Hyperons and Θ_s^+ in holographic QCD

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We revisit the holographic description of strange baryons in the context of the Sakai-Sugimoto construction, by considering the strange quark mass as heavy. Hyperons are described by a massive (K, K^*) multiplet, bound to a light-flavor instanton in bulk, much in the spirit of the Callan-Klebanov construction. The modular Hamiltonian maps onto the Landau problem, a charged particle in a two-dimensional external magnetic field, induced by the bulk Chern-Simons interaction, plus spin-orbit coupling. The ensuing holographic hyperon spectrum compares fairly with the empirical one. The holographic strange pentaquark baryon Θ_s^+ is shown to be unbound.

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I. INTRODUCTION

The holographic principle in general [1,2], and the D4-D8-D$\bar{8}$ holographic setup in particular [3], provide a framework for addressing QCD in the infrared in the double limit of a large number of colors and strong 't Hooft gauge coupling $\lambda = g_{\mathrm{YM}}^2 N_c$. It is confining and exhibits spontaneous chiral symmetry breaking geometrically. The light meson sector is well described by an effective action with manifest chiral symmetry and very few parameters, yet totally in line with more elaborate effective theories of QCD [4]. The same setup can be minimally modified to account for the description of heavy-light mesons, with manifest heavy quark symmetry [5–9].

Light and heavy-light baryons are dual to instantons and instanton-heavy meson bound states in bulk [10–15], providing a robust geometrical approach to the multibody bound state problem. The holographic construction provides a dual realization of the chiral soliton approach and its bound state variants [16,17], without the shortcomings of the derivative expansion. It is a geometrical realization of the molecular approach [18,19], without the ambiguities of the nature of the meson exchanges, and the arbitrariness in the choice of the many couplings and form factors [20]. Alternative holographic models for the description of heavy hadrons have been developed in [21,22].

Chiral symmetry constrains the light quark interactions, while heavy quark symmetry restricts the spin interactions between heavy quarks [23,24]. Both symmetries are intertwined by the phenomenon of chiral doubling [25–27] as shown experimentally in [28,29]. A theoretical approach to the multiquark states should have manifest chiral and heavy quark symmetry, a clear organizational principle in the confining regime, and should address concisely the multibody bound state problem. The holographic construction provides this framework.

In [7] two of us have analyzed the holographic baryon spectrum by considering three massless flavors u, d, s. The strange quark mass was introduced through a bulk instanton holonomy, assumed small and treated in perturbation theory. However, the strange quark mass is intermediate between heavy and light, and may require a treatment beyond perturbation theory. In this work, we will propose such a treatment.

We will consider the kaon mass as large, and identify the strangeness brane as heavy in the formulation outlined in [6,7,30]. Hyperons (baryons with strangeness -1) will be sought as bound states of massive kaons, to a bulk flavor instanton made of only the light u, d flavors. The ensuing

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modular Hamiltonian for the hyperons, will be diagonalized without recourse to perturbation theory. This construction bears much in common with the revised bound state approach for hyperons in the Skyrme model [31,32]. The result is a much improved holographic hyperon spectrum, in comparison to the one in [7].

The organization of the paper is as follows: In Sec. II, we briefly review the holographic construction, and detail the modular Lagrangian for hyperons (baryons with strangeness -1) in leading order in the heavy meson mass expansion. We retain the exact kaon mass contribution, and the nonlocal contributions stemming from the Coulomb backreaction and the bulk form of Gauss law. In Sec. III we show that the modular Hamiltonian maps onto the Landau problem of a particle in a magnetic field in two dimensions, plus spin-orbit coupling. In Sec. IV we detail the hyperon spectra, including the strange pentaquark exotic Θ_s^+, for two different approximations of the Gauss law contribution. Our conclusions are in Sec. V. A number of Appendixes are added to complement the derivations in the text.

II. THE MODULAR LAGRANGIAN

The modular Lagrangian for the holographic description of heavy-light mesons bound to a bulk flavor instanton, has been discussed in [6,7] for standard baryons, and for their exotics in [30,33–35]. Here we propose to use it for kaons, assuming the strangeness to be a heavy flavor.

A. Dirac-Born-Infeld (DBI) action

The holographic construction consists of (N_f-1)-light and 1-heavy probe branes. The light branes fuse in a hyperbolic geometry characterized by a finite size RT and a horizon at $U_{\rm KK}$. The DBI action characterizing the flavor fields in leading order in $1/\lambda$ is

$$S_{\rm DBI} \approx -\kappa \int d^4x dz {\rm Tr}(\mathbf{f}(z)\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + \mathbf{g}(z)\mathbf{F}_{\mu z}\mathbf{F}^{\nu z}), \quad (1)$$

with the warpings

$$\mathbf{f}(z) = \frac{R^3}{4U_z}, \qquad \mathbf{g}(z) = \frac{9}{8}\frac{U_z^3}{U_{\rm KK}}, \quad (2)$$

and $U_z^3 = U_{\rm KK}^3 + U_{\rm KK}z^2$, and $\kappa \equiv a\lambda N_c$ and $a = 1/(216\pi^3)$ [3]. All dimensions are in units of $M_{\rm KK}$ (Kaluza-Klein scale). The effective fields in the field strengths are [5,6]

$$\mathbf{F}_{MN} = \begin{pmatrix} F_{MN} - \Phi_{[M}\Phi_{N]}^\dagger & \partial_{[M}\Phi_{N]} + A_{[M}\Phi_{N]} \\ -\partial_{[M}\Phi_{N]}^\dagger - \Phi_{[M}^\dagger A_{N]} & -\Phi_{[M}^\dagger\Phi_{N]} \end{pmatrix}. \quad (3)$$

The matrix valued 1-form gauge field is

$$\mathbf{A} = \begin{pmatrix} A & \Phi \\ -\Phi^\dagger & 0 \end{pmatrix}. \quad (4)$$

For $N_f = 2$, the naive Chern-Simons 5-form is

$$S_{CS} = \frac{iN_c}{24\pi^2}\int_{M_5} {\rm Tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right). \quad (5)$$

For N_f coincidental branes, the Φ multiplet is massless, but for separated branes it is massive, with a generic mass term

$$\frac{1}{2}m_H^2 {\rm Tr}(\Phi_M^\dagger \Phi_M). \quad (6)$$

The value of m_H is related to the separation between the light and heavy branes, which is about the length of the heavy light (HL) string (see below).

