Searching for B_c^* via conservation laws

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To distinguish $B_c^{*+}(1^3S_1)$ and $B_c^+(1^1S_0)$ in the experiments, we propose two methods based on the conservation laws. I. From the angular momentum conservation, a nonzero helicity of J/ψ of $B_c^{(*)+} \to J/\psi\pi^+$ would be an evidence of B_c^{*+}. II. Since $B_c^+ \to B^+\phi$ is kinematically forbidden, $B_c^{*+} \to B^+\phi$ provides a clean channel to probe B_c^{*+}. Particularly, our results show that B_c^{*+} is promising to be observed at LHC via $B_c^{(*)+} \to J/\psi\pi^+$. On the other hand, we find that $\mathcal{B}(B_c^{*+} \to B^+\phi) = (7.0 \pm 3.0) \times 10^{-9}$, which is also feasible to be measured at the forthcoming experiments at HL-LHC and FCC-hh.

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I. INTRODUCTION

The B_c meson family is unique in the Standard Model (SM) as its members are composed of heavy quarks with two different flavors, beauty (b) and charm (c). The B_c mesons lie intermediate between ($c\bar{c}$) and ($b\bar{b}$) states both in mass and size, while the different quark flavors leads to much richer dynamics. On the other hand, the ground state of B_c mesons, unlike the charmonium and bottomonium, cannot annihilate into gluons or photons, providing an idea place to examine the heavy quarks. Study on the B_c mesons can deepen our understanding of both the strong and the weak interactions, revealing the underlying physics of the heavy quark dynamics. Last but not least, it provides a unique hunting ground for searching new physics beyond the SM.

The ground state of B_c meson was first observed by the CDF Collaboration at Fermilab [1] in 1998, and there have been continuous measurements on both the mass [2–4] and the lifetime [5,6] via the exclusive decay $B_c^+ \to J/\psi\pi^+$ and the semileptonic decay $B_c^+ \to J/\psi l^+\nu_l$. In 2014, the ATLAS Collaboration reported a structure with the mass of (6842 ± 9) MeV [7], which is consistent with the value predicted for $B_c(2S)$. In 2019, the excited $B_c(2^1S_0)$ was confirmed and $B_c^*(2^3S_1)$ states have been observed in the $B_c^+\pi^+\pi^-$ invariant mass spectrum by the CMS and LHCb Collaborations, with their masses determined to be (6872.1 ± 2.2) and (6841.2 ± 1.5) MeV [8,9], respectively. The $B_c(2^1S_0)^+$ decays to $B_c^+(1^1S_0)\pi^+\pi^-$ directly, and the $B_c^*(2^3S_1)^+$ state decay to $B_c^{*+}(1^3S_1)\pi^+\pi^-$ followed by $B_c^{*+}(1^3S_1) \to B_c^+(1^1S_0)\gamma$. Since the soft photon in the intermediate decay $B_c^{*+}(1^3S_1) \to B_c^+(1^1S_0)\gamma$ was not reconstructed, the mass of $B_c^*(2^3S_1)$ meson appears lower than that of $B_c(2^1S_0)$. This peculiar behaviors of the mass hierarchy makes $B_c^*(1^3S_1)$ uniquely important in studying the B_c meson family.

In the following, we will abbreviate $B_c^*(1^3S_1)$ as B_c^* so long as it does not cause confusion. Study on the B_c^* can complete the precise measurements of the spectrum of the B_c family, and the confirmation of its existence is of great importance for the understanding of strong interaction dynamics at low energy. On the mass of B_c^*, the theoretical predictions range discrepantly from 6326 to 6346 MeV [10–16], and an experimental measurement is still lacking. The dominant decay mode $B_c^* \to B_c\gamma$ has not yet been observed, partly due to the noisy soft photon background of the hadron collider. To identify B_c^* in the experiments, one of the important tasks is to distinguish them from B_c. In this study, we propose two methods based on the conservation laws:

(i) From the angular momentum conservation, the J/ψ can only possess a zero helicity from $B_c^+ \to J/\psi\pi^+$ as B_c^+ is spin-0. In contrast, the J/ψ of $B_c^{*+} \to J/\psi\pi^+$ can have either positive, zero, or negative helicities (see Fig. 1).
(ii) As $B_c^+ \to B^+\phi$ is kinematically forbidden, $B_c^{*+} \to B^+\phi$ provides a clean channel.

