Erratum: Tidal response from scattering and the role of analytic continuation [Phys. Rev. D 104, 124061 (2021)]

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The following corrections and additions are implemented and developed in the updated arXiv version [1].

I. ERRATUM

The method used in Appendix 0c to extract the finite part of the tidal term has been changed. There is a mistake in equation (A19) and therefore we employ another method. Hence, (3.52) now reads

$$E_L(\omega) = e^{i\omega t}\ell!\pi\left(\frac{\omega}{2}\right)^{\hat{d}/2+1/2+2\ell}\frac{(-1)^{\ell}2^{\ell+1}}{\Gamma(\frac{\hat{d}}{2}+\ell+1)}C^L_{\mathrm{reg}}. \tag{1.1}$$

Appendix 0c is then changed as follows.

A. Appendix 0c

In order to compute the response function we have to extract the finite part of the tidal term, $\partial_L\phi$. For that, we will directly substitute the series representation of the Bessel functions and apply the symmetric trace-free tensor derivatives and their identities. From (3.25) we obtain

$$\partial_L\phi = \sum_{k=0}^{\infty}(C^K_{\mathrm{reg}}\partial_L\partial_K\phi^{(0)}_{\mathrm{reg}} + C^K_{\mathrm{irreg}}\partial_L\partial_K\phi^{(0)}_{\mathrm{irreg}}), \tag{1.2}$$

where

$$\partial_L\partial_K\phi^{(0)}_{\mathrm{reg}} = e^{i\omega t}\sqrt{2\pi\omega}\,\partial_L\partial_K\,(r^{-\hat{d}/2}J_{\hat{d}/2}(\omega r)), \tag{1.3a}$$

$$\partial_L\partial_K\phi^{(0)}_{\mathrm{irreg}} = e^{i\omega t}\sqrt{2\pi\omega}\partial_L\partial_K(r^{-\hat{d}/2}Y_{\hat{d}/2}(\omega r)). \tag{1.3b}$$

We begin with the regular piece,

$$\partial_L\partial_K(r^{-\hat{d}/2}J_{\hat{d}/2}(\omega r)) = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!\Gamma(m+\frac{\hat{d}}{2}+1)}\left(\frac{\omega}{2}\right)^{2m+\hat{d}/2}\partial_L\partial_K(r^{2m}). \tag{1.4}$$

In order to obtain the finite part we have to take $2m=\ell+k$ derivatives. Using (A14) of [2],

$$\partial_P(r^{2j}) = 0 \quad \text{if } j = 0,1,2,\ldots,p-1, \tag{1.5}$$

implies that in order to have a nonzero result $2m=\ell+k\geq\ell$ and $2m=\ell+k\geq k$. Therefore, the only possible choice is $\ell=k$ for which $m=\ell$. Using (A13) and (A12) of [2],

$$\partial_P(r^{\kappa}) = \frac{\kappa!!}{(\kappa-2p)!!}n_P r^{\kappa-p}, \tag{1.6}$$

$$\partial_i X_P = p\delta_{i<i_p}X_{P-1>}, \tag{1.7}$$

yields

$$\partial_L \partial_L (r^{2\ell}) = (2\ell)!! \partial_L (n_L r^\ell) = \ell!(2\ell)!!. \tag{1.8}$$

Hence,

$$\underset{r\to 0}{\mathrm{FP}} \partial_L \partial_K (r^{-\hat{d}/2} J_{\hat{d}/2}(\omega r)) = \frac{\ell! 2^\ell (-1)^\ell}{\Gamma(\frac{\hat{d}}{2}+\ell+1)} \left(\frac{\omega}{2}\right)^{2\ell+\hat{d}/2}, \tag{1.9}$$

where we have used that $(2\ell)!! = 2^\ell \ell!$. Similarly, we can compute the finite part of the irregular solution. Recall that the Bessel function of the second kind reads

$$Y_{\hat{d}/2}(\omega r) = \frac{1}{\sin(\frac{\pi\hat{d}}{2})} \left[\cos\left(\frac{\pi\hat{d}}{2}\right) J_{\hat{d}/2}(\omega r) - J_{-\hat{d}/2}(\omega r)\right]. \tag{1.10}$$

