

Axial-vector transition form factors of the baryon octet to the baryon decuplet with flavor SU(3) symmetry breaking

Jung-Min Suh,^{1,*} Yu-Son Jun,^{1,†} and Hyun-Chul Kim^{1,2,‡}

¹*Department of Physics, Inha University, Incheon 22212, South Korea*

²*School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, South Korea*



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We investigate the axial-vector transition form factors of the baryon octet to the baryon decuplet within the framework of the chiral quark-soliton model, with the effects of flavor SU(3) symmetry breaking included. We consider the rotational $1/N_c$ corrections and regard the strange current quark mass as a perturbation. We compare the present results for the $\Delta \rightarrow N$ axial-vector transition with those from other models and lattice QCD. We also compute all possible axial-vector transitions from the baryon decuplet to the octet with the strangeness changed, i.e., $|\Delta S| = 1$. We obtain the value of the essential form factor C_5^A for the $\Delta \rightarrow N$ axial-vector transition at the zero momentum transfer ($Q^2 = 0$). Furthermore, the present results are in good agreement with those fitted with the T2K data. We extract the value of the axial-vector mass M_A compared to the data.

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I. INTRODUCTION

The axial-vector transitions of SU(3) baryons address multifaceted issues on strong and weak processes of hadrons. A typical axial-vector transition can be found in hyperon semileptonic decays (HSD) [1,2]. While most of the axial-vector transition constants for the baryon octet HSD were known experimentally [3], experimental evidence for the $\Xi^- \rightarrow \Xi^0 e^- \bar{\nu}_e$ decay is still elusive [4]. HSDs provide information on the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix elements $|V_{ud}|$ and $|V_{us}|$ [5,6] in addition to the pion and kaon decays [7–10]. While the CKM mixing angles extracted from HSDs can only play an auxiliary role, it is still of great importance to determine the unitarity of the CKM matrix; $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ [11–13]. HSDs also cast light on the structure of the SU(3) baryons. The experimental data on the semileptonic decay constants reveal a certain pattern of explicit flavor SU(3) symmetry breaking [14–19]. The baryon decuplet, on the other hand, decays in the baryon octet primarily through the strong interaction except for the Ω^- baryon. Nevertheless, understanding the $\Delta \rightarrow N$ axial-vector transition form factor is critical because it provides crucial

information for describing the weak single pion production ($\nu_\mu p \rightarrow \mu^- \pi^+ p$) from neutrino-nucleon scattering [20–24]. Since neutrino-nucleon scattering holds an essential clue on the neutrino oscillations, there has been a great deal of experimental programs such as the *No ν a*, MiniBooNE, T2K, NuSTEC, *Minerva*, DUNE, and SND@LHC experiments [25–32] in higher-energy regions (see also a recent review [33]). Very energetic neutrinos in future experiments such as the DUNE and SND@LHC will be available, so one can have a possible opportunity to study the structure of strange baryons in neutrino-nucleon scattering. These experiments will shed light on the axial-vector structure of nonstrange and strange baryon resonances. In addition, Alexandrou *et al.* reported the results on the $\Delta \rightarrow N$ axial-vector transition form factors based on lattice QCD [34,35]. Thus, it is of great interest to scrutinize the axial-vector transitions from the baryon decuplet to the octet, which will give multiple perspectives on the structure of baryons.

There have already been many theoretical works on the axial-vector transition form factors for the nucleon to the Δ excitation; for example, the relativistic quark model (RQM) [36–38], the isobar model (IM) [39], the nonrelativistic quark model (NRQM) [40,41], the linear σ model (LSM) and the cloudy bag model (CBM) [42], the chiral constituent quark model (χ CQM) [43], baryon chiral perturbation theory [44–47], the Barbero-Lopez-Mariano model (BLM) [48,49], the Δ -pole dominance model [50], the light cone QCD sum rule (LCSR) [51], and the nonlinear σ model (NLSM) [52]. The $\Delta \rightarrow N$ axial-vector transition form factor has often been parametrized either by the dipole-type form factor or by Adler's parametrization [20]. These parametrizations being used, the value of the axial

*suhjungmin@inha.edu

†ysjun@inha.edu

‡hchkim@inha.ac.kr

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transition mass M_A for the $N \rightarrow \Delta$ axial-vector transition can be extracted from the experimental data. Based on the ANL [21,22] and BNL data [23,24], many theoretical and experimental efforts were put on extracting the values of the $\Delta \rightarrow N$ axial vector form factor $C_5^A(0)$ and M_A ; the ranges of their values lie in 0.8–1.2 GeV and 0.8–1.0 GeV, respectively [21,22,48,49,53–55]. The off-diagonal Goldberger-Treiman (GT) relation for the $\Delta \rightarrow N$ axial-vector transition constant predicts $C_5^A(0)$ to be around 1.2 [56,57] with the experimental data on the Δ decay width considered. In this context, the deviation of the off-diagonal GT relation was also discussed and was found small [44,58]. However, Ref. [59] found the smaller value $C_5^A = 0.87 \pm 0.08$, which is more reliable for MiniBooNE and T2K experiments. In addition, the nucleon-nucleon potential such as the Bonn-Jülich potential [60] takes the smaller value of the $\pi N \Delta$ coupling constant ($f_{\pi N \Delta}^2/4\pi = 0.224$) than that derived from the Δ decay width ($f_{\pi N \Delta}^2/4\pi = 0.36$). If one uses this smaller value of $f_{\pi N \Delta}$, one would get a smaller value of C_5^A from the off-diagonal GT relation.

We also want to mention that there are only a few studies on the axial-vector transitions from the baryon decuplet to the octet. The axial-vector transition constants from the baryon decuplet to the octet with the strangeness conserved ($\Delta S = 0$) were already computed within the chiral quark-soliton model (χ QSM) [61]. Those with $|\Delta S| = 1$ were investigated in a pion mean-field approach, where all possible parameters were fixed by using the experimental data on HSD [62]. In the present work, we will extend the previous study to compute all possible axial-vector transition form factors from the baryon decuplet to the octet up to a momentum transfer $Q^2 \leq 1 \text{ GeV}^2$ within the framework of the self-consistent χ QSM with explicit flavor SU(3) symmetry breaking considered.

The χ QSM is a pion mean-field approach [63–65]. As Witten proposed [66,67], a baryon in the large N_c (the number of colors) limit emerges as a state consisting of N_c valence quarks, bound by the pion mean field, since the mesonic quantum fluctuations are suppressed by $1/N_c$. The pion mean field arises from the classical solution of the equation of motion, which can be solved self-consistently. This procedure is nothing but a Hartree approximation [68] (see also a review [69]). The presence of the N_c valence quarks makes the Dirac continuum polarized, which creates the pion mean field by which the N_c valence quarks are bound. Recently, it was shown that this mean-field approach could also describe singly heavy baryons, i.e., a singly heavy baryon as a bound state of the $N_c - 1$ valence quarks [70] (see also a recent review [71]). The classical solution obtained by this self-consistent procedure is called the classical nucleon or the chiral soliton, which needs to be quantized. While we ignore the $1/N_c$ mesonic quantum fluctuations, we have to deal with the zero modes related to continuous translational and rotational symmetries.

Since we will compute form factors of the SU(3) baryons, we have to consider the translational zero modes, which will yield the Fourier transforms, and the rotational zero modes in SU(3), with the SU(2) soliton embedded into SU(3). This embedding preserves the hedgehog symmetry of the SU(2) soliton. Assuming that the angular velocity of the soliton is slow and the mass of the strange current quark (m_s) is small, we will treat them as perturbations. Thus, we will consider the rotational $1/N_c$ and linear m_s corrections. The model has been successfully applied to various observables of the SU(3) baryons: for example, the electromagnetic structures [72–76], strange form factors [77–83], axial-vector form factors [84,85], tensor charges and corresponding form factors [82,83,86,87], semileptonic decays [18,88,89], radiative transition [90–92], the nucleon parton distributions [93–98], and the gravitational form factors [99,100] of the nucleon. In this work, we will concentrate on all possible axial-vector transitions from the baryon decuplet to the baryon octet, including both $\Delta S = 0$ and $|\Delta S| = 1$ transitions.

The present work is organized as follows: In Sec. II, we define the axial-vector transition form factors from the baryon decuplet to the baryon octet, which parametrize the matrix elements of the axial-vector current. In Sec. III, we briefly review the formalism of the χ QSM in the context of the derivation of the axial-vector transition form factors. In Sec. IV, we present their numerical results. We first discuss the effects of flavor SU(3) symmetry breaking. We then compare the numerical results with those from the lattice data. We present the results for the axial-vector transition constants and compare them with those from other theoretical works. We also provide the results for the transition radii and dipole mass that will be useful for describing hadronic processes. In the last section, we summarize the present work and draw conclusions.

II. AXIAL-VECTOR TRANSITION FORM FACTORS FROM THE BARYON OCTET TO THE BARYON DECUPLET

The axial-vector current is defined as

$$A_\mu^\chi(x) = \bar{\psi}(x) \gamma_\mu \gamma_5 \frac{\lambda^\chi}{2} \psi(x), \quad (1)$$

where λ^χ are the short-handed notation for the flavor SU(3) Gell-Mann matrices; for the strangeness-conserving ($\Delta S = 0$ or $\chi = 3, 1 \pm i2$) transitions and strangeness-changing ones ($|\Delta S| = 1$ or $\chi = 4 \pm i5$), we define $\lambda^{1 \pm i2}$ and $\lambda^{4 \pm i5}$ respectively by

$$\lambda^{1 \pm i2} = \frac{1}{\sqrt{2}}(\lambda^1 \pm i\lambda^2), \quad \lambda^{4 \pm i5} = \frac{1}{\sqrt{2}}(\lambda^4 \pm i\lambda^5). \quad (2)$$

$\psi(x)$ stands for the quark field $\psi = (u, d, s)$. Since we deal with the baryon decuplet, the Lorentz structure of the

spin-3/2 baryons should be considered [20]. This means that we have more form factors than the case of the baryon octet, which are often called the Adler form factors. Then the matrix element of the axial-vector current between the baryon decuplet and the baryon octet can be parametrized in terms of four different real form factors [101],

$$\begin{aligned} & \langle B_8(p_8, J'_3) | A_\mu^Z(0) | B_{10}(p_{10}, J_3) \rangle \\ &= \bar{u}(p_8, J'_3) \left[\left\{ \frac{C_3^{A(\chi)}(q^2)}{M_8} \gamma^\nu + \frac{C_4^{A(\chi)}(q^2)}{M_8^2} p_{10}^\nu \right\} \right. \\ & \quad \times (g_{\alpha\mu} g_{\rho\nu} - g_{\alpha\rho} g_{\mu\nu}) q^\rho + C_5^{A(\chi)}(q^2) g_{\alpha\mu} \\ & \quad \left. + \frac{C_6^{A(\chi)}(q^2)}{M_8^2} q_\alpha q_\mu \right] u^\alpha(p_{10}, J_3), \end{aligned} \quad (3)$$

where M_8 and M_{10} designate respectively the masses of the baryon octet and decuplet. $g_{\alpha\beta}$ denotes the metric tensor of Minkowski space, expressed as $g_{\alpha\beta} = \text{diag}(1, -1, -1, -1)$. In the rest frame of a decuplet baryon, p_{10}^α , p_8^α , and q^α represent respectively the momenta of a decuplet baryon, that of an octet baryon and the momentum transfer, which are written by

$$p_{10} = (M_{10}, \mathbf{0}), \quad p_8 = (E_8, -\mathbf{q}), \quad q = (\omega_q, \mathbf{q}), \quad (4)$$

with $q^2 = -Q^2 > 0$. Thus, the three-vector momentum and energy of the momentum transfer are given as

$$\begin{aligned} |\vec{q}|^2 &= \left(\frac{M_{10}^2 + M_8^2 + Q^2}{2M_{10}} \right)^2 - M_8^2 \\ \omega_q &= \left(\frac{M_{10}^2 - M_8^2 - Q^2}{2M_{10}} \right). \end{aligned} \quad (5)$$

$u^\alpha(p_{10}, J_3)$ stands for the Rarita-Schwinger spinor that describes a decuplet baryon with spin 3/2, carrying the momentum p_{10} and spin J_3 . It can be expressed by the combination of the polarization vector and the Dirac spinor, $u^\alpha(p_{10}, J_3) = \sum_{i,s} C_{1i\frac{1}{2}s}^{2J_3} \epsilon_i^\alpha(p_{10}) u_s(p_{10})$. It satisfies the Dirac equation and the auxiliary equations $p_{10\alpha} u^\alpha(p_{10}, J_3) = 0$ and $\gamma_\alpha u^\alpha(p_{10}, J_3) = 0$ [102]. $u(p_8, J'_3)$ denotes the Dirac spinor for an octet baryon.