Their responsible quark diagrams at the tree level are given in Fig. 2, where the hadronizations take place in the blue regions. As the W boson is color blind, the decays are color allowed and color suppressed, respectively.

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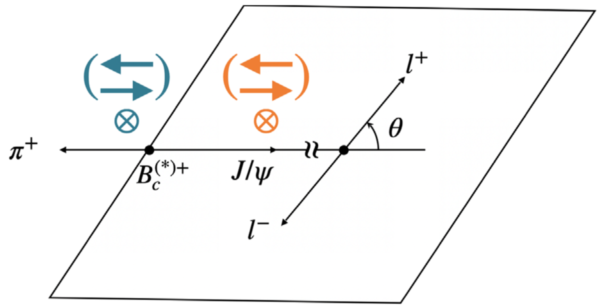

FIG. 1. The adjoint decay distributions of $B_c^{(*)+} \to \pi^+ J/\psi(\to l^+ l^-)$, where the blue and the orange represent the possible spin configuration(s) of $B_c^{(*)+}$ and J/ψ, with ⊗ indicating spin-0 at the $\vec{p}_{J/\psi}$ direction.

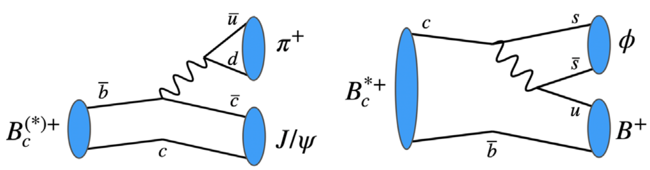

FIG. 2. The quark diagrams for $B_c^{(*)+} \to J/\psi\pi^+$ and $B_c^+ \to B^+\phi$ at the tree level.

The rest of the paper is organized as follows. The primary formulas in our calculation are presented in Sec. II. We give the numerical analysis and results in Sec. III. We conclude the study in Sec. IV.

II. HELICITY FORMALISM

To extract the helicity information of J/ψ as well as calculate the branching fractions, we give the helicity formalism of the decays in this section. The helicity information of J/ψ can be obtained from $B_c^{(*)+} \to J/\psi(\to l^- l^+)\pi^+$ with $l = e, \mu$. The advantage of the helicity analysis is that it can easily cooperate with the sequential decays and has a clear view of physical meaning [17].

Taking the initial B_c^{*+} as unpolarized, the angular distributions of $B_c^{(*)+} \to J/\psi(\to l^- l^+)\pi^+$ are given as

$$\frac{\partial\Gamma^{(*)}}{\partial\cos\theta} \propto \sum_{\lambda=\pm,0,l=\pm} |H_\lambda^{(*)} d^1(\theta)^\lambda{}_l|^2 \propto 1 - P_2 + \frac{3}{2}\alpha^{(*)} P_2, \quad (1)$$

where $H_\lambda^{(*)}$ are the helicity amplitudes with the subscripts denoting the helicity of J/ψ, $d^1(\theta)$ the Wigner d matrix for $J = 1$, θ defined in the helicity frame of J/ψ (see Fig. 1), and

$$P_2 = \frac{1}{2}(3\cos^2\theta - 1),$$
$$\alpha^{(*)} = \frac{|H_+^{(*)}|^2 + |H_-^{(*)}|^2}{|H_+^{(*)}|^2 + |H_-^{(*)}|^2 + |H_0^{(*)}|^2}. \quad (2)$$

Here, α has the physical meaning of the nonzero-polarized fraction of J/ψ. Notice that $H_\pm$ are forbidden by the angular momentum conservation, resulting in

$$\alpha = 0. \quad (3)$$

To further extract the helicity information, we define

$$\mathcal{A}^{(*)} = \frac{1}{\Gamma}\left(\int_{|\cos\theta|<x_0} \frac{\partial\Gamma^{(*)}}{\partial\cos\theta} d\cos\theta - \int_{|\cos\theta|>x_0} \frac{\partial\Gamma^{(*)}}{\partial\cos\theta} d\cos\theta\right)$$
$$= \left(3x_0 - \frac{3}{2}\right)\alpha^{(*)}, \quad (4)$$

where x_0 is chosen to satisfy

$$x_0^3 - 3x_0 + 1 = 0, \quad (5)$$

which is found to be $x_0 \approx 0.3473$.

