.

D. Results for Belinfante form

Different from the canonical form of energy-momentum and angular-momentum tensors, one can define the energy-momentum tensor in Belinfante form $\mathcal{T}^{\mu\nu}$ through the pseudogauge transformation:

$$\mathcal{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu}, \tag{92}$$

where

$$K^{\lambda\mu\nu} = \frac{1}{2}(\Sigma^{\lambda\mu\nu} - \Sigma^{\mu\lambda\nu} + \Sigma^{\nu\mu\lambda}), \tag{93}$$

and $\Sigma^{\lambda\mu\nu}$ is given by Eq. (15). The Belinfante total angular momentum reads

$$\begin{aligned} \mathcal{J}^{\alpha\mu\nu} &= J^{\alpha\mu\nu} + \partial_\rho(x^\mu K^{\rho\alpha\nu} - x^\nu K^{\rho\alpha\mu}) \\ &= x^\mu \mathcal{T}^{\alpha\nu} - x^\nu \mathcal{T}^{\alpha\mu}. \end{aligned} \tag{94}$$

It is obvious that both $\mathcal{T}^{\mu\nu}$ and $\mathcal{J}^{\alpha\mu\nu}$ are conserved.

After a short calculation, up to $\mathcal{O}(\partial^1)$, one gets [64]

$$\begin{aligned} \mathcal{T}^{\mu\nu} &= eu^\mu u^\nu + p\Delta^{\mu\nu} + \frac{1}{2}\nabla_\lambda(u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda}) \\ &= (e + \delta e_s)u^\mu u^\nu + (p + \delta\Pi_s)\Delta^{\mu\nu} + 2\delta h_s^{(\mu}u^{\nu)} \\ &\quad + (\pi^{\mu\nu} + \delta\pi_s^{\mu\nu}), \end{aligned} \tag{95}$$

where

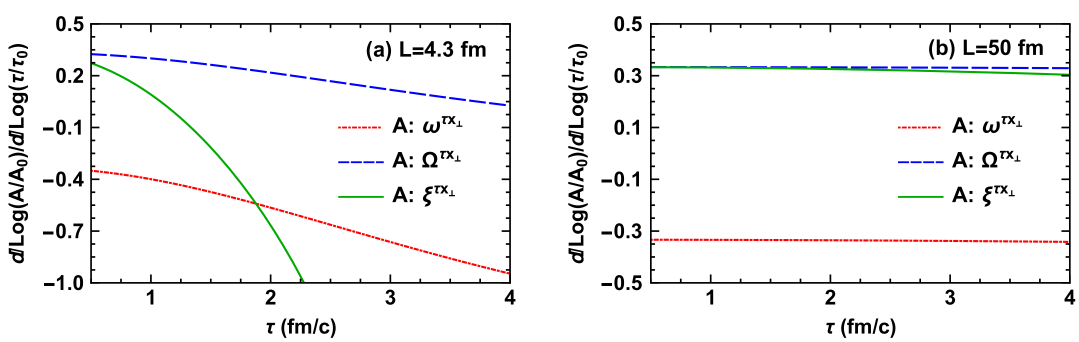

FIG. 1. We plot $d[\log(A/A_0)]/d[\log(\tau/\tau_0)]$ with $A = \Omega^{\tau x_\perp}, \xi^{\tau x_\perp}, \omega^{\tau x_\perp}$ as functions of proper time τ at $x_\perp = 0.5$ fm for (a) $L = 4.3$ fm and (b) $L = 50$ fm. The parameters are chosen as $\hat{e}_0 = 880$ at $\rho_0 = 0$ with $\eta_s = 0$, $c_0 = 11$, and $\hat{T}_0 = (\hat{e}_0/c_0)^{1/4}$ [117,118]. The red dotted, blue dashed, and green solid lines stand for the cases of $A = \Omega^{\tau x_\perp}, \xi^{\tau x_\perp}, \omega^{\tau x_\perp}$, respectively.

$$\delta e_s = -u_\mu \nabla_\lambda S^{\mu\lambda},$$
$$\delta h_s^\mu = \frac{1}{2}(\Delta^\mu_\beta \nabla_\lambda S^{\beta\lambda} - u_\beta S^{\beta\lambda}\nabla_\lambda u^\mu),$$
$$\delta \pi_s^{\mu\nu} = \nabla_\lambda (u^{\langle\mu} S^{\nu\rangle\lambda}),$$
$$\delta \Pi_s = \frac{1}{3}\Delta_{\rho\sigma}\nabla_\lambda (u^\rho S^{\sigma\lambda}) \tag{96}$$

are the spin corrections to the energy density, heat flow, shear viscous tensor, and bulk viscous pressure, respectively.