In the current work, we will concentrate on $C_5^A(q^2)$. The transition matrix element of the axial-vector current is involved in the cross section of neutrino-nucleon scattering. As discussed in many references (for example, see Refs. [49,101]), all other terms except for C_5^A are suppressed by the ratio q/M_N or q^2/M_N^2 in the case of neutrino quasielastic scattering. Thus, the value of $C_5^A(0)$ can be extracted from the neutrino scattering data. Moreover, $C_5^A(0)$ is directly connected to the strong $\pi N\Delta$ coupling constant with the Goldberger-Treiman relation [103–105]. The divergence of the axial-vector current should vanish in the chiral limit [106]

$$i\bar{u}_{B_8}(p'_8, J'_3) q_\mu \left[C_5^A(q^2) + C_6^A(q^2) \frac{q^2}{M_8^2} \right] u_{B_{10}}^\mu(p_{10}, J_3), \quad (6)$$

which yields

$$C_5^A(q^2) + C_6^A(q^2) \frac{q^2}{M_8^2} = 0. \quad (7)$$

This indicates that $C_6^A(q^2)$ must have a pole at $q^2 = 0$ because $C_5^A(0)$ does not vanish. The pole term $C_6^A(q^2)$ leads to the following structure,

$$\begin{aligned} & \bar{u}_{B_8}(p'_8, J'_3) q_\mu C_6^A(q^2) u_{B_{10}}^\mu(p_{10}, J_3) \\ & \rightarrow f_\pi \frac{g_{B_8 B_{10} M}}{M_8 + M_{10}} \bar{u}_{B_8}(p'_8, J'_3) q^\mu g_{\mu\nu} \frac{i}{q^2} u_{B_{10}}^\nu(p_{10}, J_3), \end{aligned} \quad (8)$$

where $g_{B_8 B_{10} M}$ denotes the strong coupling constant for a vertex with decuplet and octet baryons, and an octet meson. Using this relation, we find

$$\begin{aligned} & \lim_{q^2 \rightarrow 0} \left(C_5^A(q^2) + C_6^A(q^2) \frac{q^2}{M_8^2} \right) \\ &= \lim_{q^2 \rightarrow 0} \left[C_5^A(q^2) - \frac{g_{B_8 B_{10} M}}{M_8 + M_{10}} f_\pi \right] = 0, \end{aligned} \quad (9)$$

which gives the well-known Goldberger-Treiman relation (GTR) for a spin-3/2 baryon

$$C_5^A(0) = f_\pi \frac{g_{B_8 B_{10} M}}{M_8 + M_{10}}. \quad (10)$$

The meson-baryon strong coupling constants have been already investigated in this pion mean-field approach, where all dynamical parameters were fixed by the experimental data on HSDs [62]. We want to mention that the GTR has a certain discrepancy [107].

The form factors $C_5^{A,10 \rightarrow 8}(q^2)$ are determined by the transition matrix elements of the spatial component of the axial-vector current

$$\begin{aligned} & C_5^{A,10 \rightarrow 8}(q^2) \\ &= \frac{\sqrt{3}M_8}{\sqrt{E_8 + M_8}} \left[\int \frac{d\Omega_q}{4\pi} \langle B_8(p_8, J_3) | \mathbf{e}_0 \cdot \mathbf{A} | B_{10}(p_{10}, J_3) \rangle \right. \\ & \quad \left. - \sqrt{5\pi} \int \frac{d\Omega_q}{4\pi} Y_{20}(\Omega_q) \langle B_8(p_8, J_3) | \mathbf{e}_0 \cdot \mathbf{A} | B_{10}(p_{10}, J_3) \rangle \right], \end{aligned} \quad (11)$$

where $\mathbf{e}_0$ denotes the polarization vector in the spherical basis, i.e., $\mathbf{e}_0 = (0, 0, 1)$ and $\mathbf{A}$ stands for the spatial component in the vector form: $\mathbf{A} = \bar{\psi}(x) \gamma \gamma_5 \frac{\lambda}{2} \psi(x)$. We fix the third component of the spin states for the baryon octet and decuplet to be $J_3 = 1/2$ for convenience. We will

now compute these transition matrix elements in the present work.

III. AXIAL-VECTOR TRANSITION FORM FACTORS IN THE CHIRAL QUARK-SOLITON MODEL

The SU(3) χ QSM starts from the low-energy effective partition function in Euclidean space

$$\begin{aligned}\mathcal{Z}_{\chi\text{QSM}} &= \int \mathcal{D}\psi^a \mathcal{D}\psi^\dagger \mathcal{D}\pi^a \exp \left[- \int d^4x \psi^\dagger iD(\pi)\psi \right] \\ &= \int \mathcal{D}\pi \exp(-S_{\text{eff}}),\end{aligned}\quad (12)$$

where ψ and π^a represent the quark and pseudo-Nambu-Goldstone boson fields (pNG). The S_{eff} is the effective chiral action expressed as

$$S_{\text{eff}}[\pi^a] = -N_c \text{Tr} \ln D, \quad (13)$$

where Tr stands for the functional trace running over spacetime and all relevant internal spaces. The N_c is the number of colors, and $D(U)$ designates the Dirac differential operator defined by

$$D := i\cancel{\partial} + iMU\gamma_5 + i\hat{m}, \quad (14)$$

where M denotes the dynamical quark mass. Note that M is originally momentum dependent, which comes from the instanton vacuum. The momentum-dependent dynamical quark mass is originated from the quark zero mode in the presence of the instanton [108,109]. Since we use the constant dynamical quark mass in the present work, we have to introduce the regularization to tame the divergence of the quark loops. $U^{\gamma_5}(x)$ in Eq. (14) represents the SU(3) chiral field defined by

$$U^{\gamma_5}(x) := \frac{1 + \gamma_5}{2} U(x) + \frac{1 - \gamma_5}{2} U^\dagger(x) \quad (15)$$

with $U(x) = \exp(i\lambda^a \pi^a(x)/f_\pi)$. f_π is the scale factor that will be identified as the pion decay constant. $\hat{m}$ in Eq. (14) represents the current quark mass matrix given as $\hat{m} = \text{diag}(m_u, m_d, m_s)$ in flavor space. We assume isospin symmetry in this work, so that the current quark masses of the up and down quarks are set equal to each other, i.e., $m_u = m_d$ with their average mass $\bar{m} = (m_u + m_d)/2$. Then, the current quark mass matrix is written as $\hat{m} = \text{diag}(\bar{m}, \bar{m}, m_s) = \bar{m} + \delta m$. δm includes the mass of the strange current quark, which can be decomposed as

$$\delta m = m_1 \mathbf{1} + m_8 \lambda^8. \quad (16)$$

m_1 and m_8 denote the singlet and octet components of the current quark masses respectively; $m_1 = (-\bar{m} + m_s)/3$ and

$m_8 = (\bar{m} - m_s)/\sqrt{3}$. The Dirac operator (14) with γ_4 can be written as

$$\gamma_4 D = -i\partial_4 + h(U(\pi^a)) - \delta m, \quad (17)$$

where ∂_4 stands for the time derivative in Euclidean space. $h(U)$ is called the one-body Dirac Hamiltonian written as

$$h(U) = i\gamma_4 \gamma_i \partial_i - \gamma_4 M U \gamma_5 - \gamma_4 \bar{m}. \quad (18)$$

As mentioned previously, the pion mean field arises as the solution of the classical equation of motion, which is derived from $\delta S_{\text{eff}}/\delta P(r) = 0$. The equation of motion can be solved self-consistently, which resembles the Hartree approximation in many-body problems. In solving the classical equation of motion or minimizing the classical nucleon mass, one needs to find the pion field with proper symmetry. In flavor SU(2), three components of the pion field are coupled to three dimensional space, so that the pion fields are expressed in terms of the profile function $P(r)$ for the chiral soliton

$$\pi^i = n^i P(r), \quad i = 1, 2, 3, \quad (19)$$

where $n^i = x^i/r$ with $r = |\mathbf{x}|$. This expression is often called the hedgehog ansatz and the corresponding symmetry is known to be hedgehog symmetry. Since we want to keep this hedgehog symmetry of the pion field preserved [67,110] also in SU(3), we embed the SU(2) $U_{\text{SU}(2)}(x)$ field into SU(3). The SU(3) $U(x)$ field can be constructed by the trivial embedding [67]

$$U(x) = \exp(i\pi^a \lambda^a / f_\pi) = \begin{pmatrix} \exp(i\mathbf{n} \cdot \boldsymbol{\tau} P(r)/f_\pi) & 0 \\ 0 & 1 \end{pmatrix}, \quad (20)$$

where π^a are set equal to zero for $a = 4, \dots, 8$. The zero-mode quantization with this embedding will correctly yield the spectrum of the SU(3) baryons.

We can compute the matrix elements of the axial-vector current (3) by using the functional integral

$$\begin{aligned}\langle B(p', J'_3) | A_\mu^a(0) | B(p, J_3) \rangle \\ &= \frac{1}{\mathcal{Z}_{\chi\text{QSM}}} \lim_{T \rightarrow \infty} \exp \left(i p_4 \frac{T}{2} - i p'_4 \frac{T}{2} \right) \\ &\times \int d^3x d^3y \exp(-i\mathbf{p}' \cdot \mathbf{y} + i\mathbf{p} \cdot \mathbf{x}) \\ &\times \int \mathcal{D}\pi^a \int \mathcal{D}\psi \int \mathcal{D}\psi^\dagger J_B(\mathbf{y}, T/2) \psi^\dagger(0) \gamma_4 \gamma_\mu \gamma_5 \frac{\lambda^a}{2} \\ &\times \psi(0) J_B^\dagger(\mathbf{x}, -T/2) \exp \left[- \int d^4r \psi^\dagger iD(\pi^a) \psi \right],\end{aligned}\quad (21)$$

where the baryon states $|B(p, J_3)\rangle$ and $\langle B(p', J'_3)|$ are respectively written as

$$\begin{aligned}
|B(p, J_3)\rangle &= \lim_{x_4 \rightarrow -\infty} \exp(ip_4 x_4) \frac{1}{\sqrt{\mathcal{Z}_{\text{QSM}}}} \\
&\times \int d^3x \exp(i\mathbf{p} \cdot \mathbf{x}) J_B^\dagger(\mathbf{x}, x_4) |0\rangle, \\
\langle B(p', J'_3)| &= \lim_{y_4 \rightarrow \infty} \exp(-ip'_4 y_4) \frac{1}{\sqrt{\mathcal{Z}_{\text{QSM}}}} \\
&\times \int d^3y \exp(-i\mathbf{p}' \cdot \mathbf{y}) \langle 0| J_B(\mathbf{y}, y_4). \quad (22)
\end{aligned}$$

Here, $J_B(x)$ represents the Ioffe-type current that consists of the N_c valence quarks [63,111]

$$J_B(x) = \frac{1}{N_c!} \epsilon_{i_1 \dots i_{N_c}} \Gamma_{JJ_3 TT_3 Y}^{\alpha_1 \dots \alpha_{N_c}} \psi_{\alpha_1 i_1}(x) \cdots \psi_{\alpha_{N_c} i_{N_c}}(x), \quad (23)$$

with spin-flavor and color indices $\alpha_1 \cdots \alpha_{N_c}$ and $i_1 \cdots i_{N_c}$, respectively. The matrices $\Gamma_{JJ_3 TT_3 Y}^{\alpha_1 \dots \alpha_{N_c}}$ carry the spin and flavor quantum numbers of the baryon, i.e., $JJ_3 TT_3 Y$. Similarly, we can express the creation current operator $J_B^\dagger(x)$ [63,69].