The experiments of B_c^{*+} are polluted by the off-shell contributions from B_c^+ at LHC. Thus, we define the event-average $\bar{\mathcal{A}}$ as

$$\bar{\mathcal{A}} = r\mathcal{A} + r^*\mathcal{A}^* = r^*\mathcal{A}^*, \quad (6)$$

as well as the event-average nonzero-polarized fraction as

$$r\frac{\partial\Gamma}{\partial\cos\theta} + r^*\frac{\partial\Gamma^*}{\partial\cos\theta} \propto 1 - P_2 + \frac{3}{2}\bar{\alpha}P_2, \quad (7)$$

with

$$r^{(*)} = \frac{N_{B_c^{(*)}}}{N_{B_c} + N_{B_c^*}}, \quad (8)$$

where $N_{B_c^{(*)}}$ is the number of the observed events in $B_c^{(*)+} \to J/\psi(\to l^+ l^-)\pi^+$. The second equality in Eq. (6) is attributed to Eq. (3).

To get an estimation on the experiments, we calculate the amplitudes within the factorization framework. The helicity amplitudes of $B_c^{(*)+} \to J/\psi\pi^+$ are given as

$$(2\pi)^4\delta^4(p_{B_c} - p_{J/\psi} - p_\pi)H_\lambda$$
$$= i\frac{G_F}{\sqrt{2}}V_{cb}^* V_{ud} f_\pi p_\pi^\mu a_1 \langle J/\psi; p\hat{z},$$
$$J_z = \lambda|\bar{b}\gamma_\mu(1-\gamma_5)c|B_c^{(*)+}; J_z = \lambda\rangle, \quad (9)$$

where p is for the 4-momentum of the hadron in the subscript, G_F and f_π the Fermi and the pion decay constants, a_1 the effective Wilson coefficient for the color-allowed decays, J_z the angular momentum at the z direction, and $p\hat{z}$ indicates $\vec{p}_{J/\psi}//\hat{z}$.

On the other hand, the helicity amplitudes of $B_c^{*+} \to B^+\phi$ are given as

$$(2\pi)^4\delta^4(p_{B_c} - p_{B^+} - p_\phi)H_\lambda = -\frac{G_F}{\sqrt{2}}V_{cs}^* V_{su} f_\phi \epsilon_\lambda^{\mu *} a_2 \langle B^+; p\hat{z}|\bar{u}\gamma_\mu(1-\gamma_5)c|B_c^{(*)+}; J_z = -\lambda\rangle, \quad (10)$$

where a_2 is the effective Wilson coefficient for the color-suppressed decays, f_ϕ the ϕ decay constant, and $\epsilon_\lambda^{\mu *}$ the polarization 4-vector of ϕ with λ its helicity.

Finally, the decay width for B_c^+ is given as

$$\Gamma = \frac{|\vec{p}_{\rm cm}|}{8\pi M_{B_c}}|H_0|^2, \quad (11)$$

whereas the decay widths of B_c^{*+} with the daughter vector meson having λ helicity are given as

$$\Gamma_\lambda = \frac{|\vec{p}_{\rm cm}|}{24\pi M_{B_c^*}}|H_\lambda^*|^2. \quad (12)$$

The total decays widths of $B_c^* \to J/\psi\pi^+$ and $B_c^* \to B^+\phi$ can be easily obtained by adding up the contributions from $\lambda = 0, \pm$.