B. Light and heavy fields

In the coincidental brane limit, light baryons are interchangeably described as a flavor instanton or a D4 brane wrapping the S^4, with mass $M_0 = 8\pi^2\kappa$ in units of $M_{\rm KK}$. The bulk instanton is described by the O(4) gauge field

$$A_M(y) = -\bar{\sigma}_{MN}\partial_N F(y). \quad (7)$$

The labels M, N run only over $1, 2, 3, z$ unless specified otherwise. The instanton size is small with $\rho \sim 1/\sqrt{\lambda}$ [3], and it is natural to recast the DBI action using the rescaling $(x_0, x_M) \to (x_0, x_M/\sqrt{\lambda})$, $\sqrt{\lambda}\rho \to \rho$ and $(A_0, A_M) \to (A_0, \sqrt{\lambda}A_M)$. The rescaled fields satisfy the equations of motion

$$D_M F_{MN} = 0$$
$$\partial_M^2 A_0 = -\frac{1}{32\pi^2 a} F_{aMN} \star F_{aMN}. \quad (8)$$

For the heavy fields, we use the rescaling

$$(\Phi_0, \Phi_M) \to (\Phi_0, \sqrt{\lambda}\Phi_M). \quad (9)$$

The interactions between the light gauge fields (A_0, A_M) and the heavy fields (Φ_0, Φ_M) to quadratic order split to several contributions [5,6]:

$$\mathcal{L} = aN_c\lambda\mathcal{L}_0 + aN_c\mathcal{L}_1 + \mathcal{L}_{CS}. \quad (10)$$

We start by recalling the leading contributions in $1/m_H$ stemming from (10) as detailed in [6,7]. For that, we split $\Phi_M = \phi_M e^{-im_H x_0}$ for particles ($m_H \to -m_H$ for antiparticles). The leading order contribution takes the form

$$\mathcal{L}_0 = -\frac{1}{2}|f_{MN} - \star f_{MN}|^2 + 2\phi_M^\dagger(F_{MN} - \star F_{MN})\phi_N, \quad (11)$$

subject to the constraint equation $D_M\phi_M = 0$ with

$$f_{MN} = \partial_{[M}\phi_{N]} + A_{[M}\phi_{N]}, \quad (12)$$

while the subleading contributions in (10) to order $\lambda^0 m_H$ simplify to

$$\frac{\mathcal{L}_1}{aN_c} \to 4m_H\phi_M^\dagger iD_0\phi_M$$
$$\mathcal{L}_{CS} \to \frac{m_H N_c}{16\pi^2}\phi_M^\dagger \star F_{MN}\phi_N. \quad (13)$$

We identify the strange heavy-light flavor field Φ in bulk with the $(0^-, 1^-)$ kaon multiplet as $\Phi = (K, K^*)$, and proceed to bind it to a light flavor instanton as in [6,30]. The ensuing modular Lagrangian is composed of the collective variables (X_i, a_I, ρ) for the instanton collective position, SU(2) orientation and size.

In addition, when binding to the core instanton in bulk, the kaon multiplet transmutes to a two-component complex modular coordinate χ, via a zero mode. In the analysis to follow for the hyperon spectrum, this coordinate will be quantized as a boson, much in the spirit of the bound state approach in the dual analysis in [31,32]. The fermionic statistics was considered in [6,30], in the analysis of the much heavier baryons and their exotics, as it captures the key features of the heavier quark, and heavy quark symmetry.

C. Modular action

The modular action will be sought by restricting the quantum and heavy fields to the quantum moduli. More specifically, we choose to parametrize the fields using

$$A_M(t,x) = A_M^{cl}(X(t), Z(t)),$$
$$A_0(t,x) = -iV\partial_t V^{-1} \equiv \Phi$$
$$\Phi_M(t,x) = f(X(t), Z(t))\bar{\sigma}_M\chi(t). \quad (14)$$

The Φ field is parametrized as

$$\Phi = -\dot{X}_N A_N^{\rm cl} + \chi^a\Phi_a, \quad (15)$$

where Φ_a diagonalizes $D_M^{\rm cl}D_M^{\rm cl}\Phi_a$ and where

$$\chi^a = \mathrm{tr}(\tau^a \mathbf{a}^{-1}\dot{\mathbf{a}}) \quad (16)$$

are expressed in terms of the collective variables $\mathbf{a} \in \mathrm{SU}(2)$ for a rigid SU(2) rotation.

The full holographic modular Lagrangian for heavy-light kaons bound to a bulk flavor instanton, follows by inserting (14) and (15) and expanding in $1/m_H$, with the result [30] [see Eq. (23)]

$$\begin{aligned}\mathcal{L} = &+\frac{1}{2}\dot{\chi}^\dagger\dot{\chi} + \frac{3i}{\tilde{\rho}^2}\chi^\dagger\dot{\chi} - \frac{37 + 12\frac{Z^2}{\rho^2}}{192}\chi^\dagger\chi \\ &+\frac{78i}{5\tilde{\rho}^2}\chi^\dagger\tau^a\chi\chi^a - \frac{12}{5\tilde{\rho}^4}(\chi^\dagger\tau^a\chi)^2 \\ &+\left(\frac{1}{4}\frac{\dot{\rho}^2}{\rho^2} + \frac{\dot{a}_I^2}{4} + \frac{\dot{X}^2}{4\rho^2}\right)\chi^\dagger\chi - \frac{1}{2}m_H^2\chi^\dagger\chi \\ &+\mathcal{L}_{\Phi_0}[m_H] + \mathcal{L}_{\rm Coulomb},\end{aligned} \quad (17)$$

with $\tilde{\rho} = 16\pi^2 aN_c\rho$. The rescaling $\chi \to e^{im_H t}\sqrt{m_H}\chi$, with the heavy and bare mass m_H of the (K, K^*) multiplet, is subsumed.

The first two lines are standard, with the first term in the third line following from the coupling $\mathrm{tr}\Phi^2\chi^\dagger\chi$, and leading to a nonvanishing correction to the metric in the space $y_I = (\rho, \rho a_I)$.

The constraint field contribution $\mathcal{L}_{\Phi_0}[m_H]$ in the last line was analyzed in Appendixes A. 3 and A. 4 of Ref. [30], and is given by

$$\mathcal{L}_{\Phi_0}[m_H] = -\frac{1}{8}J_0^\dagger\frac{1}{-D_M^2 + m_H^2}J_0, \quad (18)$$

with the *nonlocal* source J_0 given in (A10). It follows from the Gauss constraint on the flavor gauge field in bulk, and is by far the most involved to unravel. For convenience, we detail its analysis in Appendixes A and B. Aside from the explicit mass dependence in (17), there is an implicit mass dependence in $\mathcal{L}_{\Phi_0}[m_H]$ which we have noted in the argument. Since the strange mass is intermediate between light u, d and heavy c, b, we will address the implicit mass dependence in $\mathcal{L}_{\Phi_0}[m_H]$ both in the light $m_H \to 0$, and heavy $m_H \to \infty$.

The Coulomb contribution $\mathcal{L}_{\rm Coulomb}$ was originally detailed in Appendix B in [30], and for convenience, briefly reviewed in Appendix C, with the result

$$\mathcal{L}_{\rm Coulomb} = -J_C^\dagger\frac{1}{2(-aN_c\nabla^2 + f^2\chi^\dagger\chi)}J_C. \quad (19)$$

The *nonlocal* source $J_C = (\rho^{cl} + \rho)$ is given in (C2) and (C3). Throughout, the Coulomb contribution which is small will be mostly ignored. It is a correction to be added in perturbation theory to the modular Hamiltonian, and assessed only at the end.

The holographic heavy kaon mass in the large mass limit is given by [5]

$$M_K = m_H + \frac{M_{\rm KK}}{2\sqrt{2}}, \quad (20)$$

with m_H the bare mass of the kaon doublet, and the Kaluza-Klein scale $M_{\rm KK} = 475$ MeV. In what will follow, $m_H \sim M_K$, unless specified otherwise.

