Impact of the interference between the resonance and continuum amplitudes on vector quarkonia decay branching fraction measurements

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The measurement of the branching fraction of a heavy quarkonium decaying into light hadronic final state at e^+e^- colliders is revisited. In e^+e^- annihilation experiments, background contributions from the continuum amplitude and its interference with the resonance amplitude are irreducible. These effects become more and more significant as the precision of experimental measurements improves. While the former can be easily subtracted with data taken off the resonance peak, the latter depends on the relative size and phase between the resonance and continuum amplitudes. Two ratios are defined to estimate the size of these effects, r_R^f for the ratio of the contribution of the interference term to the resonance term and r_c^f for that to the continuum term. We find that r_R^f could be as large as a few percent for narrow resonances, and both r_R^f and r_c^f could be large for broad resonances. This indicates that the interference effect is crucial for the measurements of the branching fractions aiming at the percent level or better precision and needs to be measured or estimated properly.

I. INTRODUCTION

Heavy quarkonium is a multiscale system that can be used to probe all regimes of quantum chromodynamics (QCD) [1]; thus, it provides an ideal ground test for the understanding of QCD. The test can be done by comparing various observables predicted by theory and measured by experiments. One such observable is the hadronic decay width or equivalently, the hadronic decay branching fraction, which can be examined to find the patterns and properties of quarkonium decays. In principle, the hadronic decay of heavy quarkonium can be calculated theoretically with some input from experimental measurements. However, rigorous calculations are still very limited; many of the studies of the quarkonium decays are based on phenomenological models and observations in experimental data.

An example of the quarkonia decay pattern is the long-standing puzzle in charmonium sector, namely, the "$\rho - \pi$ puzzle." In perturbative QCD (pQCD), the ratio $Q^f = \mathcal{B}[\psi(2S) \to f]/\mathcal{B}[J/\psi \to f]$ is predicted to be $Q^f = \mathcal{B}[\psi(2S) \to e^+e^-]/\mathcal{B}[J/\psi \to e^+e^-] \approx 12\%$ [2], where $\mathcal{B}[J/\psi \to f]$ and $\mathcal{B}[\psi(2S) \to f]$ are the branching fractions of J/ψ and $\psi(2S)$ decay into the same hadronic final state f. Violation of this "12% rule" was first observed by the Mark II experiment in $\rho\pi$ and $K^*\bar{K}$ decay modes [3]. It was confirmed by other experiments in more vector-pseudo-scalar (VP) decay modes as well as vector-tensor (VT) decay modes [4]. The puzzle has not been solved although many theoretical explanations have been proposed [5]. A similar rule for the ratio in bottomium sector $Q_\Upsilon^f = \mathcal{B}[\Upsilon(2S) \to f]/\mathcal{B}[\Upsilon(1S) \to f] \approx 0.77$ is expected. The test of Q_Υ^f from measurements is inconclusive [6–8].

Experimentally, vector quarkonia can be produced from e^+e^- and $p\bar{p}$ annihilation processes or heavier hadron decays; the branching fraction can be measured using subsequent decays. Among them, e^+e^- annihilation experiments are the most important contributors as they provide a clean experimental environment, and the vector quarkonia can be copiously produced at rest. Most of the hadronic decay branching fractions of charmonium states were measured with data samples collected by the CLEOc, BESII, and BESIII experiments, and the typical precision is a few tens of percent [9]. The hadronic decays of $\Upsilon(1S)$ and $\Upsilon(2S)$ were measured by the Belle and CLEO experiments [10]. The best precision is several percent. The BESIII and Belle II are currently running e^+e^- experiments. At the

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BESIII experiment, 10 billion J/ψ events and 3 billion $\psi(2S)$ events have been accumulated, and 20 fb^{-1} $\psi(3770)$ data are expected before 2024 [11]. The sizes of the data samples are at least 6 times larger than those used in previous BESIII measurements, and dozens of times larger than those in the CLEOc and BESII measurements. At the Belle II experiment, about 500 fb^{-1} data for each vector bottomnium state are planned [12], which are tens (hundreds) of times larger than those from the Belle (CLEO) experiment. Therefore, the precision of the branching fraction of vector quarkonia decay can be improved significantly.