III. NUMERICAL ANALYSIS

The meson transition matrix elements require the knowledge of the hadron wave functions. In this work, we employ the ones from the homogeneous bag model, in which the center motions of the hadrons in the original bag model are removed [18]. The bag radius (R) and the quark masses can be extracted from the mass spectra, which are found to be [19]

$$R = (2.81 \pm 0.30)\ {\rm GeV}^{-1}, \qquad M_{u,d} = 0,$$
$$M_c = 1.641\ {\rm GeV}, \qquad M_b = 5.093\ {\rm GeV}. \quad (13)$$

The details of the calculation can be found in the Appendix. In this study, f_π and f_ϕ are taken from the experiments and the Lattice QCD [20,21]

$$f_\pi = 131\ {\rm MeV}, \qquad f_\phi = (241 \pm 9)\ {\rm MeV}, \quad (14)$$

and the effective Wilson coefficients are taken to be

$$|a_1| = 1.0 \pm 0.1, \qquad |a_2| = 0.27 \pm 0.07. \quad (15)$$

The results are given in Table I, where we also include $\Gamma(B_c^{*+} \to B_c^+\gamma)$, which can be safely approximated as $1/\tau$ with τ the lifetime of B_c^{*+}. The calculated lifetime is consistent with most of the literature [15,22], but significantly smaller than the one from the nonrelativistic potential model [10], and twice larger than the one from the relativistic independent quark model [23]. Nonetheless, a large part of the uncertainties that arises from the hadron wave functions is canceled in the branching ratios of B_c^{*+}, as the lifetime is calculated under the same framework.

TABLE I. The decay widths and the branching ratios.

Channel	Helicity	Γ (eV)	$\mathcal{B}$
$B_c^+ \to J/\psi\pi^+$	H_0	$(6.3 \pm 1.3) \times 10^{-7}$	$(4.8 \pm 1.0) \times 10^{-4}$
$B_c^{*+} \to J/\psi\pi^+$	H_-	$(6.4 \pm 2.7) \times 10^{-9}$	$(1.2 \pm 0.5) \times 10^{-10}$
	H_0	$(2.8 \pm 0.6) \times 10^{-7}$	$(5.5 \pm 1.4) \times 10^{-9}$
	H_+	$(9.9 \pm 2.3) \times 10^{-7}$	$(1.9 \pm 0.5) \times 10^{-8}$
	Total	$(1.2 \pm 0.2) \times 10^{-6}$	$(2.4 \pm 0.5) \times 10^{-8}$
$B_c^{*+} \to B_c^+\gamma$	Total	53 ± 3	≈ 1
$B_c^{*+} \to B^+\phi$	Total	$(3.7 \pm 1.7) \times 10^{-7}$	$(7.0 \pm 3.0) \times 10^{-9}$

Our $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ is consistent with the relativistic quark model [24], but two times smaller compared to most of the literature [25], which can be partly attributed to that we use a smaller $|a_1|$. As our estimation is a more conservative one, the angular analysis is promising to be carried out in the experiments for there are more data points to reconstruct the distribution than we expect.

The decay of $B_c^{*+} \to B^+\phi$ is color suppressed and suffers large uncertainties from a_2 as well as $M_{B_c^*}$. In particular, as $M_{B_c^*}$ is close to the mass threshold of $B^+\phi$, the decay width can range from 0 to 10^{-6} eV, depending on $M_{B_c^{*+}}$. The dependency on $M_{B_c^{*+}}$ as well as the uncertainties caused by a_2 are plotted in Fig. 3. Taking $M_{B_c^{*+}} = 6331$ MeV, the calculated decay width is given in Table I, which is consistent with Ref. [26], within the range of the error.

From Table I, for $B_c^{*+} \to J/\psi\pi^+$ we obtain

$$\alpha^* = 0.82 \pm 0.01, \qquad \mathcal{A}^* = 0.38 \pm 0.01, \quad (16)$$

in which the theoretical uncertainty is canceled for the correlations between H_λ^*. The cross section of B_c^* meson at the LHC is expected to be $\sigma(B_c^*) = 29$ nb [27]. At an integrated luminosity of 150 ${\rm fb}^{-1}$ during LHC Run-2, 300 ${\rm fb}^{-1}$ during LHC Run-3, and 3000 ${\rm fb}^{-1}$ after High Luminosity upgrade (HL-LHC) [28], the numbers of B_c^* events are 8.7×10^9, 1.74×10^{10}, and 1.74×10^{11}, resulting in 270, 540, and 5400 events of $B_c^{*+} \to J/\psi\pi^+$, respectively. Taking the branching ratios $\mathcal{B}(J/\psi \to l^+l^-) \approx 12\%$ [20], there are expected to be 33, 65, and 650 events of $B_c^{*+} \to \pi^+ J/\psi(\to l^+l^-)$ being able to be reconstructed at LHC Run-2, LHC Run-3, and HL-LHC, respectively.