The first term is proportional to the regular solution and therefore we will focus on the second term,

$$\partial_L \partial_K (r^{-\hat{d}/2} J_{-\hat{d}/2}(\omega r)) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(m-\frac{\hat{d}}{2}+1)} \left(\frac{\omega}{2}\right)^{2m-\hat{d}/2} \partial_L \partial_K (r^{2m-\hat{d}}). \tag{1.11}$$

Now the condition to have a nonzero result reads $2m - \hat{d} = \ell + k$. Using (1.5) implies $2m - \hat{d} = \ell + k \geq \ell$ and $2m - \hat{d} = \ell + k \geq k$ and therefore $m = \hat{d}/2 + \ell$. Plugging (1.8) back into (1.11) yields

$$\underset{r\to 0}{\mathrm{FP}} \partial_L \partial_K (r^{-\hat{d}/2} J_{-\hat{d}/2}(\omega r)) = \frac{\ell! 2^\ell (-1)^\ell \cos(\frac{\pi\hat{d}}{2})}{\Gamma(\frac{\hat{d}}{2}+\ell+1)} \left(\frac{\omega}{2}\right)^{2\ell+\hat{d}/2}, \tag{1.12}$$

where given that $m = \hat{d}/2 + \ell$ is an integer and $\hat{d}$ can be odd or even,

$$(-1)^{\hat{d}/2} = \cos\left(\frac{\pi\hat{d}}{2}\right). \tag{1.13}$$

Combining (1.9) and (1.12) into (1.10) yields

$$\underset{r\to 0}{\mathrm{FP}} \partial_L \partial_K \phi^{(0)}_{\mathrm{irreg}} \propto \underset{r\to 0}{\mathrm{FP}} \partial_L \partial_K (r^{-\hat{d}/2} Y_{\hat{d}/2}(\omega r)) = 0. \tag{1.14}$$

We can now compute the frequency-dependent tidal field

$$\begin{aligned} E_L(\omega) &= \underset{r\to 0}{\mathrm{FP}} \partial_L \phi(\omega) = C^L_{\mathrm{reg}} \underset{r\to 0}{\mathrm{FP}} \partial_L \partial_L \phi^{(0)}_{\mathrm{reg}}(\omega) \\ &= e^{i\omega t} \ell! \pi \left(\frac{\omega}{2}\right)^{\hat{d}/2+1/2+2\ell} \frac{(-1)^\ell 2^{\ell+1}}{\Gamma(\frac{\hat{d}}{2}+\ell+1)} C^L_{\mathrm{reg}}. \end{aligned} \tag{1.15}$$

where we use that $\phi(\omega) = \sqrt{2\pi} e^{-i\omega t} \phi(t)$ for a fixed frequency ω.

II. ADDENDUM

We have generalized the definition of the tidal response to generic couplings. Specifically, we set the coupling constants of (3.1) and (3.8) to be the same. However, this is a particular choice and one can take into account more generic couplings. In particular we denote the coupling of the tidal action as K_Q,

$$S_{\mathrm{tidal}} = -K_{\mathrm{Q}} \int d\tau \sqrt{-u_\mu u^\mu} \sum_{\ell=0}^{\infty} \frac{1}{\ell!} Q^L \nabla_L \phi, \tag{2.1}$$

which can be set to $K_Q = 1$ without loss of generality. This introduces a prefactor in the response function,

$$F_\ell(\omega) = K_\phi \tilde{F}_\ell(\omega), \tag{2.2}$$

where $\tilde{F}_\ell(\omega)$ is the normalized response function. This subtlety leaves the results in Sec. III B unaffected but it is important when considering other, more generic coupling constants.

[1] G. Creci, T. Hinderer, and J. Steinhoff, Tidal response from scattering and the role of analytic continuation, arXiv:2108.03385.
[2] T. Hartmann, M. H. Soffel, and T. Kioustelidis, On the use of STF-tensors in celestial mechanics, Celest. Mech. Dyn. Astron. **60**, 139 (1994).