Note that a naive expansion of the Coulomb and Gauss constraint contributions in (17), as shown in Appendix D, leads to a degenerate but stable hyperon spectrum to order m_H^0, but unstable at subleading order. The unexpanded constraints produce a stable hyperon spectrum as we detail below.

D. $\mathcal{L}_{\Phi_0}[0]$ and no Coulomb

We start the analysis of (17) by considering the simple case with $m_H = 0$ *only* in the Gauss constraint or $\mathcal{L}_{\Phi_0}[0]$, and no Coulomb backreaction. Both approximations will be revisited below. With this in mind, the modular Lagrangian simplifies

$$\mathcal{L}_{\text{qua}} = \frac{1}{2}\dot{\chi}^\dagger\dot{\chi} + \frac{3i}{\tilde{\rho}^2}\chi^\dagger\dot{\chi} - \frac{37+12\frac{Z^2}{\rho^2}}{192}\chi^\dagger\chi + \frac{99i}{40\tilde{\rho}^2}\chi^\dagger\tau^a\chi\chi^a - \frac{75}{8\tilde{\rho}^4}\chi^\dagger\chi. \tag{21}$$

Note that without the spin-orbit coupling, we have

$$\mathcal{L}_0 = \frac{1}{2}\dot{\chi}^\dagger\dot{\chi} + \frac{3i}{\tilde{\rho}^2}\chi^\dagger\dot{\chi} - \frac{37+12\frac{Z^2}{\rho^2}}{192}\chi^\dagger\chi - \frac{75}{8\tilde{\rho}^4}\chi^\dagger\chi. \tag{22}$$

By setting the kaon modular variable χ as

$$\chi = \begin{pmatrix} x_1 + iy_1 \\ x_2 + iy_2 \end{pmatrix}, \tag{23}$$

(22) can be written as two harmonic oscillators coupled to magnetic field

$$\mathcal{L}_0 = \frac{1}{2}(\dot{\vec{x}}_1^{\,2} + \dot{\vec{x}}_2^{\,2}) + \omega_c(y_1\dot{x}_1 - x_1\dot{y}_1 + y_2\dot{x}_2 - x_2\dot{y}_2) - \frac{m_H^2+\Omega^2}{2}(\vec{x}_1^{\,2} + \vec{x}_2^{\,2}), \tag{24}$$

where we have defined

$$\Omega^2 = \frac{75}{4\tilde{\rho}^4} + \frac{37+6\sqrt{6}\frac{1}{\tilde{\rho}^2}}{96}, \qquad \omega_c = \frac{3}{\tilde{\rho}^2}. \tag{25}$$

This observation will be exploited next.

III. HYPERON SPECTRUM

Following on the preceding arguments, we now analyze the modular Hamiltonian stemming from (21). Without the spin-orbit contributions as we noted in (24), it maps on the well-known Landau problem in two dimensions. In this regime, the hyperons are stable but degenerate. The spin-orbit contribution modifies the potential in the holographic ρ-direction, and lifts the hyperon degeneracy.

A. Landau problem

For the modular Lagrangian (24), the pertinent Schrödinger equation reads

$$H\phi_n(x) = E\phi_n, \tag{26}$$

with the modular Hamiltonian

$$H = \frac{1}{2}D_i^\dagger D_i + \frac{\omega^2}{2}\bar{z}z, \tag{27}$$

with $z = x + iy$ and $\omega^2 = \Omega^2 + m_H^2$. The long derivative is $D_i = \partial_i - iA_i$, with the U(1) gauge field $A_i = \omega_c(y, -x)$. We now define the operators

$$\begin{aligned} a &= \frac{i}{2\sqrt{\omega_c}}(D_x - iD_y) \\ &= \frac{i}{2\sqrt{\omega_c}}(\partial_x - i\partial_y + \omega_c(x-iy)), \\ b &= \frac{-i}{2\sqrt{\omega_c}}(-\partial_x - i\partial_y - \omega_c(x+iy)), \end{aligned} \tag{28}$$

which diagonalizes the kinetic contribution

$$\frac{1}{2}D_i^\dagger D_i = \omega_c(2a^\dagger a + 1). \tag{29}$$

For the harmonic contribution, we note that

$$b^\dagger - a = -i\sqrt{\omega_c}(x - iy), \tag{30}$$

hence the Hamiltonian can be written as

$$H = \omega_c(2a^\dagger a + 1) + \frac{\omega^2}{2\omega_c}(b^\dagger - a)(b - a^\dagger). \tag{31}$$

The Hamiltonian (31) can then be diagonalized with the help of the following Bogoliubov transformation:

$$a^\dagger = \cosh\theta A^\dagger + \sinh\theta B, \tag{32}$$

$$b^\dagger = \cosh\theta B^\dagger + \sinh\theta A. \tag{33}$$

Using $[A, A^\dagger] = [B, B^\dagger] = 1$ and $[A, B] = [A, B^\dagger] = 0$, which preserves the commutation relations, we can fix the value of θ as

$$\tanh 2\theta = \frac{2\alpha}{1+2\alpha}, \qquad \alpha = \frac{\omega^2}{4\omega_c^2}. \tag{34}$$

The modular Hamiltonian without spin-orbit coupling is then diagonalized as

$$H = \frac{\Omega_+ + \Omega_-}{2} + \Omega_+ A^\dagger A + \Omega_- B^\dagger B, \tag{35}$$

with

$$\Omega_\pm = \sqrt{m_H^2 + \Omega^2 + \omega_c^2} \pm \omega_c. \tag{36}$$

In the next subsection we will explore the spin-orbital contribution.

B. Spin orbit

For fixed modular variable $\tilde{\rho}$, the holographic spectrum without spin orbit following from (35) is harmonic. Since the modular coordinate χ is quantized as a boson, the net spin and isospin of the hyperon core is determined by the instanton quantum moduli with $[IJ^P] = [\frac{1}{2}\frac{1}{2}^+]$ assignment, in the absence of spin-orbit effects. With spin-orbit contributions, the resulting hyperon states carry $[\frac{1}{2} \oplus \frac{1}{2}, (\frac{1}{2} \oplus l)^+]$ assignments, for even l. We now proceed to analyze the dynamical effects of the spin-spin and spin-orbit effects.