In our previous work [71], all of these spin corrections vanish in a Bjorken flow. Inserting our results (79) with Gubser velocity (10) into Eq. (96) yields the spin corrections to the energy density:

$$\delta e_s = 4c_1 \tau^{-1} x_\perp^{-1} L^2 (L^2 + \tau^2 - 3x_\perp^2) G(L,\tau,x_\perp)^{-3/2}, \tag{97}$$

where G and constant c_1 are given by Eqs. (77) and (69), respectively. We get the other nonvanishing spin corrections in the $(\tau, x_\perp, \phi, \eta)$ coordinates:

$$\delta h_s^\tau = 16 c_1 L^2 \tau x_\perp G(L,\tau,x_\perp)^{-2},$$
$$\delta h_s^{x_\perp} = 8 c_1 L^2 (L^2 + x_\perp^2 + \tau^2) G(L,\tau,x_\perp)^{-2},$$
$$\delta \pi_s^{\tau\tau} = -\frac{64}{3} c_1 L^2 \tau x_\perp^3 G(L,\tau,x_\perp)^{-5/2},$$
$$\delta \pi_s^{x_\perp x_\perp} = -\frac{16}{3} c_1 L^2 \tau^{-1} x_\perp (L^2 + x_\perp^2 + \tau^2)^2 G(L,\tau,x_\perp)^{-5/2},$$
$$\delta \pi_s^{\varphi\varphi} = \frac{8}{3} c_1 L^2 \tau^{-1} x_\perp^{-1} G(L,\tau,x_\perp)^{-3/2},$$
$$\delta \pi_s^{\eta\eta} = \frac{8}{3} c_1 L^2 \tau^{-3} x_\perp G(L,\tau,x_\perp)^{-3/2},$$
$$\delta \pi_s^{\tau x_\perp} = \delta \pi_s^{x_\perp \tau} = -\frac{32}{3} c_1 L^2 x_\perp^2 (L^2 + x_\perp^2 + \tau^2) G(L,\tau,x_\perp)^{-5/2},$$
$$\delta \pi_s^{\varphi\eta} = \delta \pi_s^{\eta\varphi} = \frac{1}{2} S^{\varphi\eta} \tau^{-1} (L^2 + x_\perp^2 - \tau^2) G(L,\tau,x_\perp)^{-1/2},$$
$$\delta \Pi_s = -\frac{8}{3} c_1 L^2 \tau^{-1} x_\perp G(L,\tau,x_\perp)^{-3/2}. \tag{98}$$

One may wonder why the spin correction to the bulk pressure $\delta\Pi_s$ in Eq. (98) is nonzero and may break the conformal invariance. Again, we comment that EoS (37) is the leading order one in the ultra high temperature limits and is not related to the conformal invariance directly. Therefore, there is no inconsistency between the finite $\delta\Pi_s$ and EoS (37). The $\delta\Pi_s$ comes from the pseudogauge transformation and belongs to the Belinfante energy momentum tensor $\mathcal{T}^{\mu\nu}$. In the canonical form, if the initial bulk pressure is zero, the bulk pressure is always vanishing in the presence of spin potential. Since the spin hydrodynamics in Belinfante form can be quite different from those in canonical form (one can find examples in Ref. [64]), it is not surprising that we have different bulk pressure in two forms.

Now, we turn to estimate how large these spin corrections will be. In the large L limit, we find energy density $e \sim \tau^{-4/3}$ and its spin corrections $\delta e_s \sim 1/(\tau x_\perp L^2)$. It gives $\lim_{L\to\infty} \delta e_s / e \propto \tau^{1/3}/L^2 \to 0$ at late proper time; i.e., the spin correction to the energy density δe_s is a small correction to the energy density e.

For a realistic model for Gubser flow, we choose $\hat{e}_0 = (5.55)^4$ at $\rho_0 = 0$ with $\eta_s/s = 0.268$, $c_0 = 11$, $\hat{T}_0 = (\hat{e}_0/c_0)^{1/4}$, and the parameter of characteristic length L as 4.3 fm [117,118]. Note that due to the differences of notations, η_s/s in this work is twice as much as that in Refs. [117,118]. Since the proper time is larger than 4.0 fm/c, the temperature is less than the typical freeze out temperature $T \leq 150$ MeV. We choose the range of proper time as 0.5–4.0 fm/c similar to the standard Gubser flow [117,118]. Although total energy correction $\int dx_\perp x_\perp \delta e_s$ is finite, $\delta e_s \propto x_\perp^{-1}$ may be divergent as $x_\perp \to 0$. To avoid the divergent behavior of δe, we have imposed the constraint $x_\perp > 0.5$ fm in Eq. (99). The parameter c_1 defined in Eq. (69) should not be too large due to our power counting scheme in Eq. (60) in Sec. IV C. Here, we choose $|c_1| \leq 2$ as a reasonable test. Using these parameters and Eqs. (10), (18), (37), (76), (97), (98), we obtain

$$|\delta e_s / e| < 0.1, \quad \text{at } x_\perp \in [0.5, 4.0]\ \text{fm}, \quad \tau \in [0.5, 4.0]\ \text{fm}/c, \tag{99}$$

$$|\delta \pi_s^{\mu\nu}/\pi^{\mu\nu}|, |\delta\Pi_s/p| < 0.1, \quad \text{at } x_\perp \in [0.0, 4.0]\ \text{fm}, \quad \tau \in [0.5, 4.0]\ \text{fm}/c. \tag{100}$$