The branching fractions of hadronic decays measured in e^+e^- colliders have an unavoidable background contribution, *i.e.*, the continuum process produced directly from e^+e^- annihilation, $e^+e^- \to \gamma^* \to f$. The signal events observed in an experiment contain contributions from both resonance decays and continuum production, and, more importantly, the interference between them. The importance of the continuum amplitude and the interference effect has been pointed out in Ref. [13], but in most of the measurements, this was not taken into account properly due to the absence or low statistics of data sample in the off resonance region. In principle, to determine the branching fraction of a resonance decaying into a specific final state, one needs to know the cross section of the continuum production as well as the relative phase between the resonance and continuum amplitudes. This can only be realized by measuring the cross sections at no less than three energies around the resonance peak, since we do not have reliable theoretical or experimental knowledge on the continuum cross section or the relative phase for any hadronic final state. However, this was not done in previous measurements where vector quarkonia are produced directly from e^+e^- annihilation. For most cases in which the continuum contribution was considered, it was estimated using data samples taken off the resonance and subtracted without considering the interference effect. Moreover, the possible bias from this treatment of the interference effect was not included in the systematic uncertainties. In old-generation experiments, both the statistical and systematic uncertainties are more than 10%, the interference effect may be neglected since it is not dominant. When one performs high precision measurements, this effect could be larger than many other sources of systematic uncertainties; thus, it can not be neglected.

In this paper, we first revisit the cross section formula for a hadronic final state produced in an e^+e^- collider at a resonance peak, then we quantitatively describe the importance of the continuum contribution and its interference with the resonance contribution in the branching fraction measurement. Finally, additional experimental effects from radiative correction and beam energy spread have been addressed.

II. PRODUCTION OF A HADRONIC FINAL STATE IN e^+e^- EXPERIMENT

A hadronic final state in e^+e^- colliders in the vicinity of a resonance R is produced via the coherent sum of the resonance and continuum amplitudes. If we use $a_c^f(s)$ to denote the continuum amplitude for a certain exclusive final state f, and $a_R^f(s)$ for the resonance amplitude, the cross section can be written as

$$\sigma_{\text{tot}}^f(s) = |a_c^f(s) + e^{i\varphi} \cdot a_R^f(s)|^2 \equiv \sigma_c^f(s) + \sigma_R^f(s) + \sigma_{\text{int}}^f(s), \tag{1}$$

where $\sqrt{s}$ is the center-of-mass (c.m.) energy and φ is the relative phase between the two amplitudes. We use $\sigma_c^f(s) = |a_c^f(s)|^2$, $\sigma_R^f(s) = |a_R^f(s)|^2$, and $\sigma_{\text{int}}^f(s)$ to denote the cross sections of the continuum process, the resonance process, and the interference term, respectively. The resonance amplitude $a_R^f(s)$ is parametrized as

$$a_R^f(s) = \frac{\sqrt{12\pi\Gamma_{ee}\Gamma\mathcal{B}_f}}{s - M^2 + iM\Gamma}, \tag{2}$$

where M and Γ are the mass and total width of the resonance, Γ_{ee} is the partial width of $R \to e^+e^-$, and $\mathcal{B}_f$ is the branching fraction of $R \to f$. Inserting Eq. (2) into Eq. (1), the cross section is expanded as

$$\begin{aligned}
\sigma_{\text{tot}}^f(s) = {} & \sigma_c^f(s) + \frac{12\pi\Gamma_{ee}\Gamma\mathcal{B}_f}{(s - M^2)^2 + M^2\Gamma^2} \\
& + 2\frac{\sqrt{\sigma_c^f(s)}\sqrt{12\pi\Gamma_{ee}\Gamma\mathcal{B}_f}}{(s - M^2)^2 + M^2\Gamma^2} \\
& \times [(s - M^2)\cos\varphi + M\Gamma\sin\varphi]. \tag{3}
\end{aligned}$$

If the data sample is taken at the energy of the resonance mass, i.e., $s = M^2$, Eq. (3) can be simplified as

$$\sigma_{\text{tot}}^f(s) = \sigma_c^f(s) + \frac{12\pi\mathcal{B}_{ee}\mathcal{B}_f}{M^2} + 2\frac{\sqrt{\sigma_c^f(s)}\sqrt{12\pi\mathcal{B}_{ee}\mathcal{B}_f}}{M}\sin\varphi, \tag{4}$$

where $\mathcal{B}_{ee} = \Gamma_{ee}/\Gamma$ is the branching fraction of $R \to e^+e^-$.