1. The $l = 0$ state

For $l = 0$ and $J = \frac{1}{2}$, the energy level with n $B^\dagger$ excitations is

$$E_n - E_0 = n\left(\sqrt{m_H^2 + \frac{111}{4\tilde{\rho}^4} + \frac{37 + 12\frac{Z^2}{\rho^2}}{96}} - \frac{3}{\tilde{\rho}^2}\right), \tag{37}$$

and the lowest one is archived for $n = 1$. To proceed we need to fix the ρ wave function. For that, the induced potential is given by Ω_- to which we add the harmonic oscillator potential term $\frac{1}{2}\omega_\rho^2\tilde{\rho}^2$, plus the quartic term $-\frac{3i}{5\tilde{\rho}^4}\chi^\dagger\tau^a\dot{\chi}\chi^\dagger\tau^a\chi$ as in Eq. (43) in [30]. The result is

$$V(\tilde{\rho}) = \frac{1}{2}\omega_\rho^2\tilde{\rho}^2 + \sqrt{m_H^2 + \frac{111}{4\tilde{\rho}^4} + \frac{37 + 12\frac{Z^2}{\rho^2}}{96}} - \frac{3}{\tilde{\rho}^2} + \frac{9\left(\sqrt{m_H^2 + \frac{111}{4\tilde{\rho}^4} + \frac{37+12\frac{Z^2}{\rho^2}}{96}} - \frac{3}{\tilde{\rho}^2}\right)}{5\left(m_H^2 + \frac{111}{4\tilde{\rho}^4} + \frac{37+12\frac{Z^2}{\rho^2}}{96}\right)\tilde{\rho}^4}. \tag{38}$$

We note that (38) is stable for small ρ. The additional parameter δ captures a spin-spin ordering ambiguity to be discussed below. With this in mind, and using the estimate

$$\frac{Z^2}{\rho^2} \approx \sqrt{\frac{3}{2}}\frac{1}{\tilde{\rho}^2}, \tag{39}$$

the splitting between Λ^0 and nucleon can be solved numerically for $\delta = 1$. The result is

$$M_{\Lambda^0} - M_N = 0.237 M_{\mathrm{KK}}. \tag{40}$$

For $M_{\mathrm{KK}} = 0.475$ GeV, the splitting is about 112.7 MeV, smaller than the empirical splitting of 177 MeV. This is reasonable, since the omitted Coulomb backreaction is positive (see below).

2. The $l = 2$ state

For the $l \neq 0$ cases, the quantization needs to be considered more carefully, as operator ordering issues arise. Indeed, we note that the spin operator in the Bogoliubov transformed basis, reads

$$\chi^\dagger\tau^a\chi = \frac{1}{\sqrt{m_H^2 + \Omega^2 + \omega_c^2}}(A^\dagger - B)_i\tau^a_{ij}(A - B^\dagger)_j. \tag{41}$$

The spin-spin and spin-orbit effects will be treated in first order perturbation theory. When evaluating the average of $\chi^\dagger\tau^a\chi$, one recovers the standard Schwinger representation of a $\frac{1}{2}$-spin,

$$S^a = \frac{1}{2}B_i^\dagger\tau^a_{ij}B_j, \tag{42}$$

with A_1, A_2 constructed using (x_1, y_1) and (x_2, y_2), respectively. When evaluating $(\chi^\dagger\tau^a\chi)^2$, without normal ordering, gives

$$\langle 0|B_i(A^\dagger - B)\tau^a(B^\dagger - A^\dagger) \times (A^\dagger - B)\tau^a(B^\dagger - A)B_i^\dagger|0\rangle = 12, \tag{43}$$

for $i = 1, 2$. With normal ordering, the result is different:

$$\langle 0|B_i{:}(A^\dagger - B)\tau^a(B^\dagger - A^\dagger) \times (A^\dagger - B)\tau^a(B^\dagger - A){:}B_i^\dagger|0\rangle = 6. \tag{44}$$

The normal ordering ambiguity is captured by a c-number δ. The third and perhaps most physical choice, amounts to dropping the antiparticle contribution to the spin through A, $A^\dagger$,

$$\langle 0|B_i B\tau^a B^\dagger B\tau^a B^\dagger B_i^\dagger|0\rangle = 3,$$

which is ordering free. This choice corresponds to $\delta = 1$, and will be subsumed throughout.

For $l = 2, 4, \ldots$, one has $J = (l \pm 1)/2$. We first consider the $J = (l - 1)/2$ case. Following our recent arguments in [30] [e.g., Eqs. (44) and (45)], the effective potential reads

$$
\begin{aligned}
V\left(J=\frac{l-1}{2},\tilde{\rho}\right) &= \frac{1}{2\tilde{\rho}^2}\left(l(l+2)-\frac{(l+2)\alpha N_c}{\sqrt{m_H^2+\Omega^2+\omega_c^2}\tilde{\rho}^2}+\frac{3\alpha^2N_c^2}{4(m_H^2+\Omega^2+\omega_c^2)\tilde{\rho}^4}\right)\\
&\quad+\frac{\omega_\rho^2}{2}\tilde{\rho}^2+\sqrt{m_H^2+\Omega^2+\omega_c^2}-\omega_c+\frac{9\left(\sqrt{m_H^2+\frac{111}{4\tilde{\rho}^4}+\frac{37+12\frac{Z^2}{\rho^2}}{96}}-\frac{3}{\tilde{\rho}^2}\right)}{5\left(m_H^2+\frac{111}{4\tilde{\rho}^4}+\frac{37+12\frac{Z^2}{\rho^2}}{96}\right)\tilde{\rho}^4},
\end{aligned}
\tag{45}
$$

with $\alpha=\frac{33}{10}$. The $1/m_H^2$ term due to the spin-orbit coupling is kept to maintain stability at small ρ. The change of the potential as one increases m_H tends to decrease for larger l. For $l=2$, the potentials at $m_H=2$ and $m_H=\infty$ differ moderately, but the net difference is small.

Similarly, in the $J=\frac{l+1}{2}$ case the effective potential is

$$
\begin{aligned}
V\left(J=\frac{l+1}{2},\tilde{\rho}\right) &= \frac{1}{2\tilde{\rho}^2}\left(l(l+2)+\frac{l\alpha N_c}{\sqrt{m_H^2+\Omega^2+\omega_c^2}\tilde{\rho}^2}+\frac{3\alpha^2N_c^2}{4(m_H^2+\Omega^2+\omega_c^2)\tilde{\rho}^4}\right)\\
&\quad+\frac{\omega_\rho^2}{2}\tilde{\rho}^2+\sqrt{m_H^2+\Omega^2+\omega_c^2}-\omega_c+\frac{9\left(\sqrt{m_H^2+\frac{111}{4\tilde{\rho}^4}+\frac{37+12\frac{Z^2}{\rho^2}}{96}}-\frac{3}{\tilde{\rho}^2}\right)}{5\left(m_H^2+\frac{111}{4\tilde{\rho}^4}+\frac{37+12\frac{Z^2}{\rho^2}}{96}\right)\tilde{\rho}^4}.
\end{aligned}
\tag{46}
$$

For $\delta=1$ and $m_H=M_{\mathrm{KK}}$, a numerical analysis for the hyperon states gives

$$
\begin{aligned}
&\left[J=\frac{1}{2},l=2,I=1\right]:\ M\left(\Sigma\left(1\frac{1^+}{2}\right)\right)-M_N=302\ \mathrm{MeV}\\
&\left[J=\frac{3}{2},l=2,I=1\right]:\ M\left(\Sigma\left(1\frac{3^+}{2}\right)\right)-M_N=501\ \mathrm{MeV}
\end{aligned}
\tag{47}
$$

which are to be compared to the measured values of 254 and 444 MeV. The splitting between the centroid is much more accurate:

$$
M\left(\Sigma\left(1\frac{3^+}{2}\right)\right)-M\left(\Sigma\left(1\frac{1^+}{2}\right)\right)=199\ \mathrm{MeV},
\tag{48}
$$

compared to 191 MeV, empirically.