To quantize the chiral soliton, we have to perform the functional integral over the pNG fields. Since we use the pion mean-field approximation or the saddle-point approximation, we neglect the $1/N_c$ quantum fluctuations of the pNG fields or the pion-loop corrections. However, we have to take into account the zero modes completely, which do not change the energy of the soliton. Thus, the functional integral over the U field is replaced by rotational and translational zero modes that are written as

$$\tilde{U}(\mathbf{r}, t) = A(t) U(\mathbf{r} - \mathbf{Z}(t)) A^\dagger(t), \quad (24)$$

where $A(t)$ belongs to an SU(3) unitary matrix and $\mathbf{Z}(t)$ correspond to the translational zero modes. The Dirac operator in Eq. (14) is then changed as

$$\tilde{D} = \partial_4 + h(U) + A^\dagger(t) \dot{A}(t) - i\gamma_4 \dot{\mathbf{Z}} \cdot \nabla + \gamma_4 A^\dagger(t) (\delta m) A(t), \quad (25)$$

where $A^\dagger(t) \dot{A}(t)$ is the angular velocity of the soliton $\Omega(t)$ in Euclidean space

$$A^\dagger(t) \dot{A}(t) = i\Omega = \frac{1}{2} i\Omega^a \lambda^a, \quad (26)$$

and $\dot{\mathbf{Z}}$ designates the translational velocity of the soliton

$$\dot{\mathbf{Z}} = \frac{d\mathbf{Z}}{dt}. \quad (27)$$

Then the effective action under the zero-mode quantization is expressed as

$$\begin{aligned}
\tilde{S}_{\text{eff}} &= -N_c \text{Tr} \ln[\partial_4 + A^\dagger(t) \dot{A}(t) - i\gamma_4 \dot{\mathbf{Z}} \cdot \nabla \\
&+ \gamma_4 A^\dagger(t) (\delta m) A(t) - \gamma_4 a_\mu \gamma_\mu \gamma_5 A^\dagger(t) \lambda^a A(t)], \quad (28)
\end{aligned}$$

where a_μ stands for the external axial-vector source field. Expanding the zero-mode quantized effective action in powers of angular and translational velocities that are proportional to $1/N_c$, we obtain the action as

$$\tilde{S}_{\text{eff}} \approx -N_c \text{Tr} \ln D + S_{\text{rot}}[A] + S_{\text{trans}}[\mathbf{Z}], \quad (29)$$

where

$$S_{\text{rot}}[A] = \frac{1}{2} I_{ab} \int dt \Omega^a \Omega^b, \quad S_{\text{trans}}[\mathbf{Z}] = \frac{1}{2} M_{\text{cl}} \int dt \dot{\mathbf{Z}} \cdot \dot{\mathbf{Z}}. \quad (30)$$

Here, I_{ab} is the inertial tensor for the soliton and M_{cl} is the mass of the classical soliton, which is found to be the sum of the N_c valence-quark energies and the Dirac-continuum energy; $M_{\text{cl}} = N_c E_{\text{val}} + E_{\text{sea}}$. We refer to Ref. [72] for details.

The integral over the translational zero modes yields naturally the Fourier transform, which indicates that the baryon state has the proper translational symmetry. Having performed the rotational zero-mode quantization, we can restore the rotational symmetry so that the baryon state has correct spin and flavor quantum numbers. After the zero-mode quantization, we obtain the collective Hamiltonian as follows:

$$H_{\text{coll}} = H_{\text{sym}} + H_{\text{sb}}, \quad (31)$$

where H_{coll} are decomposed into the flavor SU(3) symmetric and symmetry-breaking terms

$$\begin{aligned}
H_{\text{sym}} &= M_{\text{cl}} + \frac{1}{2I_1} \sum_{i=1}^3 \hat{J}_i^2 + \frac{1}{2I_2} \sum_{p=4}^7 \hat{J}_p^2, \\
H_{\text{sb}} &= \alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{J}_i. \quad (32)
\end{aligned}$$

Here, I_1 and I_2 stand for the moments of inertia for the soliton, which are the diagonal components of I_{ab} in Eq. (30) when $a = 1, 2, 3$ and $a = 4, \dots, 7$, respectively. The explicit expressions for them can be found in Appendix A. $D_{ab}^{(8)}$ represent SU(3) Wigner D functions. The inertial parameters α , β , and γ , which arise from the linear m_s corrections, are expressed in terms of the moments of inertia I_1 and I_2 , and the anomalous moments of inertia K_1 and K_2

$$\alpha = \left(-\frac{\Sigma_{\pi N}}{3\bar{m}} + \frac{K_2}{I_2} \right) m_s, \quad \beta = -\frac{K_2}{I_2} m_s, \quad \gamma = 2 \left(\frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s. \quad (33)$$

where $\Sigma_{\pi N}$ stands for the pion-nucleon Σ term and its expression can be found in Appendix A. K_1 and K_2 arise from the rotation of the mass term $A^\dagger(\delta m)A$ in Eq. (24) (see Ref. [112]). The corresponding expressions can also be found in Appendix A. Once the flavor SU(3) symmetry is broken, the collective wave functions of the baryon decuplet start to get mixed with states in higher representations. Thus, the states of the baryon octet and decuplet are derived by the standard second-order perturbation theory,

$$|B_{8_{1/2}}\rangle = |8_{1/2}, B\rangle + c_{10}^B |\overline{10}_{1/2}, B\rangle + c_{27}^B |27_{1/2}, B\rangle, \quad (34)$$

$$|B_{10_{3/2}}\rangle = |10_{3/2}, B\rangle + a_{27}^B |27_{3/2}, B\rangle + a_{35}^B |35_{3/2}, B\rangle, \quad (35)$$

with the mixing coefficients

$$c_{10}^B = c_{\overline{10}} \begin{bmatrix} \sqrt{5} \\ 0 \\ \sqrt{5} \\ 0 \end{bmatrix}, \quad c_{27}^B = c_{27} \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{bmatrix}, \quad (36)$$

$$a_{27}^B = a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \\ \sqrt{3/2} \\ 0 \end{bmatrix}, \quad a_{35}^B = a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2\sqrt{5/7} \\ 3\sqrt{5/14} \\ 2\sqrt{5/7} \end{bmatrix}, \quad (37)$$

respectively, in the basis $[N, \Lambda, \Sigma, \Xi]$ for the baryon octet and $[\Delta, \Sigma^*, \Xi^*, \Omega]$ for the baryon decuplet. The parameters $c_{\overline{10}}$, c_{27} , a_{27} and a_{35} are expressed in terms of α and γ .

$$c_{\overline{10}} = -\frac{I_2}{15} \left(\alpha + \frac{1}{2}\gamma \right), \quad c_{27} = -\frac{I_2}{25} \left(\alpha - \frac{1}{6}\gamma \right), \quad (38)$$

$$a_{27} = -\frac{I_2}{8} \left(\alpha + \frac{5}{6}\gamma \right), \quad a_{35} = -\frac{I_2}{24} \left(\alpha - \frac{1}{2}\gamma \right). \quad (39)$$

Each state in Eqs. (34) and (35) is given in terms of the SU(3) Wigner D functions that satisfy the quantization condition [112].

The final expression for the axial-vector transition form factors is derived as $C_5^{A,10 \rightarrow 8}$,

$$\begin{aligned} C_5^{A,10 \rightarrow 8}(Q^2) = & \frac{\langle D_{a3}^{(8)} \rangle}{3} \{ \mathcal{A}_0(Q^2) - \mathcal{A}_2(Q^2) \} + \frac{1}{3\sqrt{3}I_1} \left[\langle D_{a8}^{(8)} \hat{J}_3 \rangle + \frac{2m_s}{\sqrt{3}} K_1 \langle D_{83}^{(8)} D_{a8}^{(8)} \rangle \right] \{ \mathcal{B}_0(Q^2) - \mathcal{B}_2(Q^2) \} \\ & + \frac{d_{pq3}}{3I_2} \left[\langle D_{ap}^{(8)} \hat{J}_q \rangle + \frac{2m_s}{\sqrt{3}} K_2 \langle D_{ap}^{(8)} D_{8q}^{(8)} \rangle \right] \{ \mathcal{C}_0(Q^2) - \mathcal{C}_2(Q^2) \} - \frac{i \langle D_{a3}^{(8)} \rangle}{6I_1} \{ \mathcal{D}_0(Q^2) - \mathcal{D}_2(Q^2) \} \\ & + \frac{2m_s}{9} (\langle D_{a3}^{(8)} \rangle - \langle D_{88}^{(8)} D_{a3}^{(8)} \rangle) \{ \mathcal{H}_0(Q^2) - \mathcal{H}_2(Q^2) \} - \frac{2m_s}{9} \langle D_{83}^{(8)} D_{a8}^{(8)} \rangle \{ \mathcal{I}_0(Q^2) - \mathcal{I}_2(Q^2) \} \\ & - \frac{2m_s}{3\sqrt{3}} d_{pq3} \langle D_{ap}^{(8)} D_{8q}^{(8)} \rangle \{ \mathcal{J}_0(Q^2) - \mathcal{J}_2(Q^2) \}, \end{aligned} \quad (40)$$

where $\langle \dots \rangle$ represent the matrix elements for the SU(3) Wigner D functions between B_8 and B_{10} collective states, which are expressed in terms of the SU(3) Clebsch-Gordan coefficients. The results are explicitly given in Appendix B. $\mathcal{A}_0(Q^2), \dots, \mathcal{J}_2(Q^2)$ denote the Fourier transforms of the axial-vector transition densities, which can be found in Appendix A.

Since the matrix elements of the Wigner D functions also contain the linear m_s terms, the collective baryon states get the linear m_s corrections from those in higher representations. Thus, there are yet additional m_s corrections in addition to those shown in Eq. (40). Thus, it is more

convenient to decompose the contributions arising from flavor SU(3) symmetry breaking into two terms

$$C_5^{A,10 \rightarrow 8} = C_5^{A,10 \rightarrow 8(\text{sym})} + C_5^{A,10 \rightarrow 8(\text{op})} + C_5^{A,10 \rightarrow 8(\text{wf})}, \quad (41)$$

where $C_5^{A,10 \rightarrow 8(\text{sym})}$ denote the contributions from the SU(3) symmetric part in Eq. (40) whereas $C_5^{A,10 \rightarrow 8(\text{op})}$ and $C_5^{A,10 \rightarrow 8(\text{wf})}$ come respectively from the current quark-mass term in the effective chiral action (13) and from the collective wave functions. They are explicitly written as

$$C_5^{A,10 \rightarrow 8(\text{sym})} = \frac{\sqrt{5}}{90} \begin{pmatrix} 2 \\ -T_3 \\ -2T_3 \\ \sqrt{3} \end{pmatrix} \left[2(\mathcal{A}_0 - \mathcal{A}_2) - \frac{i(\mathcal{D}_0 - \mathcal{D}_2)}{I_1} \right] - \frac{\sqrt{5}}{90} \begin{pmatrix} 2 \\ -T_3 \\ -2T_3 \\ 1 \end{pmatrix} \frac{\mathcal{C}_0 - \mathcal{C}_2}{I_2}, \quad (42)$$

$$C_5^{A,10 \rightarrow 8(\text{op})} = \frac{\sqrt{5}m_s}{405} \left\{ \begin{pmatrix} 1 \\ -3T_3 \\ -5T_3 \\ \sqrt{3} \end{pmatrix} \left[\frac{K_1}{I_1} (\mathcal{B}_0 - \mathcal{B}_2) - (\mathcal{I}_0 - \mathcal{I}_2) \right] \right. \\ \left. + \begin{pmatrix} 7 \\ -3T_3 \\ -8T_3 \\ 4\sqrt{3} \end{pmatrix} \left[\frac{K_2}{I_2} (\mathcal{C}_0 - \mathcal{C}_2) - (\mathcal{J}_0 - \mathcal{J}_2) \right] - \begin{pmatrix} 4 \\ 0 \\ T_3 \\ \sqrt{3} \end{pmatrix} (\mathcal{H}_0 - \mathcal{H}_2) \right\}, \quad (43)$$

$$C_5^{A,10 \rightarrow 8(\text{wf})} = \frac{\sqrt{15}}{1620} a_{27} \left\{ \begin{pmatrix} 2\sqrt{2} \\ -3\sqrt{3}T_3 \\ -7\sqrt{2}T_3 \\ 0 \end{pmatrix} \left[2(\mathcal{A}_0 - \mathcal{A}_2) - \frac{i(\mathcal{D}_0 - \mathcal{D}_2)}{I_1} \right] + 4 \begin{pmatrix} \sqrt{10} \\ 0 \\ -\sqrt{2}T_3 \\ 3 \end{pmatrix} \frac{(\mathcal{C}_0 - \mathcal{C}_2)}{I_2} \right\} \\ - \frac{\sqrt{3}}{135} a_{35} \left\{ \begin{pmatrix} 5\sqrt{2} \\ 0 \\ \sqrt{10}T_3 \\ -3\sqrt{5} \end{pmatrix} \left[2(\mathcal{A}_0 - \mathcal{A}_2) - \frac{i(\mathcal{D}_0 - \mathcal{D}_2)}{I_1} \right] - \begin{pmatrix} 5\sqrt{2} \\ 0 \\ -\sqrt{10}T_3 \\ 3\sqrt{5} \end{pmatrix} \frac{(\mathcal{C}_0 - \mathcal{C}_2)}{I_2} \right\}, \quad (44)$$

where we have suppressed Q^2 dependence of C_5^A and T_3 is the third component of the isospin operator.