Based on Eq. (4), we define two ratios, r_R^f and r_c^f, representing the ratio of cross section from the interference term with respect to the resonance and continuum term, respectively,
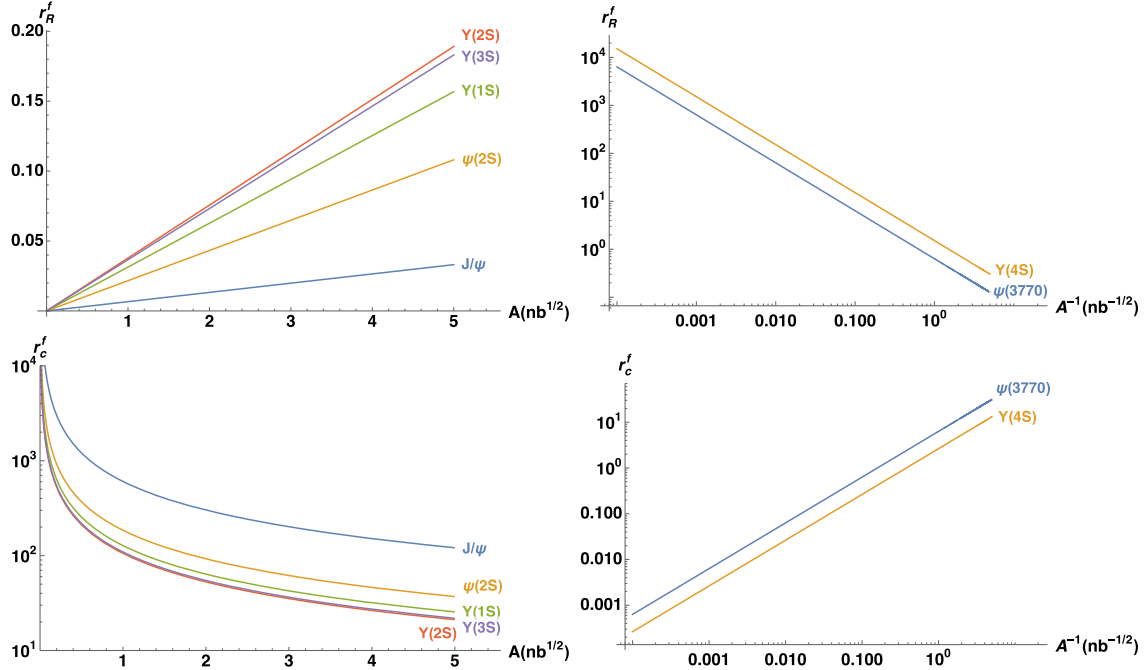
FIG. 1. The dependence of $r_R^{f\,\mathrm{max}}$ (top) and $r_c^{f\,\mathrm{max}}$ (bottom) on A for different resonances. The left column shows the result for narrow resonances where the x axis is set to A, and the right column shows the result for broad resonances where the x axis is set to A^{-1}.

$$r_R^f \equiv \frac{\sigma_{\mathrm{int}}^f(s)}{\sigma_R^f(s)} = \frac{2}{\hbar c}\sqrt{\frac{\sigma_c^f(s)}{\mathcal{B}_f}\frac{M}{\sqrt{12\pi\mathcal{B}_{ee}}}}\sin\varphi \equiv \frac{2}{\hbar c}AB\sin\varphi,$$

$$r_c^f \equiv \frac{\sigma_{\mathrm{int}}^f(s)}{\sigma_c^f(s)} = 2\hbar c\sqrt{\frac{\mathcal{B}_f}{\sigma_c^f(s)}\frac{\sqrt{12\pi\mathcal{B}_{ee}}}{M}}\sin\varphi \equiv 2\hbar c A^{-1}B^{-1}\sin\varphi.$$

$$(5)$$

Factor $A = \sqrt{\sigma_c^f(s)/\mathcal{B}_f}$ can be calculated once the cross section of the continuum process and the branching fraction are measured, and factor $B = M/\sqrt{12\pi\mathcal{B}_{ee}}$ is a constant depending on the resonance parameters. Usually the cross section is measured in the unit of barn and M in the unit of GeV/c^2, so the conversion constant $\hbar c$ is added in the denominator (numerator) for r_R^f (r_c^f). From Eq. (5), it is obvious that the magnitudes of the ratios reach maxima when the relative phase φ is $\pm 90°$. The dependence of $r_R^{f\,\mathrm{max}}$ and $r_c^{f\,\mathrm{max}}$ on A is illustrated in Fig. 1 for narrow resonances and broad resonances separately.