We comment that Eqs. (97) and (98) are the evidence to show that spin corrections in the Belinfante form of spin hydrodynamic exist. These spin corrections are expected in Ref. [64].

V. CONCLUSION

In this work we have obtained the analytical solutions for the dissipative spin hydrodynamics with radial expansion in a Gubser flow.

After a short review on the standard Gubser flow, we briefly discuss the relativistic dissipative spin hydrodynamics in the Minkowski space-time $\mathbb{R}^{3,1}$ and extend the main equations to the $dS_3 \times \mathbb{R}$ space-time under Weyl rescaling. Unfortunately, we find that there are extra contributions $\propto \hat{\nabla}_\mu \tau$ from Weyl rescaling to both the energy momentum conservation Eq. (23) and angular momentum conservation Eq. (25). We emphasize that the energy-momentum conservation equation is no longer conformal invariant in the current work due to its antisymmetric components. For simplicity, we drop the bulk viscous pressure and $\hat{q}^\mu$. We further assume the transport coefficients $\hat{\eta}_s/\hat{s}$ and $\hat{\gamma}/\hat{s}$ are small constants similar to the ordinary Gubser flow.

We then discuss the thermodynamic relations (36) and the equations of state (37), (38) in the $dS_3 \times \mathbb{R}$ space-time. For convenience, we introduce a dimensionless scalar $\bar{\omega}^2$ in Eq. (39). Next, we derive the special configuration for the fluid, in which the fluid velocity in a Gubser flow holds. Fortunately, the extra terms from Weyl rescaling $\sim\hat{\nabla}_\mu\tau$ in energy momentum and angular momentum conservation equations vanish in this configuration. In the power series expansion of small $\bar{\omega}^2, \hat{\eta}_s/\hat{s}, \hat{\gamma}/\hat{s}$, we have derived the analytic solutions for the dissipative spin hydrodynamics in the $dS_3 \times \mathbb{R}$ space-time. The evolution of energy density and spin density $\hat{S}^{\rho\theta}, \hat{S}^{\varphi\eta}$ are shown in Eqs. (66), (67), (68) in $dS_3 \times \mathbb{R}$ space-time. We also transform these physical quantities back to the Minkowski space-time $\mathbb{R}^{3,1}$.

Our main results for the energy density e and spin density $S^{0x}, S^{0y}, S^{xz}, S^{yz}$ in Minkowski space-time $\mathbb{R}^{3,1}$ are given by Eqs. (76), (80)–(83). There are two remarkable differences between the solutions found here and in a Bjorken flow [71]. The first thing is that the solutions in a Gubser flow provide the additional information for transverse expansion of the systems, which is missing in a Bjorken flow [71]. Meanwhile, now we have derived four nonzero components $S^{0x}, S^{0y}, S^{xz}, S^{yz}$ in spin density tensor in current work, while we only have one nonzero component S^{xy} in Bjorken flow [71]. It indicates that our current findings are not a simple extension of Bjorken flow.

In large L and small η/s, γ/s limits we find that $e \propto \tau^{-4/3}, T \propto \tau^{-1/3}, S^{\mu\nu} \propto L^{-2}\tau^{-1}, \omega^{\mu\nu} \propto L^{-2}\tau^{-1/3}$, which are similar to the behavior in Bjorken expansion [71]. Moreover, $S^{\mu\nu}$ decays much faster in the cases of a finite L and with nonzero dissipative effects than in large L and perfect fluid limit.

In our model, we find that in large L and small η/s, γ/s limits the thermal shear tensor $\xi^{\mu\nu}$ may be more important than spin potential $\omega^{\mu\nu}$ and thermal vortical tensor $\Omega^{\mu\nu}$, but in finite L case their evolution behavior depends on the parameters L strongly. We can not naively drop any one of them in the modified Cooper-Frye formula. To clarify it, we need further studies based on spin hydrodynamics in the future.

At the end, after the pseudogauge transformation, we discuss the results for spin hydrodynamics in the Belinfante form. We observe that the spin corrections to the energy density and other dissipative terms do not vanish in a Gubser flow, which is quite different with the results in a Bjorken flow.

Our analytic solutions also provide the test beds for the future numerical simulations of relativistic dissipative spin hydrodynamics.

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