IV. HYPERON SPECTRUM REVISITED

We now consider the hyperon spectrum with spin-orbit effect, but with $\mathcal{L}_{\Phi_0}[m_H]$ in the opposite limit of large m_H for comparison. The details of $\mathcal{L}_{\Phi_0}[m_H]$ are presented in Appendix A, including its closed form results in the heavy mass limit.

A. Without Coulomb

In this case the potentials in the holographic ρ-direction are modified as follows:

$$
V_{l=0}(\tilde{\rho})=\frac{1}{2}\omega_\rho^2\tilde{\rho}^2+\left(m_H^2+\frac{9}{\tilde{\rho}^4}\left(1+\frac{4.11}{m_H^2\tilde{\rho}^2}\right)+\frac{37+12\frac{Z^2}{\rho^2}}{96}\right)^{\frac{1}{2}}-\frac{3}{\tilde{\rho}^2}+\frac{9\left(\left(m_H^2+\frac{9}{\tilde{\rho}^4}(1+\frac{4.11}{m_H^2\tilde{\rho}^2})+\frac{37+12\frac{Z^2}{\rho^2}}{96}\right)^{\frac{1}{2}}-\frac{3}{\tilde{\rho}^2}\right)}{5\left(m_H^2+\frac{9}{\tilde{\rho}^4}(1+\frac{4.11}{m_H^2\tilde{\rho}^2})+\frac{37+12\frac{Z^2}{\rho^2}}{96}\right)\tilde{\rho}^4}
\tag{49}
$$

for $l=0$, and for $l=2$

$$
\begin{aligned}
V_l\left(J=\frac{l-1}{2},\tilde{\rho}\right) &= V_{l=0}(\tilde{\rho})+\frac{1}{2(1+\frac{1}{2m_H\tilde{\rho}^2})\tilde{\rho}^2}\left(l(l+2)-\frac{(l+2)\tilde{\alpha}N_c}{\sqrt{m_H^2+\tilde{\Omega}^2+\omega_c^2}\tilde{\rho}^2}+\frac{3\tilde{\alpha}^2N_c^2}{4(m_H^2+\tilde{\Omega}^2+\omega_c^2)\tilde{\rho}^4}\right)\\
V_l\left(J=\frac{l+1}{2},\tilde{\rho}\right) &= V_{l=0}(\tilde{\rho})+\frac{1}{2(1+\frac{1}{2m_H\tilde{\rho}^2})\tilde{\rho}^2}\left(l(l+2)+\frac{(l)\tilde{\alpha}N_c}{\sqrt{m_H^2+\tilde{\Omega}^2+\omega_c^2}\tilde{\rho}^2}+\frac{3\tilde{\alpha}^2N_c^2}{4(m_H^2+\tilde{\Omega}^2+\omega_c^2)\tilde{\rho}^4}\right)
\end{aligned}
\tag{50}
$$

with

$$\tilde{\alpha} = \frac{13}{10} + \frac{162}{35\tilde{m}_H^2\tilde{\rho}^4},$$
$$\tilde{\Omega}^2 = \frac{37 + 6\sqrt{6}\frac{1}{\tilde{\rho}^2}}{96} + \frac{9 * 4.11}{m_H^2\tilde{\rho}^2}. \quad (51)$$

Also, there is a modification to the curvature in the ρ-direction. We should also include the leading warping contribution at large m_H, the details of which are identical to those presented in [30]. With this in mind, the hyperon spectrum is now given by

$$\left[J = \frac{1}{2}, l = 0, I = 0\right]: M(\Lambda) - M_N = 68.1 \text{ MeV},$$
$$\left[J = \frac{1}{2}, l = 2, I = 1\right]: M\left(\Sigma\left(1\frac{1^+}{2}\right)\right) - M_N = 289 \text{ MeV}$$
$$\left[J = \frac{3}{2}, l = 2, I = 1\right]: M\left(\Sigma\left(1\frac{3^+}{2}\right)\right) - M_N = 400 \text{ MeV} \quad (52)$$

The $J = \frac{1}{2}$ Σ state is pushed up, and the $J = \frac{3}{2}$ Σ state is pushed down, with a split in the centroid

$$\frac{M_\Sigma(1\frac{1}{2}^+) + M_\Sigma(1\frac{3}{2}^+)}{2} - M_N = 344 \text{ MeV}, \quad (53)$$

which is close to the empirical value of 349 MeV.

B. With Coulomb

As we indicated earlier, throughout we assumed $m_H \sim M_{\mathrm{KK}}$ in (20). Here we correct for this shortcoming, with

$$m_H = 0.68 M_{\mathrm{KK}} \quad (54)$$

and $M_{\mathrm{KK}} = 475$ MeV fixed by the light baryon spectrum [14]. M_{KK} is fixed in [30], and m_H is fixed in a way to minimize the Λ mass.

Also, the neglected Coulomb contribution can be estimated in perturbation theory, and in the heavy meson mass limit, it is about

$$V_{\mathrm{C}} \approx \frac{83}{30\tilde{\rho}^2}. \quad (55)$$

To derive this result, one simply needs to notice that the total potential in the heavy quark limit is $V_{\mathrm{total}} = \frac{Q}{\rho^2} = -\frac{7}{30\tilde{\rho}^2}$ [see Eq. (40) in Ref. [6] for the definition of Q], among them $-\frac{3}{\tilde{\rho}^2}$ is due to the $-\omega_c$ contribution in Eq. (37), and the remaining $V_{\mathrm{total}} + \omega_c$ is the Coulomb contribution in the infinite heavy quark limit. This Coulomb potential provides for an upper bound estimate.

With this in mind, the modified hyperon masses are

$$\left[J = \frac{1}{2}, l = 0, I = 0\right]: M(\Lambda) - M_N = 214 \text{ MeV},$$
$$\left[J = \frac{1}{2}, l = 2, I = 1\right]: M\left(\Sigma\left(1\frac{1^+}{2}\right)\right) - M_N = 368 \text{ MeV}$$
$$\left[J = \frac{3}{2}, l = 2, I = 1\right]: M\left(\Sigma\left(1\frac{3^+}{2}\right)\right) - M_N = 460 \text{ MeV}. \quad (56)$$

The experimental values are 177, 254 and 440 MeV respectively, with 37, 133 and 20 MeV differences.

Using the corrected value of m_H above, and the upper estimate for the Coulomb correction, in Table I, we collect all hyperon masses for the three approximations presented earlier. The chief observation is that the large mass analysis without Coulomb corrections appears closer to the empirical values of the lowest three empirical hyperons, without any adjustable parameter. These results are to be compared to those reported by Callan and Klebanov using the Skyrme model [31,32], with also no Coulomb corrections.

We recall that in the present holographic construction, the relative splitting between the hyperons, and the splitting of the hyperon centroid from the nucleon, which eliminate much of the uncertainty in M_{KK}, are in remarkable agreement with the empirically reported splittings.