III. NARROW RESONANCES

Resonances with masses below open heavy flavor threshold, such as J/ψ, $\psi(2S)$, $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$, are narrow, and the production cross section in e^+e^- collision is much larger than the continuum cross section. Using M, Γ, and Γ_{ee} values from PDG [10], the total cross sections for the five resonances at peak positions can be calculated with

$$\sigma_R(s) = \frac{12\pi\Gamma_{ee}\Gamma}{(s-M)^2 + M^2\Gamma^2} = \frac{12\pi\mathcal{B}_{ee}}{M^2}. \qquad (6)$$

The inclusive hadronic cross sections of the continuum process can be estimated by using the $\mathcal{R}$ value $\mathcal{R}(s) = \sigma_c(e^+e^- \to \mathrm{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ and $\sigma(e^+e^- \to \mu^+\mu^-) = (4\pi\alpha^2)/(3s)$. With the $\mathcal{R}$ values from the latest BESIII measurement in the vicinity of J/ψ and $\psi(2S)$ [14], and from PDG for $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ [10], the inclusive hadronic cross sections of the continuum process at the five resonances peak positions are calculated and listed in Table I, together with the cross sections of the resonance process and the B factor. The absolute value of σ_c is 3 or 4 orders of magnitude smaller than σ_R, while σ_{int} can be sizeable depending on the relative phase φ, as shown in the following example. So for narrow resonances, r_R^f will be used to characterize the size of the interference effect.

We use the branching fraction measurement of J/ψ and $\psi(2S) \to \Lambda\bar{\Lambda}$ as an example to estimate the size of the interference effect. The branching fractions of $J/\psi \to \Lambda\bar{\Lambda}$

TABLE I. The cross sections of the resonances (σ_R), the continuum process (σ_c), and the factor B at the resonance peak positions.

$\sqrt{s}$ (GeV)	$M_{J/\psi}$	$M_{\psi(2S)}$	$M_{\Upsilon(1S)}$	$M_{\Upsilon(2S)}$	$M_{\Upsilon(3S)}$	$M_{\psi(3770)}$	$M_{\Upsilon(4S)}$
σ_R (nb)	91,404	8,562	4,069	2,796	2,984	9.9	1.7
σ_c (nb)	20.6	15.4	3.4	3.1	2.9	19.0	2.8
B (GeV/c^2)	2.06	6.74	9.78	11.8	11.4	198	473

TABLE II. The measured branching fractions ($\mathcal{B}^f$), the estimated cross sections from continuum contribution (σ_c^f), and the maximum value of r_R^f at J/ψ and $\psi(2S)$ peak position.

R	$\mathcal{B}[R \to \Lambda\bar{\Lambda}](10^{-4})$	$\sigma_c^{\Lambda\bar{\Lambda}}$ (nb)	$r_R^{f\,\max}$ (%)
J/ψ	$19.43 \pm 0.03 \pm 0.33$	1.22×10^{-2}	1.7
$\psi(2S)$	$3.97 \pm 0.02 \pm 0.12$	0.57×10^{-3}	2.6

and $\psi(2S) \to \Lambda\bar{\Lambda}$ were reported by the BESIII experiment using 1.3×10^9 J/ψ and 4.5×10^8 $\psi(2S)$ events, respectively [15]. A data sample at $\sqrt{s} = 3.08$ GeV ($\sqrt{s} = 3.65$ GeV) was collected with an integrated luminosity of 30 pb^{-1} (44 pb^{-1}) to study the contribution from continuum process. No events passed the event selection from the sample collected at $\sqrt{s} = 3.08$ GeV, and only six events survived from the sample collected at $\sqrt{s} = 3.65$ GeV, which accounted for 0.34% of the signal events selected from the $\psi(2S)$ sample. So in both cases, the continuum contribution was neglected, and the branching fractions were determined. The measured branching fractions are summarized in Table II. Lately, the BESIII experiment measured the cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ using data samples collected at c.m. energies from 3.51 to 4.60 GeV [16]. It is found that the cross section can be described with a power-law function, $C \cdot (M_{\psi(3770)}^2/s)^n$, and the two parameters are determined to be $C = 379 \pm 22$ fb and $n = 8.8 \pm 0.4$. Using the central values of these parameters, the continuum cross sections ($\sigma_c^{\Lambda\bar{\Lambda}}$) at J/ψ and $\psi(2S)$ peak positions are obtained and listed in Table II. Inserting all the numbers into Eq. (5), the maximum values of r_R^f can be read from Fig. 1 and are listed in the last column of Table II, which are 1.7% and 2.6% for J/ψ and $\psi(2S)$, respectively; these can be compared with the quoted total systematic uncertainties of 1.7% and 2.8%.