By choosing $6400\ {\rm MeV} > M(J/\psi\pi^+) > 6325\ {\rm MeV}$ with $M(J/\psi\pi^+)$ the invariant mass of $J/\psi\pi^+$, most of the off-shell contribution from B_c^+ would be filtered, in which N_{B_c} is expected to be less than 20 at running LHC from the Fig. 1 of Ref. [29].[1] Thus, N_{B_c} and $N_{B_c^*}$ can be safely taken as equal in the simulation.

[1] In fact, at the bottom right figure, there appears to have a little bump around 6340 MeV.

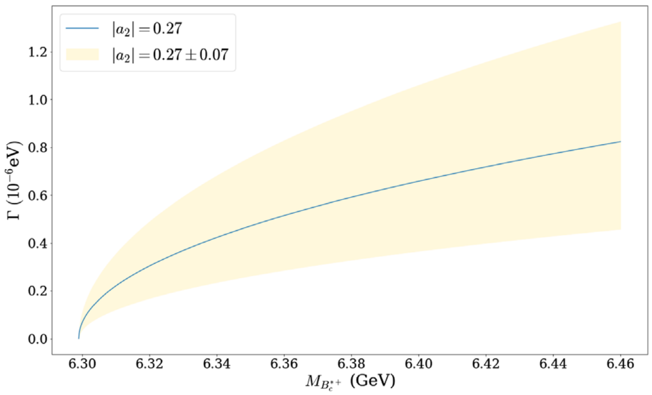

FIG. 3. $\Gamma(B_c^{*+} \to B^+\phi)$ versus $M_{B_c^{*+}}$, where the yellow region covers the uncertainty of a_2.

We generate the pseudodata based on the experimental conditions at LHC Run-2, LHC Run-3, and HL-LHC. The off-shell contributions from B_c^+ are also included with $N_{B_c} = N_{B_c^*}$ as discussed in the previous paragraph. The numbers of the events are plotted against $\cos\theta$ in Fig. 4, and the numerical results of $\bar{\alpha}$ and $\bar{\mathcal{A}}$ are given in Table II. Our analysis show that there would be a $1.5\sigma 0$ signal of nonzero $\bar{\mathcal{A}}$ at LHC Run-2, and a 5σ signal at HL-LHC, which would be a solid evidence of B_c^*.

TABLE II. The $\bar{\alpha}$ and $\bar{\mathcal{A}}$ fitted from the pseudodata in Fig. 4 with statistical uncertainties.

	$N_{B_c} = N_{B_c^*}$	$\bar{\alpha}$	$\bar{\mathcal{A}}$
LHC Run-2	33	0.48 ± 0.39	0.15 ± 0.10
LHC Run-3	65	0.43 ± 0.26	0.17 ± 0.07
HL-LHC	650	0.44 ± 0.10	0.19 ± 0.03

On the other hand, at the forthcoming experiments at FCC-hh [30], the number of B_c^* events are expected to be 10^{12}. Hence, there would be about 2000 and 10^5 $B_c^{*+} \to B^+\phi$ events at HL-LHC and FCC-hh, respectively, which would be sufficient for the experiments to determine the mass.

IV. CONCLUSIONS

Utilizing the conservation laws, we propose two novel methods for distinguishing B_c^* and B_c in the experiments. The calculated branching fractions of $B_c^+ \to J/\psi\pi^+$ and $B_c^{*+} \to B^+\phi$ are compatible with the literature, indicating