Since we have assumed isospin symmetry, we can find the isospin relations for the axial-vector transition form factors as follows [18]:

$$\begin{aligned} (\Delta^+ \rightarrow p) &= (\Delta^0 \rightarrow n) = -\frac{1}{\sqrt{3}}(\Delta^{++} \rightarrow p) = \frac{1}{\sqrt{3}}(\Delta^- \rightarrow n) = (\Delta^0 \rightarrow p) = -(\Delta^+ \rightarrow n) \\ \sqrt{2}(\Sigma^{*+} \rightarrow \Sigma^+) &= -\sqrt{2}(\Sigma^{*-} \rightarrow \Sigma^-) = (\Sigma^{*0} \rightarrow \Sigma^+) = (\Sigma^{*-} \rightarrow \Sigma^0) = (\Sigma^{*+} \rightarrow \Sigma^0) = (\Sigma^{*0} \rightarrow \Sigma^-) \\ (\Sigma^{*0} \rightarrow \Lambda) &= \frac{1}{\sqrt{2}}(\Sigma^{*-} \rightarrow \Lambda) = -\frac{1}{\sqrt{2}}(\Sigma^{*+} \rightarrow \Lambda) \\ (\Xi^{*0} \rightarrow \Xi^0) &= -(\Xi^{*-} \rightarrow \Xi^-) = \frac{1}{2}(\Xi^{*-} \rightarrow \Xi^0) = \frac{1}{2}(\Xi^{*0} \rightarrow \Xi^-) \\ (\Delta^{++} \rightarrow \Sigma^+) &= \frac{\sqrt{3}}{\sqrt{2}}(\Delta^+ \rightarrow \Sigma^0) = \sqrt{3}(\Delta^0 \rightarrow \Sigma^-) \\ (\Sigma^{*0} \rightarrow p) &= \frac{1}{\sqrt{2}}(\Sigma^{*-} \rightarrow n) \\ (\Sigma^{*0} \rightarrow \Xi^0) &= \sqrt{2}(\Sigma^{*0} \rightarrow \Xi^-) \\ (\Xi^{*0} \rightarrow \Sigma^+) &= \sqrt{2}(\Xi^{*-} \rightarrow \Sigma^0), \end{aligned} \quad (45)$$

where $(B_{10} \rightarrow B_8)$ denote the axial vector form factors $C_5^{A,B_{10} \rightarrow B_8}$. We also find several sum rules between those form factors,

$$\begin{aligned} (\Delta^+ \rightarrow p) &= -\frac{1}{\sqrt{6}}(\Xi^{*-} \rightarrow \Lambda) - \frac{1}{\sqrt{2}}(\Xi^{*-} \rightarrow \Sigma^0) + \frac{4}{\sqrt{3}}(\Sigma^{*0} \rightarrow \Lambda) + (\Xi^{*-} \rightarrow \Xi^0) \\ (\Sigma^{*+} \rightarrow \Sigma^+) &= -\frac{1}{\sqrt{3}}(\Sigma^{*0} \rightarrow \Lambda) + \frac{1}{\sqrt{6}}(\Xi^{*-} \rightarrow \Lambda) + \frac{1}{\sqrt{2}}(\Xi^{*-} \rightarrow \Sigma^0) \\ (\Sigma^{*0} \rightarrow \Lambda) &= \frac{\sqrt{3}}{\sqrt{2}}(\Sigma^{*0} \rightarrow p) + \frac{\sqrt{3}}{2\sqrt{2}}(\Xi^{*-} \rightarrow \Sigma^0) - \frac{1}{2\sqrt{2}}(\Xi^{*-} \rightarrow \Lambda) - \frac{\sqrt{3}}{2}(\Xi^{*-} \rightarrow \Xi^0) \end{aligned}$$

$$\begin{aligned}
(\Sigma^{*0} \rightarrow p) &= -\frac{1}{2}(\Xi^{*-} \rightarrow \Sigma^0) + \frac{\sqrt{3}}{2}(\Xi^{*-} \rightarrow \Lambda) + \frac{1}{\sqrt{6}}(\Omega^- \rightarrow \Xi^0) \\
(\Xi^{*-} \rightarrow \Xi^0) &= \frac{\sqrt{2}}{\sqrt{3}}(\Xi^{*-} \rightarrow \Lambda) - \frac{\sqrt{2}}{\sqrt{3}}(\Sigma^{*0} \rightarrow \Lambda) + \frac{1}{\sqrt{3}}(\Omega^- \rightarrow \Xi^0) \\
(\Delta^+ \rightarrow \Sigma^0) &= -(\Xi^{*-} \rightarrow \Sigma^0) + \frac{1}{\sqrt{3}}(\Xi^{*-} \rightarrow \Lambda).
\end{aligned} \tag{46}$$

IV. RESULTS AND DISCUSSION

Before we compute the axial-vector transition form factors of the baryon decuplet, we first discuss how the parameters are fixed. In the χ QSM, there are four different parameters: the dynamical quark mass M , the cutoff mass Λ in the regularization functions, the strange current quark mass m_s , and the average of the up and down current quarks $\bar{m} = 6.131$ MeV, as mentioned in Sec. III. $\bar{m}$ is determined by reproducing the physical value of the pion mass, $m_\pi = 140$ MeV. The strange current quark mass is usually fixed by the kaon mass, $m_K = 495$ MeV. Its value is obtained to be 150 MeV. However, we use a slightly larger value $m_s = 180$ MeV, which describes the mass spectra of the baryon octet and decuplet [69,112]. The cutoff mass Λ is determined by the pion decay constant $f_\pi = 93$ MeV. On the other hand, the dynamical quark mass M is a free parameter in the χ QSM but is also fixed by reproducing the electric charge radius of the proton [72], i.e., the corresponding value of M is $M = 420$ MeV. We use exactly the same values of these parameters in the present work. As shown in Eq. (11), C_5^A involves the octet mass M_8 . The baryon masses in the χ QSM also include the rotational $1/N_c$ and m_s corrections. If we turn off all the corrections, the baryon masses become the classical nucleon mass M_{cl} or the soliton mass, which is proportional to N_c . To be

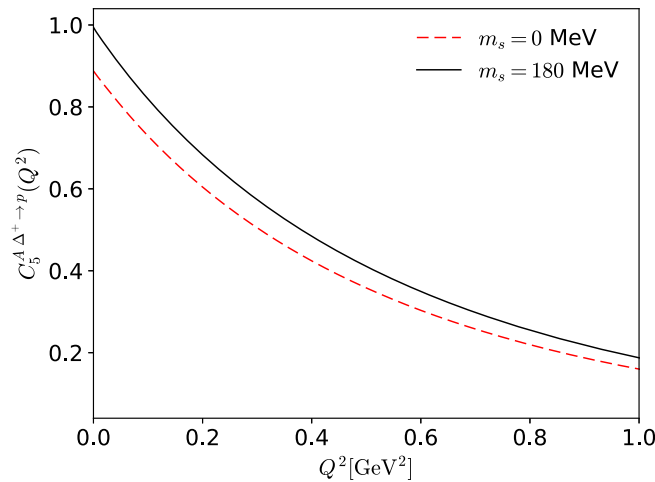


FIG. 1. Effects of the explicit flavor SU(3) symmetry breaking on the axial-vector transition form factors $C_5^{A, \Delta^+ \to p}(Q^2)$ for the $\Delta^+ \rightarrow p$ transition. The solid curve draws the total result whereas the dashed one depicts the result without the m_s corrections.

theoretically more consistent, we will take M_{cl} instead of a octet baryon mass [113,114]. In fact, the numerical results are improved by considering M_{cl} in place of M_8 by around 10%. Similar effects can be seen in the calculation of the magnetic dipole moments of the SU(3) baryons.

We first examine the effects of flavor SU(3) symmetry breaking on the axial-vector transition form factor for the $\Delta^+ \rightarrow p$ transition. In Fig. 1, we draw the results for the $\Delta^+ \rightarrow p$ axial-vector transition form factors. The solid curve depicts the total result, whereas the dashed one draws that with the effects of the explicit flavor SU(3) symmetry breaking turned off. The corrections from the linear m_s contribute to $C_5^{A, \Delta^+ \to p}(Q^2)$ by about 10%, as expected. As discussed already in Ref. [78], the effects of the explicit flavor SU(3) symmetry breaking range in magnitude from 5% to 15%, depending on the decay modes. So, the linear m_s corrections are also marginal in the case of the $\Delta^+ \rightarrow p$ axial-vector transition form factors.

A few works computed theoretically the axial-vector transition form factors [41–43]. So, we first compare the current result for $C_5^{A, \Delta \to N}(Q^2)$ with those from other models as shown in Fig. 2. The solid curve draws the present result,

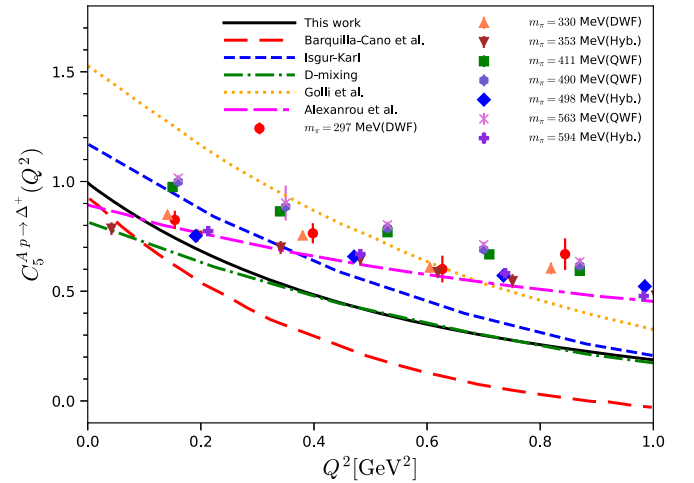


FIG. 2. Numerical results of the $C_5^{A(3) B_{10} \to B_8}(Q^2)$ for the transition from the Δ^+ isobar to the proton in comparison with those from other models. The solid curve draws the present result, whereas long-dashed, dashed, dot-dashed, dotted ones are taken from Refs. [41–43]. The present results are also compared with the data taken from lattice QCD [34,35]. The dot-dot-dashed curve depicts a fit to a monopole form of the quenched lattice data (see Fig. 17 in Ref. [34]).

whereas the long-dashed one is taken from Ref. [43], in which the chiral constituent quark model was used. In Ref. [43], the effective Hamiltonian was constructed by introducing a confinement potential, a one-gluon exchange potential, and a one-pion exchange potential. Because of the one-pion exchange potential, the model is called the chiral constituent quark model. Since the nucleon and Δ states are constructed in terms of five harmonic oscillator bases, the nonvalence-quark contributions are expressed by states corresponding to $qqqq\bar{q}$ component. We want to mention that the decomposition of the Fock space in quantum field theory can only rigorously be performed in the light cone basis [115]. Note that they use the empirical value of the axial transition mass $M_A \approx 1.28$ GeV as an input, whereas it is predicted in the present work. The result of $C_5^A(0)$ in Ref. [43] is completely determined by the one-body axial-vector current, while

the exchange-current contributions are almost canceled by each other. Thus, the value of $C_5^A(0)$ is obtained to be $C_5^A(0) = 0.93$, which is very similar to the present result; $C_5^A(0) = 0.994$. On the other hand, the Q^2 dependence of C_5^A from Ref. [43] is quite different from the present one, as shown in Fig. 2. That, from Ref. [43], falls off much faster than the present result as Q^2 increases. The dashed and dot-dashed ones are obtained from Ref. [41]. Apart from the explicit forms of the potential, the model is similar to that used in Ref. [43]. In Ref. [41], three different schemes were employed. The result in the short-dashed curve, which was denoted by the Isgur-Karl (IK) model, was obtained by using the parameters given in Refs. [116,117]. As shown in Fig. 2, the result of C_5^A is larger and decreases faster than the present one as Q^2 increases. On the other hand, the result of $C_5^A(0)$ from the D -state mixing model depicted in the

TABLE I. Numerical results for the triplet axial-vector transition constant $C_5^{AB_{10} \rightarrow B_8}(0)$ with $|\Delta S| = 0$ in comparison with those from lattice QCD (LQCD) [35], the relativistic quark models (RQM) [36–38], the isobar model [39], the nonrelativistic quark model (NRQM) [41], the linear σ -model (LSM) and the cloudy bag model (CBM) [42], the chiral constituent quark model (χ CQM) [43], the relativistic baryon-chiral perturbation theory (RBCPT) [45,46], the Barbero-Lopez-Mariano approach [48,49], Graczyk *et al.* [56], Hernandez *et al.* [59], the light cone QCD sum rule (LCSR) [51], and the nonlinear σ model [52]. We also compare the present results fitted to the T2K experimental data(T2K) [28].