C. Exotics

This approach extends to light multiquark exotics with open or hidden strangeness, much like the heavier multiquark exotics with open or hidden charm and bottom discussed in [30,33–35]. In particular, an estimate of the mass of the strange pentaquark Θ_s^+ (the exotic $uudd\bar{s}$) is

TABLE I. Hyperon and exotic spectrum.

B	IJ^P	l	n_ρ	n_z	Mass (small)	Mass (small with Coulomb)	Mass (large)	Mass (large with Coulomb)	Exp-MeV
Λ	$0\frac{1}{2}^+$	0	0	0	962	1182	974	1152	1115
Σ	$1\frac{1}{2}^+$	2	0	0	1134	1315	1149	1306	1192
	$1\frac{3}{2}^+$	2	0	0	1346	1472	1254	1398	1387
Θ_s^+	$0\frac{1}{2}^+$	0	0	0	⋯	1617	⋯	1599	

given in Table I. The mass of about 1600 MeV stems mainly from the Ω_+ frequency (antiparticle) which is $\frac{6}{\bar{\rho}^2}$ higher than the Ω_- (particle). (Recall that the effective magnetic field induced by the bulk Chern-Simons interaction is repulsive for particles, and attractive for antiparticles.) An additional repulsion of about $\frac{3}{\bar{\rho}^2}$ stems from the Coulomb backreaction in the heavy mass estimate. A Θ_s^+ of about 1600 MeV lies above the nK^+ threshold of 1434 MeV, and is unstable against strong decay. This result is consistent with the fact that the proposed Θ_s^+ state [36–39] is insofar unaccounted for experimentally.

V. CONCLUSIONS

In the holographic construction presented in [5–7], heavy hadrons are described in bulk using a set of degenerate N_f light D8-$\overline{\text{D8}}$ branes plus one heavy probe brane in the cigar-shaped geometry that spontaneously breaks chiral symmetry. This construction enforces both chiral and heavy-quark symmetry and describes well the low-lying heavy-light mesons, baryons and multiparticle exotics [30,33–35]. Heavy hadrons whether standard or exotics, are composed of heavy-light mesons bound to a core instanton in bulk.

In [7] the analysis of the hyperon spectrum was carried to order m_H^0 where spin effects are absent. In this analysis, we have now carried the analysis at next to leading order in $1/m_H$ where the spin-orbit and spin corrections are manifest. In contrast to [7], the modular fields were quantized as bosons and not fermions. The quantized Hamiltonian describes a particle in an external two-dimensional magnetic field, with spin-orbit coupling.

The hyperon spectrum with the Gauss constraint treated in both the heavy and light kaon mass limit shows very small changes. It is in overall agreement with the empirical hyperon spectrum, and is much improved in comparison to the analysis in [7], where the strange mass was analyzed perturbatively. This construction allows for the description of multiquark exotics with strangeness, and shows that the contentious exotic Θ_s^+ is unbound. In a way, this construction should be regarded as the dual of the improved Callan-Klebanov construction for hyperons, as bound kaon skyrmions [31,32].

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APPENDIX A: DERIVATION OF $\mathcal{L}_{\Phi_0}[m_H]$

For a generic kaon mass of order m_H, we must include its contribution in the Gauss law constraint as captured by the time component $\Phi_{M=0}$ of the heavy-light vector field. This is the most difficult term to unravel to order $1/m_H$. For that, we recall from Appendix A. 3 in [30] that the constraint equation for Φ_0 is

$$\begin{aligned}(-D_M^2+m_H^2)\Phi_0+2F_{M0}\Phi_M\\ -\frac{i}{16\pi^2 a}F_{PQ}(\partial_P+A_P)\Phi_Q=0,\end{aligned}\tag{A1}$$

after using the self-dual condition for F. Using the standard relations for $\bar{\sigma}_{MN}$, we have for the last two contributions in (A1)

$$\begin{aligned}F_{PQ}\partial_P\Phi_Q&=\frac{6\rho^2}{(X^2+\rho^2)^2}\frac{1}{r}\frac{df}{dr}\bar{\sigma}\cdot X\chi,\\ F_{PQ}A_P\Phi_Q&=-\frac{6\rho^2}{(X^2+\rho^2)^3}f\bar{\sigma}\cdot X\chi.\end{aligned}\tag{A2}$$

For the first contribution in (A1) we have

$$\begin{aligned}F_{M0}\Phi_M=&\frac{6f}{(X^2+\rho^2)^2}(\rho^2\bar{\sigma}\cdot\dot{X}+\bar{\sigma}\cdot X\rho\dot{\rho})\chi\\ &+\chi^a D_M\Phi^a\bar{\sigma}_M\chi f\end{aligned}\tag{A3}$$

with

$$\Phi^a=\frac{1}{2(X^2+\rho^2)}\bar{\sigma}\cdot X\tau^a\sigma\cdot X,\tag{A4}$$

or more explicitly

$$\chi^a D_M\Phi^a\bar{\sigma}_M\chi f=\frac{3\rho^2 f}{(X^2+\rho^2)^2}\bar{\sigma}\cdot X\tau^a\chi\chi^a.\tag{A5}$$

Inserting (A5) and (A6) into (A1) we have

$$(-D_M^2+m_H^2)\Phi_0+J_0=0,\tag{A6}$$

with

$$\begin{aligned}J_0=&\frac{12f}{(X^2+\rho^2)^2}(\rho^2\bar{\sigma}\cdot\dot{X}+\bar{\sigma}\cdot X\rho\dot{\rho})\chi+\frac{6f\rho^2}{(X^2+\rho^2)^2}\bar{\sigma}\cdot X\tau^a\chi\chi^a\\ &+\frac{3i}{2\pi^2 a}\frac{\rho^2 f}{(X^2+\rho^2)^3}\bar{\sigma}\cdot X\chi+\frac{2f}{r}\frac{\partial\hat{A}_0}{\partial r}\bar{\sigma}\cdot X\chi\end{aligned}\tag{A7}$$

the source for Φ_0

$$\mathcal{L}_{\Phi_0}=\frac{1}{8}\int d^4X J_0^\dagger(X)\Phi_0(X).\tag{A8}$$

In this equation the Abelian part of F_{N0} has been included. Since

$$\frac{1}{r}\frac{\partial\hat{A}_0}{\partial r}=\frac{i}{4\pi^2 a}\frac{1}{(X^2+\rho^2)^2}\left(1+\frac{2\rho^2}{X^2+\rho^2}\right)\tag{A9}$$

one finally has

$$J_0=\frac{12f}{(X^2+\rho^2)^2}(\rho^2\bar\sigma\cdot\dot X+\bar\sigma\cdot X\rho\dot\rho)\chi+\frac{6\rho^2 f}{(X^2+\rho^2)^2}\bar\sigma\cdot X\tau^a\chi\chi^a$$
$$+\frac{i}{2\pi^2 a}\frac{f}{(X^2+\rho^2)^2}\left(1+\frac{5\rho^2}{X^2+\rho^2}\right)\bar\sigma\cdot X\chi. \tag{A10}$$