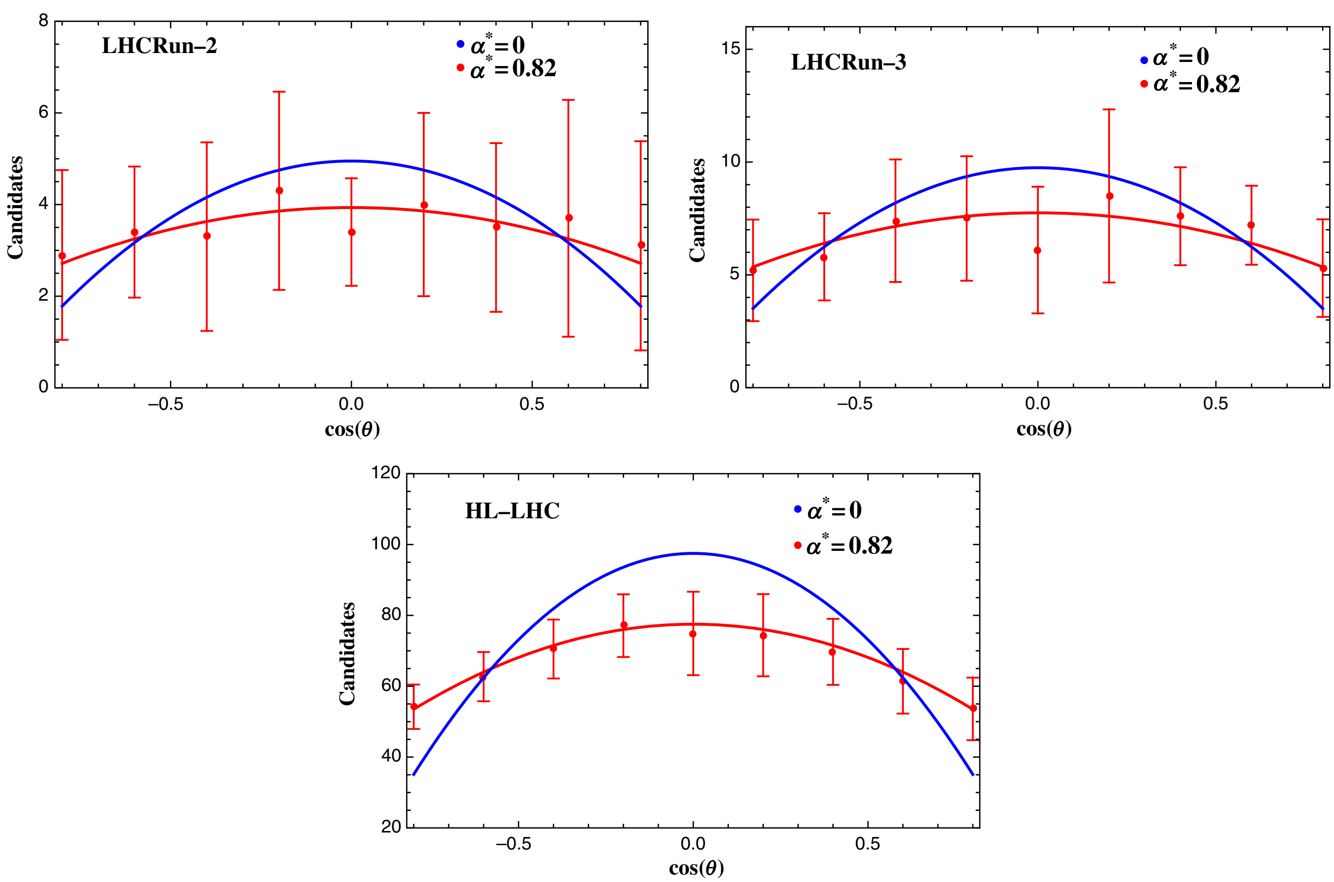

FIG. 4. The numbers of the observed events of $B_c^{(*)} \to J/\psi(\to l^+l^-)\pi^+$ plotted against $\cos\theta$. The red points with statistical uncertainties are the pseudodata generated by the Monte Carlo method for $\alpha^* = 0.82$, and the blue and the red lines are drawn with $\alpha^* = 0$ and $\alpha^* = 0.82$ in Eq. (7), respectively.

that our analysis is reliable. The nonzero polarized fraction of J/ψ from $B_c^{(*)+} \to J/\psi\pi^+$ has been found to be $\alpha = 0(0.82)$. Furthermore, the branching fractions of $B_c^{*+} \to J/\psi\pi^+$ and $B_c^{*+} \to \phi\pi^+$ have been obtained as $(2.4 \pm 0.5) \times 10^{-8}$ and $(7.0 \pm 3.0) \times 10^{-9}$, respectively. To calculate the lifetime of B_c^*, we have found that $\Gamma(B_c^{*+} \to B_c^+\gamma) = (53 \pm 3)$ eV with the homogeneous bag model, consistent with most of the literature.