We find that the interference effect is surprisingly large compared with the precision that current experiments can reach for the two decays mentioned above. Since the hadronic decays of J/ψ and $\psi(2S)$ have similar branching fractions and the continuum cross sections of many final states are at a few to a few tens of picobarn level, we expect a similar size of interference effect in other decay modes. In Fig. 1 we show $r_R^{f\,\max}$ for $A = 0 \to 5$: this should cover most of the decay modes of these narrow resonances. Our result indicates that the interference may change the measured branching fractions by subpercent to more than 10% depending on different final states and different resonances, and the effect is more prominent for bottomonium states. So this effect must be considered in evaluating the systematic uncertainties if it cannot be fully taken into account in the measurement of the decay branching fractions of narrow charmonium and bottmonium states.

IV. BROAD RESONANCES

For resonances located above open heavy flavor threshold, such as the $\psi(3770)$ and $\Upsilon(4S)$, the total cross sections of the resonance production and continuum process are of the same level of magnitude, as shown in Table I. However, the dominant decay modes of these resonances are open heavy flavor final states; the decay into light hadronic final states is suppressed according to the OZI rule [17] and is only a tiny fraction of the total decay, $(7_{-8}^{+9})\%$ for the $\psi(3770)$ and $< 4\%$ for the $\Upsilon(4S)$ [10]. According to the available searches from previous experiments [18–26], we have good reason to believe that the total decay rate to light hadrons should be at 1% level for both $\psi(3770)$ and $\Upsilon(4S)$. So the ratio of the cross sections of the resonance decay and continuum production is at 0.5% level. It is worth noticing that the estimation of the total decay rate to light hadrons for $\psi(3770)$ (1%) agrees with the theoretical prediction given in Ref. [27]. This is the reason we use r_c^f to characterize the size of the interference effect in the case of broad resonances. In this case, the interference term introduces a deviation of the cross section measured at the resonance peak from the smooth (almost constant) continuum cross sections in the vicinity of the resonance. The scale of the deviation depends on the factor A defined in Eq. (5) for a certain resonance and the relative phase between the two amplitudes. The maximum deviation $r_c^{f\,\max}$ is displayed in Fig. 1.

The minor deviations, either higher or lower than the continuum cross sections, observed in Refs. [18,21–24] may have indicated nonzero $\psi(3770)$ decays into light hadron final states, but more data are needed to confirm the observations. In any case, the way of calculating the (upper limits of the) branching fractions without considering the interference as in the previous measurements is incorrect, especially when the cross section at the resonance peak is lower than that off the resonance, which reveals a destructive interference between the continuum and resonance amplitudes. The measurements of $e^+e^- \to p\bar{p}$ [19] and $p\bar{p}\pi^0$ [20] in the vicinity of the $\psi(3770)$ peak show this effect and evidence for $\psi(3770)$ decays to these final states.

There are also cases where the branching fractions of resonance decay are large, and the cross sections from the continuum process, the resonance decay, and the interference term are comparable. In these cases, all three components are essential, and special precautions should be taken to obtain the correct branching fraction of the resonance decay.