$C_5^{AB_{10} \rightarrow B_8}(0)$	$\Delta^+ \rightarrow p$	$\Sigma^{*+} \rightarrow \Sigma^+$	$\Sigma^{*0} \rightarrow \Lambda$	$\Xi^{*-} \rightarrow \Xi^-$
$m_s = 0$ MeV	0.888	-0.443	0.765	0.412
$m_s = 180$ MeV	0.994	-0.446	0.840	0.425
LQCD [35] ($m_\pi = 297$ MeV)	$0.944 \pm 0.058^{a,b}$	...	...	...
LQCD [35] ($m_\pi = 330$ MeV)	$0.970 \pm 0.030^{a,b}$	...	...	...
LQCD [34] ($m_\pi = 353$ MeV)	$0.750 \pm 0.019^{a,b}$	...	...	...
LQCD [34] ($m_\pi = 411$ MeV)	$0.906 \pm 0.015^{a,b}$	...	...	...
LQCD [34] ($m_\pi = 490$ MeV)	$0.930 \pm 0.014^{a,b}$	...	...	...
LQCD [34] ($m_\pi = 498$ MeV)	$0.864 \pm 0.032^{a,b}$	...	...	...
LQCD [34] ($m_\pi = 563$ MeV)	$0.952 \pm 0.016^{a,b}$	...	...	...
LQCD [34] ($m_\pi = 594$ MeV)	$0.883 \pm 0.022^{a,b}$	...	...	...
RQM1 [36]	0.97	...	...	...
RQM2 [37]	0.83	...	...	...
RQM3 [38]	0.97	...	...	...
Fogli <i>et al.</i> [39]	1.18	...	...	...
Liu <i>et al.</i> [41]	1.17	...	...	...
LSM [42]	1.53	...	...	...
CBM [42]	0.81	...	...	...
χ CQM [43]	0.93	...	...	...
RBCPT1 [45]	1.16	...	...	...
Barbero <i>et al.</i> [48,49]	1.35	...	...	...
Graczyk <i>et al.</i> [56]	1.19 ± 0.08	...	...	...
Hernandez <i>et al.</i> [59]	0.867 ± 0.075	...	...	...
LCSR [51]	1.14 ± 0.20	...	...	...
Alvarez-Ruso <i>et al.</i> [52]	1.12 ± 0.11	...	...	...
RBCPT2 [46]	1.17 ± 0.02	...	...	...
T2K(Prefit) [28]	0.96 ± 0.15	...	...	...
T2K(Postfit) [28]	0.98 ± 0.06	...	...	...

^aSince the expressions for the axial-vector transition constants in Ref. [35] are different from the present one by -1 , we have considered this factor for comparison.

^bIn Ref. [34], these values are extrapolated ones obtained by using the dipole parametrization.

TABLE II. Numerical results for the axial transition mass in comparison with the lattice data [34,35], that extracted from the Argonne National Laboratory (ANL) data [21,22], CERN BEBC data [55], the Brookhaven National Laboratory(BNL) data [24,56], MiniBooNE data [26], and T2K fitted results [28]. We also compare the present results with those from other works [52–54]. We use the dipole-type form factor for parametrization A. Parametrization B corresponds to Alder’s parametrization [20].

M_A [GeV]	$\Delta^+ \rightarrow p$	$\Sigma^{*+} \rightarrow \Sigma^+$	$\Sigma^{*0} \rightarrow \Lambda$	$\Xi^{*0} \rightarrow \Xi^0$
Parametrization A	0.863	1.03	1.03	1.35
Parametrization B	1.17	1.32	1.31	1.47
LQCD [35] ($m_\pi = 297$ MeV) (dipole)	1.699 ± 0.170	...	...	...
LQCD [35] ($m_\pi = 329$ MeV) (dipole)	1.588 ± 0.070	...	...	...
LQCD [34] ($m_\pi = 353$ MeV) (dipole)	2.202 ± 0.113	...	...	...
LQCD [34] ($m_\pi = 411$ MeV) (dipole)	1.534 ± 0.036	...	...	...
LQCD [34] ($m_\pi = 490$ MeV) (dipole)	1.537 ± 0.033	...	...	...
LQCD [34] ($m_\pi = 498$ MeV) (dipole)	1.892 ± 0.101	...	...	...
LQCD [35] ($m_\pi = 563$ MeV) (dipole)	1.544 ± 0.032	...	...	...
LQCD [34] ($m_\pi = 594$ MeV) (dipole)	1.924 ± 0.085	...	...	...
Fogli <i>et al.</i> [39]	0.75	...	...	...
ANL [21]	0.93 ± 0.11	...	...	...
BEBC [55]	0.85 ± 0.10	...	...	...
Rein <i>et al.</i> [53]	0.95	...	...	...
BNL [24]	$1.28^{+0.08}_{-0.10}$	...	...	...
Lalakulich <i>et al.</i> [54] ^a	1.05	...	...	...
Lalakulich <i>et al.</i> [54] ^b	0.95	...	...	...
Hernandez <i>et al.</i> [59]	0.985 ± 0.082	...	...	...
Graczyk <i>et al.</i> [56]	0.94 ± 0.04	...	...	...
MiniBooNE [26]	1.35 ± 0.17	...	...	...
Alvarez-Ruso <i>et al.</i> [52]	0.954 ± 0.063	...	...	...
T2K(Prefit) [28]	1.20 ± 0.03	...	...	...
T2K(Postfit) [28]	1.13 ± 0.08	...	...	...

^aThey use the parametrization form as $C_5^A(Q^2) = \frac{C_5^A(0)}{(1+Q^2/M_A^2)^2} \frac{1}{1+2Q^2/M_A^2}$.

^bThey use the parametrization form as $C_5^A(Q^2) = \frac{C_5^A(0)}{(1+Q^2/M_A^2)^2} (\frac{1}{1+Q^2/3M_A^2})^2$.

dot-dashed curve is smaller than the present one. However, its Q^2 dependence is milder than that from the present one as well as that from the IK model. In Ref. [42], the linear sigma model and the cloudy bag model were employed. The dotted curve in Fig. 2 illustrates the result from the linear sigma model. The value of $C_5^A(0)$ from Ref. [42] is quite overestimated in comparison with the fitted results from the T2K experiment [28].

In Fig. 2, the dot-dot-dashed curve illustrates a fit to a monopole form of the quenched lattice data [34,35]. It tends to fall off relatively slower than those of other models and that of the present one. It is well known that the lattice calculations with the *unphysical* pion mass produce in general hadronic form factors that fall off very slowly as Q^2 increases. Considering the picture that the pion fields govern the structure of the nucleon and Δ in outer parts, one can understand that the smaller pion mass renders the sizes of N and Δ smaller than physical ones. The result of the current work for $C_5^{A,\Delta^+ \rightarrow p}$ is in good agreement with the lattice one as will be shown explicitly in Table I.

In Table I, we list the values of $C_5^A(0)$ for four different axial-vector transitions, with and without the effects of

explicit SU(3) symmetry breaking. One can quickly obtain the values of C_5^A for all other channels from the isospin relations given in Eq. (45). Since there are many results for the $\Delta^+ \rightarrow p$ axial-vector transition derived from other works, we compare the current results with them. As already

TABLE III. Numerical results for the axial transition radius in comparison with those from various approaches; baryon chiral perturbation theory(BCPT) [44,47], the chiral constituent quark model (χ CQM) [43], nonrelativistic quark potential model [41] with two different methods (Isgur-Karl and D-mixing), and lattice QCD [34].

	$\Delta^+ \rightarrow p$	$\Sigma^{*+} \rightarrow \Sigma^+$	$\Sigma^{*0} \rightarrow \Lambda$	$\Xi^{*0} \rightarrow \Xi^0$
$\langle r^2 \rangle_{B_{10}B_8}$ [fm ²]	0.542	0.452	0.452	0.345
BCPT1 [44]	0.424–0.498	...	...	...
BCPT2 [47]	0.345	...	...	...
χ CQM [43]	0.59	...	...	...
Isgur-Karl [41]	0.32	...	...	...
D-mixing [41]	0.30	...	...	...
Lattice QCD [34]	0.18	...	...	...

discussed in Fig. 1, the effects of the explicit SU(3) symmetry breaking on the $\Delta \rightarrow N$ transition are about 10%. While the contribution of the linear m_s corrections to $C_5^{A\Sigma^0 \rightarrow \Lambda}(0)$ is similar to that of the $\Delta \rightarrow N$ transition, the effects of explicit SU(3) symmetry breaking are almost negligible. The final result for $C_5^{A\Delta \rightarrow N}(0)$ is obtained to be 0.994, which is in good agreement with the T2K data [28]. Those from Refs. [36,38,42,43,59] are also in good agreement with the T2K data. That from Ref. [37] is underestimated but those from Refs. [39,41,42,45,46,48,49,51,52,56] yield larger values than the fitted results from the T2K data.

The axial-transition form factors can be parametrized in terms of the axial transition mass M_A . Two different parametrizations are used, i.e., dipole-type parametrization,

$$C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_A^2)^2} \quad (47)$$

and the Adler's one,

$$C_5^A(Q^2) = \frac{C_5^A(0)[1 + aQ^2/(b + Q^2)]}{(1 + Q^2/M_A^2)^2}, \quad (48)$$

where a and b are fixed respectively to be $a = -1.2$ and $b = 2.0$. We use both the parametrizations and call the first one parametrization A and the second one parametrization B. In Table II, we list the present results for the axial transition mass in the case of the $\Delta S = 0$ axial-vector transitions. In general, the value of M_A from parametrization A is smaller than that from parametrization B. The lattice calculations use the dipole-type parametrization, while many works employ Adler's one. The present result (parametrization A) for the $\Delta^+ \rightarrow p$ transition is much smaller than those from lattice QCD. One can easily understand this difference, since the results for $C_5^{A,\Delta^+ \rightarrow p}$ from the lattice data fall off much slower than the present one. The result for $M_A(\Delta^+ \rightarrow p)$ is in good agreement with the fitted results from the T2K. However, there is a caveat; if one computes the axial transition mean-square radius for the $\Delta^+ \rightarrow p$ decay by using the dipole-type and Adler-type parametrizations, we find $\langle r^2 \rangle_{\Delta^+ p} = 0.531 \text{ fm}^2$ and $\langle r^2 \rangle_{\Delta^+ p} = 0.470 \text{ fm}^2$, respectively. It indicates that the dipole-type parametrization yields a closer value of $\langle r^2 \rangle_{\Delta^+ p}$ to the model result (see Table III, where we give the following value, $\langle r^2 \rangle_{\Delta^+ p} = 0.542 \text{ fm}^2$).