To solve (A6), we need the massive spin-0 propagator in the instanton background,

$$G_2(X,Y)=\langle X|\frac{1}{-D_M^2+m_H^2}|Y\rangle, \tag{A11}$$

in terms of which the Gauss law constraint yields the modular Lagrangian contribution (A6) in the form

$$\mathcal{L}_{\Phi_0}=-\frac{1}{8}\int d^4Xd^4YJ_0^\dagger(X)\langle X|\frac{1}{-D_M^2+m_H^2}|Y\rangle J_0(Y). \tag{A12}$$

APPENDIX B: EXPANSION OF $\mathcal{L}_{\Phi 0}[m_H]$

The spin-0 Greens function (A11) is not known for arbitrary m_H, except for $m_H=0$. Here, we provide a general expression for the different modular contributions in $\mathcal{L}_{\Phi 0}[m_H]$, and then specialize to the two extreme cases of $m_H=0$ and large m_H, for which analytical expressions can be obtained. More specifically, we have

$$\mathcal{L}_{\Phi_0}=-\chi^\dagger\chi\left(\frac{\alpha}{\rho^2}\dot X^2+\frac{\beta}{\rho^2}(\dot\rho^2+\rho^2 a_I^2)\right)$$
$$+\frac{i\gamma}{\tilde\rho^2}\chi^\dagger\tau^a\chi\chi^a-\frac{\delta}{\tilde\rho^4}\chi^\dagger\chi, \tag{B1}$$

with the coefficients

$$\alpha=\frac{12^2}{4}\int d^4Xd^4Y\frac{f(X)g_1(X,Y)f(Y)}{(X^2+1)^2(Y^2+1)^2},$$
$$\beta=\frac{12^2}{16}\int d^4Xd^4Y\frac{f(X)g_2(X,Y)f(Y)}{(X^2+1)^2(Y^2+1)^2},$$
$$\gamma=\frac{48N_c}{8}\int d^4Xd^4Y\frac{f(X)}{(X^2+1)^2}\left(1+\frac{5}{X^2+1}\right)\frac{g_2(X,Y)f(Y)}{(Y^2+1)^2},$$
$$\delta=\frac{64N_c^2}{16}\int d^4Xd^4Y\frac{f(X)}{(X^2+1)^2}\left(1+\frac{5}{X^2+1}\right)g_2(X,Y)$$
$$\times\frac{f(Y)}{(Y^2+1)^2}\left(1+\frac{5}{Y^2+1}\right). \tag{B2}$$

Here the scalar functions trace over the spin-0 propagator

$$g_1(X,Y)=\mathrm{tr}G_2(X,Y),$$
$$g_2(X,Y)=\mathrm{tr}(\sigma\cdot XG_2(X,Y)\bar\sigma\cdot Y), \tag{B3}$$

after the rescaling $\rho\to 1$ and $m_H\to m_H\rho^2$. For $m_H=0$, the expressions will be quoted explicitly below. For m_H large, the spin-0 propagator is zero-mode free, and can be approximated by its free part,

$$G_2(X,Y)\to\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik\cdot(X-Y)}}{k^2+m_H^2}. \tag{B4}$$

Using the Fourier transforms

$$\frac{f(X)}{(X^2+1)^2}=\int\frac{d^4k}{(2\pi)^4}g_1(k)e^{ik\cdot X},$$
$$\frac{f(X)}{(X^2+1)^2}\left(1+\frac{5}{X^2+1}\right)=\int\frac{d^4k}{(2\pi)^4}g_2(k)e^{ik\cdot X}, \tag{B5}$$

we have

$$g_1(k)=\frac{4\sqrt{2}\pi}{15}e^{-|k|}(1+|k|),$$
$$g_2(k)=\frac{4\sqrt{2}\pi}{105}e^{-|k|}(5|k|^2+22|k|+22), \tag{B6}$$

so that

$$\alpha=\frac{12^2}{2}\int\frac{d^4k}{(2\pi)^4}\frac{|g_1(k)|^2}{k^2+\tilde m_H^2},$$
$$\beta=\frac{12^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{|\nabla g_1(k)|^2}{k^2+\tilde m_H^2},$$
$$\gamma=\frac{48N_c}{4}\int\frac{d^4k}{(2\pi)^4}\frac{\nabla g_1(k)\cdot\nabla g_2(k)}{k^2+\tilde m_H^2},$$
$$\delta=\frac{64N_c^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{|\nabla g_2(k)|^2}{k^2+\tilde m_H^2}. \tag{B7}$$

Large m_H:

$$\alpha=\frac{12^2}{\pi^2\tilde m_H^2}\int d^4X\frac{1}{(X^2+1)^7}=\frac{24}{5\tilde m_H^2},$$
$$\beta=\frac{12^2}{4\pi^2\tilde m_H^2}\int d^4X\frac{X^2}{(X^2+1)^7}=\frac{6}{5\tilde m_H^2},$$
$$\gamma=\frac{48N_c}{2\pi^2\tilde m_H^2}\int d^4X\frac{X^2}{(X^2+1)^7}\left(1+\frac{5}{X^2+1}\right)=\frac{54N_c}{35\tilde m_H^2},$$
$$\delta=\frac{64N_c^2}{4\pi^2\tilde m_H^2}\int d^4X\frac{X^2}{(X^2+1)^7}\left(1+\frac{5}{X^2+1}\right)^2=\frac{146N_c^2}{35\tilde m_H^2}. \tag{B8}$$

This will actually contribute to order $\frac{1}{m_H^3}$ after the rescaling in χ.

Zero m_H:

$$\alpha = \beta = \frac{1}{4}, \qquad \gamma = \frac{N_c}{2}, \qquad \delta = \frac{25N_c^2}{24}. \tag{B9}$$

APPENDIX C: COULOMB CORRECTION

Here we provide a complete treatment of the Coulomb backreaction contribution (with more details in Appendix B in [30]). After rescaling the U(1) flavor gauge field in bulk $A_0 \to iA_0$, the Lagrangian for A^0 reads

$$\mathcal{L}[A_0] = \frac{aN_c}{2}(\vec{\nabla}A_0)^2 + \frac{f^2}{2}\chi^\dagger\chi A_0^2 + A_0(\rho^{cl} + \rho), \tag{C1}$$

where ρ^{cl} is the classical source (without the modular field χ)

$$\rho^{cl} = aN_c\nabla^2 A_0^{cl} = -\frac{3N_c}{\pi^2}\frac{\rho^4}{(x^2+\rho^2)^4} \tag{C2}$$

and ρ the quantum source with the modular field

$$\rho = \frac{f^2}{2}i(\chi^\dagger\dot{\chi} - \dot{\chi}^\dagger\chi) + \frac{3}{16\pi^2 a}\frac{2\rho^2 - X^2}{(X^2+\rho^2)^2}f^2\chi^\dagger\chi. \tag{C3}$$

Note that the contribution

$$\begin{aligned}&\frac{3}{16\pi^2 a}\frac{2\rho^2 - X^2}{(X^2+\rho^2)^2}f^2\chi^\dagger\chi \\ &\quad = \frac{3}{16\pi^2 a}\frac{f^2\rho^2}{(X^2+\rho^2)^2}\chi^\dagger\chi + \frac{3}{64\pi^2 a}\partial_N\left(\frac{x_N f^2}{(x^2+\rho^2)}\right)\chi^\dagger\chi\end{aligned} \tag{C4}$$

originates solely from the Chern-Simons term in bulk.