Again, we take $\psi(3770) \to \Lambda\bar{\Lambda}$ as an example. The total cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ at the $\psi(3770)$ peak is measured to be (562 ± 42) fb [16]. The cross section of the continuum process at $\psi(3770)$ peak position is estimated as 379 fb using the same formula as used in Sec. III. Inserting the numbers into Eq. (4), the nominal value of the branching fraction of $\psi(3770) \to \Lambda\bar{\Lambda}$ is calculated to be in

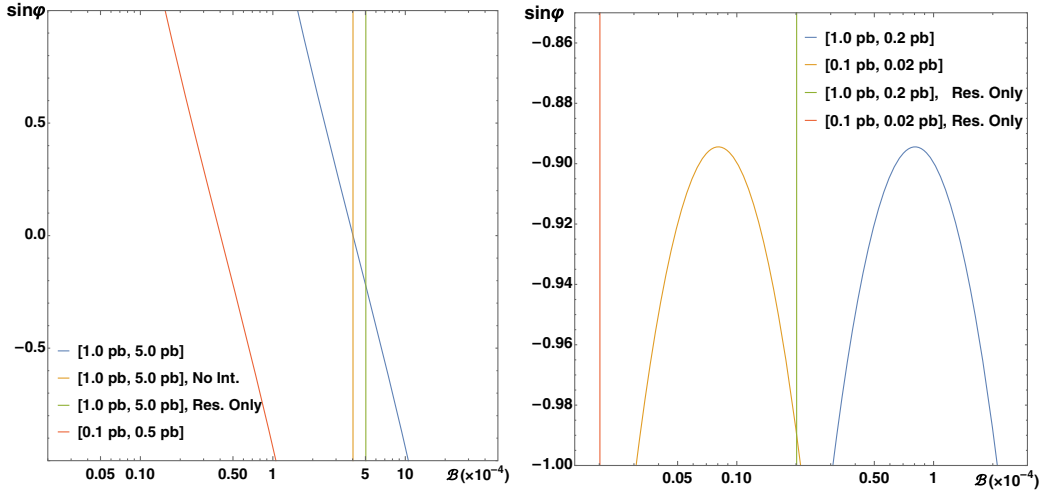FIG. 2.   The two-dimensional functions of $\mathcal{B}^f$ and $\sin \varphi$ with different σ_c^f and σ_{tot}^f values estimated at the $\psi(3770)$ nominal mass. In the plots, the vertical lines represent the branching fraction calculated with interference term neglected (No Int.) or both the interference and continuum terms neglected (Res. Only).

the range of $[1.79 \times 10^{-6}, 1.88 \times 10^{-4}]$, depending on the value of φ. If the interference term is simply neglected, the branching fraction is 1.8×10^{-5}, and could be very different from the true value depending on the unknown φ. In Ref. [16], the cross section line shape is fitted with the coherent sum of $\psi(3770)$ resonance and a power-law continuum term, and the relative phase is determined. In this case, the branching fraction can be determined exactly. The central value is $\mathcal{B}_{\text{con.}} = 2.4 \times 10^{-5}$ or $\mathcal{B}_{\text{des.}} = 1.44 \times 10^{-4}$ [16]. These are well covered by our estimated range above.

As mentioned earlier, the interference term will introduce a deviation, either positive or negative, to the continuum cross section at the resonance peak position. In both cases, neglecting the interference term, as has been done in previous measurements [21–26], will lead to imprecise branching fractions, as discussed in Ref. [28]. If σ_{tot}^f is larger than σ_c^f, the true branching fraction determined with an interference effect taken correctly into account could be larger or smaller than the one determined by simply subtracting the continuum contribution, depending on the value of φ. If σ_{tot}^f is smaller than σ_c^f, neglecting the interference term only will result in an unphysical branching fraction value, while neglecting both will lead to a smaller branching fraction. In Fig. 2, two-dimensional functions of $\mathcal{B}^f$ and $\sin \varphi$ with typical $[\sigma_c^f, \sigma_{\text{tot}}^f]$ values at $\psi(3770)$ peak position are shown, the cases where σ_{tot}^f are larger or smaller than σ_c^f are displayed separately. In both cases, the vertical lines in the plots represent the branching fractions when the interference term or both the interference and continuum terms are neglected. Although the absolute difference between the branching fractions calculated with or without the interference and continuum contribution depends on the relative phase, it is very significant in most of the parameter space.

V. ADDITIONAL EXPERIMENTAL EFFECTS

In the above discussion, the radiative correction and energy spread of the colliding beams are not considered to make the discussions clear and simple. We prove here that these two effects do not affect the conclusions above.