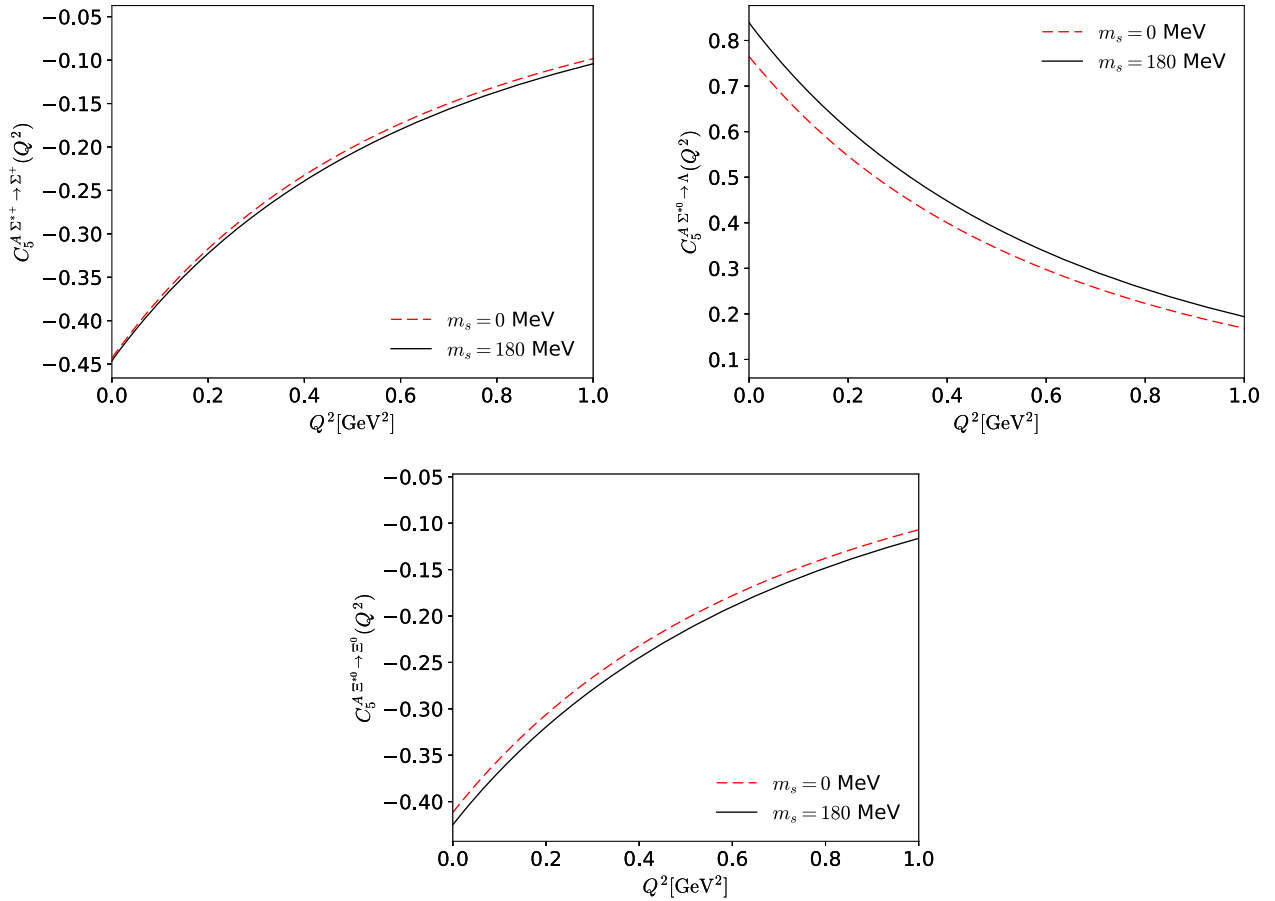


FIG. 3. Effects of the explicit flavor SU(3) symmetry breaking on $C_5^{A(3)\Sigma^{++} \rightarrow \Sigma^+}(Q^2)$ (upper left panel), $C_5^{A(3)\Sigma^0 \rightarrow \Lambda}(Q^2)$ (upper right panel), and $C_5^{A(3)\Sigma^0 \rightarrow \Xi^0}(Q^2)$ (lower panel). Notations are the same as in Fig. 1.

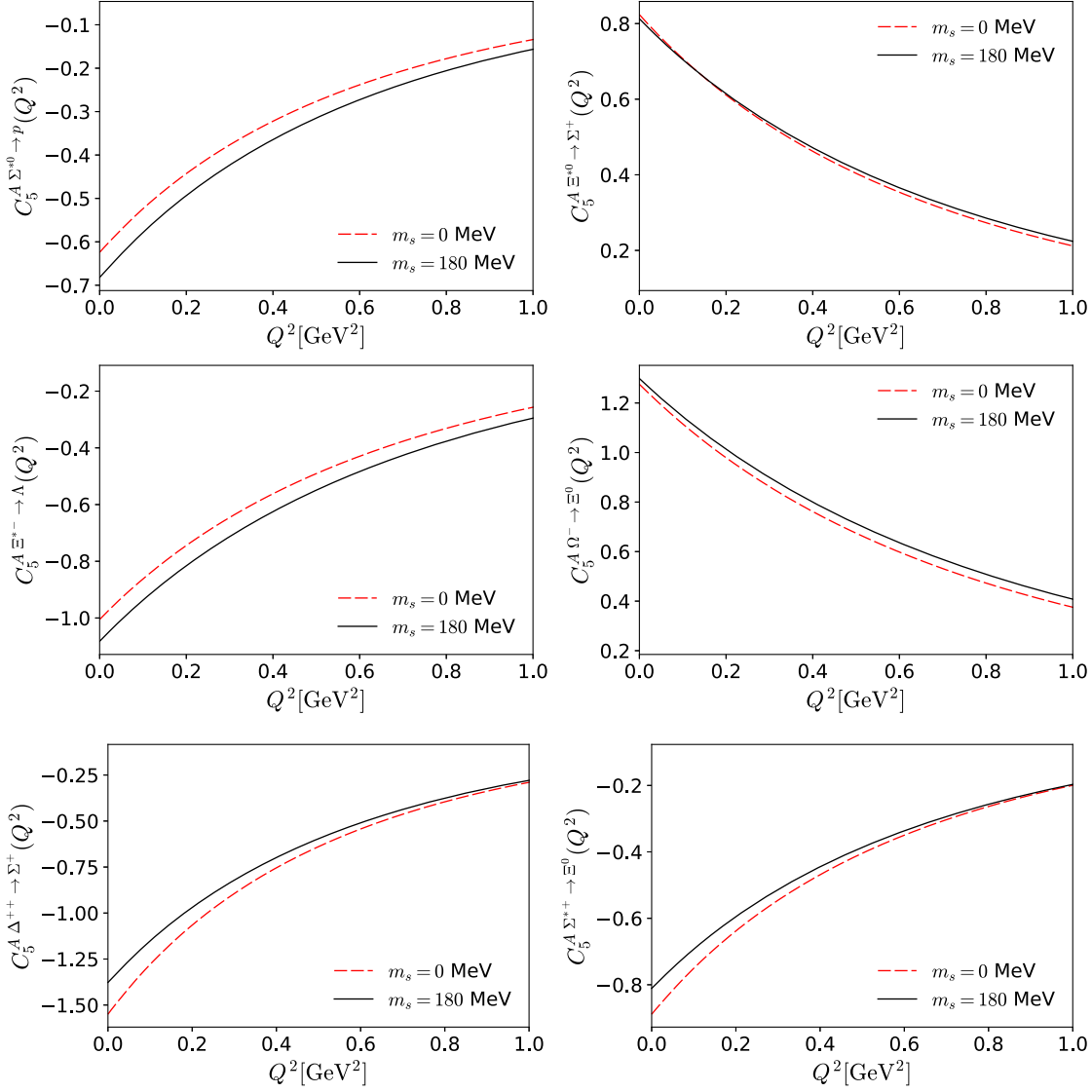


FIG. 4. Effects of the explicit flavor SU(3) symmetry breaking on $C_5^{AB_{10} \rightarrow B_8}(Q^2)$ with $|\Delta S| = 1$. Notations are the same as in Fig. 1.

The mean-square radii for the $B_{10} \rightarrow B_8$ axial vector transitions give information on the behaviors of the corresponding form factors in the vicinity of $Q^2 = 0$, since they are defined by

$$\langle r^2 \rangle_{B_{10}B_8} = -6 \frac{dC_5^A(Q^2)}{dQ^2} \Big|_{Q^2=0}. \quad (49)$$

Table III lists the results for the $\langle r^2 \rangle_{B_{10}B_8}$. In the second column, we compare the current result for $\langle r^2 \rangle_{\Delta N}$ with those from other works and found that the present result is in agreement with that from the χ CQM [43] whereas it is smaller than the other works. Note that as the strangeness $|S|$ increases, the magnitudes of the axial-transition radii are reduced. So, we have the inequality relation

$$\langle r^2 \rangle_{\Delta N} > \langle r^2 \rangle_{\Sigma^* \Sigma} > \langle r^2 \rangle_{\Xi^* \Xi}. \quad (50)$$

Figure 3 draws the axial-vector transition form factors $C_5^{A(3)\Sigma^{*+} \rightarrow \Sigma^+}(Q^2)$, $C_5^{A(3)\Sigma^{*0} \rightarrow \Lambda^0}(Q^2)$, and $C_5^{A(3)\Xi^{*0} \rightarrow \Xi^0}(Q^2)$. The effects of flavor SU(3) symmetry breaking on $C_5^{A(3)\Sigma^{*+} \rightarrow \Sigma^+}$ and $C_5^{A(3)\Xi^{*0} \rightarrow \Xi^0}$ are negligibly small (below 4%), whereas they contribute to $C_5^{A(3)\Sigma^{*0} \rightarrow \Lambda^0}$ by about 10%. Thus, the effects of flavor SU(3) symmetry breaking are overall marginal on the axial-vector transition form factors. In Fig. 4, we illustrate the axial-vector transition form factors $C_5^{AB_{10} \rightarrow B_8}$ with strangeness changed. These transitions accompany the kaons in neutrino-nucleon scattering to preserve strangeness. The results again show that the effects of flavor SU(3) symmetry breaking contribute to $C_5^{AB_{10} \rightarrow B_8}$ at most by about 10%. In Tables IV and V, we list the numerical results for $C_5^{AB_{10} \rightarrow B_8}(0)$, the corresponding axial transition mass M_A and axial transition radii. We compare the results for $C_5^{AB_{10} \rightarrow B_8}(0)$ with those obtained

TABLE IV. Numerical results for $C_5^{AB_{10} \rightarrow B_8}(0)$ in comparison with those from the general framework of a chiral soliton model (χ SM) [18]. We use the dipole-type form factor for parametrization A. Parametrization B corresponds to Alder's parametrization [20].

$C_5^{AB_{10} \rightarrow B_8}(0)$	$\Sigma^{*0} \rightarrow p$	$\Xi^{*0} \rightarrow \Sigma^+$	$\Xi^{*-} \rightarrow \Lambda$	$\Omega^- \rightarrow \Xi^0$
$m_s = 0$ MeV	-0.624	0.824	-1.01	1.28
$m_s = 180$ MeV	-0.682	0.813	-1.08	1.30
χ SM [18]	-0.675 ± 0.002	0.954 ± 0.003	-1.169 ± 0.004	1.653 ± 0.006
M_A [GeV] (A)	1.25	1.38	1.37	1.57
M_A [GeV] (B)	1.32	1.50	1.49	1.67
$\langle r^2 \rangle$ [fm ²] (dipole)	0.375	0.338	0.342	0.297

TABLE V. Numerical results for $C_5^{AB_{10} \rightarrow B_8}(0)$ in comparison with those from the general framework of a chiral soliton model (χ SM) [18]. We use the dipole-type form factor for parametrization A. Parametrization B corresponds to Alder's parametrization [20].