We have shown that $B_c^{*+} \to B^+\phi$ would be promising to be measured at HL-LHC as well as FCC-hh. To examine the feasibilities of the measurements, we have conducted simulations based on the experimental conditions. Remarkably, we have shown that the helicity analysis on $B_c^{(*)+} \to J/\psi\pi^+$ is ready to be performed at LHC. Thus, we urge the experimentalists to probe the angular distributions of $B_c^{(*)+} \to J/\psi(\to l^+l^-)\pi^+$ in the region of $M(J/\psi\pi^+) >$ 6325 MeV, which can be served as an evidence of $B_c^{(*)+}$.

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APPENDIX: THE BARYON WAVE FUNCTIONS

Here, we give the meson wave functions of the homogeneous bag model, which are used in the calculation of the transition matrix elements in the main text. In the original version of the bag model, both the asymptotic freedom and the confinement of the QCD are described by the bag radius, R. The quarks are confined in the bag but moving freely within it, satisfying the free Dirac equation

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \quad \text{for } r < R. \tag{A1}$$

For low-lying hadrons, we can take the wave functions to be spherical, and we arrive at

$$\psi(x)_q = \phi_q(\vec{x})e^{-iE_q t} = N\begin{pmatrix} \omega_{q+}j_0(p_q r)\chi \\ i\omega_{q-}j_1(p_q r)\hat{r}\cdot\vec{\sigma}\chi \end{pmatrix} \times e^{-iE_q t} \quad \text{for } r < R, \tag{A2}$$

where q is the quark flavor, N the normalizing constant, χ the two component spinor, p_q the magnitude of the 3-momentum, and $\omega_{q\pm} = \sqrt{1 \pm m_q/E_q}$ with E_q the quark energy. The antiquark wave functions are obtained by taking the charge conjugate.

At the boundary of the bag the current shall vanish, which give us the boundary condition, read as

$$\hat{r}\cdot(\bar{\psi}\vec{\gamma}\psi) = 0, \quad \text{at } |\vec{x}| = R. \tag{A3}$$

In analogy to the familiar infinite square well, p_q is quantized, satisfying

$$\tan(p_q R) = \frac{p_q R}{1 - m_q R - E_q R}. \tag{A4}$$

We concern the low-lying hadrons only and therefore take the minimum of p_q. At the massless and the heavy quark limits we have

$$\lim_{m_q R\to 0} p_q R = 2.0428, \qquad \lim_{m_q R\to\infty} p_q R = \pi, \tag{A5}$$

respectively. A meson can be constructed by confining a quark and an antiquark to a same bag. By considering the bag energy, zero point energy, and the interaction between quarks, the bag model can successfully explain most of the low-lying hadron masses as well as the ratios of the magnetic dipole moments [19].

However, despite the success on the hadron masses, the wave functions of the bag model are problematic when it comes to decays. As the description of a static bag is essentially localized, the hadron wave function can not be the momentum eigenstates, and thus the transition matrix elements cannot be consistently calculated. This problem has been resolved with the linear superposition of infinite bags by one of the authors (Liu), and with it the experimental branching ratios of $\Lambda_b \to \Lambda_c^+\pi^+$ and $\Lambda_b \to p\pi^+$ can be well explained [18].

In the homogeneous bag model, the meson wave functions at rest are given as

$$\Psi(x_{q_1}, x_{q_2}) = \mathcal{N}\int d^3\vec{x}\phi_{q_1}(\vec{x}_{q_1} - \vec{x})\phi_{q_2}^c(\vec{x}_{q_2} - \vec{x})e^{-i(E_{q_1}t_{q_1} + E_{q_2}t_{q_2})}, \tag{A6}$$

where $\mathcal{N}$ is the normalizing constant, and c in the superscript denotes the charge conjugate. The wave function in Eq. (A6) is manifestly invariant under the space translation and therefore describes a meson at rest. The wave functions with nonzero momenta can be easily obtained by Lorentz boost.