Taken these effects into account, the observed cross section can be written as

$$\sigma_{\text{exp}}^f(s) = \int_0^{1-s_m/s} dx \int_0^\infty d\sqrt{s'} F(x, s') \cdot \sigma^f(s'(1-x)) \cdot G(\sqrt{s}, \sqrt{s'}). \quad (7)$$

Here, $x = 1 - s_{\text{eff}}/s$ and $\sqrt{s_{\text{eff}}}$ represents the effective c.m. energy after losing energy due to photon emission, $\sqrt{s_m}$ is the cutoff of $\sqrt{s_{\text{eff}}}$ for the final state system and should be at least as large as the invariant mass of the final state system, $\sqrt{s}$ is the nominal c.m. energy, and $\sqrt{s'}$ is the actual c.m. energy, which differs from the nominal one due to beam energy spread. The radiator function $F(x, s)$ is calculated with a precision of 0.1% in Ref. [29]. $G(\sqrt{s}, \sqrt{s'}) = \frac{1}{\sqrt{2\pi}\Delta} e^{-\frac{(\sqrt{s}-\sqrt{s'})^2}{2\Delta^2}}$ is the beam energy spread function where Δ stands for the c.m. energy spread.

At the BESIII experiment, the Δ values for J/ψ, $\psi(2S)$, and $\psi(3770)$ are 0.8, 1.3, and 1.4 MeV, respectively. At the Belle II experiment, the typical Δ value is about $\sqrt{2} \cdot 5$ MeV. Using Eq. (7) and replacing σ^f with Eq. (3), the dependence of r_R^f and r_c^f on φ after considering radiative correction and energy spread can be obtained. The results shown in Figs. 3 and 4 are calculated with the integral over dx done in the range of $[0, 0.2]$ and the continuum cross section $\sigma_c^f(s)$ assumed to be proportional to $1/s$.
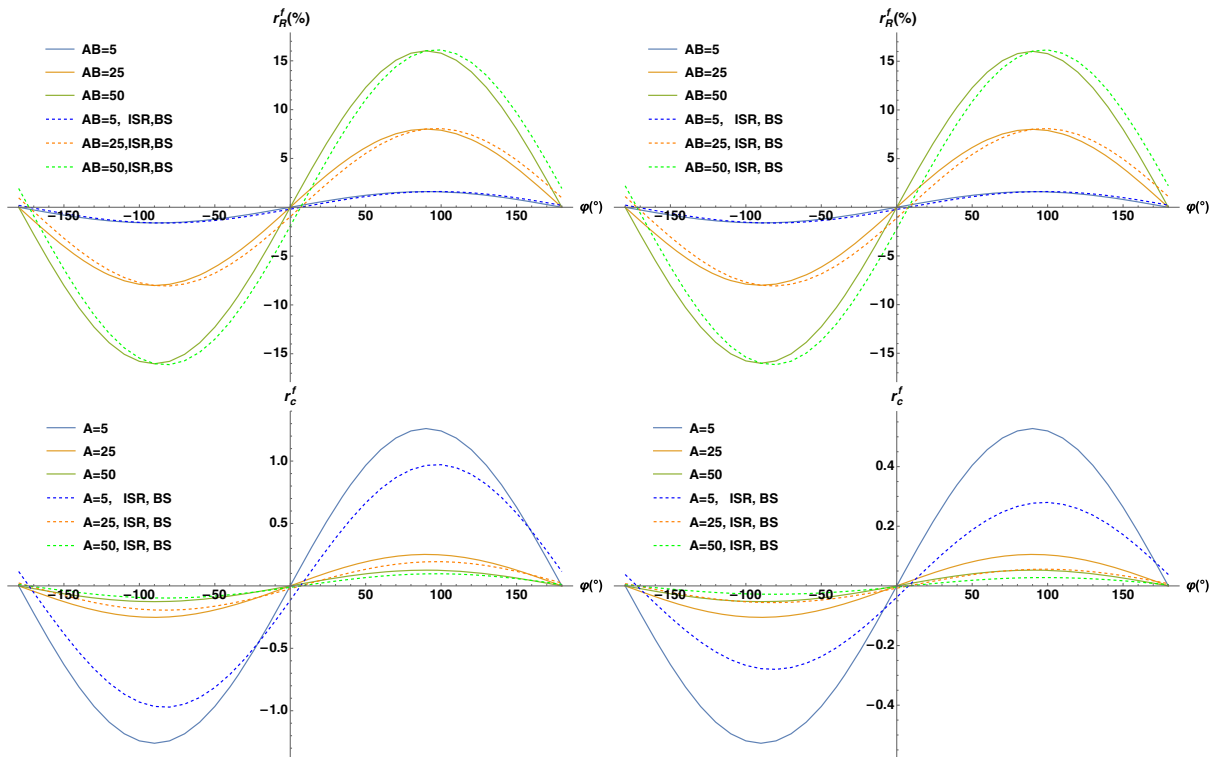
FIG. 3. The dependence of r_R^f (top row) and r_c^f (bottom row) on φ with different AB before (solid lines) and after (dashed lines) considering radiative correction (ISR) and beam energy spread (BS). The top left plot is at J/ψ peak position, the top right is at $\Upsilon(1S)$ peak position, the bottom left is at $\psi(3770)$ peak position, and the bottom right is at $\Upsilon(4S)$ peak position.