$C_5^{AB_{10} \rightarrow B_8}(0)$	$\Delta^{++} \rightarrow \Sigma^+$	$\Sigma^{*+} \rightarrow \Xi^0$
$m_s = 0$ MeV	-1.55	-0.889
$m_s = 180$ MeV	-1.38	-0.811
χ SM [18]	-1.547 ± 0.006	-0.928 ± 0.004
M_A [GeV] (A)	1.14	1.27
M_A [GeV] (B)	1.22	1.37
$\langle r^2 \rangle$ [fm ²] (dipole)	0.409	0.368

from the chiral soliton model [18], where all the dynamical parameters given in the present work were fixed by using the experimental data on hyperon semileptonic decays [3]. The uncertainties of the results from Ref. [18] reflect the experimental errors. Except for $C_5^{A\Sigma^{*0} \rightarrow p}(0)$, the current results are slightly underestimated but are qualitatively in agreement, compared with those from Ref. [18]. The results for the axial transition radii indicate that as the strangeness $|S|$ increases, the values of $\langle r^2 \rangle_{B_{10}B_8}$ are lessened,

$$\begin{aligned} \langle r^2 \rangle_{\Delta^{++}p} &> \langle r^2 \rangle_{\Sigma^{*0}p} > \langle r^2 \rangle_{\Sigma^{*+}\Xi^0} > \langle r^2 \rangle_{\Xi^{*0}\Sigma^+} \\ &> \langle r^2 \rangle_{\Xi^{*-}\Lambda^0} > \langle r^2 \rangle_{\Omega^-\Xi^0}. \end{aligned} \quad (51)$$

V. SUMMARY AND CONCLUSION

In the present work, we aimed at investigating the axial-vector transition form factors for the transitions from the baryon decuplet to the baryon octet within the framework of the SU(3) self-consistent chiral quark-soliton model. We considered the rotational $1/N_c$ corrections and the effects of flavor SU(3) symmetry breaking, dealing with the strange current quark perturbatively. We found that the linear m_s corrections are marginal and even tiny to be neglected, depending on the transition modes. We first compared the results for the axial-vector $\Delta^+ \rightarrow p$ transition form factors

with those from lattice QCD and other models and phenomenological analyses. We obtained the axial-vector transition form factor for the $\Delta^+ \rightarrow p$ transition at $Q^2 = 0$ as $C_5^A(0) = 0.994$. We derived the axial transition mass with the dipole-type parametrization as $M_A = 0.863$ GeV whereas we got $M_A = 1.17$ with Adler's parametrization. $\Delta^+ \rightarrow p$ transition form factor at $Q^2 = 0$ is in good agreement with the lattice data and the fitted results from the T2K data. Since the axial transition mass plays a critical role in understanding the neutrino-proton interaction such as $\nu p \rightarrow \mu^- p \pi^+$, we used the dipole-type and Adler's parametrizations for the $\Delta^+ \rightarrow p$ form factor. We obtained $M_A = 1.17$ GeV with Adler's parametrization employed. This result is in good agreement with the fitting of the T2K data. We then computed the radius squared for the $\Delta^+ \rightarrow p$ transition. The result is larger than those from other works but is in agreement with that from Ref. [43]. We also obtained the axial-vector form factors for other transition modes, including the strangeness-changing transitions. We found that the values of the axial transition radii decrease as the strangeness of the transition modes increases. So far, we are not able to conclude whether this tendency is model-independent. One can extend the present theoretical framework to investigate the axial-vector transition form factors of the singly heavy baryons. The corresponding works are under way.

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APPENDIX A: EXPLICIT EXPRESSIONS FOR THE MOMENTS AND ANOMALOUS OF INERTIA, THE πN SIGMA TERM, AND THE FORM FACTORS

In this appendix we present the explicit expressions for the moments and anomalous moments of inertia, the πN sigma term, and the Q^2 -dependent functions given in Eqs. (40). The moments of inertia I_1, I_2 are expressed as

$$\begin{aligned}
I_1 &= N_c \delta^{ij} \left(\frac{1}{2} \sum_{\epsilon_n \neq \epsilon_v} \frac{1}{\epsilon_n - \epsilon_v} \langle v | \tau^i | n \rangle \langle n | \tau^j | v \rangle + \frac{1}{4} \sum_{n,m}^{n \neq m} \langle n | \tau^i | m \rangle \langle m | \tau^j | n \rangle \mathcal{R}_3(\epsilon_n, \epsilon_m) \right) \\
I_2 &= N_c \left(\frac{1}{4} \sum_{\epsilon_n^0} \frac{1}{\epsilon_n^0 - \epsilon_v} \langle n^0 | v \rangle \langle v | n^0 \rangle + \frac{1}{4} \sum_{n,m^0}^{n \neq m^0} \langle m^0 | n \rangle \langle n | m^0 \rangle \mathcal{R}_3(\epsilon_n, \epsilon_{m^0}) \right), \tag{A1}
\end{aligned}$$

and the anomalous moments of inertia are written by

$$\begin{aligned}
K_1 &= N_c \delta^{ij} \left(\frac{1}{2} \sum_{\epsilon_n \neq \epsilon_v} \frac{1}{\epsilon_n - \epsilon_v} \langle v | \tau^i | n \rangle \langle n | \gamma^0 \tau^j | v \rangle + \frac{1}{4} \sum_{n,m}^{n \neq m} \langle n | \tau^i | m \rangle \langle m | \gamma^0 \tau^j | n \rangle \mathcal{R}_5(\epsilon_n, \epsilon_m) \right) \\
K_2 &= N_c \left(\frac{1}{4} \sum_{\epsilon_n^0} \frac{1}{\epsilon_n^0 - \epsilon_v} \langle n^0 | v \rangle \langle v | \gamma^0 | n^0 \rangle + \frac{1}{4} \sum_{n,m^0}^{n \neq m^0} \langle m^0 | \gamma^0 | n \rangle \langle n | m^0 \rangle \mathcal{R}_5(\epsilon_n, \epsilon_{m^0}) \right). \tag{A2}
\end{aligned}$$

The πN sigma term is expressed as

$$\Sigma_{\pi N} = -N_c \left(1 - \frac{1}{\sqrt{3}} D_{88}^{(8)} \right) \left[\langle v | \gamma^0 | v \rangle + \sum_n \langle n | \gamma^0 | n \rangle \mathcal{R}_1(\epsilon_n) - \text{vacuum subtraction} \right]. \tag{A3}$$

$\mathcal{A}_0(Q^2), \dots, \mathcal{J}_0(Q^2)$ are defined by

$$\begin{aligned}
\mathcal{A}_0(Q^2) &= N_c \mathcal{M} \int d^3 r j_0(Q|\mathbf{r}|) \left[\phi_{\text{val}}^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \phi_{\text{val}}(\mathbf{r}) + \sum_n \phi_n^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \phi_n(\mathbf{r}) \mathcal{R}_1(E_n) \right], \\
\mathcal{B}_0(Q^2) &= N_c \mathcal{M} \int d^3 r j_0(Q|\mathbf{r}|) \left[\sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) \boldsymbol{\sigma} \phi_n(\mathbf{r}) \cdot \langle n | \boldsymbol{\tau} | \text{val} \rangle - \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \boldsymbol{\sigma} \phi_m(\mathbf{r}) \cdot \langle m | \boldsymbol{\tau} | n \rangle \mathcal{R}_5(E_n, E_m) \right], \\
\mathcal{C}_0(Q^2) &= N_c \mathcal{M} \int d^3 r j_0(Q|\mathbf{r}|) \left[\sum_{n_0 \neq \text{val}} \frac{1}{E_{\text{val}} - E_{n_0}} \phi_{\text{val}}^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \phi_{n_0}(\mathbf{r}) \langle n_0 | \text{val} \rangle - \sum_{n,m_0} \phi_n^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \phi_{m_0}(\mathbf{r}) \langle m_0 | n \rangle \mathcal{R}_5(E_n, E_{m_0}) \right], \\
\mathcal{D}_0(Q^2) &= N_c \mathcal{M} \int d^3 r j_0(Q|\mathbf{r}|) \left[\sum_{n \neq \text{val}} \frac{\text{sgn}(E_n)}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) (\boldsymbol{\sigma} \times \boldsymbol{\tau}) \phi_n(\mathbf{r}) \cdot \langle n | \boldsymbol{\tau} | \text{val} \rangle \right. \\
&\quad \left. + \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \boldsymbol{\sigma} \times \boldsymbol{\tau} \phi_m(\mathbf{r}) \cdot \langle m | \boldsymbol{\tau} | n \rangle \mathcal{R}_4(E_n, E_m) \right], \\
\mathcal{H}_0(Q^2) &= N_c \mathcal{M} \int d^3 r j_0(Q|\mathbf{r}|) \left[\sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \langle n | \gamma^0 | \text{val} \rangle + \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \phi_m(\mathbf{r}) \langle m | \gamma^0 | n \rangle \mathcal{R}_2(E_n, E_m) \right], \\
\mathcal{I}_0(Q^2) &= N_c \mathcal{M} \int d^3 r j_0(Q|\mathbf{r}|) \left[\sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) \boldsymbol{\sigma} \phi_n(\mathbf{r}) \cdot \langle n | \gamma^0 \boldsymbol{\tau} | \text{val} \rangle + \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \boldsymbol{\sigma} \phi_m(\mathbf{r}) \cdot \langle m | \gamma^0 \boldsymbol{\tau} | n \rangle \mathcal{R}_2(E_n, E_m) \right], \\
\mathcal{J}(Q^2) &= N_c \mathcal{M} \int d^3 r j_0(Q|\mathbf{r}|) \left[\sum_{n_0 \neq \text{val}} \frac{N_c}{E_{\text{val}} - E_{n_0}} \phi_{\text{val}}^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \phi_{n_0}(\mathbf{r}) \langle n_0 | \gamma^0 | \text{val} \rangle \right. \\
&\quad \left. + N_c \sum_{n,m_0} \phi_n^\dagger(\mathbf{r}) \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \phi_{m_0}(\mathbf{r}) \langle m_0 | \gamma^0 | n \rangle \mathcal{R}_2(E_n, E_{m_0}) \right], \tag{A4}
\end{aligned}$$

where $\mathcal{M}$ is defined by

$$\mathcal{M} = \sqrt{\frac{3M_8}{E_8 + M_8}}. \tag{A5}$$

The regularization functions are defined as

$$\begin{aligned}
\mathcal{R}_1(E_n) &= \frac{-E_n}{2\sqrt{\pi}} \int_0^\infty \phi(u) \frac{du}{\sqrt{u}} e^{-uE_n^2}, \\
\mathcal{R}_2(E_n, E_m) &= \frac{1}{2\sqrt{\pi}} \int_0^\infty \phi(u) \frac{du}{\sqrt{u}} \frac{E_m e^{-uE_m^2} - E_n e^{-uE_n^2}}{E_n - E_m}, \\
\mathcal{R}_4(E_n, E_m) &= \frac{1}{2\pi} \int_0^\infty du \phi(u) \int_0^1 d\alpha e^{-\alpha u E_m^2 - (1-\alpha) u E_n^2} \frac{(1-\alpha)E_n - \alpha E_m}{\sqrt{\alpha(1-\alpha)}}, \\
\mathcal{R}_5(E_n, E_m) &= \frac{\text{sgn}(E_n) - \text{sgn}(E_m)}{2(E_n - E_m)}.
\end{aligned} \tag{A6}$$

Here, $|\text{val}\rangle$ and $|n\rangle$ denote the quark states in the valence and Dirac continuum with the corresponding eigenenergies E_{val} and E_n of the one-body Dirac Hamiltonian $h(U)$, respectively.