By demanding the normalization condition

$$\langle p|p'\rangle = 2p^0(2\pi)^3\delta^3(\vec{p} - \vec{p}'), \tag{A7}$$

we find

$$\frac{1}{\mathcal{N}^2} = 2M\int d^3\vec{x}_\Delta \prod_{i=1,2} d^3\vec{x}_{q_i}^r \phi_{q_i}^\dagger\left(\vec{x}_{q_i}^r + \frac{1}{2}\vec{x}_\Delta\right)\phi_{q_i}\left(\vec{x}_{q_i}^r - \frac{1}{2}\vec{x}_\Delta\right), \tag{A8}$$

with p and M the hadron momentum and mass, respectively.

With the wave functions, the meson transition matrix elements can be computed straightforwardly. For simplicity we take $B_c^{(*)-} \to J/\psi\pi^-$ as an example. The results of $B_c^{(*)+} \to J/\psi\pi^+$ can be obtained by taking the CP conjugate as CP is conserved in the $b \to c$ transition. The transition matrix elements read as

$$\int \langle J/\psi|\bar{c}\gamma^\mu b(x)e^{ip_\pi x}|B_c^{(*)-}\rangle d^4x = \mathcal{Z}\int d^3\vec{x}_\Delta V^\mu(\vec{x}_\Delta)D_c(\vec{x}_\Delta),$$
$$\int \langle J/\psi|\bar{c}\gamma^\mu\gamma_5 b(x)e^{ip_\pi x}|B_c^{(*)-}\rangle d^4x = \mathcal{Z}\int d^3\vec{x}_\Delta A^\mu(\vec{x}_\Delta)D_c(\vec{x}_\Delta), \tag{A9}$$

with

$$\mathcal{Z} = (2\pi)^4\delta^4(p_{B_c^{(*)}} - p_{J/\psi} - p_\pi)\mathcal{N}_{B_c^{(*)}}\mathcal{N}_{J/\psi},$$
$$D_c(\vec{x}_\Delta) = \sqrt{1-v^2}\int d^3\vec{x}\phi_c^\dagger\left(\vec{x}+\frac{1}{2}\vec{x}_\Delta\right)\phi_c\left(\vec{x}-\frac{1}{2}\vec{x}_\Delta\right)e^{-2iE_c\vec{v}\cdot\vec{x}},$$
$$V^\mu(\vec{x}_\Delta) = \int d^3\vec{x}\phi_c^\dagger\left(\vec{x}+\frac{1}{2}\vec{x}_\Delta\right)\gamma^0\gamma^\mu\phi_b\left(\vec{x}-\frac{1}{2}\vec{x}_\Delta\right)e^{i(M_{J/\psi}+M_{B_c^{(*)}}-E_c-E_b)\vec{v}\cdot\vec{x}},$$
$$A^\mu(\vec{x}_\Delta) = \int d^3\vec{x}\phi_c^\dagger\left(\vec{x}+\frac{1}{2}\vec{x}_\Delta\right)\gamma^0\gamma^\mu\gamma_5\phi_b\left(\vec{x}-\frac{1}{2}\vec{x}_\Delta\right)e^{i(M_{J/\psi}+M_{B_c^{(*)}}-E_c-E_b)\vec{v}\cdot\vec{x}}, \tag{A10}$$

Here, the calculation is taken at the Briet frame where B_c^- and J/ψ have the velocity $-\vec{v}$ and $\vec{v}$, respectively. Although the derivation is quite tedious (see Ref. [18] for an example), their physical meaning can be easily understood:

(i) $\mathcal{Z}$ is the overall normalizing constant along with the momentum conservation.

(ii) D_c is the overlapping coefficient attributed by the spectator quark between the initial and the final states. Note that their centers of the bags are separated at a distance of $\vec{x}_\Delta$.

(iii) $V^\mu(A^\mu)$ is the matrix element of the (axial) vector current at the quark level, where the centers of the bags are separated at a distance of $\vec{x}_\Delta$.

Here, the exponential in the integrals would oscillate violently at large velocity, causing a suppression that is a punishment for not being at the same speed. The matrix elements of $B_c^{*+} \to B^+\phi$ can be calculated in the same manner.

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