The r_R^f distributions with different AB at J/ψ and $\Upsilon(1S)$ masses and r_c^f distributions with different A (B set to values from Table I) at $\psi(3770)$ and $\Upsilon(4S)$ masses are shown in Fig. 3. The distributions for other resonances are similar. In both r_R^f and r_c^f distributions, there is a shift along φ that is caused by radiative correction. For a narrow resonance, the size of r_R^f is almost the same before and after taking radiative correction and energy spread into account. For broad resonances, the two effects reduce the maximum of r_c^f by 23% for $\psi(3770)$ at the BESIII experiment and 47% for $\Upsilon(4S)$ at the Belle II experiment. The size of the reduction depends on the resonance parameters, and Δ, does not depend on the value of A. Figure 4 shows the ratio of r_c^f calculated with or without radiative correction and energy spread, $r_c^{f\prime}/r_c^f$, as a function of Δ/Γ.

VI. SUMMARY

In this paper, the importance of the interference between the continuum and resonance amplitudes in the measurement of branching fractions of the decays of vector quarkonia at e^+e^- colliders is discussed. The exact formula that can be used to estimate the size of the interference effect is investigated. Although the absolute contribution from the continuum process is negligible for narrow resonances, the interference contribution can be at a few percent level, depending on the final states. Whereas for the broad resonances, the interference effect could be even more significant and needs to be taken into account properly to avoid wrong interpretation of the data.

With the currently available data samples at the BESIII experiment and the expected data samples at the Belle II experiment, the precision of the branching fractions is expected to be at a few percent or better level,
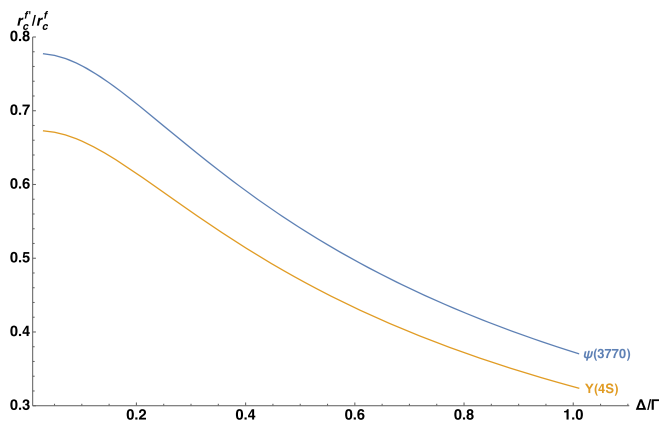

FIG. 4. The reduction of r_c^f from the radiative correction and energy spread for $\psi(3770)$ and $\Upsilon(4S)$ as a function of Δ/Γ, illustrated by $r_c^{f\prime}/r_c^f$ with $r_c^{f\prime}$ stands for that after considering the two effects.

so mishandling the interference effect will lead to systematic bias with a much larger size compared to the statistical uncertainty and comparable to or even larger than all the other sources of the systematic uncertainties. The optimal solution to this problem is to change the data taking strategy: accumulate data at no less than three different energies in the vicinity of a resonance, and measure the continuum and the resonance decay amplitudes together with the relative phase between them.

At colliders planned for the future, such as the super-tau-charm factories STCF in China [30] and SCT in Russia [31], and the super-J/ψ factory [32], the design luminosity is expected to be hundreds of times higher than the current tau-charm factory. It will be even more crucial to handle the interference properly as the precision of the measurements can be further improved with a few orders of magnitude larger data samples.

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