$\mathcal{A}_2(Q^2), \dots, \mathcal{J}_2(Q^2)$ are defined by

$$\begin{aligned}
\mathcal{A}_2(Q^2) &= N_c \mathcal{M} \int d^3r j_2(Q|\mathbf{r}) \left[\phi_{\text{val}}^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \phi_{\text{val}}(\mathbf{r}) + \sum_n \phi_n^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \phi_n(\mathbf{r}) \mathcal{R}_1(E_n) \right], \\
\mathcal{B}_2(Q^2) &= N_c \mathcal{M} \int d^3r j_2(Q|\mathbf{r}) \left[\sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \phi_n(\mathbf{r}) \cdot \langle n | \boldsymbol{\tau} | \text{val} \rangle \right. \\
&\quad \left. - \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \phi_m(\mathbf{r}) \cdot \langle m | \boldsymbol{\tau} | n \rangle \mathcal{R}_5(E_n, E_m) \right], \\
\mathcal{C}_2(Q^2) &= N_c \mathcal{M} \int d^3r j_2(Q|\mathbf{r}) \left[\sum_{n_0 \neq \text{val}} \frac{1}{E_{\text{val}} - E_{n_0}} \phi_{\text{val}}^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \phi_{n_0}(\mathbf{r}) \langle n_0 | \text{val} \rangle \right. \\
&\quad \left. - \sum_{n,m_0} \phi_n^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \phi_{m_0}(\mathbf{r}) \langle m_0 | n \rangle \mathcal{R}_5(E_n, E_{m_0}) \right], \\
\mathcal{D}_2(Q^2) &= N_c \mathcal{M} \int d^3r j_2(Q|\mathbf{r}) \left[\sum_{n \neq \text{val}} \frac{\text{sgn}(E_n)}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \times \boldsymbol{\tau} \phi_n(\mathbf{r}) \cdot \langle n | \boldsymbol{\tau} | \text{val} \rangle \right. \\
&\quad \left. + \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \times \boldsymbol{\tau} \phi_m(\mathbf{r}) \cdot \langle m | \boldsymbol{\tau} | n \rangle \mathcal{R}_4(E_n, E_m) \right], \\
\mathcal{H}_2(Q^2) &= N_c \mathcal{M} \int d^3r j_2(Q|\mathbf{r}) \left[\sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \langle \mathbf{r} | n \rangle \langle n | \gamma^0 | \text{val} \rangle \right. \\
&\quad \left. + \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \phi_m(\mathbf{r}) \langle m | \gamma^0 | n \rangle \mathcal{R}_2(E_n, E_m) \right], \\
\mathcal{I}_2(Q^2) &= N_c \mathcal{M} \int d^3r j_2(Q|\mathbf{r}) \left[\sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \phi_n(\mathbf{r}) \cdot \langle n | \gamma^0 \boldsymbol{\tau} | \text{val} \rangle \right. \\
&\quad \left. + \frac{1}{2} \sum_{n,m} \phi_n^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \phi_m(\mathbf{r}) \cdot \langle m | \gamma^0 \boldsymbol{\tau} | n \rangle \mathcal{R}_2(E_n, E_m) \right], \\
\mathcal{J}_2(Q^2) &= N_c \mathcal{M} \int d^3r j_2(Q|\mathbf{r}) \left[\sum_{n_0 \neq \text{val}} \frac{N_c}{E_{\text{val}} - E_{n_0}} \phi_{\text{val}}^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \phi_{n_0}(\mathbf{r}) \langle n_0 | \gamma^0 | \text{val} \rangle \right. \\
&\quad \left. + N_c \sum_{n,m_0} \phi_n^\dagger(\mathbf{r}) \{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \}_1 \cdot \boldsymbol{\tau} \phi_{m_0}(\mathbf{r}) \langle m_0 | \gamma^0 | n \rangle \mathcal{R}_2(E_n, E_{m_0}) \right].
\end{aligned} \tag{A7}$$

APPENDIX B: MATRIX ELEMENTS OF THE SU(3) WIGNER D FUNCTION

In Tables VI–XI to XI, we list the results for the matrix elements of the relevant collective operators, which are required for the calculation of the axial-vector transition form factors.

TABLE VI. The matrix elements of the single and double Wigner D functions when $a = 3$.

$B_{10} \rightarrow B_8$	$\Delta \rightarrow N$	$\Sigma^* \rightarrow \Sigma$	$\Xi^* \rightarrow \Xi$	$\Sigma^* \rightarrow \Lambda$
$\langle B_8 D_{33}^{(8)} B_{10} \rangle$	$\frac{2\sqrt{5}}{15}$	$-\frac{\sqrt{5}}{15} T_3$	$-\frac{2\sqrt{5}}{15} T_3$	$\frac{\sqrt{15}}{15}$
$\langle B_8 D_{38}^{(8)} \hat{J}_3 B_{10} \rangle$	0	0	0	0
$\langle B_8 d_{bc3} D_{3b}^{(8)} \hat{J}_c B_{10} \rangle$	$-\frac{\sqrt{5}}{15}$	$\frac{\sqrt{5}}{30} T_3$	$\frac{\sqrt{5}}{15} T_3$	$-\frac{\sqrt{15}}{30}$
$\langle B_8 D_{83}^{(8)} D_{38}^{(8)} B_{10} \rangle$	$\frac{\sqrt{5}}{90}$	$-\frac{\sqrt{5}}{30} T_3$	$-\frac{\sqrt{5}}{18} T_3$	$\frac{\sqrt{15}}{90}$
$\langle B_8 D_{88}^{(8)} D_{33}^{(8)} B_{10} \rangle$	$\frac{2\sqrt{5}}{45}$	0	$\frac{\sqrt{5}}{90} T_3$	$\frac{\sqrt{15}}{90}$
$\langle B_8 d_{bc3} D_{8c}^{(8)} D_{3b}^{(8)} B_{10} \rangle$	$\frac{7\sqrt{15}}{270}$	$-\frac{\sqrt{15}}{90} T_3$	$-\frac{4\sqrt{15}}{135} T_3$	$\frac{2\sqrt{5}}{45}$

TABLE VII. The transition matrix elements of the single Wigner D function operators coming from the 27-plet component of the baryon wave functions when $a = 3$.

$B_{10} \rightarrow B_8$	$\Delta \rightarrow N$	$\Sigma^* \rightarrow \Sigma$	$\Xi^* \rightarrow \Xi$	$\Sigma^* \rightarrow \Lambda$
$\langle B_{27} D_{33}^{(8)} B_{10} \rangle$	$\frac{\sqrt{30}}{135}$	$-\frac{\sqrt{5}}{30} T_3$	$-\frac{7\sqrt{30}}{270} T_3$	$\frac{2\sqrt{15}}{135}$
$\langle B_{27} D_{38}^{(8)} \hat{J}_3 B_{10} \rangle$	0	0	0	0
$\langle B_{27} d_{ab3} D_{3a}^{(8)} J_b B_{10} \rangle$	$\frac{2\sqrt{30}}{135}$	$-\frac{\sqrt{5}}{15} T_3$	$-\frac{7\sqrt{30}}{135} T_3$	$\frac{4\sqrt{15}}{135}$
$\langle B_8 D_{33}^{(8)} B_{27} \rangle$	$\frac{2\sqrt{6}}{27}$	0	$-\frac{2\sqrt{30}}{135} T_3$	$\frac{2\sqrt{15}}{45}$
$\langle B_8 D_{38}^{(8)} \hat{J}_3 B_{27} \rangle$	0	0	0	0
$\langle B_8 d_{ab3} D_{3a}^{(8)} J_b B_{27} \rangle$	$\frac{\sqrt{6}}{27}$	0	$-\frac{\sqrt{30}}{135} T_3$	$\frac{\sqrt{15}}{45}$

TABLE VIII. The matrix elements of the single and double Wigner D functions for $a = 4 + i5$.

$B_{10} \rightarrow B_8$	$\Delta^{++} \rightarrow \Sigma^+$	$\Delta^+ \rightarrow \Lambda$	$\Sigma^{*+} \rightarrow \Xi^-$
$\langle B_8 D_{p3}^{(8)} B_{10} \rangle$	$-\sqrt{\frac{2}{15}}$	0	$-\frac{\sqrt{2}}{3\sqrt{5}}$
$\langle B_8 D_{p8}^{(8)} \hat{J}_3 B_{10} \rangle$	0	0	0
$\langle B_8 d_{bc3} D_{pb}^{(8)} \hat{J}_c B_{10} \rangle$	$\frac{1}{\sqrt{30}}$	0	$\frac{1}{3\sqrt{10}}$
$\langle B_8 D_{83}^{(8)} D_{p8}^{(8)} B_{10} \rangle$	$\frac{1}{6\sqrt{30}}$	0	$\frac{1}{9\sqrt{10}}$
$\langle B_8 D_{88}^{(8)} D_{p3}^{(8)} B_{10} \rangle$	$\frac{1}{6\sqrt{30}}$	0	$\frac{1}{9\sqrt{10}}$
$\langle B_8 d_{bc3} D_{8c}^{(8)} D_{pb}^{(8)} B_{10} \rangle$	$\frac{\sqrt{2}}{9\sqrt{5}}$	0	$\frac{1}{9\sqrt{30}}$

TABLE IX. The transition matrix elements of the single Wigner D functions coming from the 27-plet component of the baryon wave functions for $a = 4 + i5$.

$B_{10} \rightarrow B_{27}$	$\Delta^{++} \rightarrow \Sigma^+$	$\Delta^+ \rightarrow \Lambda$	$\Sigma^{*+} \rightarrow \Xi^-$
$\langle B_{27} D_{p3}^{(8)} B_{10} \rangle$	$-\frac{1}{6\sqrt{30}}$	0	$-\frac{1}{9\sqrt{15}}$
$\langle B_{27} D_{p8}^{(8)} \hat{J}_3 B_{10} \rangle$	0	0	0
$\langle B_{27} d_{ab3} D_{pa}^{(8)} J_b B_{10} \rangle$	$-\frac{1}{3\sqrt{30}}$	0	$-\frac{2}{9\sqrt{15}}$
$\langle B_8 D_{p3}^{(8)} B_{27} \rangle$	$\frac{2}{9}$	0	$\frac{2\sqrt{2}}{9\sqrt{5}}$
$\langle B_8 D_{p8}^{(8)} \hat{J}_3 B_{27} \rangle$	0	0	0
$\langle B_8 d_{ab3} D_{pa}^{(8)} J_b B_{27} \rangle$	$\frac{1}{9}$	0	$\frac{\sqrt{2}}{9\sqrt{5}}$

TABLE X. The matrix elements of the single and double Wigner D functions for $a = 4 - i5$.

$B_{10} \rightarrow B_8$	$\Sigma^{*0} \rightarrow p$	$\Xi^{*0} \rightarrow \Sigma^+$	$\Xi^{*-} \rightarrow \Lambda$	$\Omega^- \rightarrow \Xi^0$
$\langle B_8 D_{\Xi-3}^{(8)} B_{10} \rangle$	$\frac{1}{3\sqrt{5}}$	$-\frac{\sqrt{2}}{3\sqrt{5}}$	$\frac{1}{\sqrt{15}}$	$-\sqrt{\frac{2}{15}}$
$\langle B_8 D_{\Xi-8}^{(8)} \hat{J}_3 B_{10} \rangle$	0	0	0	0
$\langle B_8 d_{bc3} D_{\Xi-b}^{(8)} \hat{J}_c B_{10} \rangle$	$-\frac{1}{6\sqrt{5}}$	$\frac{1}{3\sqrt{10}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{\sqrt{30}}$
$\langle B_8 D_{83}^{(8)} D_{\Xi-8}^{(8)} B_{10} \rangle$	$-\frac{1}{9\sqrt{5}}$	$-\frac{1}{18\sqrt{10}}$	$-\frac{1}{4\sqrt{15}}$	0
$\langle B_8 D_{88}^{(8)} D_{\Xi-3}^{(8)} B_{10} \rangle$	$\frac{1}{18\sqrt{5}}$	$\frac{1}{9\sqrt{10}}$	0	$\frac{1}{2\sqrt{30}}$
$\langle B_8 d_{bc3} D_{8c}^{(8)} D_{\Xi-b}^{(8)} B_{10} \rangle$	$-\frac{1}{9\sqrt{15}}$	$\frac{\sqrt{5}}{18\sqrt{6}}$	$-\frac{1}{12\sqrt{5}}$	$\frac{1}{6\sqrt{10}}$

TABLE XI. The transition matrix elements of the single Wigner D functions coming from the 27-plet component of the baryon wave functions for $a = 4 - i5$.

$B_{10} \rightarrow B_{27}$	$\Sigma^{*0} \rightarrow p$	$\Xi^{*0} \rightarrow \Sigma^+$	$\Xi^{*-} \rightarrow \Lambda$	$\Omega^- \rightarrow \Xi^0$
$\langle B_{27} D_{\Xi-3}^{(8)} B_{10} \rangle$	$-\frac{2\sqrt{2}}{9\sqrt{15}}$	$\frac{\sqrt{2}}{9\sqrt{5}}$	$-\frac{1}{3\sqrt{15}}$	$\frac{1}{6\sqrt{5}}$
$\langle B_{27} D_{\Xi-8}^{(8)} \hat{J}_3 B_{10} \rangle$	0	0	0	0
$\langle B_{27} d_{ab3} D_{\Xi-a}^{(8)} J_b B_{10} \rangle$	$-\frac{4\sqrt{2}}{9\sqrt{15}}$	$\frac{2\sqrt{2}}{9\sqrt{5}}$	$-\frac{2}{3\sqrt{15}}$	$\frac{1}{3\sqrt{5}}$
$\langle B_8 D_{\Xi-3}^{(8)} B_{27} \rangle$	$\frac{2}{9\sqrt{5}}$	$\frac{2}{9\sqrt{15}}$	$\frac{\sqrt{2}}{3\sqrt{5}}$	0
$\langle B_8 D_{\Xi-8}^{(8)} \hat{J}_3 B_{27} \rangle$	0	0	0	0
$\langle B_8 d_{ab3} D_{\Xi-a}^{(8)} J_b B_{27} \rangle$	$\frac{1}{9\sqrt{5}}$	$\frac{1}{9\sqrt{15}}$	$\frac{1}{3\sqrt{10}}$	0

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