Given the action for A_0, at the minimum we have

$$\mathcal{L}_{\mathrm{Coulomb}} = -J_C\frac{1}{2(-aN_c\nabla^2 + f^2\chi^\dagger\chi)}J_C, \tag{C5}$$

with $J_C = (\rho^{cl} + \rho)$, which is a complicated function of the scalar $\chi^\dagger\chi$. More importantly, it yields always a positive mass correction. Note that the f^2/m_H term in the denominator plays the role of a screening mass, which can be made more manifest through a coordinate transformation.

For a general analysis of the Coulomb correction, we need the Green's function in the background field,

$$G_1(X,Y) = \langle X|\frac{1}{-aN_c\nabla^2 + f^2\chi^\dagger\chi}|Y\rangle. \tag{C6}$$

In the text, we provide an estimate of this contribution in perturbation theory, with the replacement $\chi^\dagger\chi \to 1$, for a single bound kaon.

APPENDIX D: NAIVE $1/m_H$ ANALYSIS

In (17) both the Gauss law constraint and the Coulomb backreaction are complicated functions of the modular coordinate χ and m_H. Naively, a standard quantum analysis would require expanding them in $1/m_H$. This expansion leads to an unstable hyperon spectrum at next-to-leading order, as we now demonstrate. In a way the charge constraint and screening should not be expanded, to guarantee quantum stability.

Consider (17) with all terms expanded to order $\mathcal{O}(1/m_H^2)$:

$$\mathcal{L} = \mathcal{L}_{\mathrm{quadratic}} + \mathcal{L}_{\mathrm{int}}, \tag{D1}$$

where the quadratic part reads

$$\begin{aligned}\mathcal{L}_{\mathrm{quadratic}} = {}& i\chi^\dagger\dot{\chi} + \frac{1}{2m_H}\dot{\chi}^\dagger\dot{\chi} + \frac{9}{2\tilde{\rho}^2}\chi^\dagger\chi + \frac{9}{2m_H\tilde{\rho}^2}j \\ &+ \frac{78}{5m_H\tilde{\rho}^2}i\chi^\dagger\tau^a\chi\chi^a + \frac{102}{5m_H\tilde{\rho}^4}\chi^\dagger\chi \\ &- \frac{37 + 12\frac{Z^2}{\rho^2}}{192 m_H}\chi^\dagger\chi,\end{aligned} \tag{D2}$$

and the "high-order contribution" $\mathcal{L}_{\mathrm{int}}$ reads

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} = {}& -\frac{12}{5m_H\tilde{\rho}^4}\vec{S}^2 - \frac{2}{3\tilde{\rho}^2}n^2 + \frac{1}{m_H\tilde{\rho}^4}\left(-\frac{56}{5}n^2 + \frac{4}{3}n^3 - \frac{4}{3}jn\tilde{\rho}^2\right) \\ &+ \frac{1}{m_H^2\tilde{\rho}^6}\left(-\frac{128n^4}{45} + \frac{376n^3}{15} - \frac{4017n^2}{70}\right. \\ &\left. - jn\tilde{\rho}^2\left(\frac{56}{5} - \frac{8}{3}n\right) - \frac{2}{3}j^2\tilde{\rho}^4\right),\end{aligned} \tag{D3}$$

with

$$j = \frac{i}{2}(\chi^\dagger\dot{\chi} - \dot{\chi}^\dagger\chi), \qquad n = \chi^\dagger\chi. \tag{D4}$$

We now focus on the quadratic part, by replacing $\chi \to e^{im_H t}\sqrt{m_H}$, so that

$$\begin{aligned}\mathcal{L}_{\mathrm{quadratic}} = {}& \frac{1}{2}\dot{\chi}^\dagger\dot{\chi} + \frac{9i}{2\tilde{\rho}^2}\chi^\dagger\dot{\chi} - \frac{m_H^2}{2}\chi^\dagger\chi \\ &+ \left(\frac{102}{5\tilde{\rho}^4} - \frac{37 + 12\frac{Z^2}{\rho^2}}{192}\right)\chi^\dagger\chi + \frac{78i}{5\tilde{\rho}^2}\chi^\dagger\tau^a\chi\chi^a.\end{aligned} \tag{D5}$$

Again, this can be interpreted as a system with two harmonic oscillators in a ρ dependent background magnetic field, coupled with each other by the spin-orbital term. In terms of (23), we have

$$\mathcal{L} = \frac{1}{2}(\dot{\vec{x}}_1^{\,2} + \dot{\vec{x}}_2^{\,2}) + \frac{9}{2\tilde{\rho}^2}(y_1\dot{x}_1 - x_1\dot{y}_1 + y_2\dot{x}_2 - x_2\dot{y}_2) - \frac{m_H^2 + \Omega^2(\rho)}{2}(\vec{x}_1^{\,2} + \vec{x}_2^{\,2}) + \text{spin orbit}, \tag{D6}$$

with $\vec{x}_1 = (x_1, y_1)$, $\vec{x}_2 = (x_2, y_2)$ and

$$\Omega^2(\rho) = -\frac{204}{5\tilde{\rho}^4} + \frac{37 + 12\frac{Z^2}{\rho^2}}{96}. \tag{D7}$$

We proceed to quantize (D6) in the Born-Oppenheimer approximation. We fix y_I and Z and first quantize $\vec{x}_1$ and $\vec{x}_2$. This is justified in the large m_H limit, where χ is fast moving at frequency m_H, while the other degrees of freedom are slow moving with a typical frequency $\omega_y = \frac{1}{\sqrt{6}}M_{\rm KK}$.

We first look at the $l = 0$ state where the spin-orbit coupling vanishes. In this case $\vec{x}_1$ and $\vec{x}_2$ decouple, and we have two identical harmonic oscillators in the background field:

$$\vec{A} = \omega_c(y, -x), \qquad \omega_c = \frac{9}{2\tilde{\rho}^2}. \tag{D8}$$

This is the famed Landau problem, with a spectrum

$$E = \left(n_+ + \frac{1}{2}\right)\Omega_+ + \left(n_- + \frac{1}{2}\right)\Omega_-, \tag{D9}$$

with

$$\Omega_\pm = \sqrt{m_H^2 + \Omega^2 + \omega_c^2} \pm \omega_c. \tag{D10}$$

At large m_H, one has

$$\Omega_\pm = m_H \pm \omega_c + \frac{\Omega^2(\rho) + \omega_c^2}{2m_H} + \mathcal{O}\left(\frac{1}{m_H^2}\right). \tag{D11}$$

Clearly, the $\pm$ solutions can be interpreted as particle/antiparticles. To leading order in $\mathcal{O}(1/m_H)$, the two frequencies agree with the case where χ is quantized as a fermion. Unfortunately,

$$\Omega^2(\rho) + \omega_c^2 = \frac{81}{4\tilde{\rho}^4} - \frac{204}{5\tilde{\rho}^4} < 0, \tag{D12}$$

indicating an instability at the quadratic order. We conclude that the screening effect in the Coulomb part should not be expanded, as it causes a charge instability.

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